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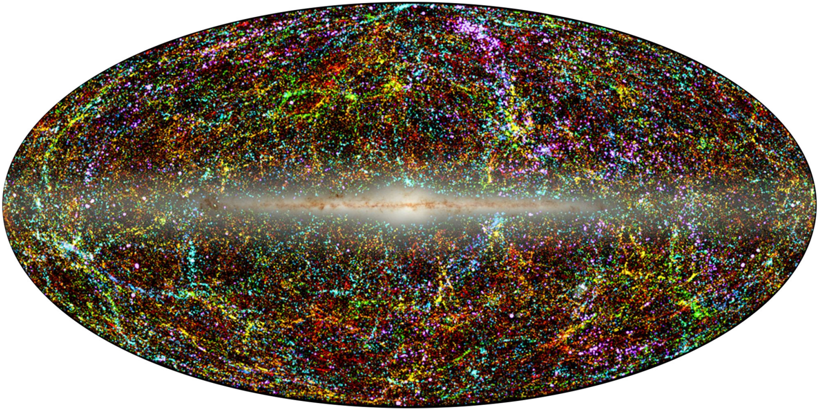
A microscopic approach to cosmic structure formation

Matthias Bartelmann
with
Felix Fabis, Daniel Berg, Elena Kozlikin, Robert Lilow, Carsten Littek, Celia Viermann

Heidelberg University, ZAH, ITA

CERN CKC TH institute – The big bang and the little bangs – August 2016

Problems in cosmic structure formation



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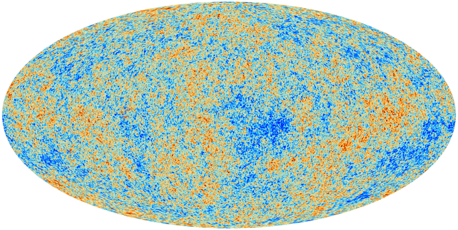

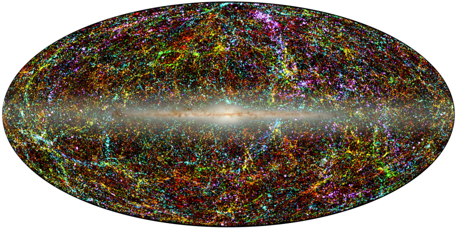

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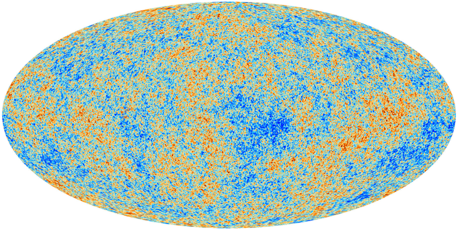

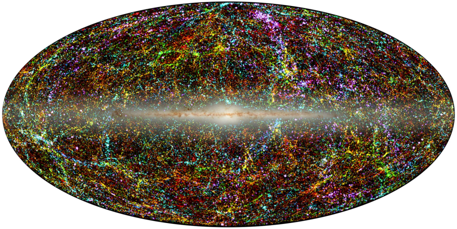

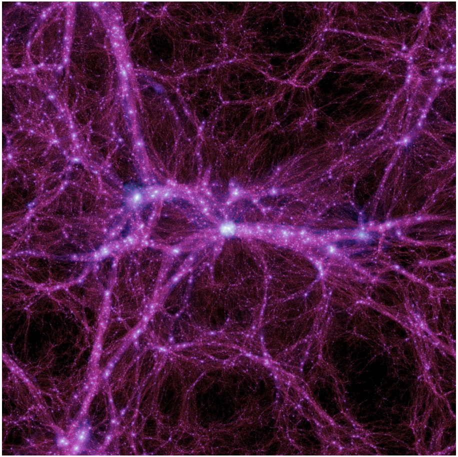

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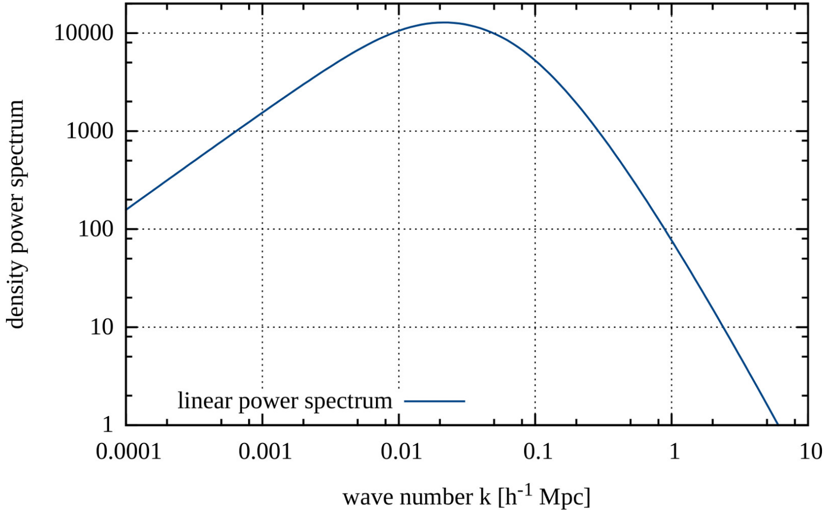

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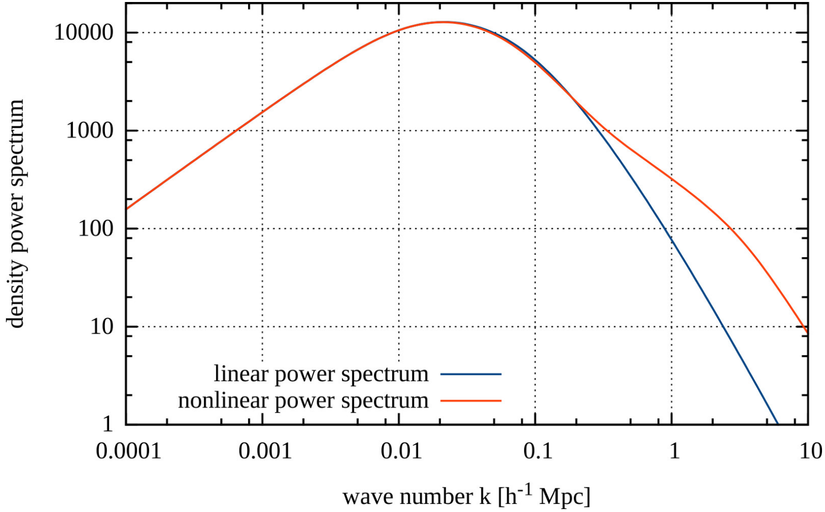

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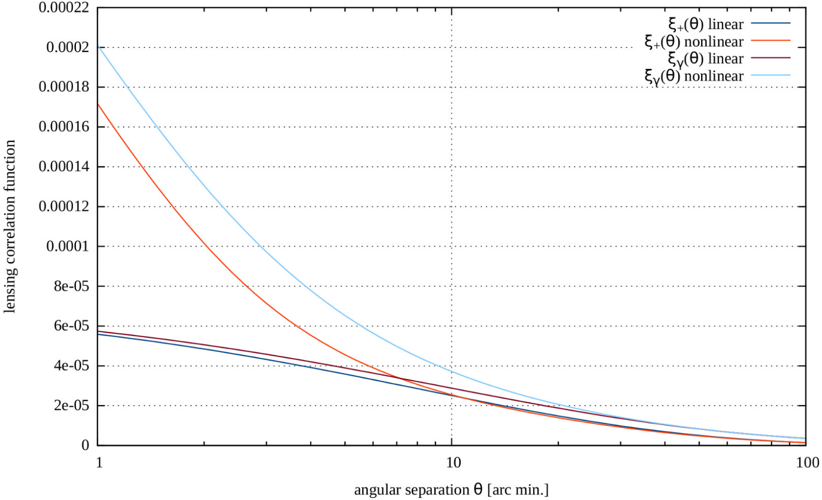

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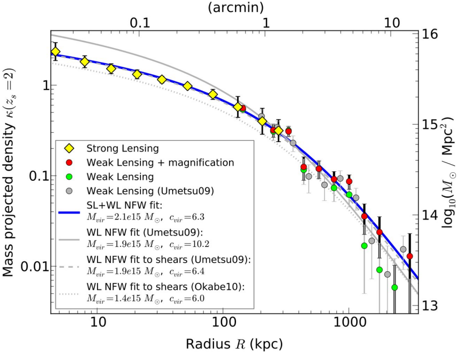

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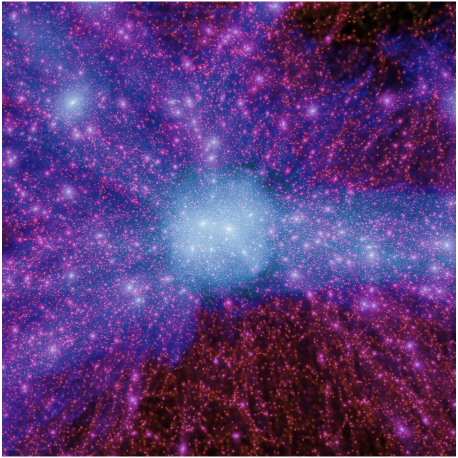

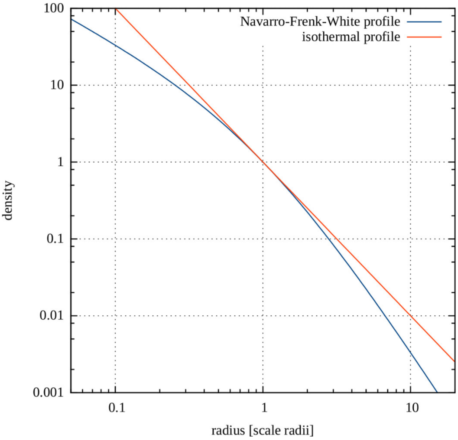

Cosmological perturbation theory

Conventional:

- Hydrodynamical equations:

$$\dot{\delta} + \vec{\nabla} \cdot \vec{u} = 0$$
$$\dot{\vec{u}} + 2H\vec{u} = -\vec{\nabla}\phi$$
$$\vec{\nabla}^2\phi = 4\pi G\bar{\rho}\delta$$

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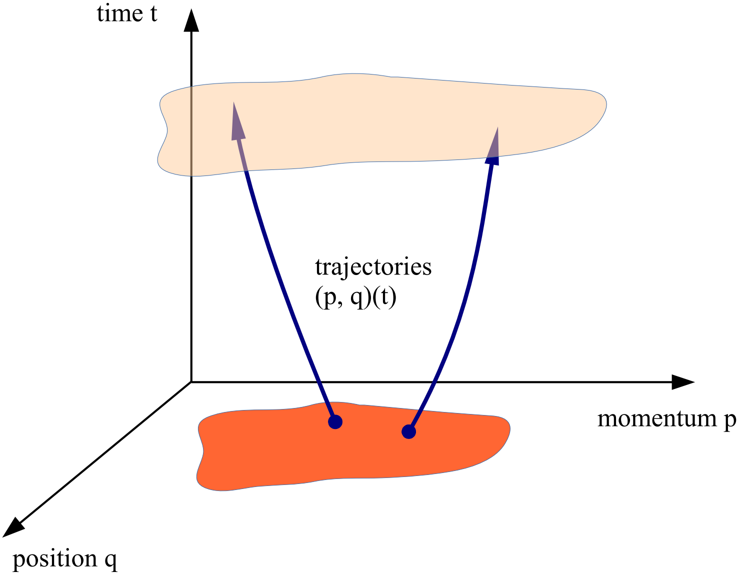
New approach:

- Non-equilibrium statistics of N classical particle trajectories
- Describe particle ensemble by partition sum (generating functional) Z
- Derive statistical properties by functional derivatives

Phase-space trajectories



Phase-space trajectories

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- Classical particles follow Hamiltonian equations of motion,

$$\dot{q} = \frac{\partial \mathcal{H}}{\partial p}\,, \qquad \dot{p} = -\frac{\partial \mathcal{H}}{\partial q}\,, \qquad x = (q, p)$$

- Trajectories are described by (retarded) Green's function $G_\mathrm{R}(t, t')$,

$$\bar{x}(t) = \underbrace{G_\mathrm{R}(t, 0)x^{(\mathrm{i})}}_{\text{free motion}} - \underbrace{\int_0^t G_\mathrm{R}(t, t')K(t')\mathrm{d}t'}_{\text{interaction}}$$

- In static space, with potential v,

$$G_\mathrm{R}(t, t') = \begin{pmatrix} 1 & (t - t')/m \\ 0 & 1 \end{pmatrix}, \qquad K = \begin{pmatrix} 0 \\ \nabla v \end{pmatrix}$$

Principle

Equilibrium thermodynamics:

$$Z_{gc} = \int \mathrm{d}\Gamma \, \mathrm{e}^{-\beta(H(x)-\mu N)}$$

$$\langle N \rangle = \frac{1}{\beta} \frac{\partial \ln Z_{gc}}{\partial \mu}$$

Principle

Non-equilibrium, statistical quantum field theory:

$$Z_0[J] = \mathrm{Tr} \int \mathcal{D}\varphi \, \mathrm{e}^{\mathrm{i} \int (\mathcal{L}_0 + J\varphi) \mathrm{d}^4 x}$$

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Non-equilibrium, statistical theory for classical degrees of freedom:

$$Z_0[J, K] = \int \mathrm{d}\Gamma^{(\mathrm{i})} \, \mathrm{e}^{\mathrm{i} \int \langle J, \bar{x} \rangle \mathrm{d}t}$$

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$$\langle \rho(1)\rho(2) \rangle = \frac{\delta}{\mathrm{i}\delta H_\rho(1)} \frac{\delta}{\mathrm{i}\delta H_\rho(2)} Z$$

Some detail

- Generating functional:

$$Z[H,J,K] = \mathrm{e}^{\mathrm{i}\hat{S}}\mathrm{e}^{\mathrm{i}H\hat{\Phi}} \int \mathrm{d}\Gamma^{(\mathrm{i})}\mathrm{e}^{\mathrm{i}\int \langle J,\bar{x}\rangle \mathrm{d}t}$$

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- Interaction operator:

$$\hat{S}_{\mathrm{I}} = -\int \mathrm{d}1 \int \mathrm{d}2 \frac{\delta}{\delta H_B(1)} v(12) \frac{\delta}{\delta H_\rho(2)}$$

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- Perturbation theory: $\mathrm{e}^{\mathrm{i}\hat{S}} = 1 + \mathrm{i}\hat{S} - \frac{1}{2}\hat{S}^2 + \ldots$

Some detail

- Generating functional:

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Specialisation to cosmology

1. Choose initial phase-space measure

$$\mathrm{d}\Gamma^{(\mathrm{i})} = P(q,p)\,\mathrm{d}^{3N}q\,\mathrm{d}^{3N}p$$

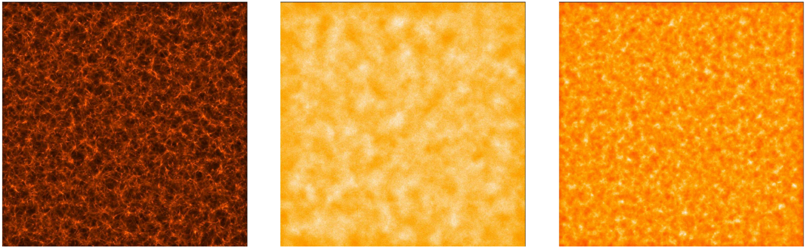
   fully specified by initial power spectrum

2. Change time coordinate

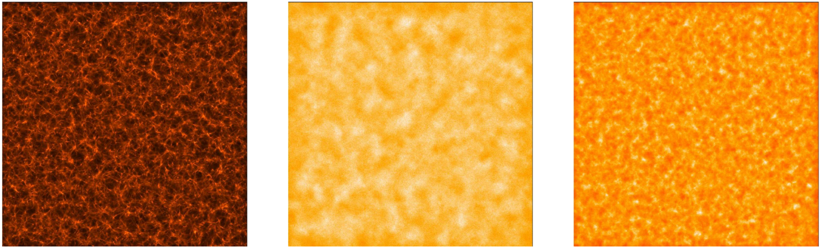
$$t \to \tau = D_+(t) - D_+(t_\mathrm{i})$$

3. Adapt Green's function to expanding universe

Improved Zel'dovich trajectories



Improved Zel'dovich trajectories



$$\vec{q}(\tau) = \vec{q}^{\,(\mathrm{i})} + \vec{p}^{\,(\mathrm{i})} \int_0^{\tau} \exp\left(h(\tau')\right) \mathrm{d}\tau'$$

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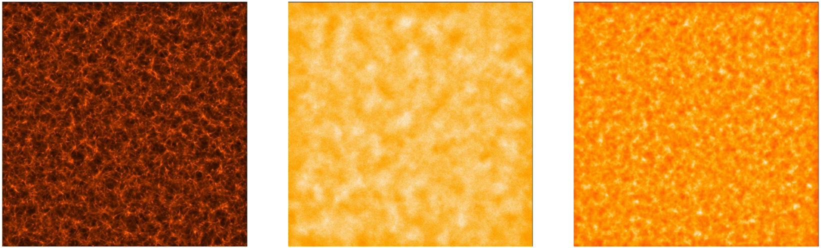

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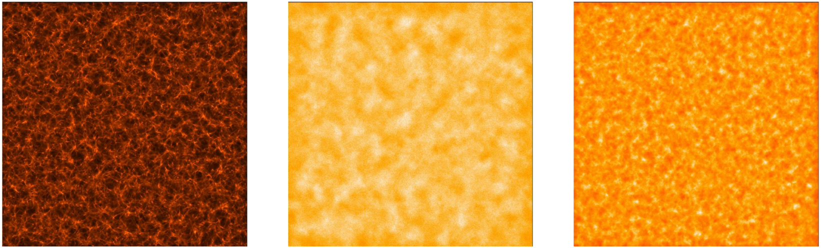

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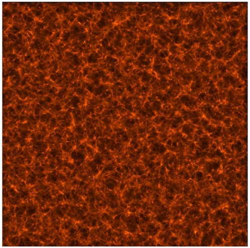

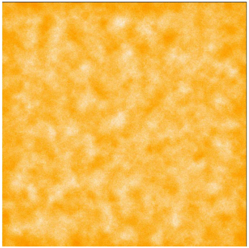

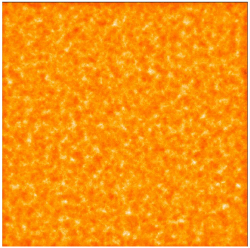

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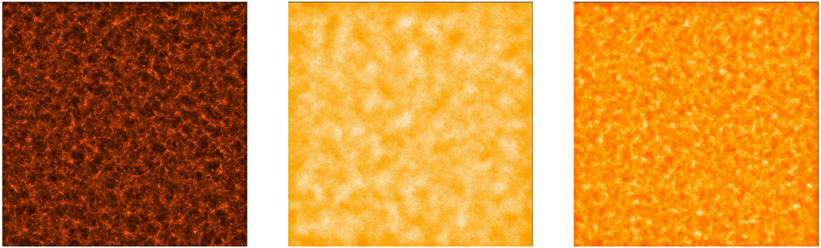

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$$\hat{v}(k) = -\frac{3}{2k^2}\frac{a}{g^2} \propto a^{-2} \text{ (EdS)}$$

Density power spectrum

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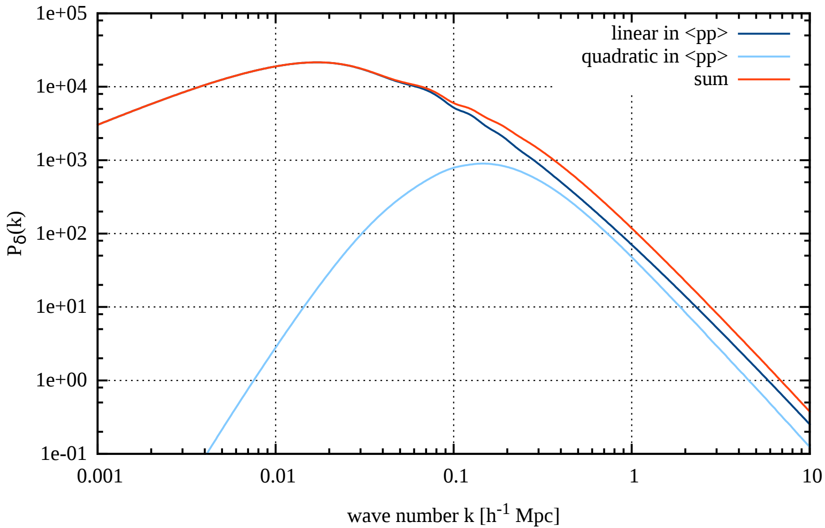
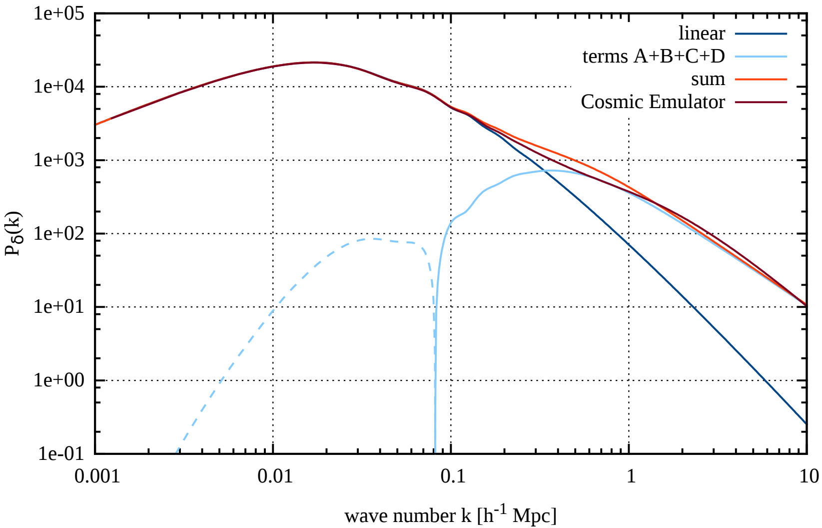


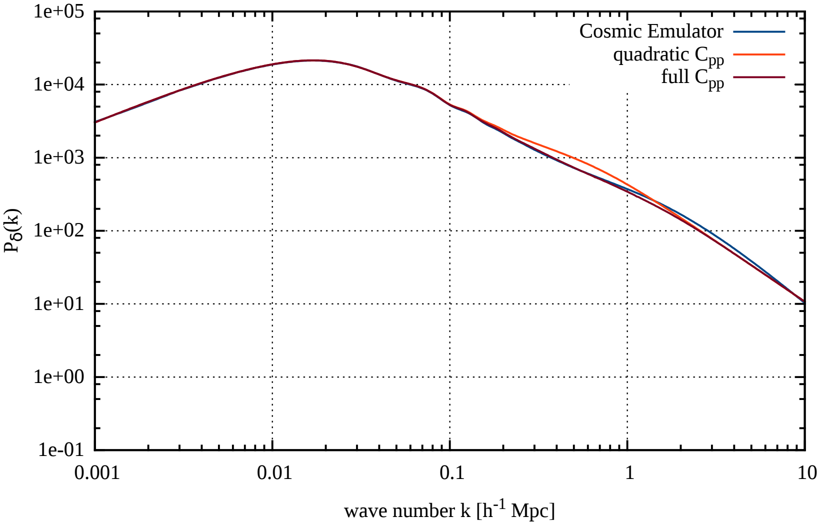
Improved Zel'dovich trajectories, no interaction

Density power spectrum



Improved Zel'dovich trajectories plus first-order interaction

Density power spectrum



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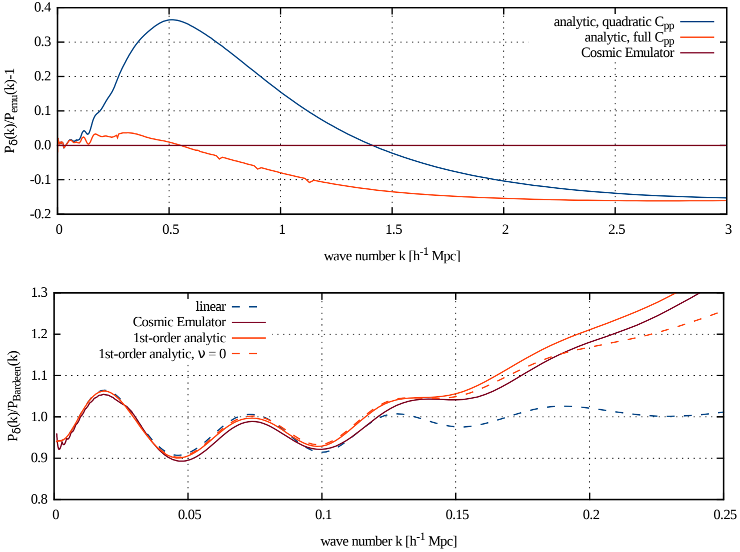

Improved Zel'dovich trajectories plus first-order interaction

Summary

1. Non-equilibrium statistical field theory for dark-matter particles set up
2. Hamiltonian equations of motion, simple Green's function
3. Expansion parameter is deviation from unperturbed (improved Zel'dovich) trajectories
4. n-point statistics for collective fields obtained by functional derivatives
5. First-order perturbation theory reproduces numerical results already quite well

Further applications