Particle production and equilibration rates: the case of right-handed neutrinos in the broken phase

Jacopo Ghiglieri, AEC ITP, University of Bern

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tBBatLB-NEPiCaiHIC, CERN, July 13 2016

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An excellent password!

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Introduction

- There is a host of problems in bangs of all sizes where some fields/particles are weakly coupled to a thermal bath and hence out of equilibrium with it
- One can then study their thermal production and equilibration rates
- Classic examples
  - Photons/dileptons in heavy ion collision
  - Thermal relics (DM, right-handed neutrinos...) in the EU
- A well-defined Thermal Field Theory problem

Thermal production

- Assume an equilibrated hot bath (QGP, early universe) with its internal coupling g and a particle ϕ, weakly coupled (coupling h) to other d.o.f.s, so that ϕ is not in equilibrium

$$\mathcal{L} = \mathcal{L}_\phi + h\phi^* J + h^* J^* \phi + \mathcal{L}_{\rm bath}$$

J built of bath operators

- With a simple derivation one obtains that the rate (per unit volume) is proportional to a thermal average of a JJ correlator

$$\frac{d\Gamma_\phi}{d^3k} = \frac{|h|^2}{2E_k}\Pi^<(k) = \frac{|h|^2}{2E_k}\int d^4X e^{iK\cdot X}\,{\rm Tr}\,\rho_{\rm bath}J(0)J(x)$$

- The expression is LO in h but to all orders in g

Example: photon production

- In the case of photon/dilepton production then $J = J^{\mu}_{EM}$, $h = e$ and $g = g_s$
- At leading order in QED and to all orders in QCD the photon and dilepton rates are given by

$$\frac{d\Gamma_{\gamma}(k)}{d^3k} = -\frac{\alpha}{4\pi^2 k}\int d^4X e^{iK\cdot X}\mathrm{Tr}\rho J^{\mu}(0)\, J_{\mu}(X)$$

$$\frac{d\Gamma_{l+l-}(k)}{dk^0 d^3k} = -\frac{\alpha^2}{6\pi^3 K^2}\int d^4X e^{iK\cdot X}\mathrm{Tr}\rho J^{\mu}(0)\, J_{\mu}(X)$$

The ingredients

$$\Pi^<(K) \equiv \int d^4X e^{iK\cdot X} \mathrm{Tr} \rho J^\mu(0)\, J_\mu(X)$$

- electromagnetic current *J*: how the d.o.f.s couple to photons
- density operator ϱ. In the equilibrium (possibly just local) approximation it becomes the thermal density $\rho \propto e^{-\beta H}$ and the whole thing a thermal average
- The action *S*: how the d.o.f.s propagate and interact

Right-handed neutrinos

Right-handed neutrinos

- Sterile right-handed neutrinos with O(GeV) masses could play a big role in cosmology

- They can generate a lepton asymmetry through oscillations, which can then be converted to a baryon asymmetry (sphalerons): **leptogenesis**
  Akhmedov Rubakov Smirnov **PRL81** (1998), Asaka Shaposhnikov **PLB620** (2005)

- If a large lepton asymmetry ($n_l \gg n_B$) persists to T~1 GeV it can be resonantly converted to keV scale sterile, dark matter neutrinos
  Shi Fuller **PRL82** (1999)

Motivation

- Computing reliably the lepton asymmetry in a specific scenario is usually challenging (CP violation, oscillations, plasma physics)
- On the other hand, establishing whether an existing asymmetry gets *washed out* allows to put constraints (or rule out) scenarios
- In this talk: three related quantities that govern the dynamics of GeV scale sterile neutrinos and of the lepton asymmetry in the broken phase

In this talk

- Three related quantities
  - Production rate of RHNs
  - Equilibration rate of RHNs
  - Washout rate of the lepton asymmetry
- In the broken phase 5 GeV$<T<$160 GeV
- GeV scale RHNs ($M \ll T$). Relevance for the SHiP experiment at CERN

JG Laine **JCAP07** (2016)

In this talk

- The considered range fills a gap in the literature where, as I will show, these rates peak

- Previous calculations in the symmetric phase for all kinematic ranges, and deep in the broken phase T<5 GeV

  Salvio Lodone Strumia (2011), Laine Schröder (2012), Biondini Brambilla Escobedo Vairo (2012), Garbrecht Glowna Herranen (2013), Laine (2013), Anisimov Besak Bödeker (2010-12), Ghisoiu Laine (2014)
  Asaka Laine Shaposhnikov (2006), JG Laine (2015), Venumadhav Cyr-Racine Abazajian Hirata (2015)

Overview

Right-handed neutrinos

- Add 3 sterile, Majorana neutrinos coupling to the three active lepton flavours and the Higgs field

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}\sum_I \bar{N}_I (i\gamma^\mu\partial_\mu - M_I) N_I - \sum_{I,a} (\bar{N}_I\, h_{Ia} \tilde{\phi}^\dagger a_L\, l_a + \bar{l}_a\, a_R\, \tilde{\phi}\, h^*_{Ia} N_I)$$

- At T>5 GeV all SM leptons are effectively massless and the information we need is encoded in the spectral function of this SM two-point function

$$\Pi_{\rm E}(K) \equiv {\rm Tr}\Big\{ i\not{K} \int_0^{1/T} {\rm d}\tau \int_{\mathbf{x}} e^{iK\cdot X} \Big\langle (\tilde{\phi}^\dagger a_L\, l)(X)\, (\bar{l}\, a_R\, \tilde{\phi})(0) \Big\rangle_T \Big\}$$

$$\rho(K) \equiv {\rm Im}\, \Pi_{\rm E}(K)|_{k_n \to -i(k_0 + i\epsilon)}$$

Thermal average of SM operators coupling to RHNs

Right-handed neutrinos

$$\Pi_{\rm E}(K) \equiv \mathrm{Tr}\Big\{ i\not{K} \int_0^{1/T} \mathrm{d}\tau \int_{\mathbf{x}} e^{iK\cdot X} \Big\langle (\tilde{\phi}^\dagger a_L\, l)(X)\, (\bar{l}\, a_R\, \tilde{\phi})(0) \Big\rangle_T \Big\}$$

$$\rho(K) \equiv \mathrm{Im}\, \Pi_{\rm E}(K)|_{k_n \to -i(k_0 + i\epsilon)}$$

- RHN equilibration rate

$$\dot{f}_{I\mathbf{k}} = \gamma_{I\mathbf{k}}\left(n_{\rm F}(E_I) - f_{I\mathbf{k}}\right) + \mathcal{O}\left[\left(n_{\rm F} - f_{I\mathbf{k}}\right)^2, n_a^2\right]$$

$$\gamma_{I\mathbf{k}} = \sum_a \frac{|h_{Ia}|^2 \rho(K)}{E_I} + \mathcal{O}(h^4)$$

Approach to equilibrium of the RHN phase space distribution (on-shell RHNs, $E_I = (\mathbf{k}^2 + M^2)^{1/2}$)

Bödeker Sangel Wörmann **PRD93** (2015)

Right-handed neutrinos

$$\Pi_{\rm E}(K) \equiv {\rm Tr}\Big\{ i\not{K} \int_0^{1/T} {\rm d}\tau \int_{\mathbf{x}} e^{iK\cdot X} \Big\langle (\tilde{\phi}^\dagger a_L\, l)(X)\, (\bar{l}\, a_R\, \tilde{\phi})(0) \Big\rangle_T \Big\}$$

$$\rho(K) \equiv {\rm Im}\, \Pi_{\rm E}(K)|_{k_n \to -i(k_0 + i\epsilon)}$$

- RHN production rate when $f_{I\mathbf{k}} \ll n_{\rm F}$

$$\dot{n}_I = \sum_a \int \frac{d^3k}{(2\pi)^3} \frac{2 n_{\rm F}(E_I) |h_{Ia}|^2 \rho(K)}{E_I} + \mathcal{O}(h^4, n_I)$$

$$n_I = 2 \int \frac{d^3k}{(2\pi)^3} f_{I\mathbf{k}}$$

Growth of the RHN number density far from equilibrium

Right-handed neutrinos

$$\Pi_{\mathrm{E}}(K) \equiv \mathrm{Tr}\left\{ i\not{K} \int_0^{1/T} \mathrm{d}\tau \int_{\mathbf{x}} e^{iK\cdot X} \left\langle (\tilde{\phi}^\dagger a_L\, l)(X)\, (\bar{l}\, a_R\, \tilde{\phi})(0) \right\rangle_T \right\}$$

$$\rho(K) \equiv \mathrm{Im}\, \Pi_{\mathrm{E}}(K)|_{k_n \to -i(k_0 + i\epsilon)}$$

- Washout rate for the lepton number for flavour *a*

$$\dot{n}_a = -\gamma_{ab} n_b + \mathcal{O}[n_a(n_{\mathrm{F}} - f_{I\mathbf{k}}), n_a^3]$$

$$\gamma_{ab} = -\sum_I \int \frac{d^3k}{(2\pi)^3} \frac{2n'_{\mathrm{F}}(E_I)|h_{Ia}|^2 \rho(K)}{E_I} \Xi^{-1}_{ab} + \mathcal{O}(h^4)$$

Depends on the susceptibility $\Xi_{ab} = \partial n_a / \partial \mu_b|_{\mu_b = 0}$
not diagonal because of charge neutrality constraints

Bödeker Laine **JCAP05** (2014)

Computing ρ

- In the broken phase the Higgs e.v. $v>0$. We consider the parametric range $T\gtrsim v$, so that thermal masses ($O(gT)$) and Higgs mechanism masses ($O(gv)$) are of the same order. In practice

$$30\,\mathrm{GeV} \lesssim T \lesssim 160\,\mathrm{GeV}$$

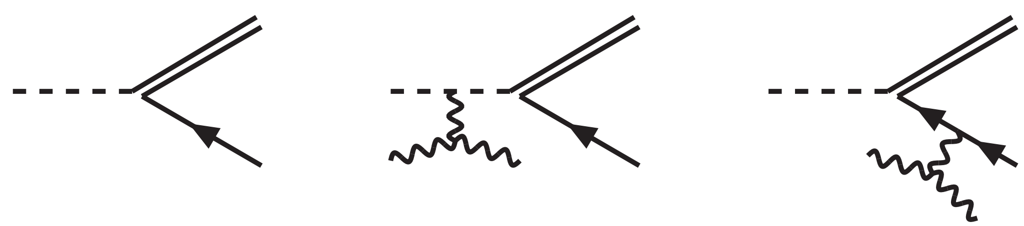
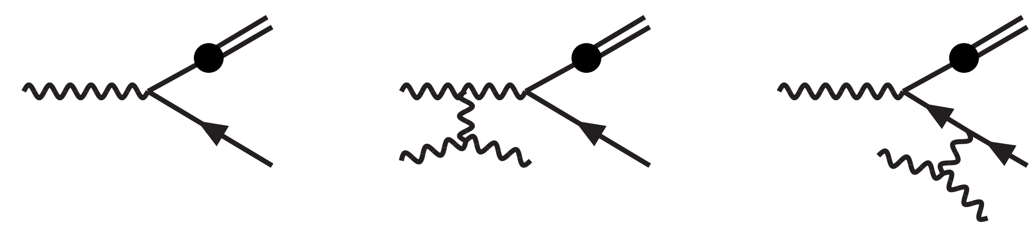
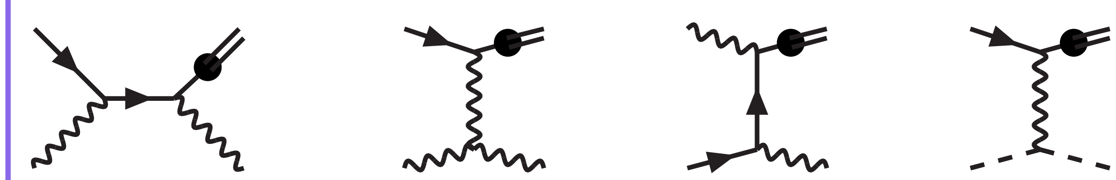
  where $g=(g_1,g_2,h_t,\lambda^{1/2})$ (parametrically equivalent)

  - In this region $M_I\lesssim gT$

- We also consider $m_W\gtrsim\pi T$ to cover the low-temperature region down to 5 GeV

$$\Pi_{\mathrm{E}}(K) \equiv \mathrm{Tr}\left\{ i\not{K} \int_0^{1/T} \mathrm{d}\tau \int_{\mathbf{x}} e^{iK\cdot X} \left\langle (\tilde{\phi}^{\dagger} a_L\, l)(X)\, (\bar{l}\, a_R\, \tilde{\phi})(0) \right\rangle_T \right\}$$

- The Higgs doublet can be a propagating d.o.f. (Higgs or Goldstone) or an expectation value insertion. Distinction into direct and indirect processes

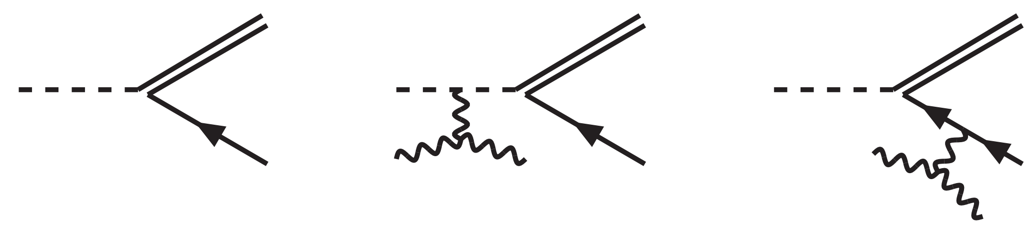
	Direct	Indirect
1↔2	 and others, and crossings	 and others, and crossings
2↔2	 and others, and crossings	 and others, and crossings

- Only the sum is gauge invariant. Feynman R_ξ gauge simplest

- Direct processes give $\varrho \sim g^2 T^2$. Indirect processes can have a near-resonant enhancement (hold on)

The computation

Direct 1↔2 processes



- Since all masses are $O(gT)$, tree level processes (if possible) are $\sim m^2 \sim g^2T^2$ and collinear
- Long formation times $O(1/g^2T)$) imply that soft scatterings, at rate g^2T, need to be resummed to all orders ⇒ Landau-Pomeranchuk-Migdal (LPM) effect
  Long QCD history (BDMPS, AMY). Introduced for RHNs in the *symmetric phase* in Anisimov Besak Bödeker **JCAP03** (2011), Besak Bödeker **JCAP03** (2012), Ghisoiu Laine **JCAP12** (2014)

Symmetric phase LPM

- In the **symmetric phase**

$$\rho(K)^{\rm LPM} = \frac{1}{16\pi}\int_{-\infty}^{\infty} {\rm d}\omega \left[1 - n_{\rm F}(\omega) + n_{\rm B}(k_0-\omega)\right] \frac{k_0}{k_0-\omega}$$

$$\times \lim_{\mathbf{y}\to\mathbf{0}} \; 4 \left\{ \frac{M^2}{k_0^2} {\rm Im}\left[g\;\;(\mathbf{y})\right] + \frac{1}{\omega^2}{\rm Im}\left[\nabla_\perp \cdot \mathbf{f}\;\;(\mathbf{y})\right]\right\}$$

- The functions **f** and g encode the resummed soft interactions through

$$\hat{H} \;\equiv\; -\frac{M^2}{2k_0} + \frac{m_l^2 - \nabla_\perp^2}{2\omega} + \frac{m_\phi^2 - \nabla_\perp^2}{2(k_0-\omega)} - i\,\Gamma(y)$$

$$(\hat{H} + i0^+)\, g(\mathbf{y}) \;=\; \delta^{(2)}(\mathbf{y})\,, \quad (\hat{H}+i0^+)\,\mathbf{f}(\mathbf{y}) \;=\; -\nabla_\perp \delta^{(2)}(\mathbf{y})$$

where m_l and m_ϕ are the thermal masses of leptons and scalars and the soft interactions are ($m_{{\rm E}i}$ screening masses)

$$\Gamma(y) = \frac{T}{8\pi}\sum_{i=1}^{2} d_i\, g_i^2 \left[\ln\left(\frac{m_{{\rm E}i}y}{2}\right) + \gamma_{\rm E} + K_0\left(m_{{\rm E}i}y\right)\right]$$

Symmetric phase LPM

- In **QCD** (photon/dilepton production)

$$\rho(K)^{\mathrm{LPM}} = \frac{N_c}{\pi}\int_{-\infty}^{\infty} \mathrm{d}\omega \big[1 - n_{\mathrm{F}}(\omega) - n_{\mathrm{F}}(k_0-\omega)\big]$$
$$\times \lim_{\mathbf{y}\to\mathbf{0}} \left\{ \frac{M^2}{k_0^2}\mathrm{Im}\left[g(\mathbf{y})\right] + \left(\frac{1}{2\omega^2} + \frac{1}{2(k_0-\omega)^2}\right)\mathrm{Im}\left[\nabla_\perp \cdot \mathbf{f}(\mathbf{y})\right]\right\}$$

- The functions **f** and *g* encode the resummed soft interactions through

$$\hat{H} \;\equiv\; -\frac{M^2}{2k_0} + \frac{m_q^2 - \nabla_\perp^2}{2\omega} + \frac{m_q^2 - \nabla_\perp^2}{2(k_0-\omega)} - i\,\Gamma(y)$$

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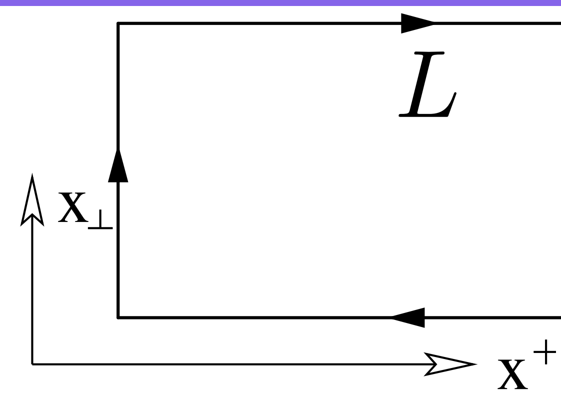
where m_q is the thermal mass of quarks and the soft interactions are (m_{D} SU(3) screening mass)

$$\Gamma(y) = \frac{g^2 C_F T}{2\pi}\left[\ln\left(\frac{m_D y}{2}\right) + \gamma_E + K_0(m_D y)\right]$$

The soft interactions



$$\propto e^{-\Gamma(y)L}$$

BDMPS-Z, Wiedemann, Casalderrey-Solana Salgado, D'Eramo Liu Rajagopal, Benzke Brambilla Escobedo Vairo

- All points at spacelike or lightlike separation, only preexisting correlations
- Soft contribution becomes Euclidean! Caron-Huot **PRD79** (2008)
  - Can be "easily" computed in perturbation theory
  - Possible lattice QCD measurements Laine Rothkopf **JHEP1307** (2013) Panero Rummukainen Schäfer **PRL112** (2014)

Euclideanization of light-cone soft physics

- For $t/x_z = 0$: equal time Euclidean correlators.

$$G_{rr}(t=0,\mathbf{x}) = \sum\!\!\!\!\!\!\!\int_p G_E(\omega_n,p)e^{i\mathbf{p}\cdot\mathbf{x}}$$

Caron-Huot **PRD79** (2009)

Euclideanization of light-cone soft physics

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$$G_{rr}(t=0,\mathbf{x}) = \sum\!\!\!\!\!\!\!\int_p G_E(\omega_n,p)e^{i\mathbf{p}\cdot\mathbf{x}}$$

- Consider the more general case $|t/x^z|<1$

$$G_{rr}(t,\mathbf{x}) = \int dp^0 dp^z d^2p_\perp e^{i(p^z x^z + \mathbf{p}_\perp\cdot\mathbf{x}_\perp - p^0x^0)}\left(\frac{1}{2}+n_{\rm B}(p^0)\right)(G_R(P)-G_A(P))$$

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- Consider the more general case $|t/x^z| < 1$

$$G_{rr}(t,\mathbf{x}) = \int dp^0 dp^z d^2p_\perp e^{i(p^z x^z + \mathbf{p}_\perp\cdot\mathbf{x}_\perp - p^0 x^0)} \left(\frac{1}{2} + n_{\mathrm{B}}(p^0)\right) \left(G_R(P) - G_A(P)\right)$$

- Change variables to $\tilde{p}^z = p^z - p^0(t/x^z)$

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- Retarded functions are analytical in the upper plane in any timelike or lightlike variable => G_R analytical in p^0

Caron-Huot **PRD79** (2009)

Euclideanization of light-cone soft physics

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$$G_{rr}(t,\mathbf{x}) = T\sum_n \int dp^z d^2p_\perp e^{i(p^z x^z + \mathbf{p}_\perp\cdot\mathbf{x}_\perp)} G_E(\omega_n,p_\perp,p^z + i\omega_n t/x^z)$$

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- Soft physics dominated by *n=0* (and *t*-independent) =>EQCD!

Caron-Huot **PRD79** (2009)

Euclideanization of light-cone soft physics

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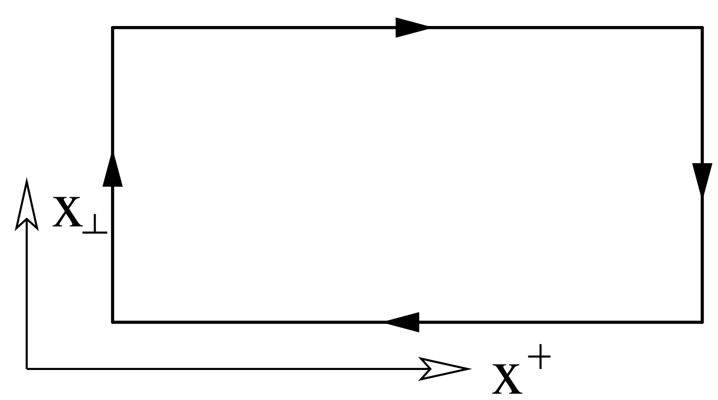
- Retarded functions are analytical in the upper plane in any timelike or lightlike variable => G_R analytical in p^0

$$G_{rr}(t,\mathbf{x})_{\mathrm{soft}} = T\int d^3p\, e^{i\mathbf{p}\cdot\mathbf{x}}\, G_E(\omega_n=0,\mathbf{p})$$

- Soft physics dominated by $n=0$ (and t-independent) =>EQCD!

Caron-Huot **PRD79** (2009)

Euclideanization of light-cone soft physics



$$\propto e^{-\Gamma(y)L}$$

- At leading order

$$\Gamma(y) \propto T \int \frac{d^2 q_\perp}{(2\pi)^2} \left(1 - e^{i\mathbf{x}_\perp \cdot \mathbf{q}_\perp}\right) G_E^{++}(\omega_n = 0, q_z = 0, q_\perp) = T \int \frac{d^2 q_\perp}{(2\pi)^2} \left(1 - e^{i\mathbf{x}_\perp \cdot \mathbf{q}_\perp}\right) \left(\frac{1}{q_\perp^2} - \frac{1}{q_\perp^2 + m_D^2}\right)$$

- Agrees with the earlier sum rule in Aurenche Gelis Zaraket **JHEP0205** (2002)

Symmetric phase LPM

- In the **symmetric phase**

$$\rho(K)^{\rm LPM} = \frac{1}{16\pi}\int_{-\infty}^{\infty} {\rm d}\omega \big[1 - n_{\rm F}(\omega) + n_{\rm B}(k_0-\omega)\big]\,\frac{k_0}{k_0-\omega}$$

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Broken phase LPM

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$$\times \lim_{\mathbf{y}\to\mathbf{0}} \; 4 \left\{ \frac{M^2}{k_0^2} \mathrm{Im}\left[g \;\; (\mathbf{y})\right] + \frac{1}{\omega^2} \mathrm{Im}\left[\nabla_\perp \cdot \mathbf{f} \;\; (\mathbf{y})\right] \right\}$$

- Broken electroweak symmetry implies
  - Broken degeneracy of scalar masses $m_\phi^2 \to \mathrm{diag}(m_{\phi_0}^2, m_{\phi_3}^2, m_{\phi_1}^2)$
  - Soft interactions become sensitive to “vacuum” masses and to the electromagnetic charges

Broken phase LPM

$$\rho(K)^{\rm LPM} = \frac{1}{16\pi}\int_{-\infty}^{\infty} {\rm d}\omega \left[1 - n_{\rm F}(\omega) + n_{\rm B}(k_0 - \omega)\right] \frac{k_0}{k_0 - \omega}$$
$$\times \lim_{\mathbf{y}\to\mathbf{0}} \sum_{\mu=0}^{3} \left\{ \frac{M^2}{k_0^2} {\rm Im}\left[g_\mu(\mathbf{y})\right] + \frac{1}{\omega^2} {\rm Im}\left[\nabla_\perp \cdot \mathbf{f}_\mu(\mathbf{y})\right] \right\}$$

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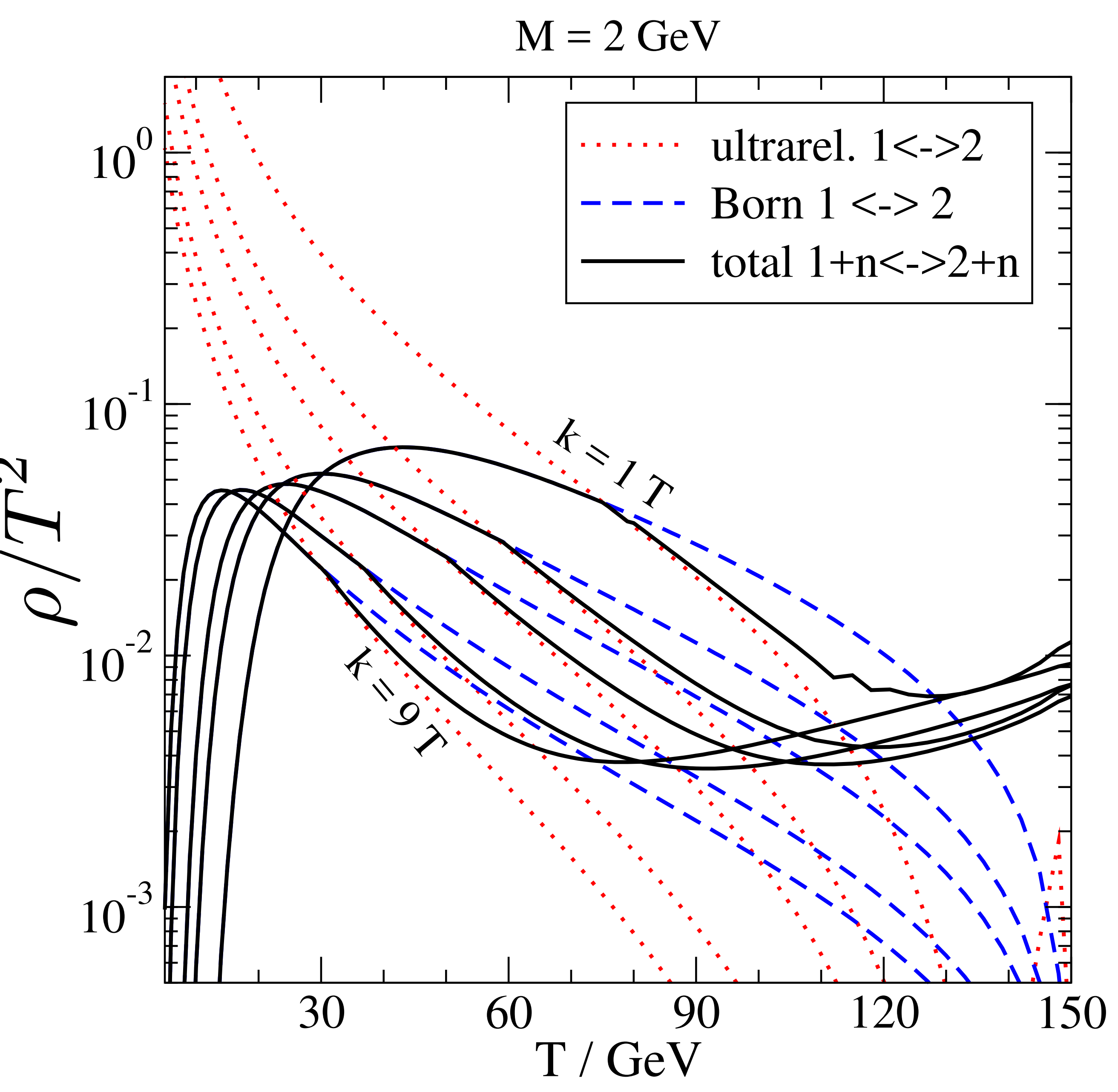
$\Rightarrow$ Matrix structure between the $\nu\phi_0$, $\nu\phi_3$ and $e\phi_\pm$ states

$$\Gamma_{3\times 3} = \begin{pmatrix} 2\Gamma_W(0) + \Gamma_Z(0) & -\Gamma_Z(y) & -2\Gamma_W(y) \\ -\Gamma_Z(y) & 2\Gamma_W(0) + \Gamma_Z(0) & -2\Gamma_W(y) \\ -\Gamma_W(y) & -\Gamma_W(y) & 2\Gamma_W(0) + \Gamma_{Z'}(0) - \Gamma_{Z'}(y) \end{pmatrix}$$

Direct 1↔2 processes

- Red: tree level processes with collinear ($m \ll T$) approx. Unphysical growth at low T
- Blue: full tree level with $m_l=0$, proper m_ϕ, accurate at low T
- Black: full solution of the LPM equations at high T, manually switched to blue at low T. **Final 1↔2 result**



Direct 2↔2 processes

- As long as all external state masses are $O(gT)$ or $O(gv)$ they can be neglected at leading order ($O(g^2T^2)$). Hence, no change with respect to the symmetric phase evaluation in Besak Bödeker **JCAP03** (2012)

$$\int_{\mathrm{ph.\,space}} f(p)f(p')(1\pm f(k'))|\mathcal{M}|^2\delta^4(P+P'-K-K')$$

- Phase space convolution of statistical functions and matrix elements. HTL resummation needed for soft fermion exchange. Analiticity arguments lead to a simple form for the soft part of the result Besak Bödeker **JCAP03** (2012) JG Hong Lu Kurkela Moore Teaney **JHEP05** (2013)

Direct 2↔2 processes

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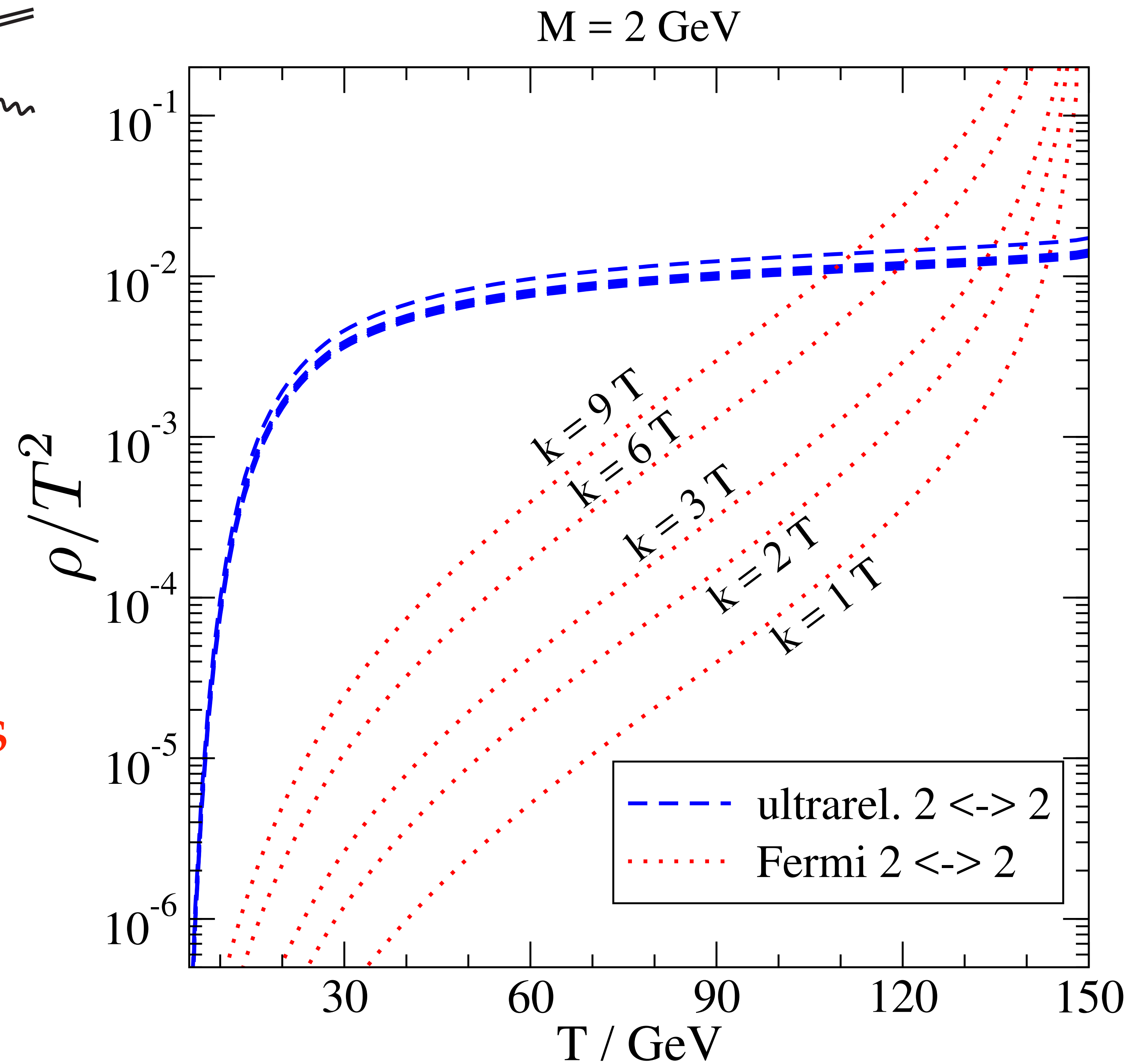
- At low $T<m_W$ initial state bosons (scalar or gauge) are very massive. We switch off the rate at low T by multiplying it for the W boson susceptibility

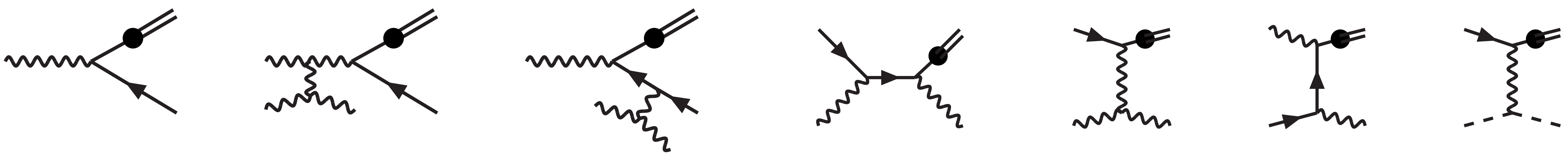
- The formally leading-order contribution at low T is scalar-mediated scatterings off b quarks. We find it is however negligible

Direct 2↔2 processes

- Besak-Bödeker rate times the *W* boson susceptibility
- Scalar-mediated scatterings off *b* quarks in the Fermi limit



Indirect processes

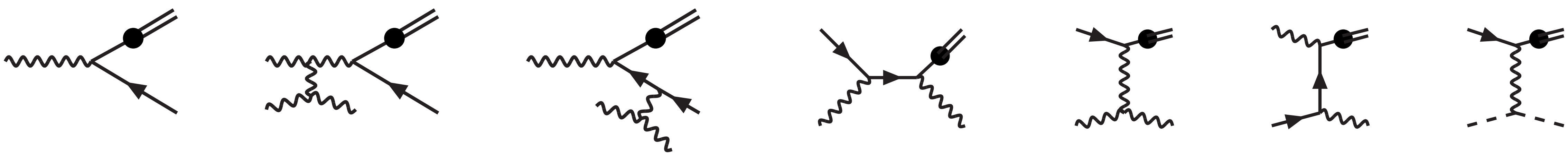


- In the indirect case ρ is directly proportional to the spf of active neutrinos, i.e.

$$\rho(K)^{\text{indir.}} = \frac{v^2}{2}\frac{M^2\, 2K\cdot \mathrm{Im}\,\Sigma}{(M^2 + 2K\cdot \mathrm{Re}\,\Sigma)^2 + 4(K\cdot \mathrm{Im}\,\Sigma)^2}$$

- Real part of the active neutrino self- energy
    - At high T $\;2K\cdot \mathrm{Re}\,\Sigma = -m_l^2 \sim g^2T^2$
    - At low T (positive) matter potential
    - (Broad) resonance

Indirect processes



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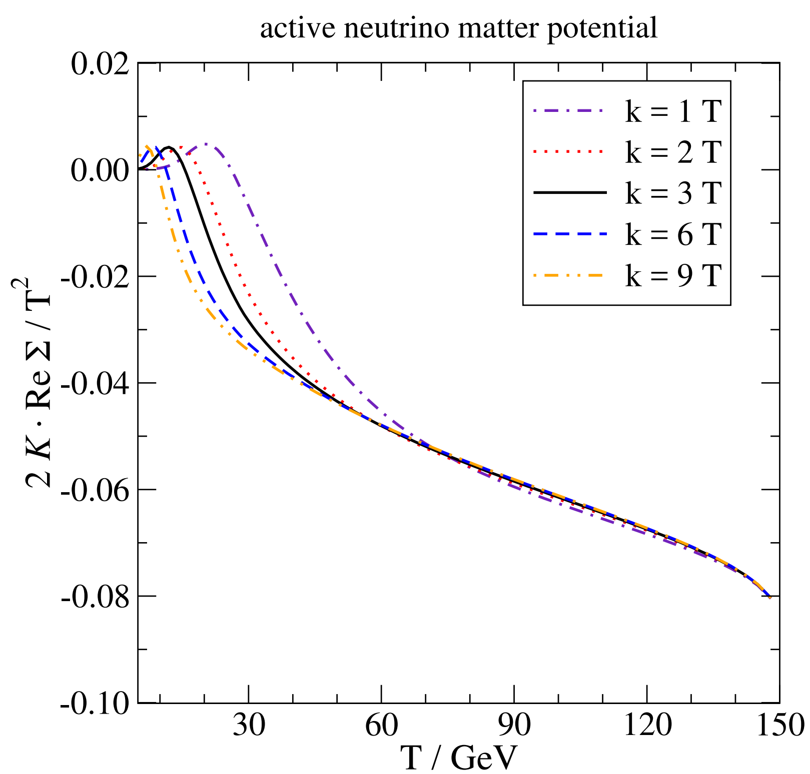
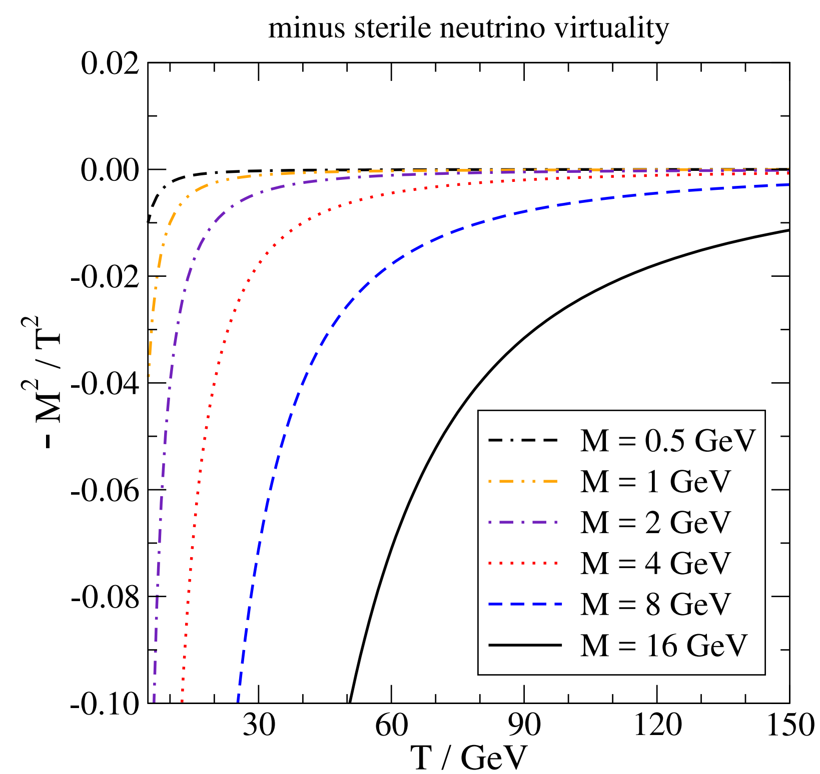
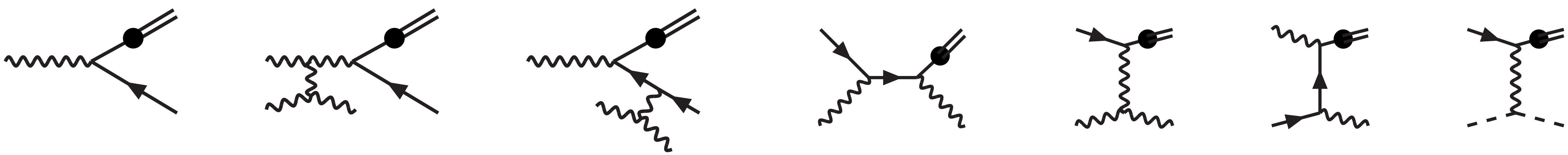




Indirect processes



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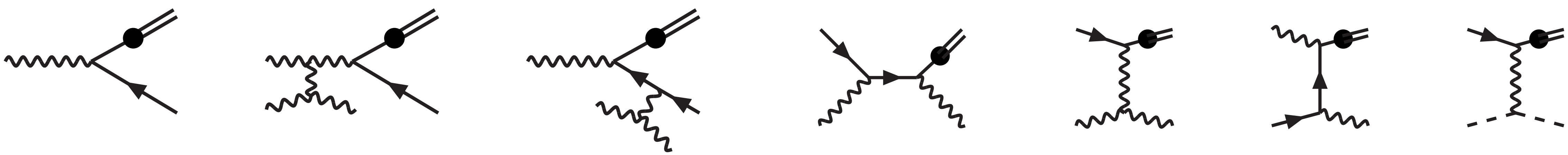
- Imaginay part of the active neutrino self- energy: active neutrino width $2K \cdot \mathrm{Im}\,\Sigma = k_0 \Gamma$
  - At high T dominated by soft 2↔2 scatterings. $\Gamma \sim g^2 T$ and thus (for $M \sim gT$) $\rho \sim v^2$
  - At low T dominated by 1↔2 decays of gauge bosons

Indirect processes



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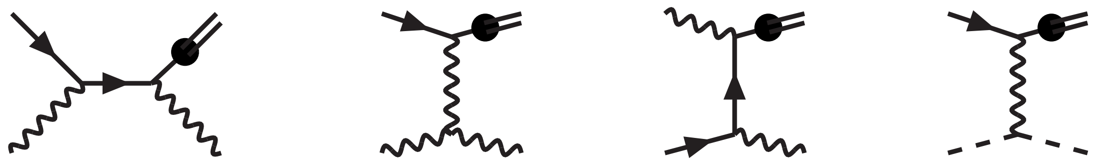
$$\rho(K)^{\text{indir.}} = \frac{v^2}{2} \frac{M^2 \, 2K \cdot \mathrm{Im}\,\Sigma}{(M^2 + 2K \cdot \mathrm{Re}\,\Sigma)^2 + 4(K \cdot \mathrm{Im}\,\Sigma)^2}$$

- Real part of the active neutrino self- energy
- Imaginary part of the active self energy
- Medium-modified mixing angle squared

$$\theta^2_{\text{med}} = \frac{v^2}{2} \frac{M^2}{(M^2 + 2K \cdot \mathrm{Re}\,\Sigma)^2 + 4(K \cdot \mathrm{Im}\,\Sigma)^2}$$

Indirect 2↔2 processes

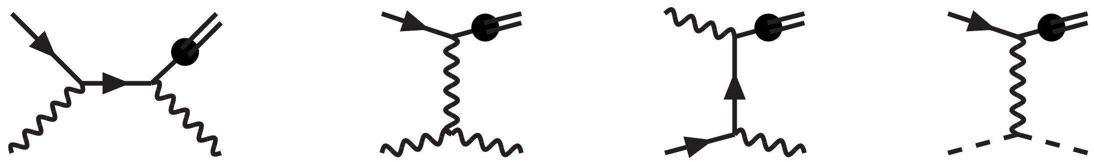


- Naively $\Gamma \sim g^4 T$, but soft ($Q \sim gT$) t-channel gauge boson scatterings have a large enhancement. Need to resum the "vacuum" masses and Hard Thermal Loops

- Euclideanization (Caron-Huot **PRD82** (2008)) still applicable. In the W exchange case
$$\Gamma_W^{\rm soft} = \frac{g_2^2 T}{4\pi}\int_0^\infty dq_\perp\, q_\perp \left[\frac{1}{q_\perp^2 + m_W^2} - \frac{1}{q_\perp^2 + m_W^2 + m_{E2}^2}\right]$$
transverse Euclidean propagator (vacuum mass only) - longitudinal propagator (vacuum and SU(2) screening mass)

- Z exchange more complicated (mixing of $SU(2)_L$ and $U(1)_Y$) but conceptually the same

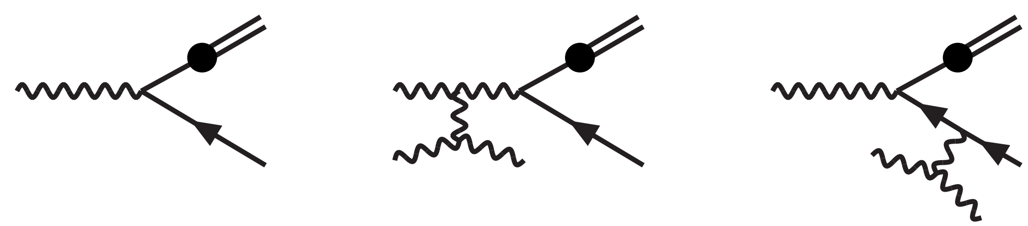
Indirect 2↔2 processes



$$\Gamma_W^{\rm soft} = \frac{g_2^2 T}{4\pi}\int_0^\infty dq_\perp\, q_\perp \left[\frac{1}{q_\perp^2 + m_W^2} - \frac{1}{q_\perp^2 + m_W^2 + m_{E2}^2}\right]$$

- At low T these approximations are inaccurate, they don't go into the Fermi limit
- We replace them with the Fermi limit results from Asaka Laine Shaposhnikov **JHEP01** (2007) (in a more compact form, as the masses of all scatterers are negligible for T>5 GeV)

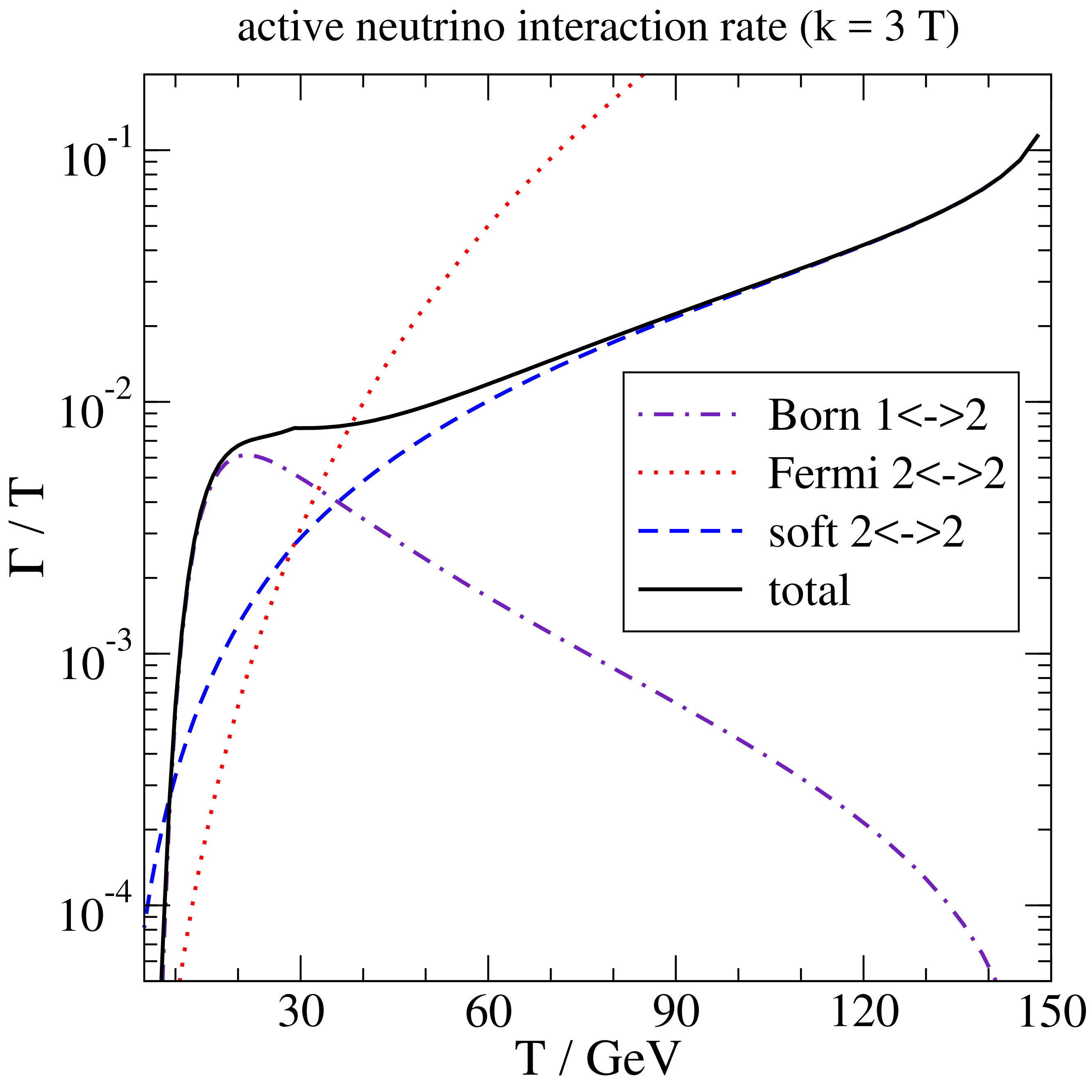
Indirect 1↔2 processes



- At **high** ***T*** they are very similar to the direct 1↔2 processes, with the scalar replaced by a gauge boson and the coupling $h \to g$. Hence $k_0\Gamma \sim g^2m^2 \sim g^4T^2$ and thus **negligible** w.r.t. the indirect 2↔2 processes
- At **low** ***T*** the LPM effect becomes negligible. The Born-level decays of gauge bosons into leptons $k_0\Gamma \sim g^2m^2$ become the **leading contribution**, also w.r.t the indirect 2↔2 processes

Indirect processes

- Soft 2↔2 scatterings, leading at high T
- 2↔2 scatterings in the Fermi limit, accurate but subleading at low T
- Born 1↔2 rate, leading at low T, inaccurate but negligible at high T
- **Total:** 1↔2 + the appropriate (smallest) 2↔2



Results

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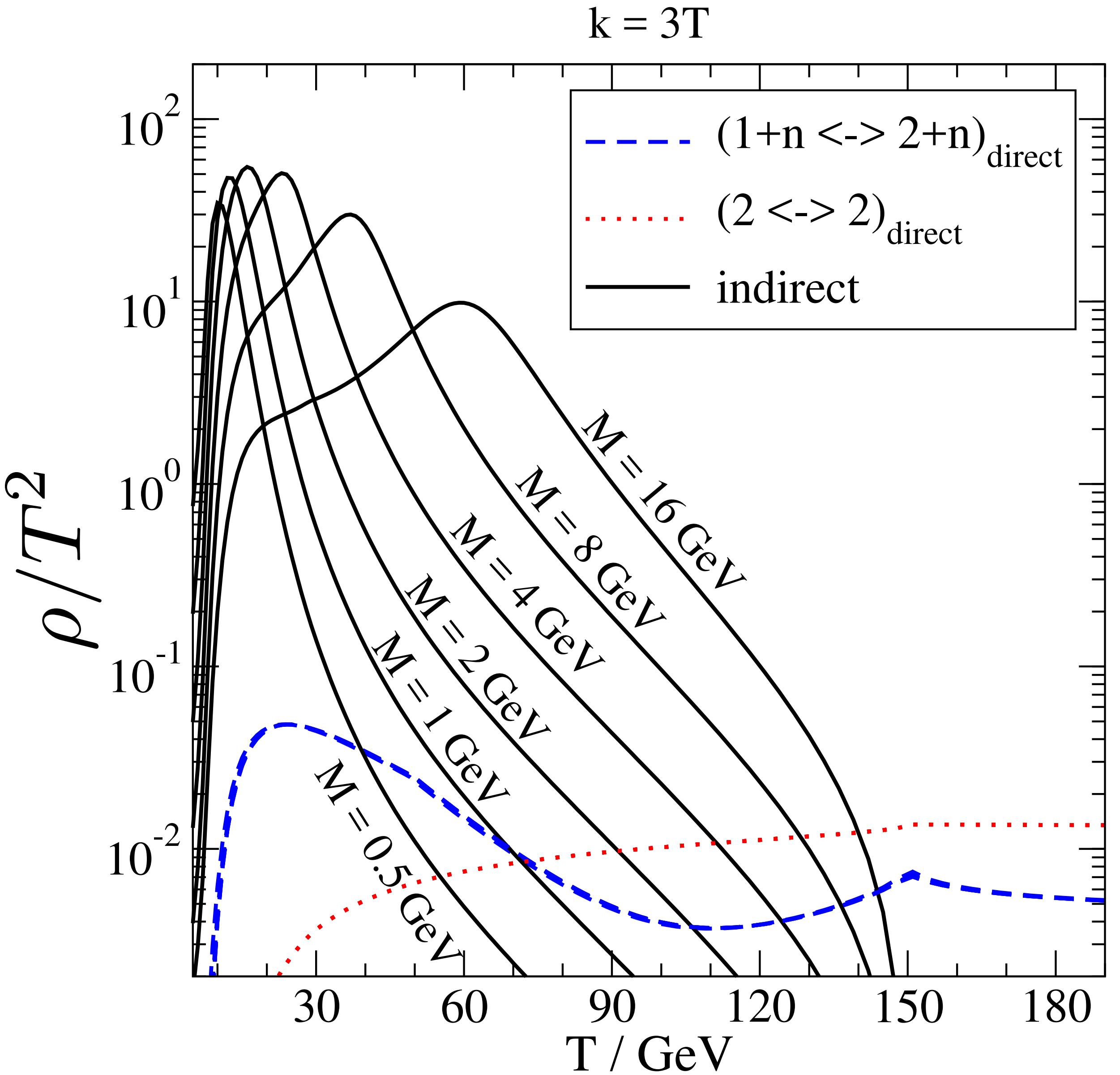
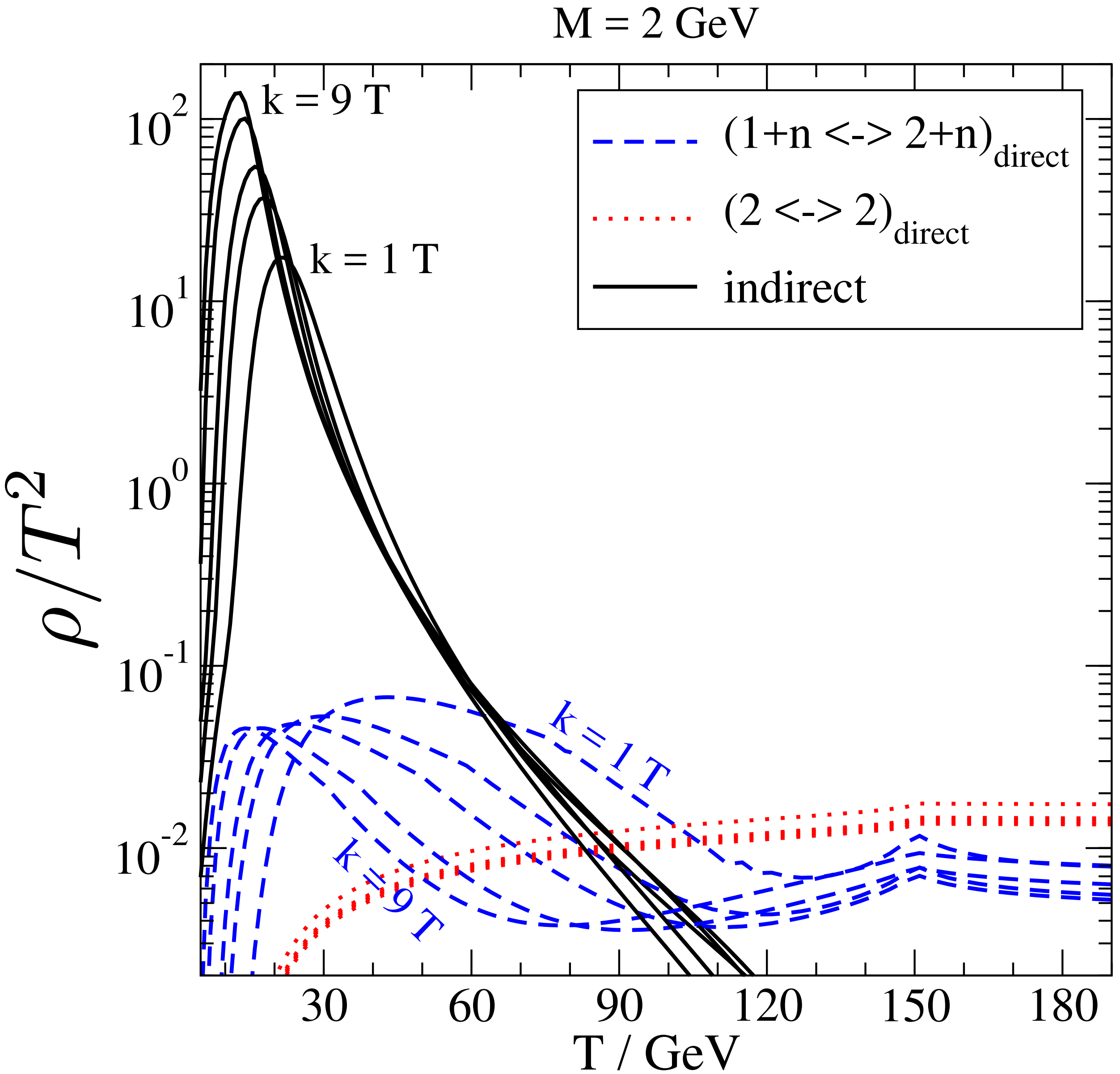
- Indirect processes rapidly dominate and peak at low T (in our 1-loop parameter fixing $T_{EW} \approx 150$ GeV)





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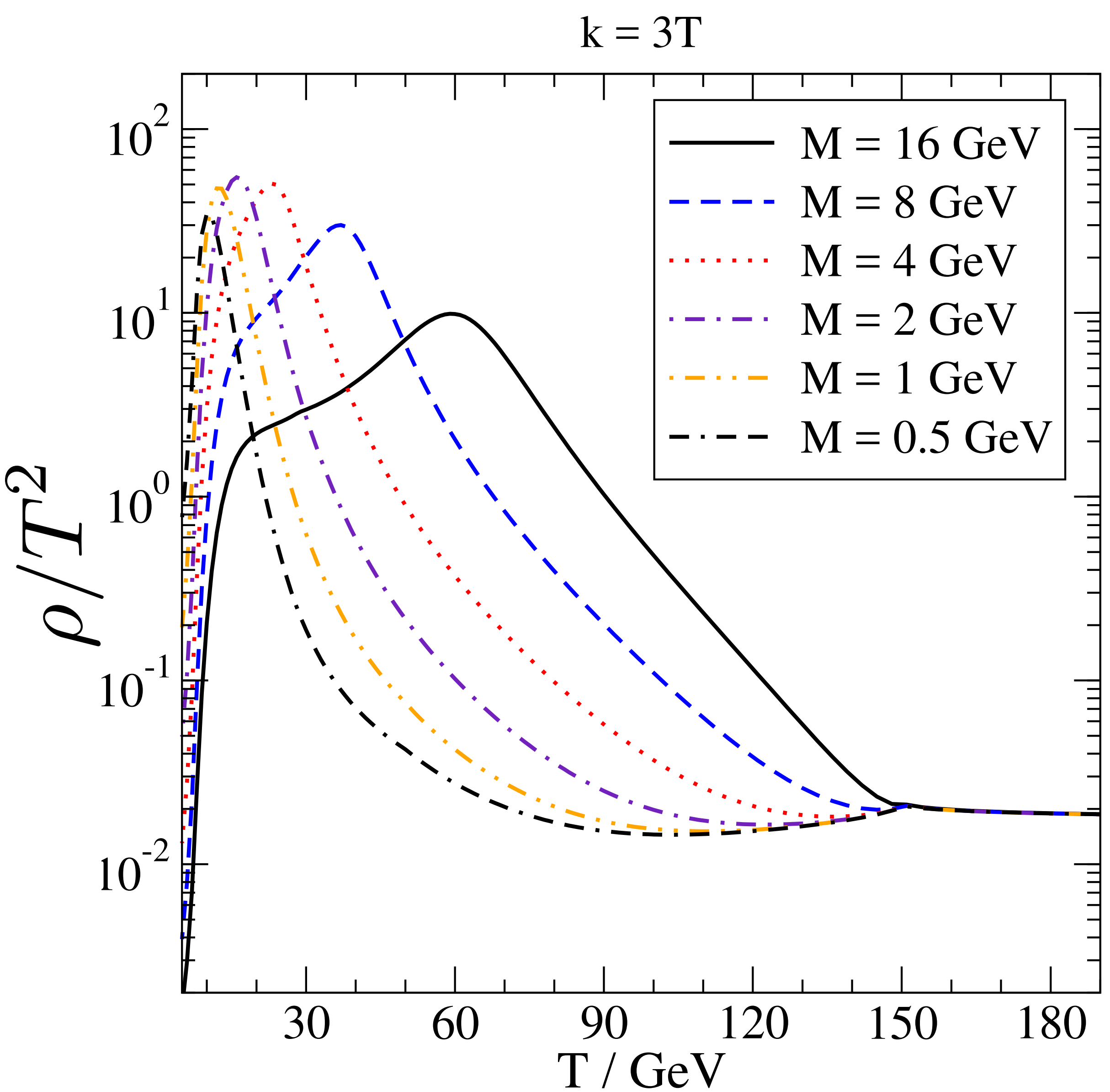
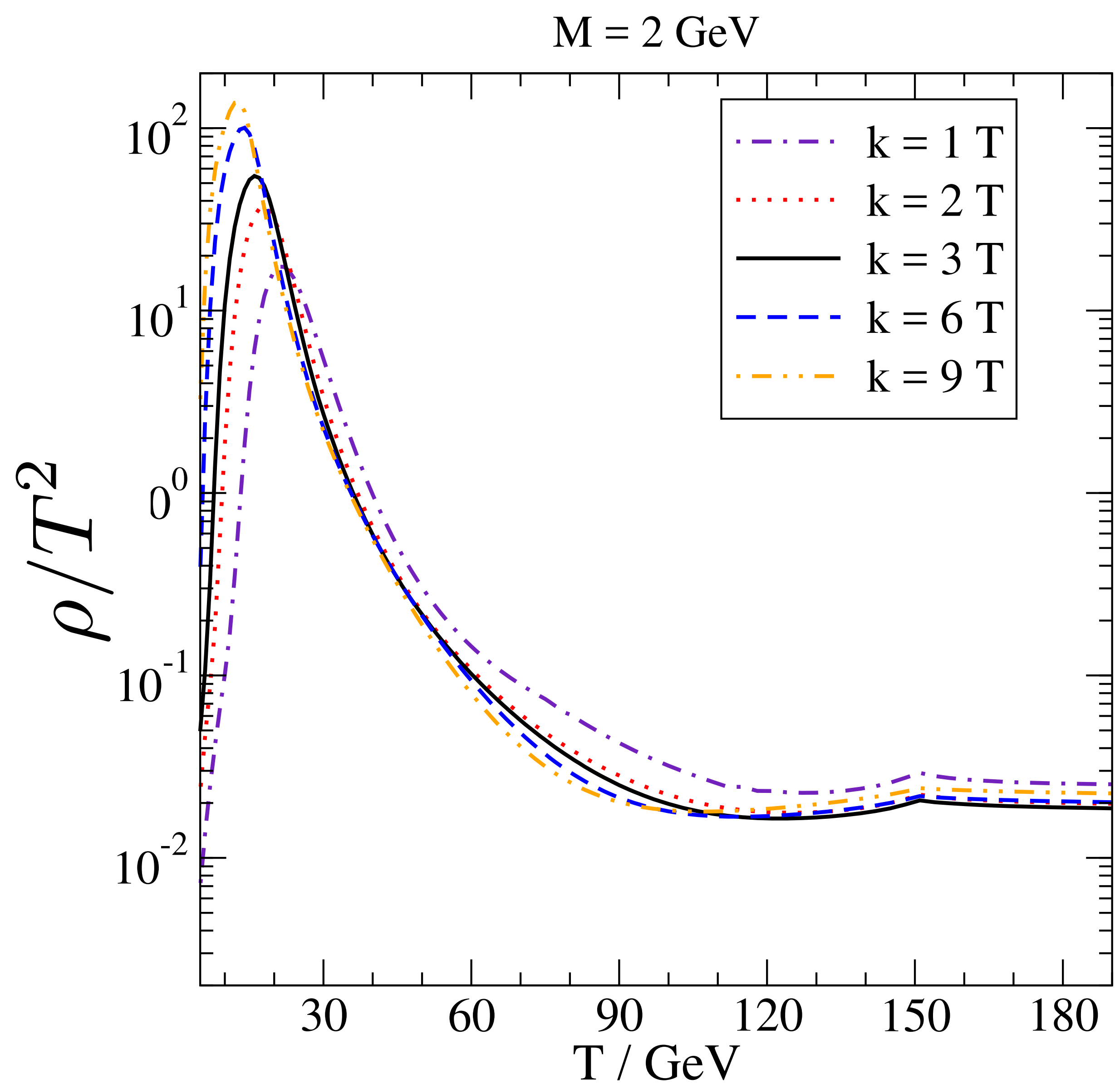




Cosmological implications

- Compare the equilibration and washout rates to the Hubble rate

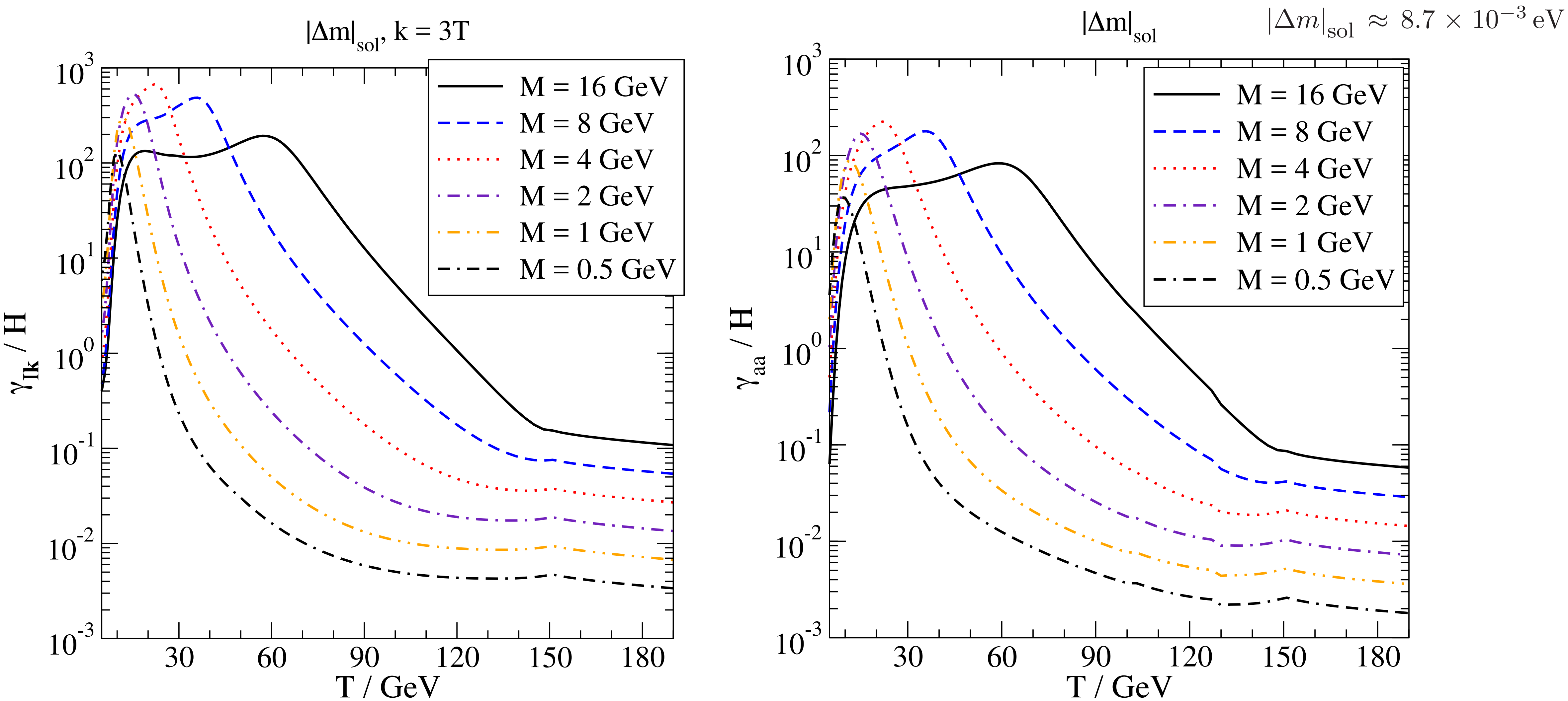
$$\gamma_{I\mathbf{k}} = \sum_a \frac{|h_{Ia}|^2 \rho(K)}{E_I}$$

$$H = \sqrt{\frac{8\pi e}{3 m_{\rm Pl}^2}}$$

$$\gamma_{ab} = -\sum_I \int \frac{d^3k}{(2\pi)^3} \frac{2 n_{\rm F}'(E_I) |h_{Ia}|^2 \rho(K)}{E_I} \Xi_{ab}^{-1}$$

- Fix the RHNs Yukawa couplings in a seesaw scenario with hierarchical neutrinos, with only one Yukawa coupling contributing to a given mass difference $|\Delta m| = |h_{Ia}|^2 v^2/(2M)$

Cosmological implications



- Leptogenesis possible because no equilibrium at T≳130 GeV
- Resonant generation of keV scale RHNs hindered by washout at T≲30 GeV. Fine-tuned windows still possible

Summary

- We have studied the dynamics of **GeV scale right-handed neutrinos** with Yukawa couplings to SM leptons and scalars in the **broken phase in the early universe**

- We have determined the **equilibration, production and washout rates at leading order** for 5 GeV $< T <$ 160 GeV

- In the broken phase **these rates peak at T~10-30 GeV**, due to the **efficient, resonance-like indirect processes**, with consequences for leptogenesis and keV scale dark matter

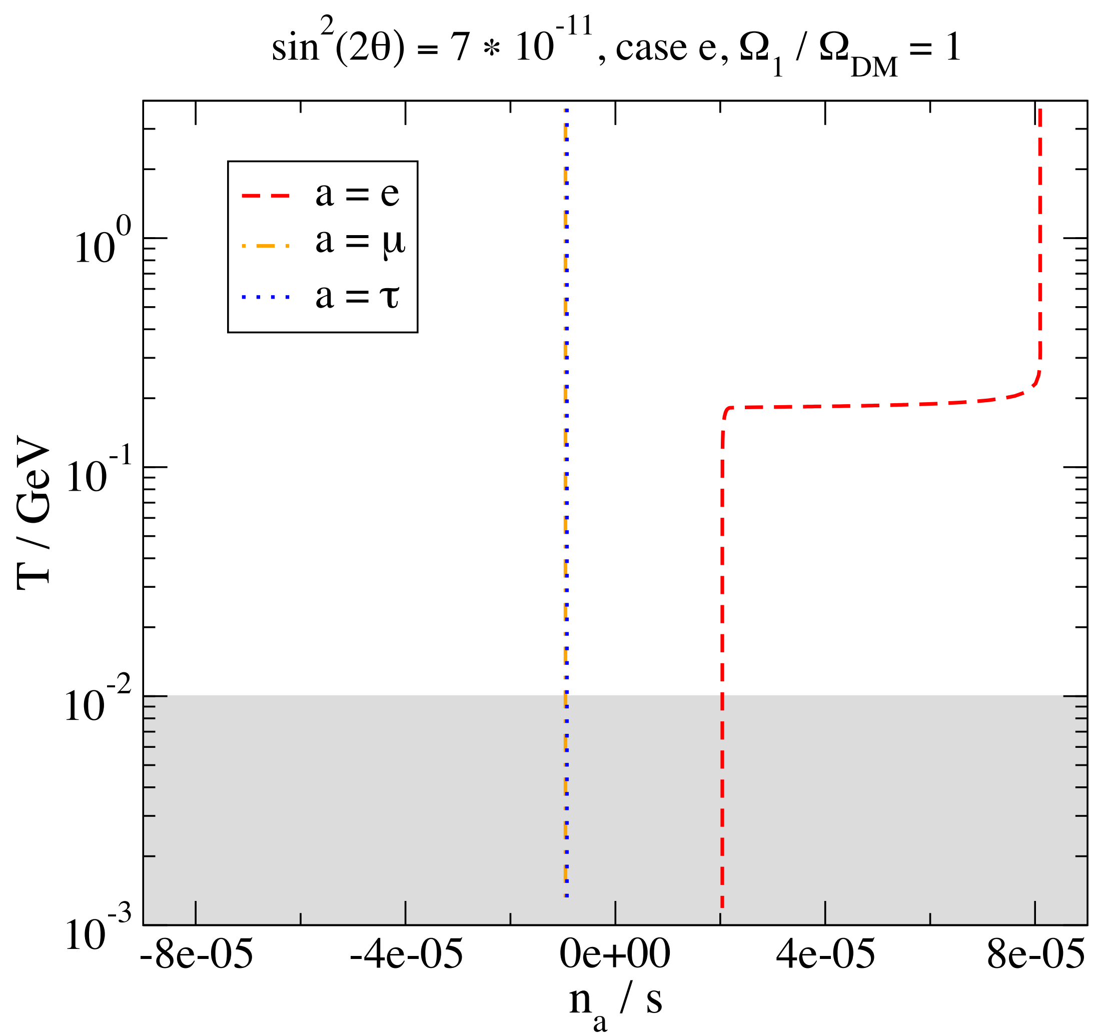
- Advancements in the understanding of HTLs and LPM in the broken phase
  Spectra available for download at
  http://www.laine.itp.unibe.ch/production-midT/

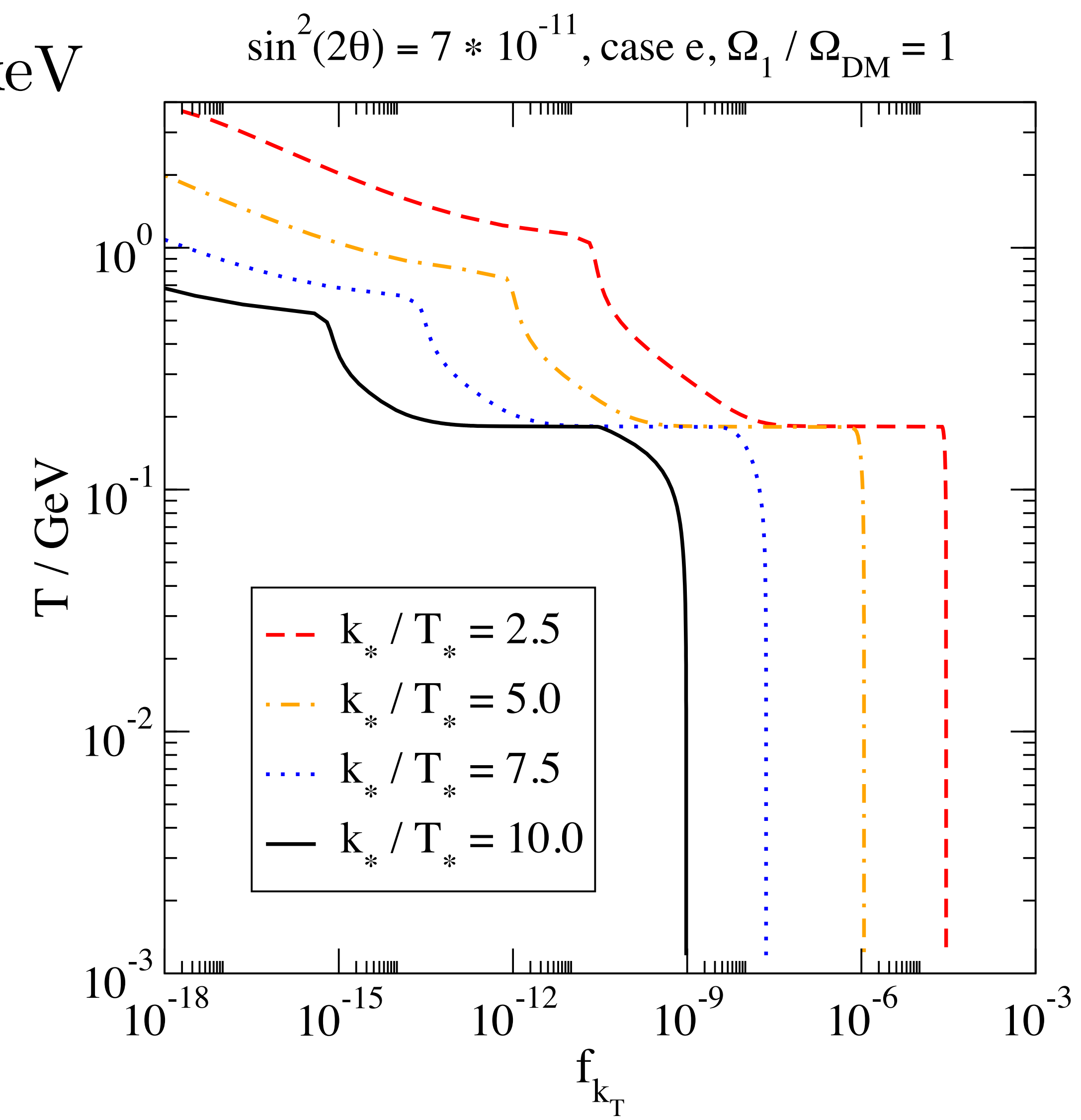
Backup

Resonant sterile neutrino production

- Resonant production of right-handed neutrinos: a non-zero lepton asymmetry (left) creates a resonance that efficiently converts it into right-handed neutrino (DM) abundance (right) JG Laine **JHEP1511** (2015)

$m_N = 7.1$ keV





Broken phase LPM

$$\Gamma_{3\times 3} = \begin{pmatrix} 2\Gamma_W(0)+\Gamma_Z(0) & -\Gamma_Z(y) & -2\Gamma_W(y) \\ -\Gamma_Z(y) & 2\Gamma_W(0)+\Gamma_Z(0) & -2\Gamma_W(y) \\ -\Gamma_W(y) & -\Gamma_W(y) & 2\Gamma_W(0)+\Gamma_{Z'}(0)-\Gamma_{Z'}(y) \end{pmatrix}$$

- In the Z and photon exchanges ($Z' \equiv$ combination that couples to charged left-handed leptons) the mixing of $SU(2)_L$ and $U(1)_Y$ components is different in the longitudinal and transverse gauge bosons exchanges

$$\Gamma_Z(y) \equiv \frac{(g_1^2+g_2^2)T}{4}\int_{\mathbf{q}_\perp} e^{i\mathbf{q}_\perp\cdot\mathbf{y}}\left[\frac{1}{q_\perp^2+m_Z^2} - \frac{\cos^2(\theta-\tilde\theta)}{q_\perp^2+m_{\tilde Z}^2} - \frac{\sin^2(\theta-\tilde\theta)}{q_\perp^2+m_{\tilde Q}^2}\right],$$

$$\Gamma_{Z'}(y) \equiv \frac{(g_1^2+g_2^2)T}{4}\int_{\mathbf{q}_\perp} e^{i\mathbf{q}_\perp\cdot\mathbf{y}}\left[\frac{\cos^2(2\theta)}{q_\perp^2+m_Z^2} + \frac{\sin^2(2\theta)}{q_\perp^2} - \frac{\cos^2(\theta+\tilde\theta)}{q_\perp^2+m_{\tilde Z}^2} - \frac{\sin^2(\theta+\tilde\theta)}{q_\perp^2+m_{\tilde Q}^2}\right]$$

$$m_{\widetilde W}^2 \equiv m_W^2 + m_{\mathrm{E2}}^2\,, \quad m_{\tilde Z}^2 \equiv m_+^2\,, \quad m_{\tilde Q}^2 \equiv m_-^2\,,$$

$$m_\pm^2 \equiv \frac{1}{2}\left\{m_Z^2 + m_{\mathrm{E1}}^2 + m_{\mathrm{E2}}^2 \pm \sqrt{\sin^2(2\theta)m_Z^4 + [\cos(2\theta)m_Z^2 + m_{\mathrm{E2}}^2 - m_{\mathrm{E1}}^2]^2}\right\}$$

$$\sin(2\tilde\theta) \equiv \frac{\sin(2\theta)m_Z^2}{\sqrt{\sin^2(2\theta)m_Z^4 + [\cos(2\theta)m_Z^2 + m_{\mathrm{E2}}^2 - m_{\mathrm{E1}}^2]^2}}$$