The equation of state and the sphaleron rate in the Standard Model

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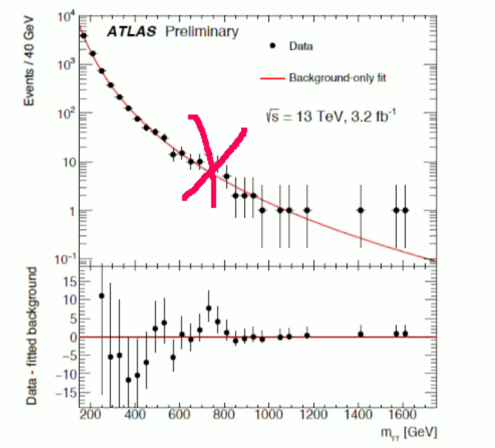
D'Onofrio, Rummukainen, Tranberg arXiv:1404.3565
D'Onofrio, Rummukainen, arXiv:1508.07161

Big Bang and the little bangs, CERN 8/2016

The “Bump”

- The “bump” has ceased to be (. . . or is it just resting?)
- the Standard Model rules (still)



Phase transitions in the Standard Model

- *No* phase transitions in the Standard Model (at $\mu = 0$)
  - QCD and EW "phase transitions" are cross-overs
- Many BSM models have a first order EW phase transition $\mapsto$
  - EW baryogenesis (*Kainulainen*)
  - Gravitational waves (*Weir*)

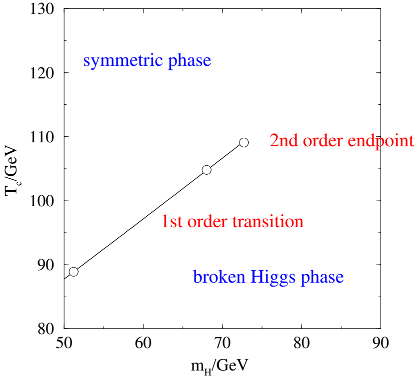
In this talk:

- Precision results of EW:
  - Pseudocritical temperature
  - Equation of state - “softness”, width of the cross-over
  - Sphaleron rate
- Why study these?
  - it’s the Standard.
  - Background to cosmological applications, e.g. leptogenesis
  - Precision properties of physics

Phase diagram of the electroweak SM

- After lots of activity on and off the lattice:

$\rightarrow$ No phase transition at all, smooth "cross-over" for $m_{\rm Higgs} \gtrsim 72\,$GeV



[Kajantie,Laine,K.R.,Shaposhnikov,Tsypin 95–98]

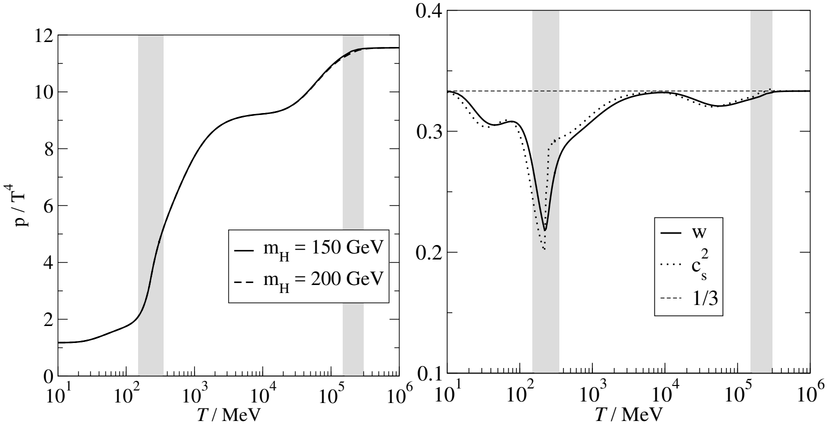
see also

[Csikor,Fodor, Heitger]

[Gürtler,Ilgenfritz,Schiller,Strecha]

Overall EOS

[Laine, Schröder 2006]



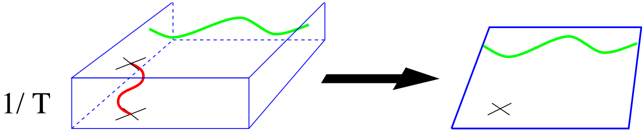
- Perturbation theory + Lattice QCD + Hadron RG
- Here EW transition featureless

Equation of state and the pseudocritical temperature

3d effective theory

- Tool for perturbative and lattice computations
- Modes $p > g^2 T$ are perturbative (at weak coupling): can be integrated out in stages:

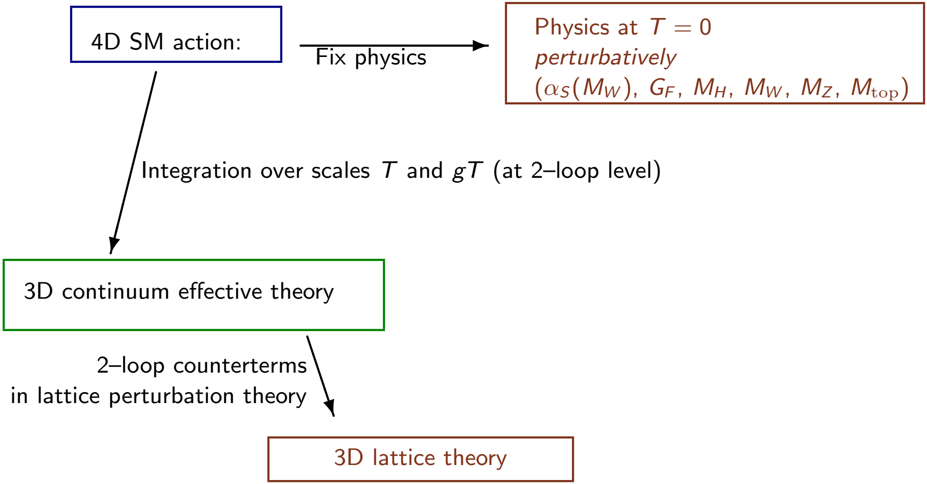
  1. $p \gtrsim T$: fermions, non-zero Matsubara frequencies
  $\rightarrow$ 3d theory (dimensional reduction)



  2. Electric modes $p \sim gT$

- Obtain a "magnetic theory" for modes $p \lesssim g^2 T$. Fully contains the non-perturbative thermal physics.

3d effective theory

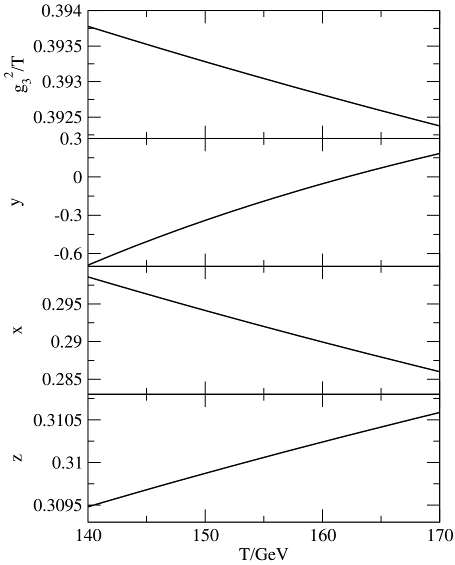


3d effective Lagrangian:

$$L = \frac{1}{4} F^a_{ij} F^a_{ij} + \frac{1}{4} B_{ij} B_{ij} + (D_i\phi)^\dagger D_i\phi + m_3^2 \phi^\dagger\phi + \lambda_3 (\phi^\dagger\phi)^2$$

- SU(2) + U(1) gauge + Higgs
- Parameters:

$$g_3^2 \sim g_W^2 T$$
$$x \equiv \lambda_3 / g_3^2$$
$$y \equiv m_3^2 / g_3^4$$
$$z \equiv g_3'^2 / g_3^2$$



Calculating the EOS

- Developed by Laine and Meyer 2015
- Higgs mass parameter ν and T are the only dimensionful parameters
- For dimensional reasons:

$$\frac{p(T)}{T^4} = \frac{p_{\rm R}}{T^4}\left(\frac{\bar\mu}{T}, \frac{\nu^2(\bar\mu)}{T^2}, g^2(\bar\mu)\right) - \frac{p_{\rm 0R}(\bar\mu, \nu^2(\bar\mu), g^2(\bar\mu))}{T^4}$$

  $\bar\mu$: $\overline{\rm MS}$ scale, $p_{\rm 0R}$: zero-temperature pressure
  $g^2(\bar\mu)$: all dimensionless couplings in the SM
- *Interaction measure:*

$$\begin{aligned}\Delta &= T\frac{\mathrm{d}}{\mathrm{d}T}\frac{p(T)}{T^4} \\ &= -\frac{\partial(p_{\rm R}/T^4)}{\partial \ln(\bar\mu/T)} - \frac{2\nu^2[Z_m\langle\phi^\dagger\phi\rangle_{\rm 4d}]_{\rm R}}{T^4} + \frac{4p_{\rm 0R}}{T^4} \\ &\equiv \Delta_1 + \Delta_2 + \Delta_3\end{aligned}$$

- Applying 3d effective theory: $\Delta_2 = \Delta_2^A + \Delta_2^B\,\frac{\langle\phi^\dagger\phi\rangle_{\rm 3D}}{T}$

Calculating the EOS

- Thus, $\Delta = \Delta_1 + \Delta_3 + \Delta_2^A + \delta_2^B \frac{\langle \phi^\dagger \phi \rangle_{\rm 3D}}{T}$
- All Δ_i's computed perturbatively, uncertainty $O(g^6)$ [Laine, Meyer]
- On the other hand, $\langle \phi^\dagger \phi \rangle_{\rm 3D}$ can be perturbatively computed in the symmetric phase and in the broken phase
- Expansions fail near $m^2/g_3^4 \equiv y \sim 0$ – i.e. just at the cross-over

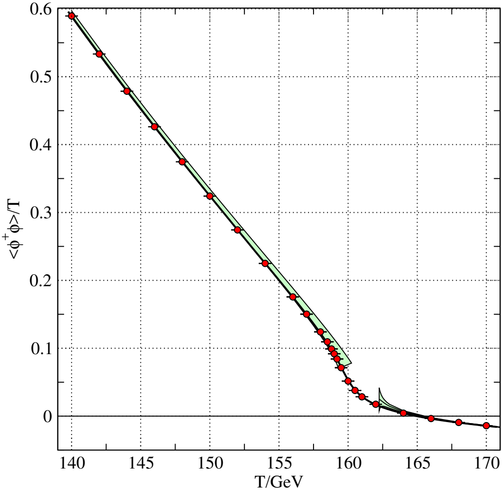
$\Rightarrow$ Non-perturbative evaluation of $\langle \phi^\dagger \phi \rangle$ needed $\Rightarrow$ Lattice simulations

- Using thermodynamic identities, other quantities can be obtained: p, e, heat capacity C_V, speed of sound c_s ...
- Laine and Meyer did the analysis, but using poor quality lattice data (provided by me et al.).

$\Rightarrow$ Revisit with new simulations

Higgs field expectation value

- Red dots: 3D lattice (continuum limit)
- Green bands:
  - Broken phase: 2-loop Coleman-Weinberg [Kajantie et al 95]
  - Symmetric phase: 3-loop [Laine and Meyer 2015]
- Agreement remarkably good away from the cross-over
- P.T. does not converge near the cross-over



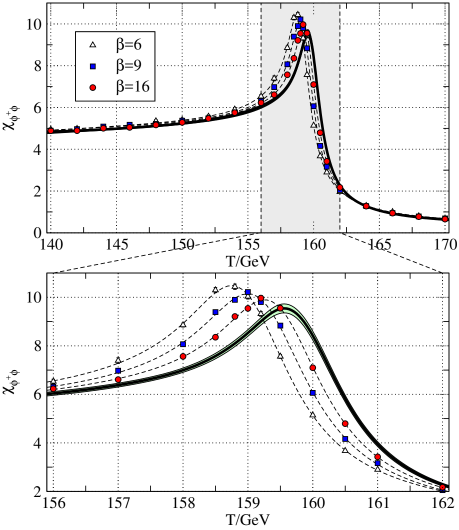
Higgs susceptibility & pseudocritical temperature

- Define pseudocritical T: maximum location of the $\langle\phi^\dagger\phi\rangle$ susceptibility $\chi_{\phi^\dagger\phi}$:

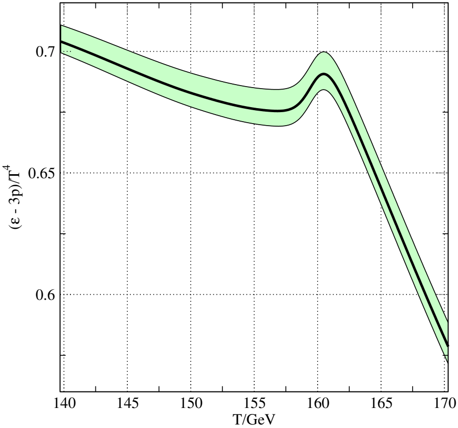
  $T_c = 159.6 \pm 0.1 \pm 1.5$ GeV

- 1st error: lattice errors
- 2nd error: estimate of the systematic uncertainty in 3d action

Right: continuum extrapolation of χ

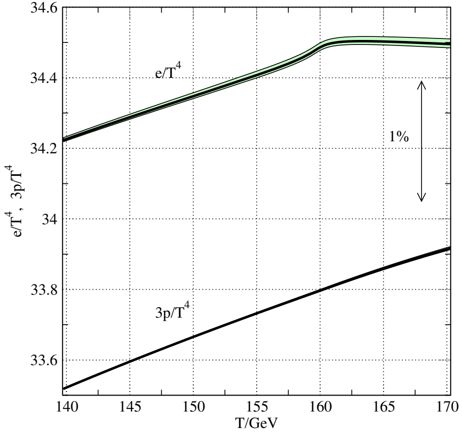


Interaction measure



- Convert $\langle \phi^\dagger \phi \rangle$ to interaction measure $\Delta = (\epsilon - 3p)/T^4$
- Band: lattice errors + variation $\bar{\mu} = (0.5 \ldots 2)\pi T$.
- Non-trivial feature between broken phase (low T) and symmetric phase (high T) behaviour
- Width of the cross-over region $\sim 2 \ldots 3$ GeV

Pressure and energy density

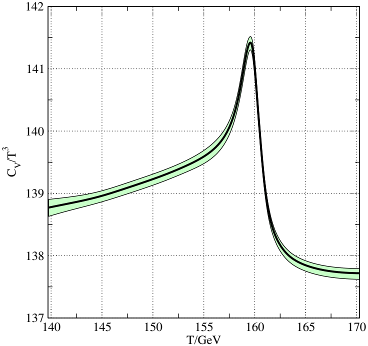


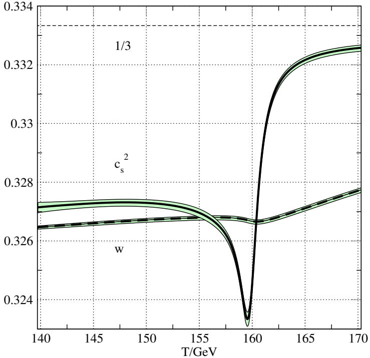
- Pressure:

$$\frac{p(T)}{T^4} - \frac{p(T_0)}{T_0^4} = \int_{T_0}^{T} dT' \frac{\Delta(T')}{T'}$$

- Use reference temperature $T_0 = 140\,\mathrm{GeV}$,
- pert. pressure $p(T_0)/T_0^4 = 11.173$ [Laine, Schröder 2006]
- Energy density $e = \Delta + 3p$
- $p(T_0)$ has $\sim 1\%$ uncertainty $\rightarrow$ uncertainty for e and p

More thermodynamics



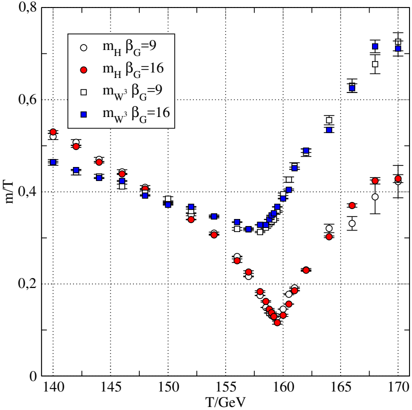


- Heat capacity $C_V = e'(T)$
- Speed of sound: $c_s^2 = p'/e'$
- EOS parameter $w = p/e$

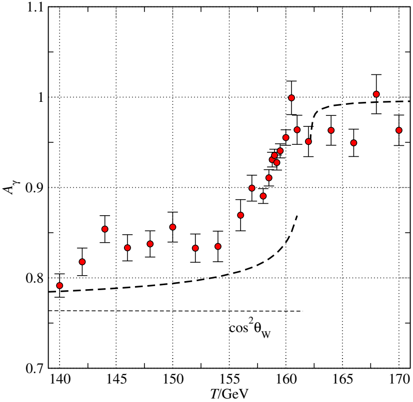
Cross-over well defined, but very soft!

Masses

Higgs and W^3 screening masses



Effective $\cos^2 \theta_{\mathrm{Weinberg}}$



Sphaleron rate

Background

- Anomaly: baryon number B and gauge topology are connected:

$$\Delta B = \Delta L = 3\Delta N_{\mathrm{CS}} = \frac{3}{32\pi^2}\int_0^t dt \int dV F^{\mu\nu}\tilde{F}_{\mu\nu}$$

- Baryogenesis
- Rate in thermal equilibrium:

$$\Gamma = \lim_{V,t\to\infty} \frac{\langle(\Delta N_{\mathrm{CS}}(t))^2\rangle}{Vt}$$

- In the symmetric phase, $\Gamma \propto \alpha_W^5 T^4$ [Arnold, Son, Yaffe 97] or rather $\Gamma \propto \alpha_W^5 \log(1/\alpha_W) T^4$ [Bödeker 98]
- In the broken phase the rate is exponentially suppressed
- Turning off of the rate is important for some baryogenesis scenarios

Calculation of the sphaleron rate

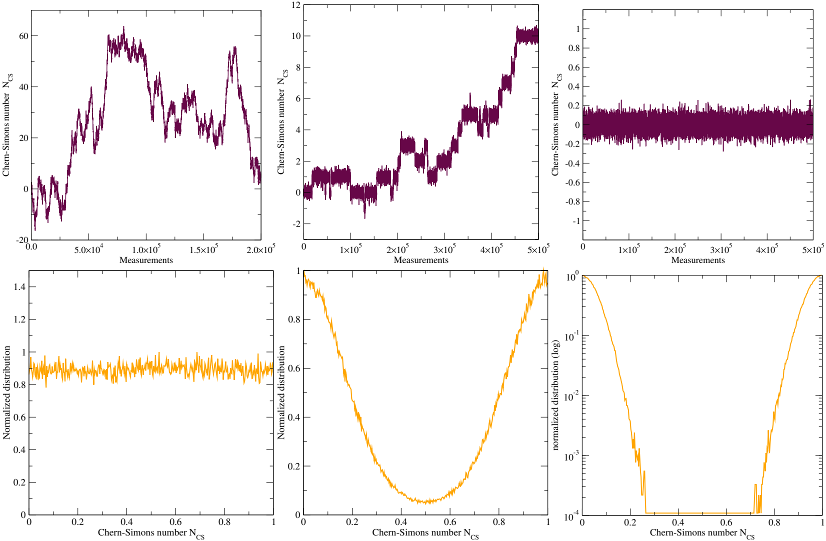
Non-perturbative $\Rightarrow$ real-time lattice simulations
Several methods:

- Classical EOM [Ambjorn, Krasnitz 95; + many]
  - UV modes (HTL) make result lattice spacing depedent [Arnold 97]
- Classical + HTL effective theories [Moore, Hu, Muller 97; Bodeker, Moore, K.R 99]
  - More control over UV modes, no continuum limit
  - Used also in studies of plasma instabilities in HIC
- Bödeker method: [Bödeker 98], heat bath version [Moore 98]
  - Fully dissipative evolution of soft ($g^2 T$) modes
  - ✓ Exact to leading log order $\log(1/g^2)$, $\exists$ continuum limit, very simple to use
  - Same "action" and cont. limit as in 3D thermo simulation

$\exists$ lot of lattice results in pure gauge, few in broken EW phase.
Here: physical Higgs mass

Evolution of $N_{\rm CS}$

Symmetric $T = 152$GeV, broken $T = 145$GeV, deeply broken $T = 140$GeV (with $m_H = 113$GeV) [D'Onofrio et al 12]



Multicanonical evolution

- Deep in the broken phase the barrier between vacua is large:

⇒ Rate becomes very small

  - Too small to be measured with std. method
  - But still physically relevant!

- Rate can me measured with **multicanonical** methods, which enables to overcome potential barriers in simulations
- For broken phase sphaleron rate, method developed by [Moore 99]

Result: sphaleron rate

Symmetric phase:

$$\frac{\Gamma}{T^4} = (8.0 \pm 1.3) \times 10^{-7} \approx (18 \pm 3)\alpha_W^5$$

Broken phase: parametrized as

$$\log \frac{\Gamma}{T^4} = (0.83 \pm 0.01)\frac{T}{\text{GeV}} - (147.7 \pm 1.9)$$

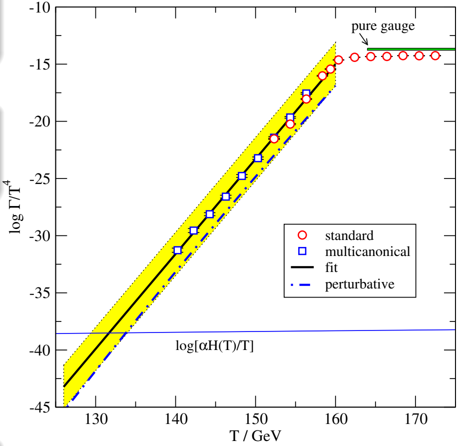
Errors dominated by systematics

Cosmology: Hubble cooling $\dot{T} = -HT$
Freeze-out temperature T_* from

$$\frac{\Gamma(T_*)}{T_*^3} = \alpha H(T_*)$$ with $\alpha \approx 0.1015$ [Burnier et al 05]

Baryon number freeze-out

$$T_* = 131.7 \pm 2.3 \text{ GeV}$$



Conclusions

- Standard Model equation of state solved to 1% level
- Pseudocritical temperature $T_c = 159.6 \pm 0.1 \pm 1.5$ GeV
- Cross-over weak but well defined
- Width of the cross-over region 2-3 GeV
- Baryon number freeze-out $T_* = 131.7 \pm 2.3$ GeV
- Symmetric phase sphaleron rate $\Gamma / T^4 = (18 \pm 3)\alpha_W^5$
- Broken phase rate can be parametrized as $\log \Gamma / T^4 = (0.83 \pm 0.01)\, T/\text{GeV} - (147.7 \pm 1.9)$
- ... can be fed in to e.g. some leptogenesis scenarios