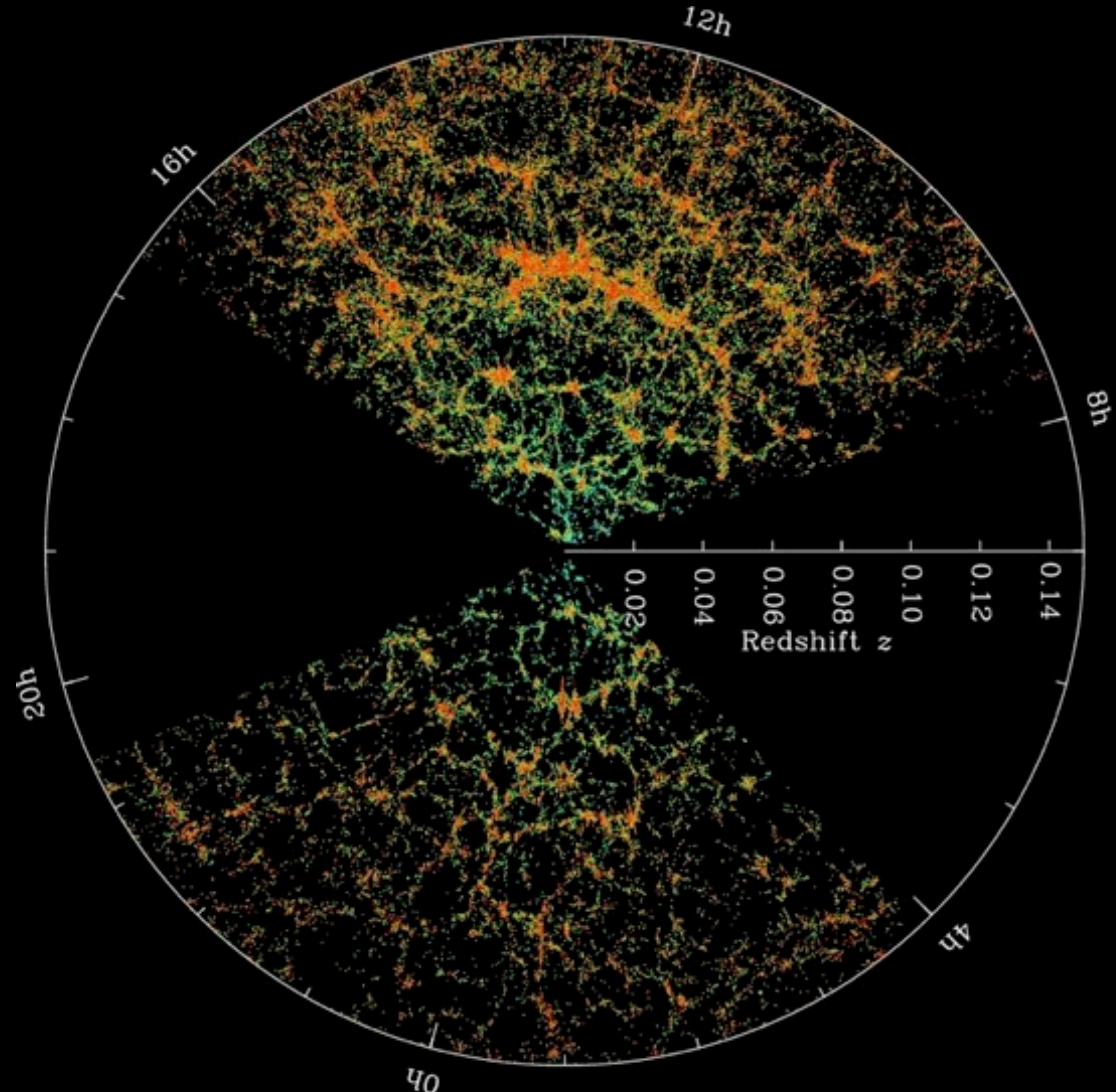
FOUND in TRANSLATION: Applying the methods of QFT to Cosmic Structure

Sergey Sibiryakov

CERN, August 2016

The beautiful Universe of SDSS



Existing galaxy surveys:

SDSS

DARK ENERGY SURVEY

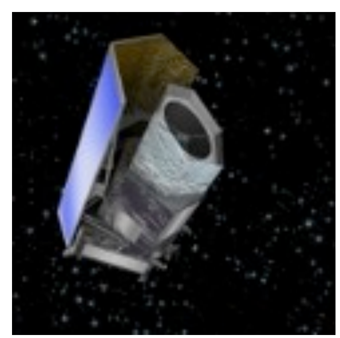
2MASS
2 MICRON ALL SKY SURVEY

PAU

Future surveys:

LSST

Euclid



ENERGY SPECTROSCOPIC INSTRUMENT

SKA
SQUARE KILOMETRE ARRAY

Physics with LSS

- primordial non-gaussianity

  ➡ interactions in the inflationary sector

- baryon acoustic oscillations = standard ruler in the Universe

  ➡ dark energy equation of state

- evolution of perturbations

  ➡ neutrino mass

  properties of dark matter (e.g. fifth force, WDM) and dark energy (e.g. clustering)

Primordial non-gaussianity

gaussian random field: $\langle \delta_\rho(k_1)\delta_\rho(k_2)\rangle = P(k_1)\delta(k_1+k_2)$

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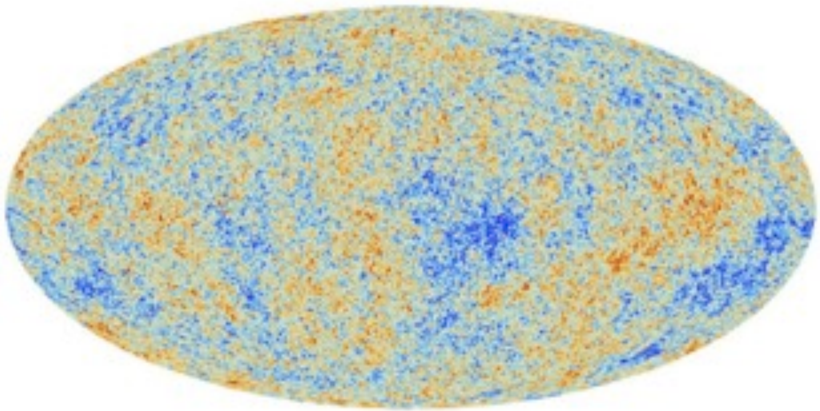
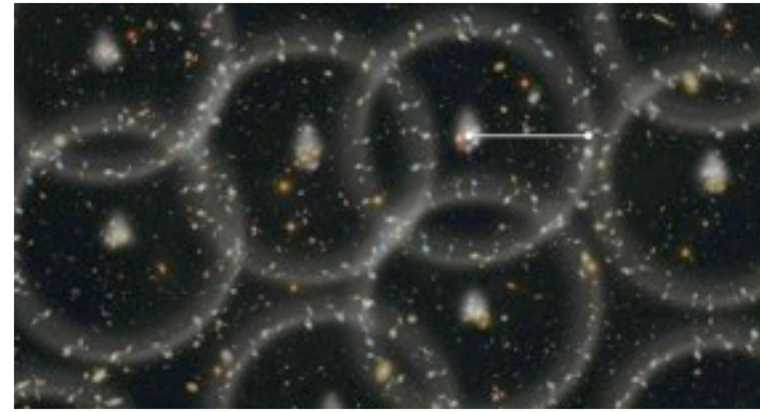
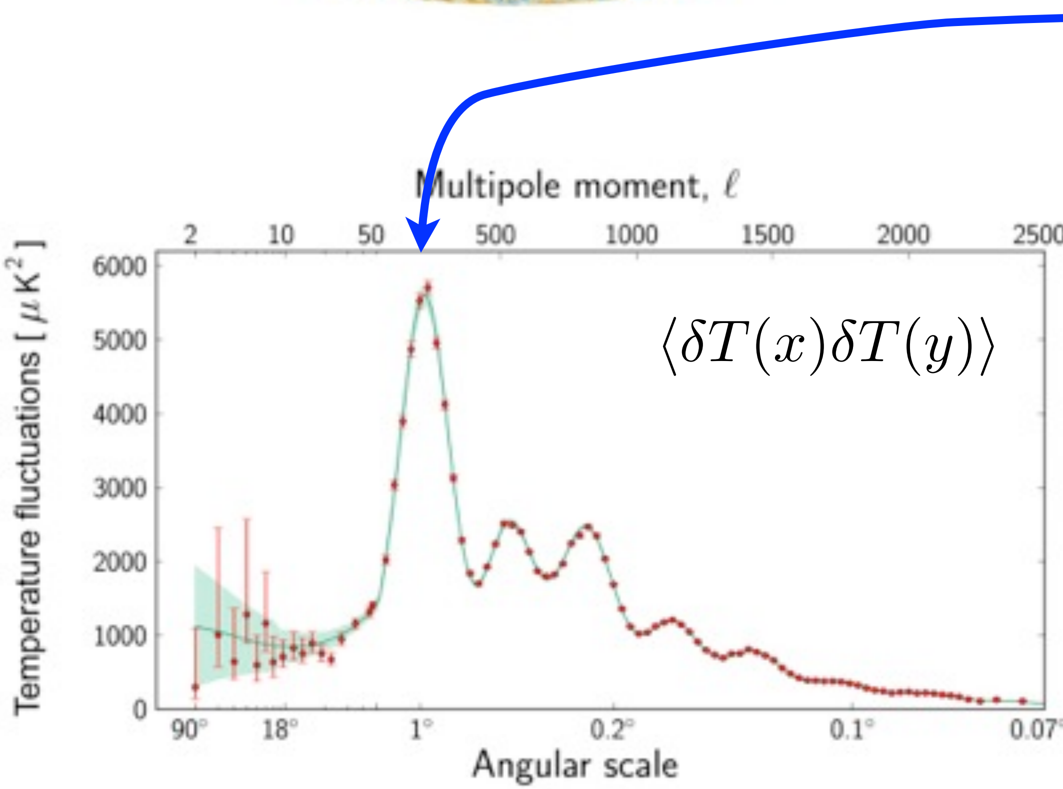
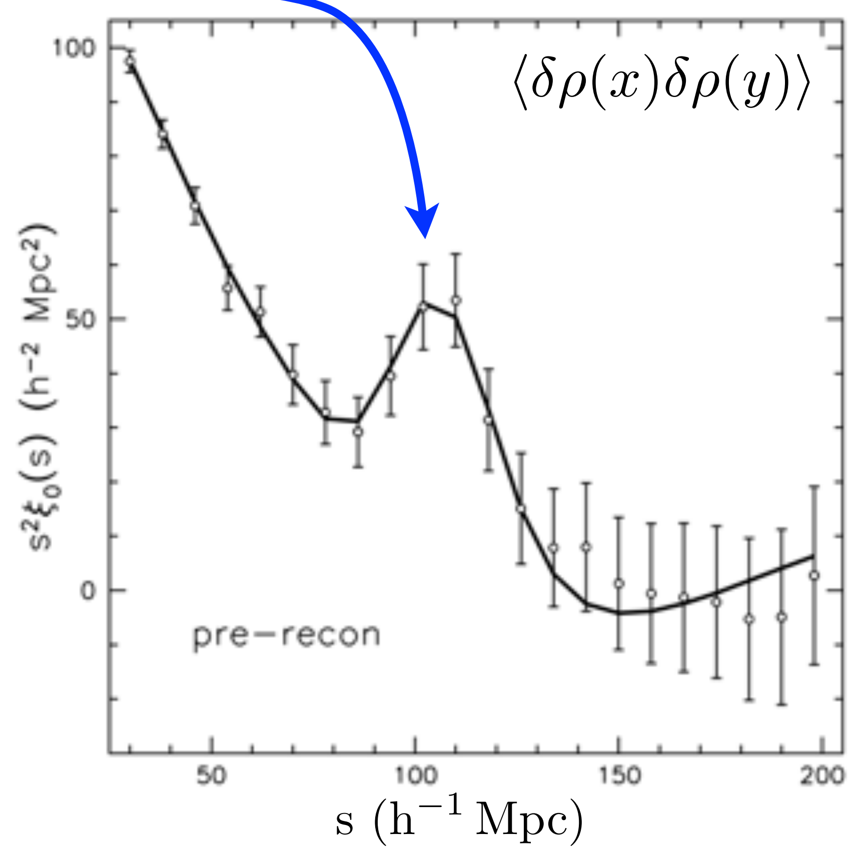
$f_{NL} \sim 1$ naturally appears in extended inflationary models (multiple fields, extended kinetic action, ...)

Baryon acoustic oscillations









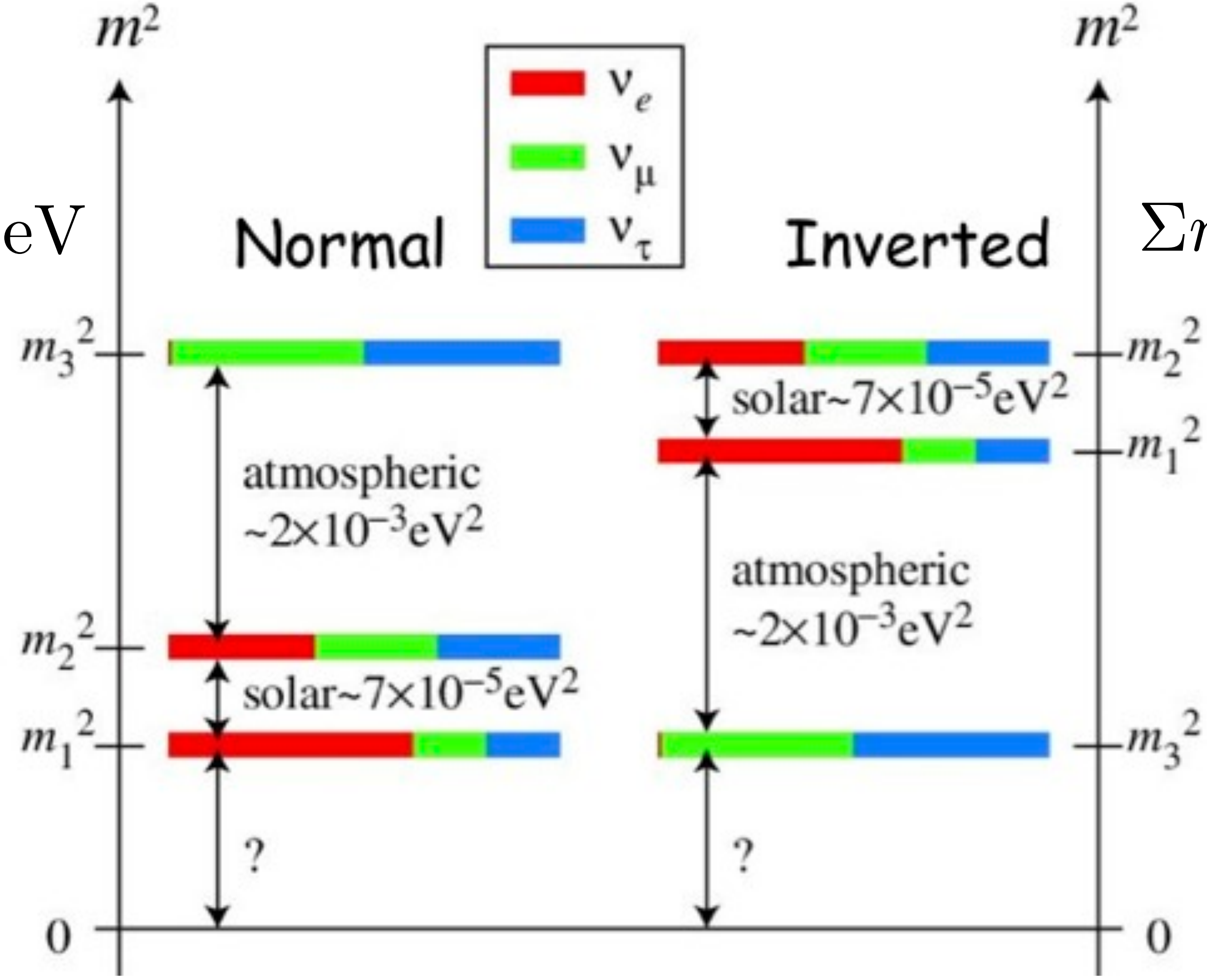
Planck collaboration

Anderson et al. (BOSS collaboration)

Neutrino mass: current status

$\Sigma m_\nu > 0.05$ eV

$\Sigma m_\nu > 0.1$ eV



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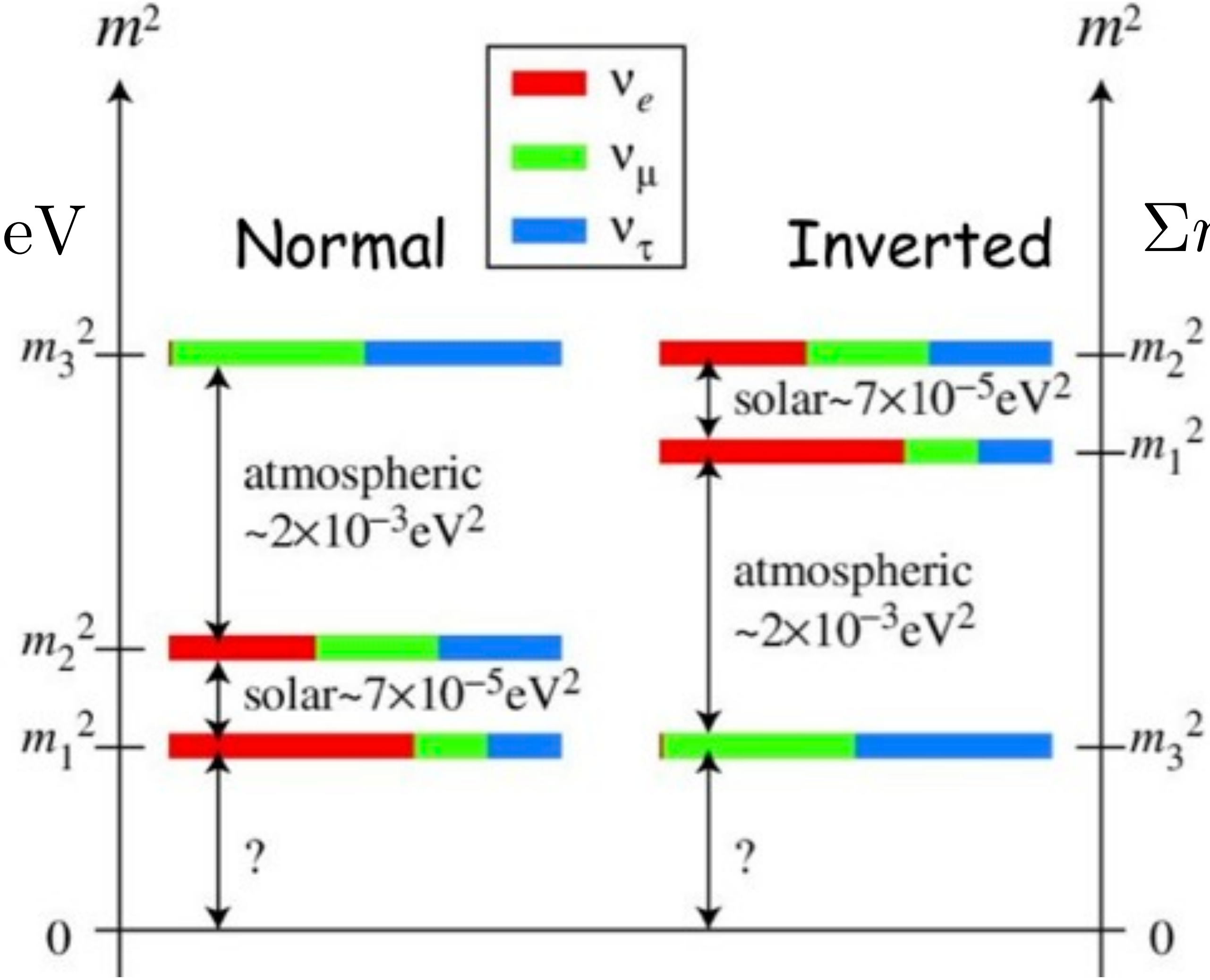

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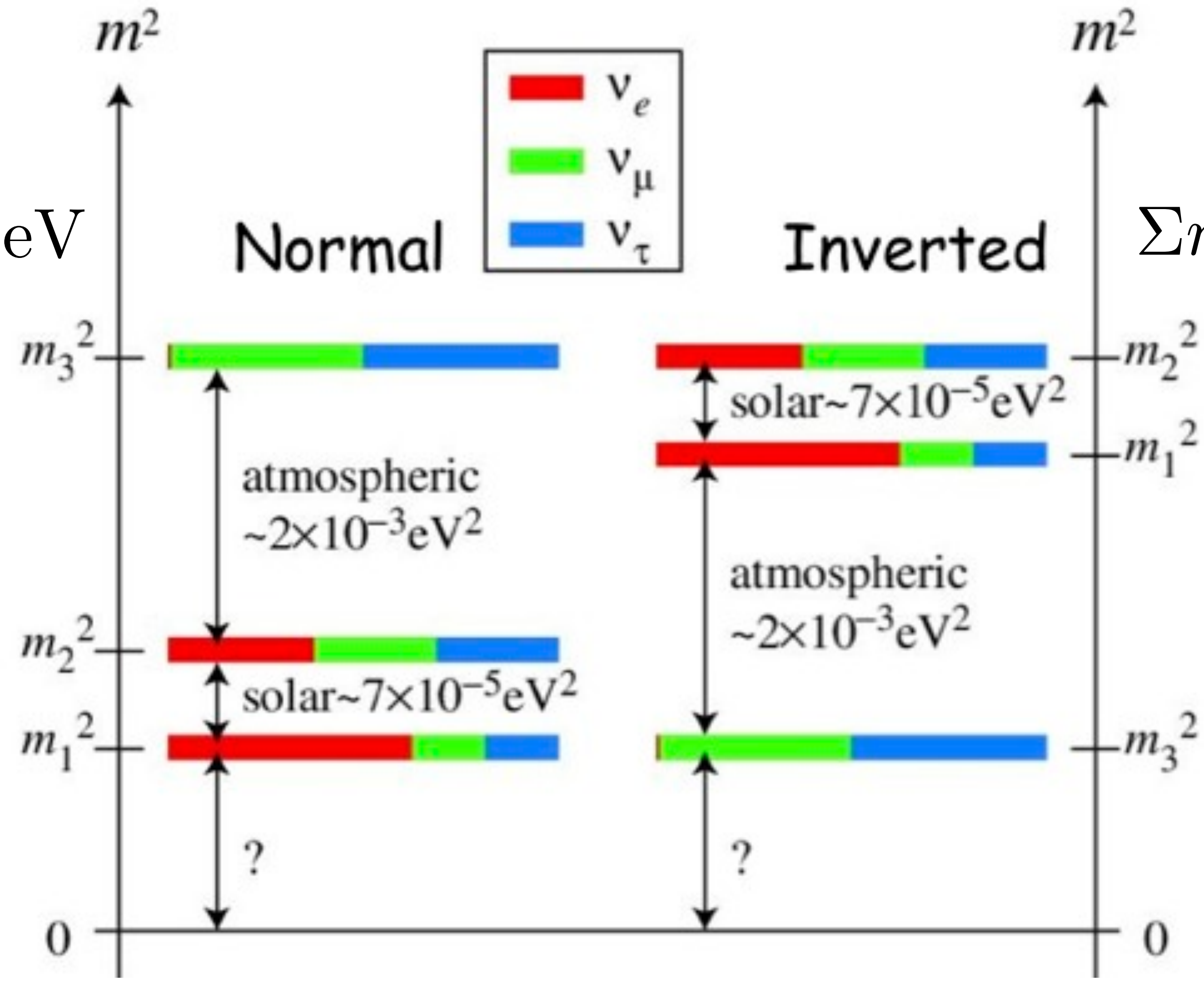
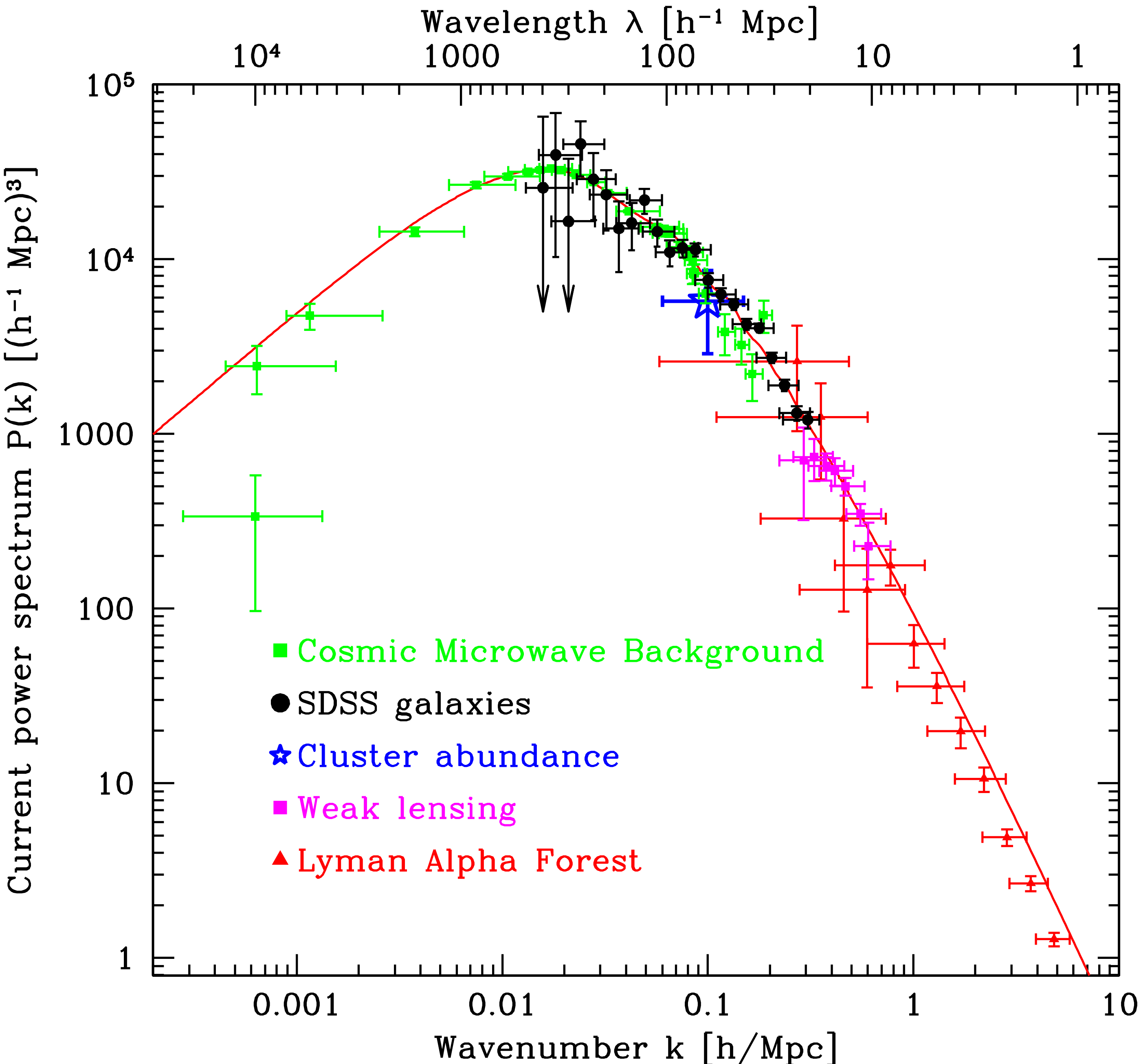


$\Sigma m_{\nu_e} < 2$ eV (tritium decay)

$\Sigma m_\nu < 0.23$ eV (Planck 2015)

< 0.12 eV (CMB + Lyα) *Palanque-Delabrouile et al. (2015)*

About scales



Tegmark et al. (2004)

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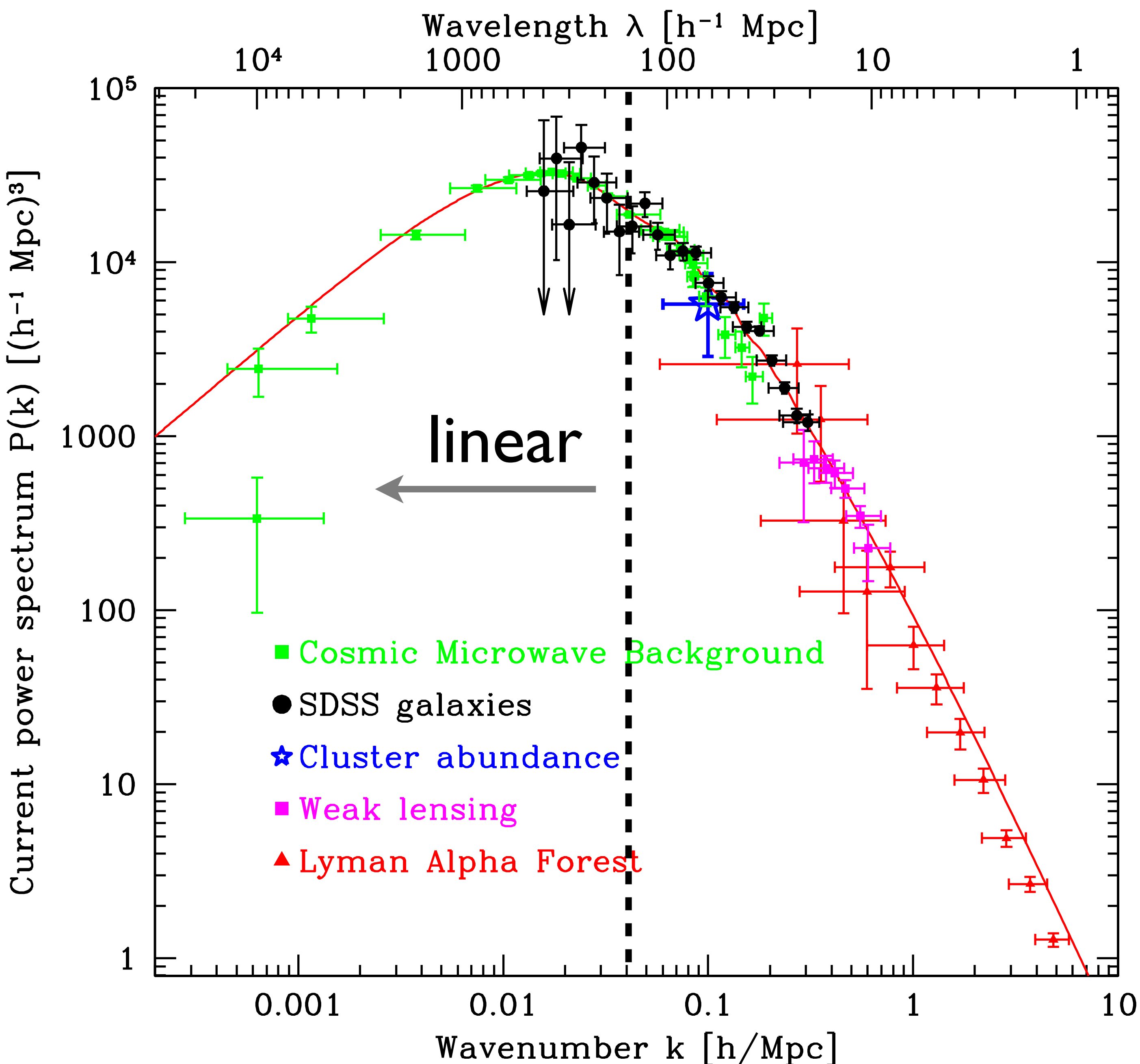

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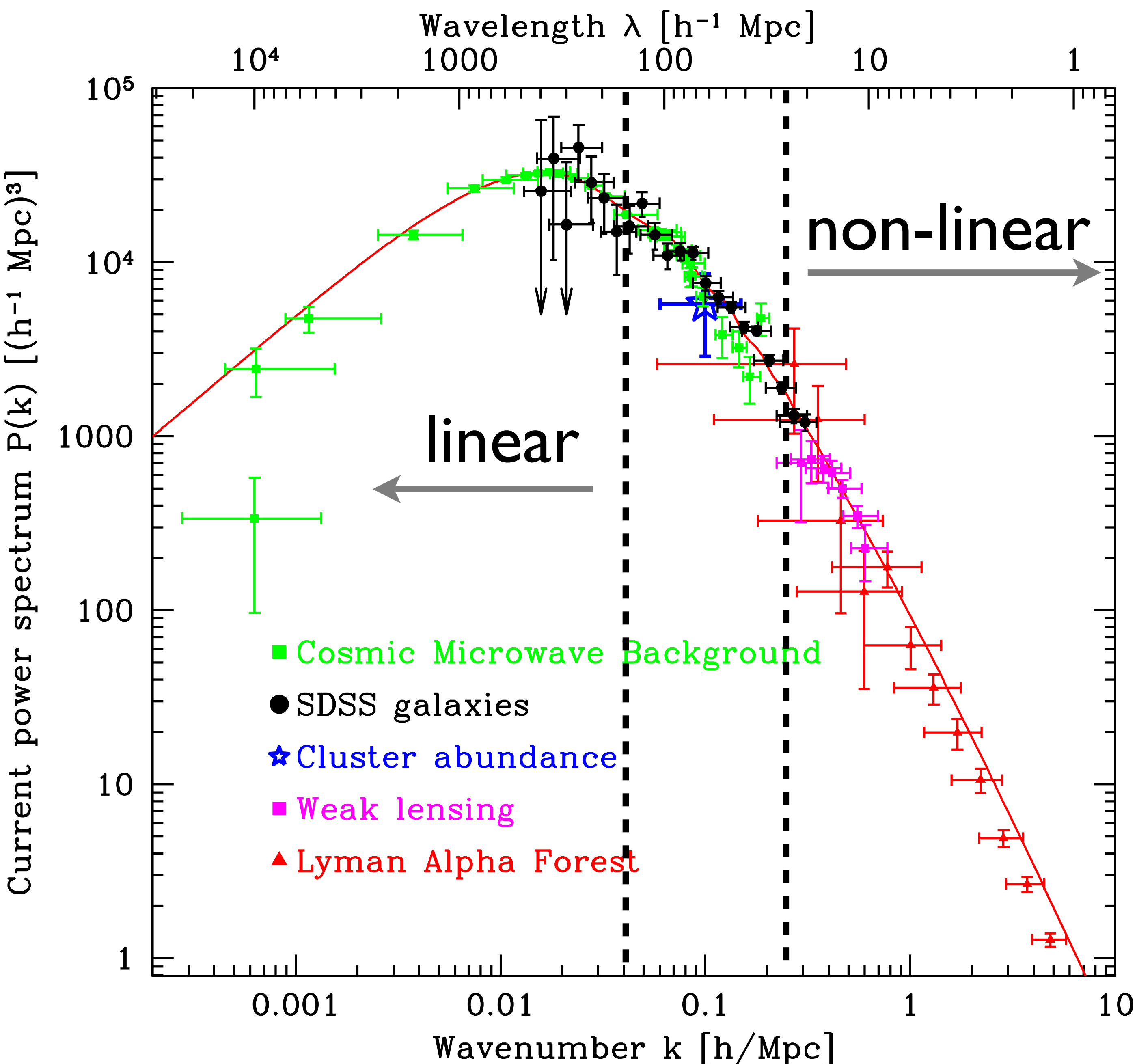

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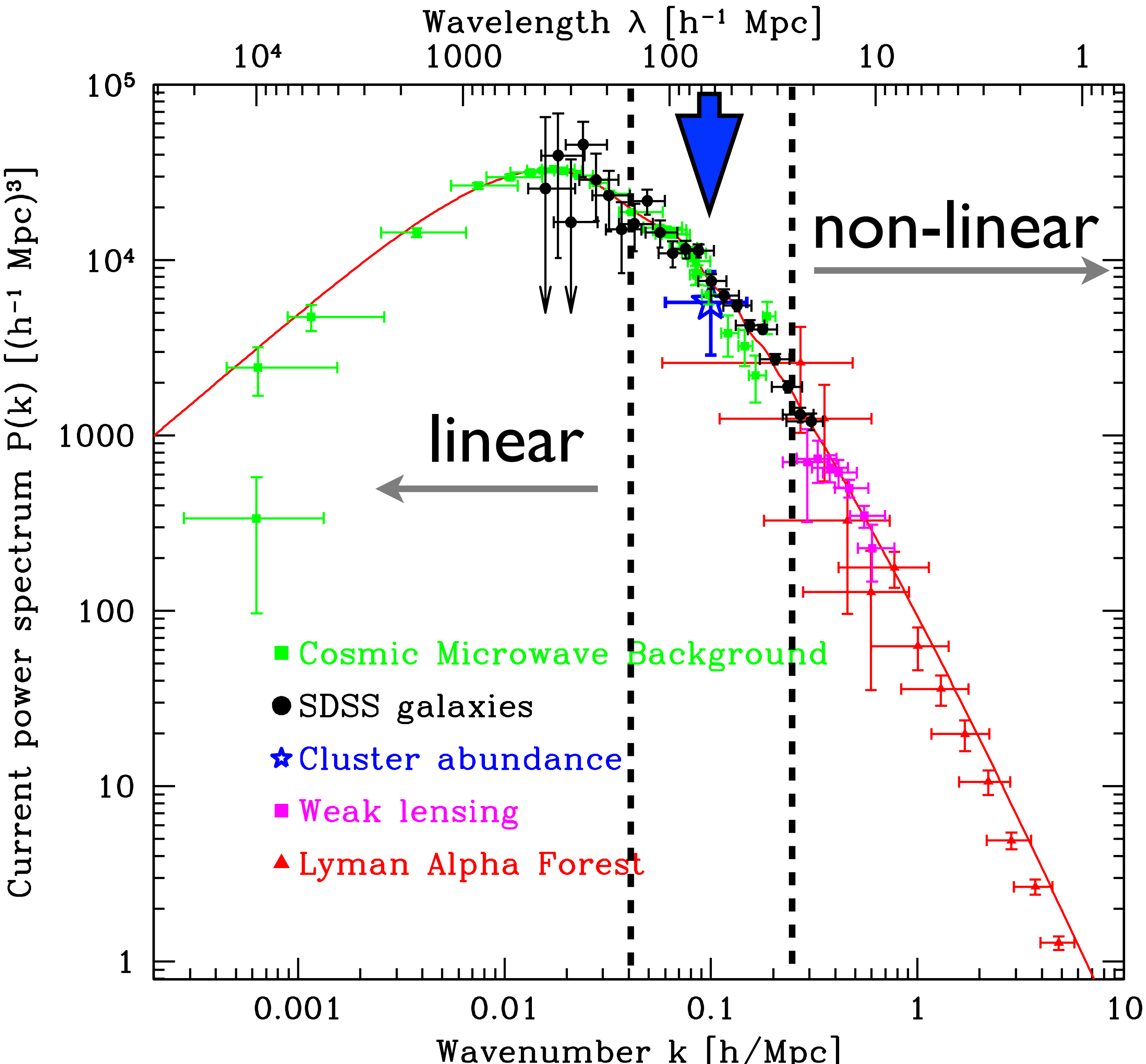

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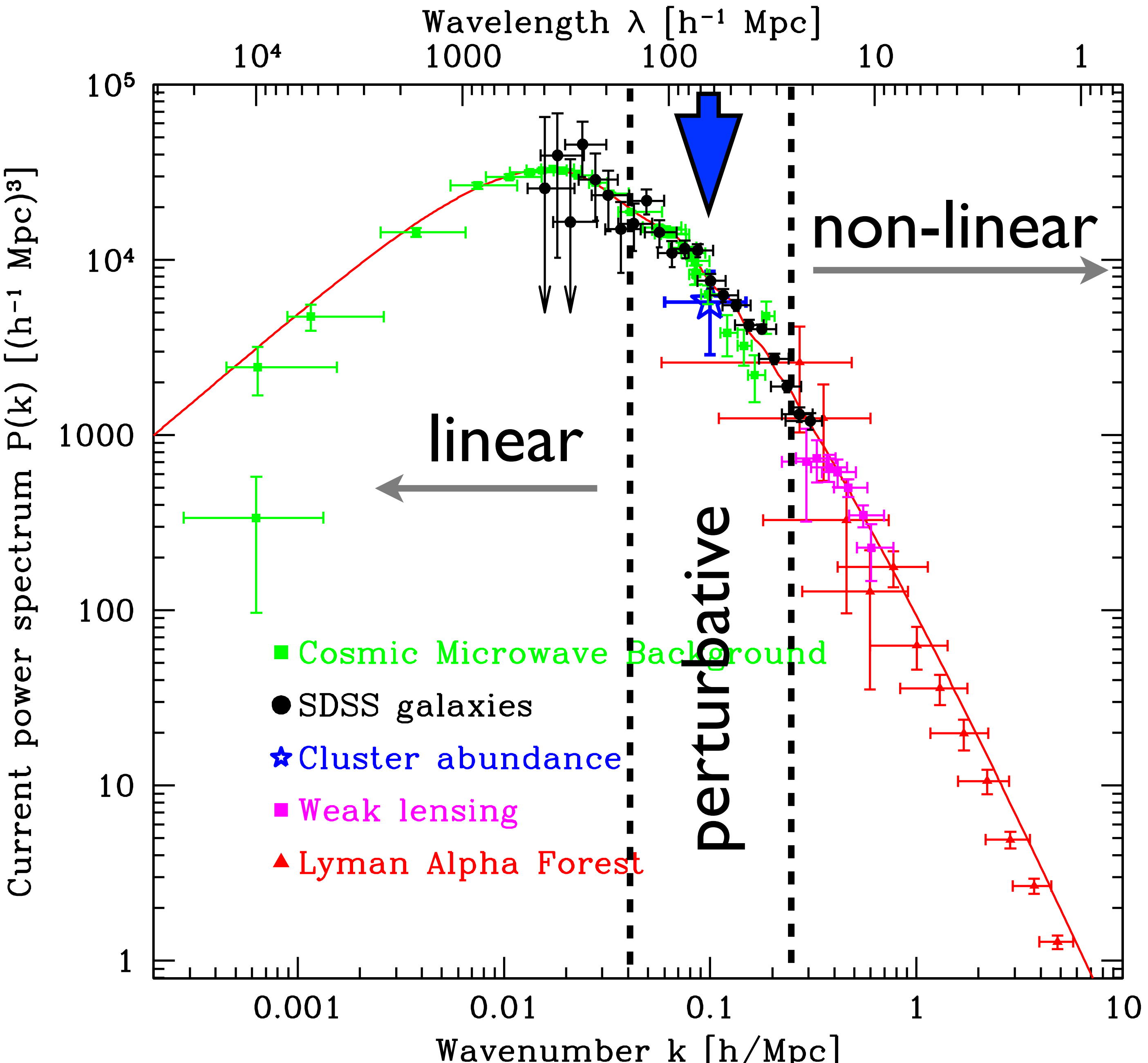

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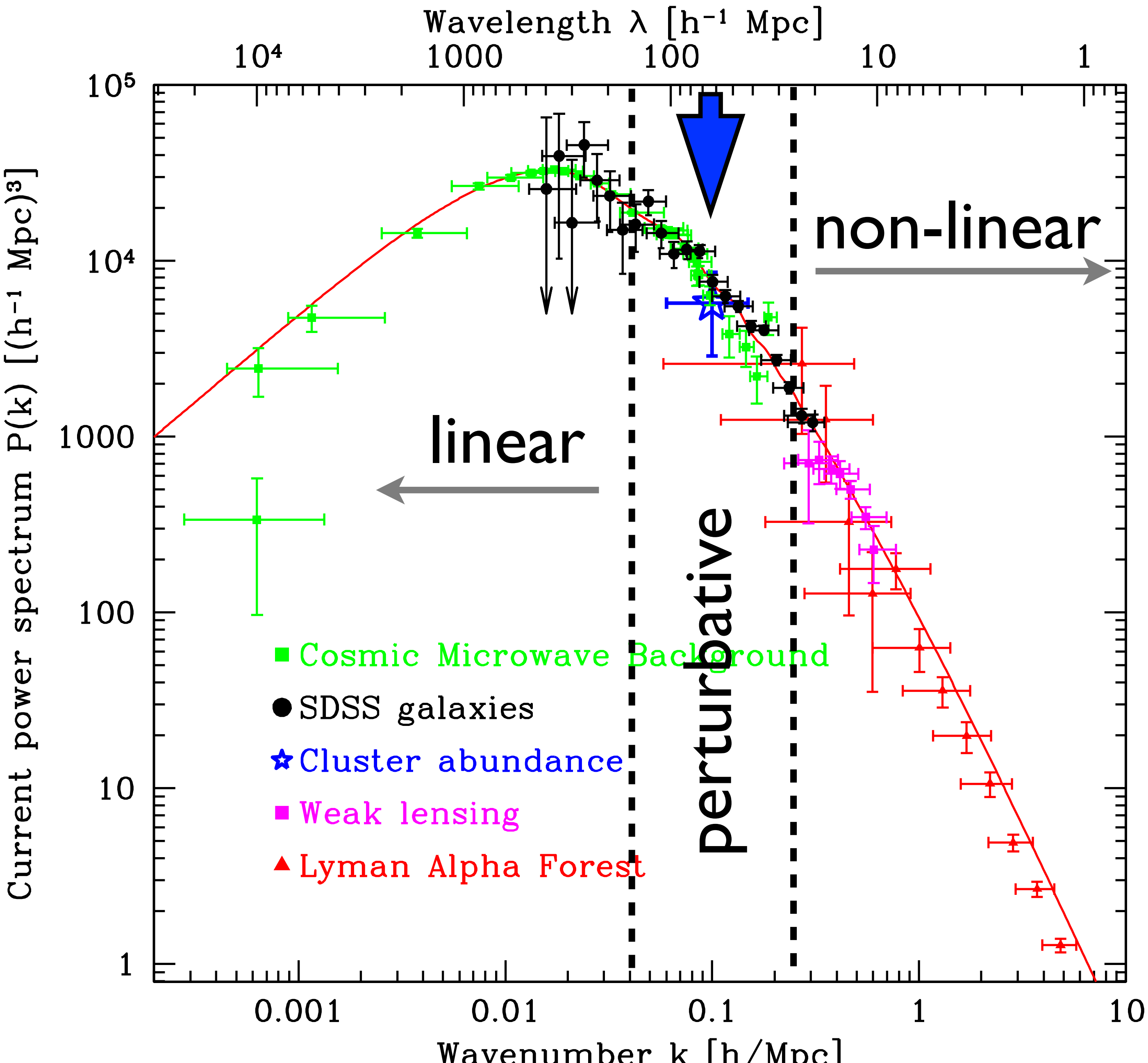

statistical error

$$\propto (k_{max})^{-3/2}$$

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Challenges to theorists

The fundamental description is known (?): collisionless particles interacting through gravity

Vlasov -- Poisson system for the distribution function $f(\mathbf{x}, \mathbf{v}, t)$

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} - \nabla \phi \cdot \frac{\partial f}{\partial \mathbf{v}} = 0 \ , \qquad \nabla^2 \phi = 4\pi G \int f \ d^3\mathbf{v}$$

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  - – costly, the theory parameters cannot be changed easily
- analytical perturbative methods at $k \lesssim 0.3 \, \mathrm{h}^{-1}\mathrm{Mpc}$

A formula is worth a million pictures !

Juan Maldacena

in "The symmetry and simplicity of the laws of physics"

Simplifying the problem

Newtonian approximation at $l \ll H^{-1} \sim 10^4\ \mathrm{Mpc}$

DM particles move by $u\, H^{-1} \sim 10\ \mathrm{Mpc}$

10^{-3}

⇒ fluid description at $l \gg 10\ \mathrm{Mpc}$

$$\frac{\partial \delta_\rho}{\partial \tau} + \nabla \left[(1 + \delta_\rho) \mathbf{u} \right] = 0$$

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vorticity decays at linear level ➡ work with $\theta \propto \nabla \cdot \mathbf{u}$

Standard perturbation theory (SPT)

$$\dot{\delta}_\rho(k) - \theta(k) = \int d^3q \; \alpha(q, k-q) \; \theta(q)\delta_\rho(k-q)$$

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$\psi^{(1)} =$  ψ_0

evolution from τ to τ_0

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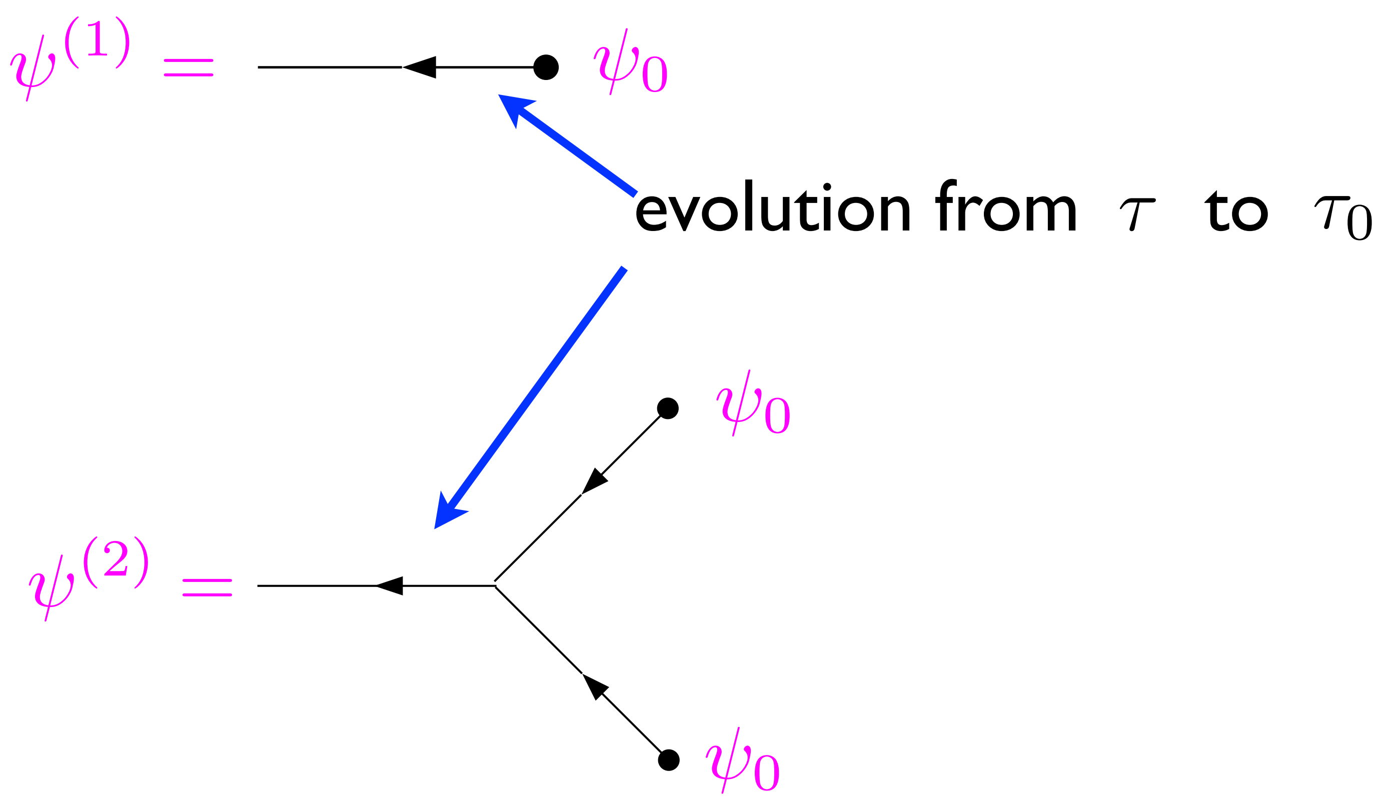

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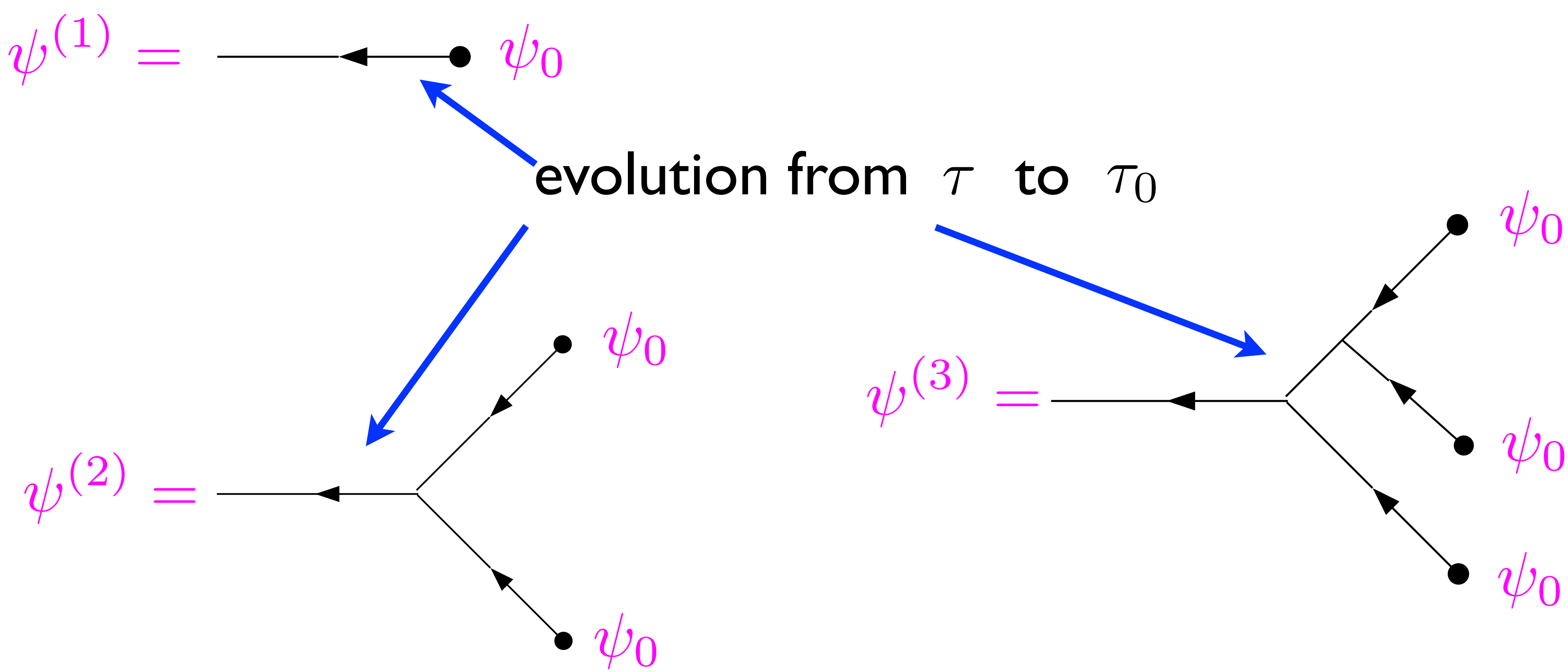
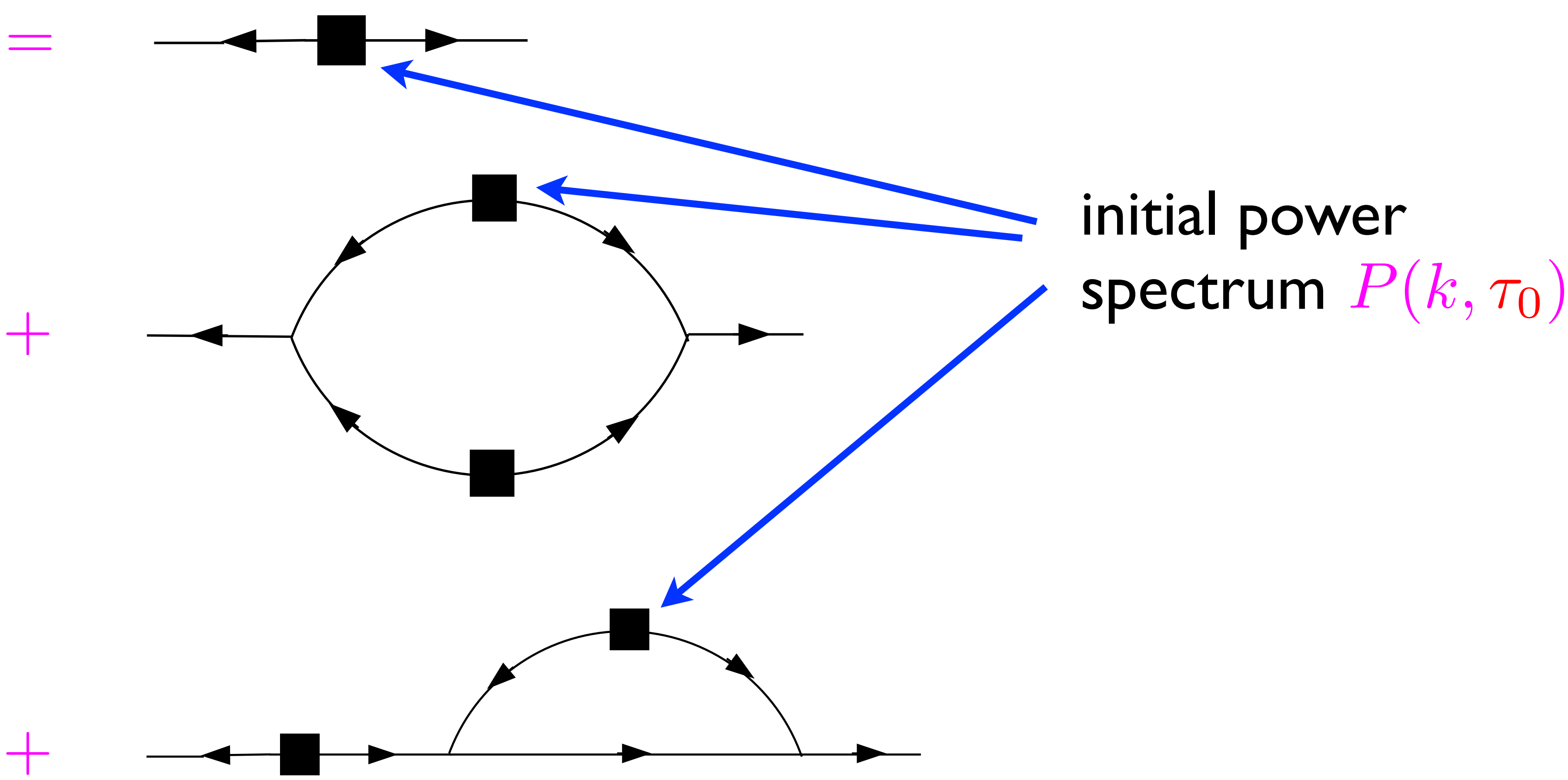


Average over the ensemble of initial conditions:

$$\langle\psi(k_1,\tau)\psi(k_2,\tau)\rangle = \langle\psi^{(1)}\psi^{(1)}\rangle + \langle\psi^{(2)}\psi^{(2)}\rangle + 2\langle\psi^{(1)}\psi^{(3)}\rangle + \ldots =$$



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"Infrared" Kernels α, β in the e.o.m.'s behave as $1/q$

➡ individual loop diagrams diverge at small momenta

When summed, the divergences cancel in equal-time correlators

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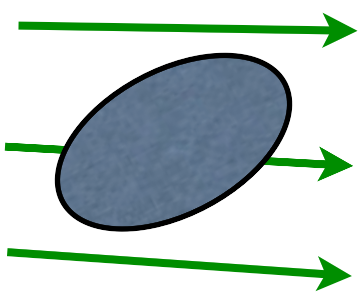

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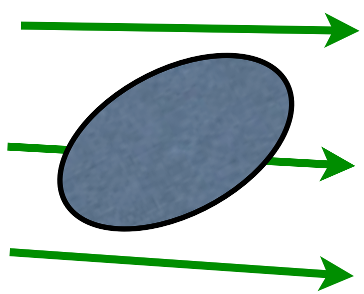

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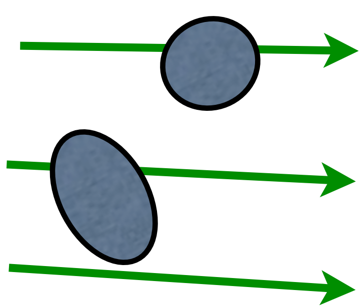

two overdensities will move (almost) identically → cancellation in equal-time correlators

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➡ EFT of LSS

Baumann, Nicolis, Senatore, Zaldarriaga (2010)
Carrasco, Hertzberg, Senatore (2012)
Pajer, Zaldarriaga (2013)

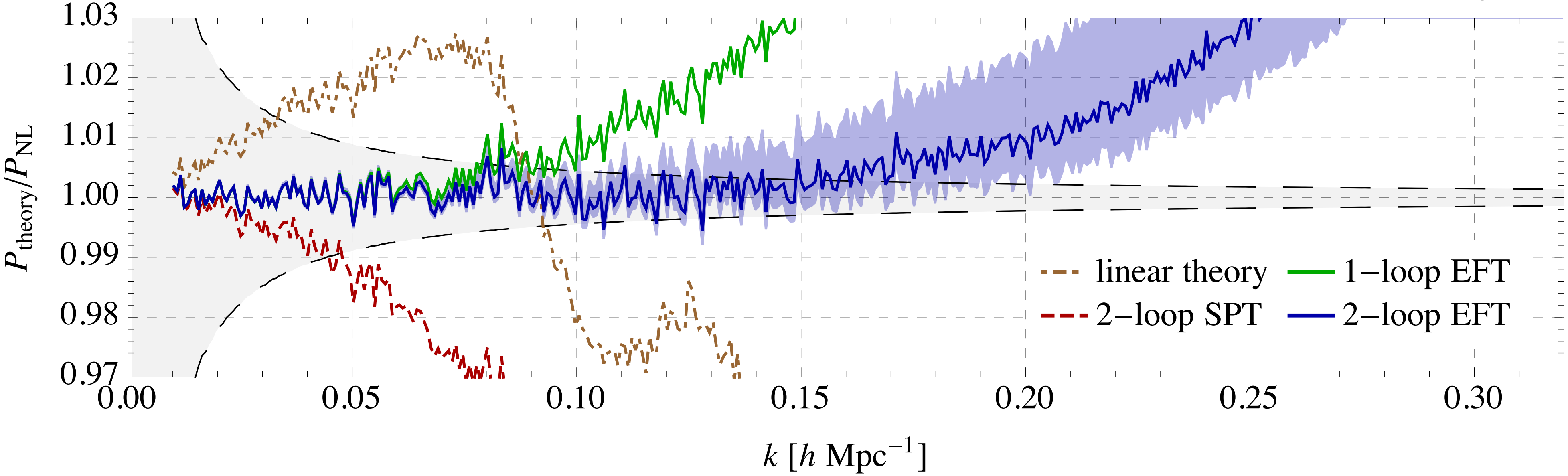
+ many more

$$\dot{u}^i + \mathcal{H}u^i + u^j \nabla_j u^i + \nabla\phi = -\frac{1}{\rho}\partial_j \tau^{ij}$$

$$\tau^{ij}_{vis} + \tau^{ij}_{stoch}$$

$$\tau^{ij}_{vis} = -c_s^2 \delta^{ij}\delta_\rho + \tilde{c}\delta^{ij}\Delta\delta_\rho + c_1\delta^{ij}(\Delta\phi)^2 + c_2\partial^i\partial^j\phi\Delta\phi + c_3\partial^i\partial_k\phi\partial^j\partial_k\phi + \dots$$

from Foreman, Perrier, Senatore (2015)



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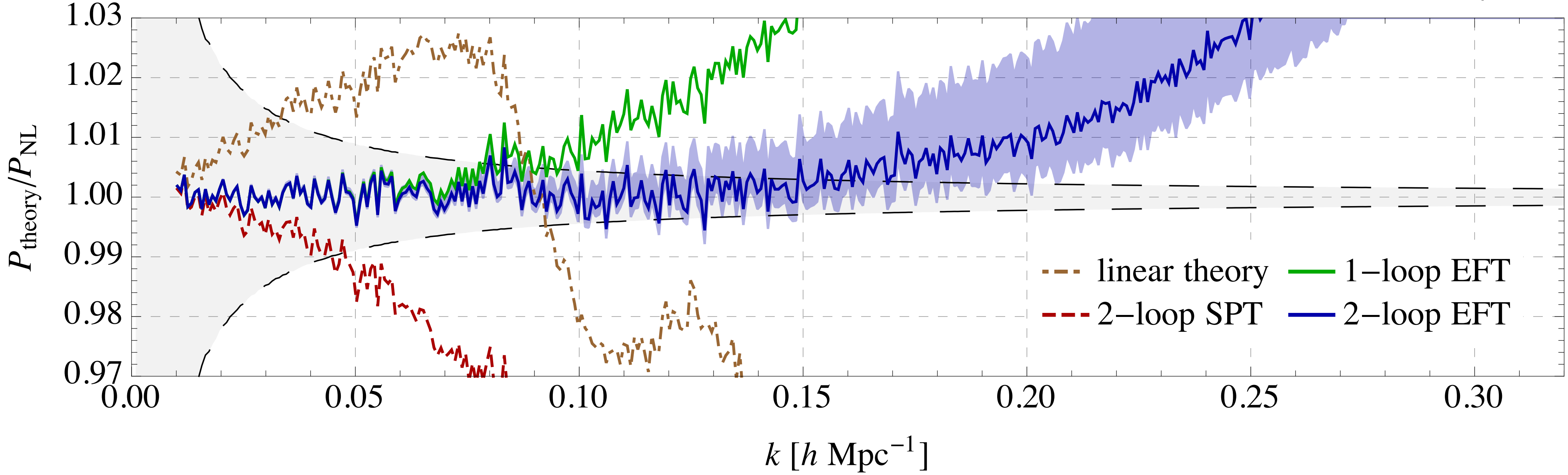

Complications:

- coefficients of the counterterms have non-local time-dependence

  Abolhasani, Mirbabayi, Pajer (2015)

- treatment of stochastic terms is unclear

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To clear up

➡ use the methods of QFT (statistical mechanics)

Example: resummation of IR divergences in QED is clearly separated from UV renormalization

TSPT: time-sliced perturbation theory

Valageas (2004)
Blas, Garny, Ivanov, S.S. (2015,2016)

Main ideas: Focus on equal-time correlators

Instead of evolving fields, evolve the probability distribution function

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$$\dot{\psi} = \Omega\psi + \sum_{m=2} \frac{A_m}{m!}\psi^m \quad \Rightarrow \quad \psi(\tau;\psi_0)$$

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$$\Gamma_0[\psi_0] = \frac{\psi_0^2}{2P}$$

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TSPT: $$\int d\psi \, e^{-\Gamma[\psi;\tau]}\psi^2 \qquad \Gamma[\psi;\tau] = \sum_n \frac{\Gamma_n(\tau)}{n!}\,\psi^n$$

Two integrals must coincide

➡ equation for the "vertices"

$$\frac{d}{d\tau}\left(d\psi e^{-\Gamma[\psi;\tau]}\right) = 0$$

➡ $$\dot{\Gamma}_n = -n\Omega\Gamma_n - \underbrace{\sum_{m=2}^{n} C_n^m A_m \Gamma_{n-m+1}}_{} + A_{n+1}$$

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The same logic for fields in space with the substitution:
integral $\Longrightarrow$ path integral

Generating functional for cosmological correlators

$$Z[J, J_\delta; \tau] = \int [\mathcal{D}\theta] \exp\Big\{ -\Gamma[\theta;\tau] + \int \theta J + \int \delta_\rho[\theta;\tau] J_\delta \Big\}$$

$$\Gamma = \frac{1}{2}\int \frac{\theta^2}{\hat{P}(k)} + \sum_{n=3}^{\infty} \frac{1}{n!} \int \Gamma_n(\tau)\theta^n$$

$$\delta_\rho = \sum_{n=1}^{\infty} \frac{1}{n!} \int K_n(\tau)\theta^n$$

TSPT - 3d Euclidean QFT vocabulary:

- Γ --- 1PI effective action
- δ_ρ --- composite source
- τ --- external parameter

Advantages

- For gaussian initial conditions the time dependence factorize

$$\Gamma = \frac{1}{g^2(\tau)}\bar{\Gamma}$$

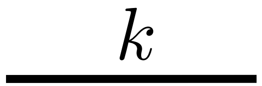
effective coupling constant

NB. For primordial NG

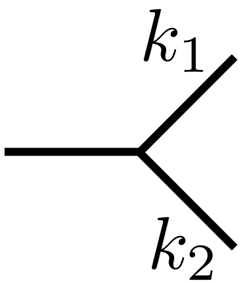
$$\Gamma = \frac{1}{g^2}\bar{\Gamma} + \frac{1}{g^3}\hat{\Gamma}$$

$\sim f_{NL} g_0$

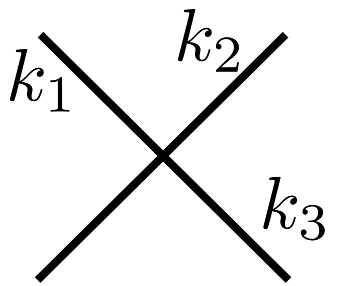
- Simplified diagrammatic technique



$$= g^2 \bar{P}(k)$$



$$= \frac{1}{g^2} \bar{\Gamma}_3(k_1, k_2)$$



$$= \frac{1}{g^2} \bar{\Gamma}_4(k_1, k_2, k_3)$$

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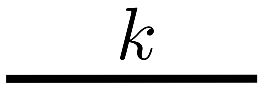

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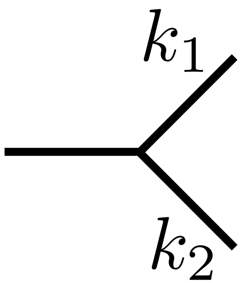

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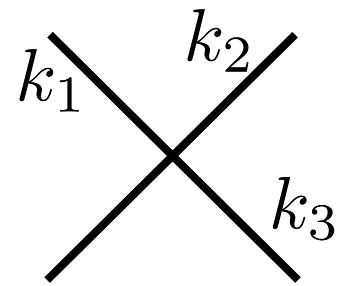

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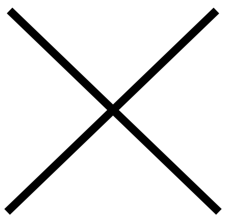
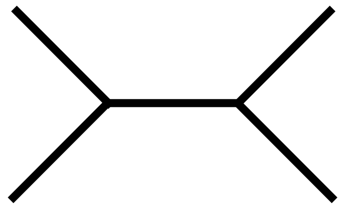
$\langle \theta\theta\theta\theta \rangle =$  + 

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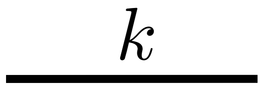

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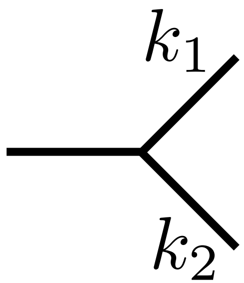

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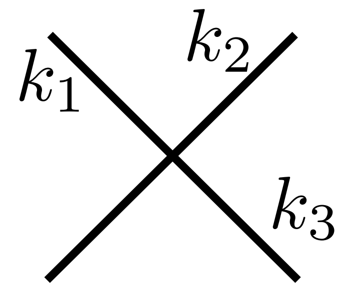

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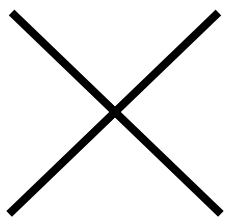
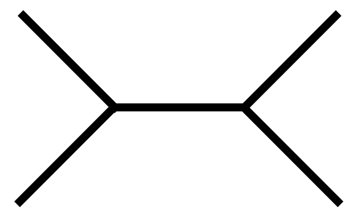
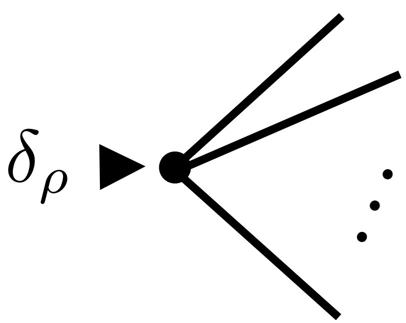
$\langle \theta\theta\theta\theta \rangle =$  + 



$= K_n(k_1, k_2, \ldots, k_n)$

IR safety

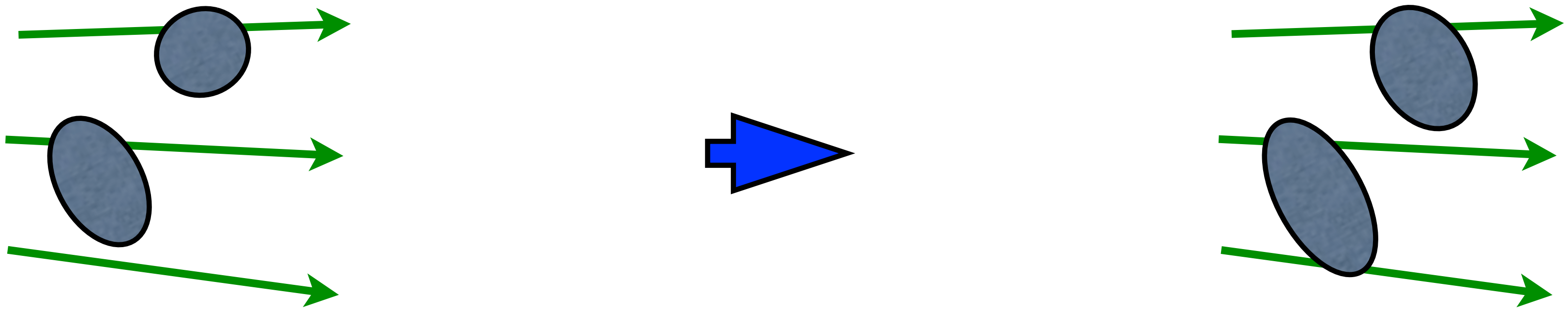
All Γ_n, K_n are finite for soft momenta

$$\lim_{\epsilon \to 0} \Gamma_n(k_1, \ldots, k_l, \epsilon q_1, \ldots, \epsilon q_{n-l}) < \infty$$

→ no IR divergences in the individual loop diagrams

NB. Can be related to the equivalence principle / Galilean invariance of Γ through Ward identities

IR enhanced effects due to flow gradients



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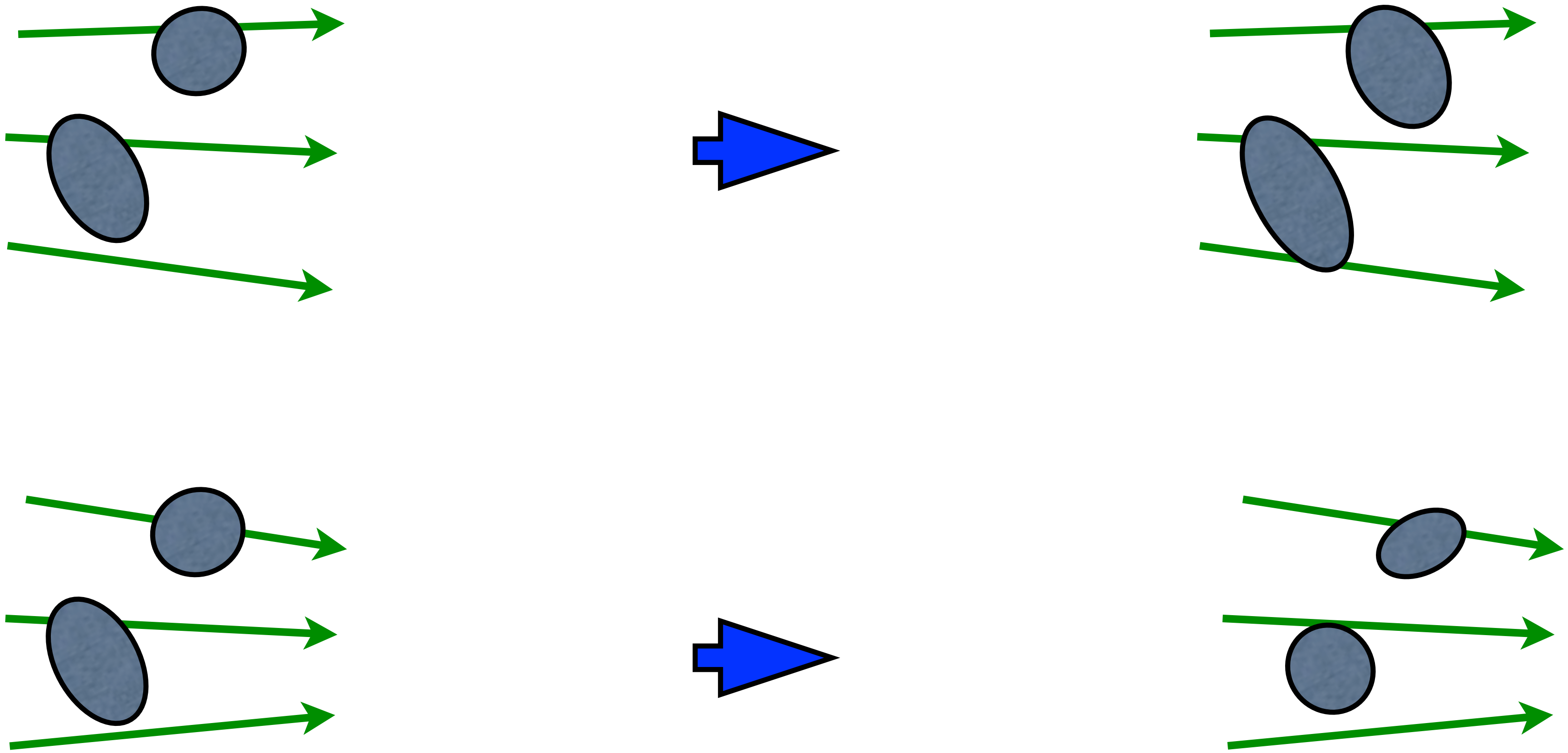

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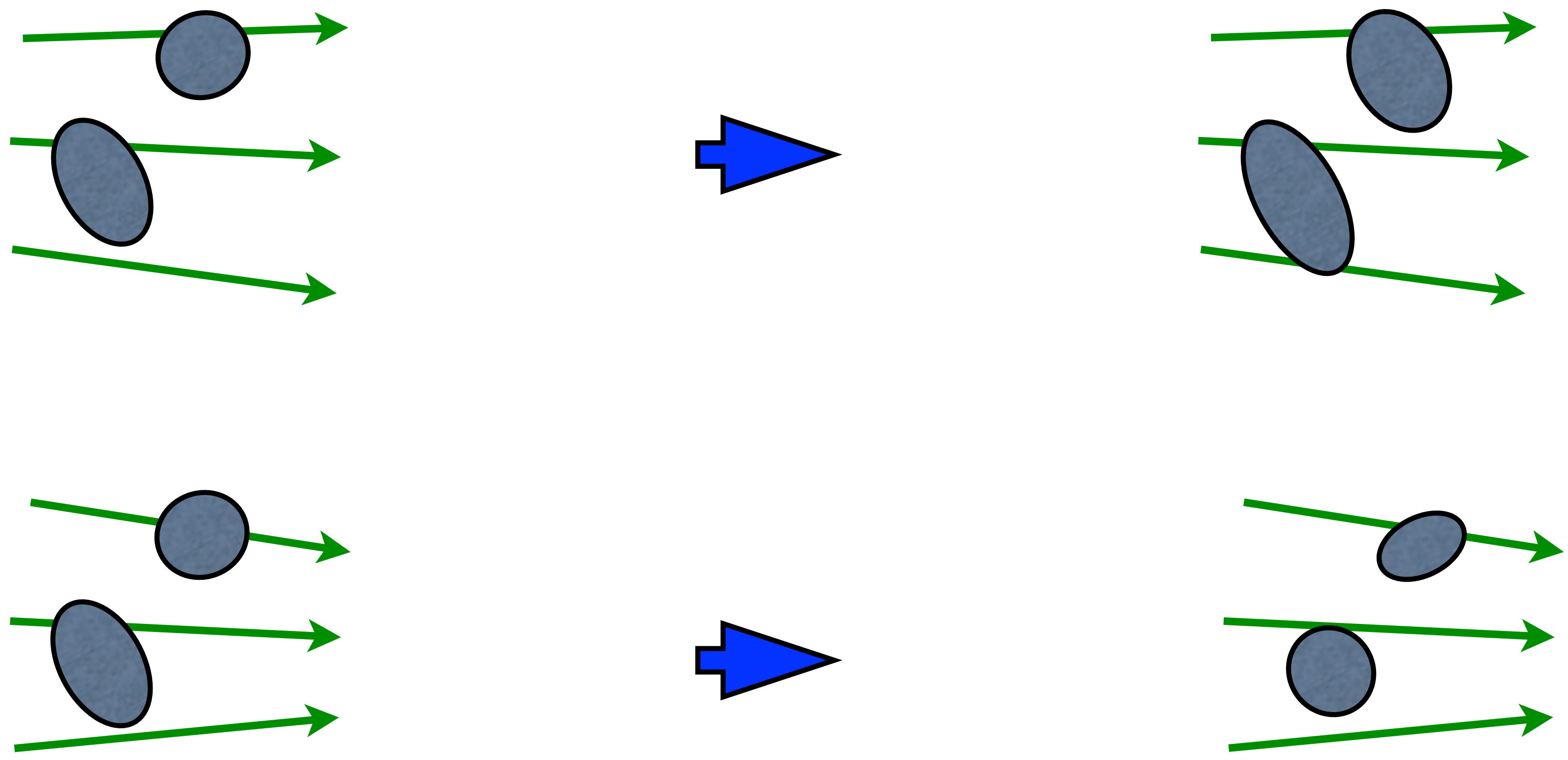

smearing of features in PS and other correlation functions

IR resummation

In TSPT large IR contributions can be systematically resummed

Step 1: smooth + wiggly decomposition

$$P(k) = P_s(k) + P_w(k) \quad \Rightarrow \quad \Gamma(k) = \Gamma_s(k) + \Gamma_w(k)$$

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Step I: smooth + wiggly decomposition

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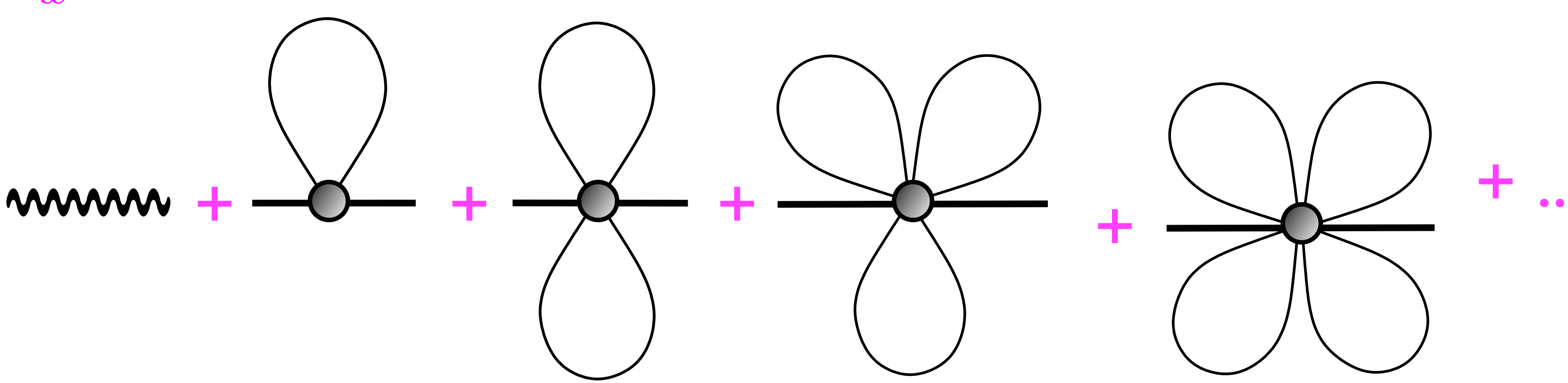
Step II: identification of leading diagrams correcting the wiggly part daisies

$$P_w^{\text{dressed}} =$$



Step III: add the smooth part

$$P(k) = P_s(k) + \mathrm{e}^{-k^2\Sigma_L^2} P_w(k)$$

BAO wavelength

$$\Sigma_L^2 = \frac{4\pi}{3}\int_0^{k_L} dq\, P_s(q)\, \left(1 - j_0(qr_s) + 2j_2(qr_s)\right)$$

Baldauf et al. (2015)

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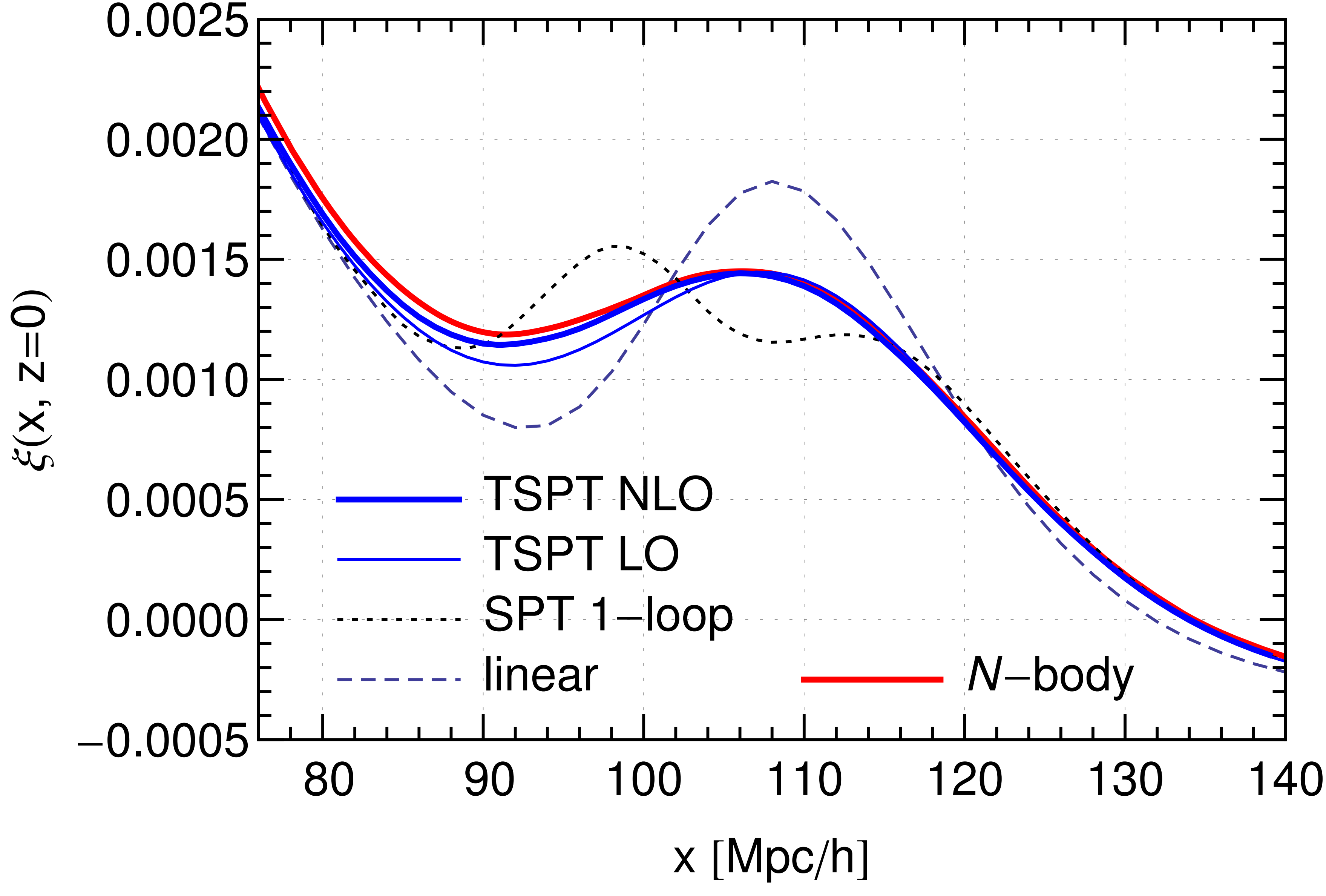
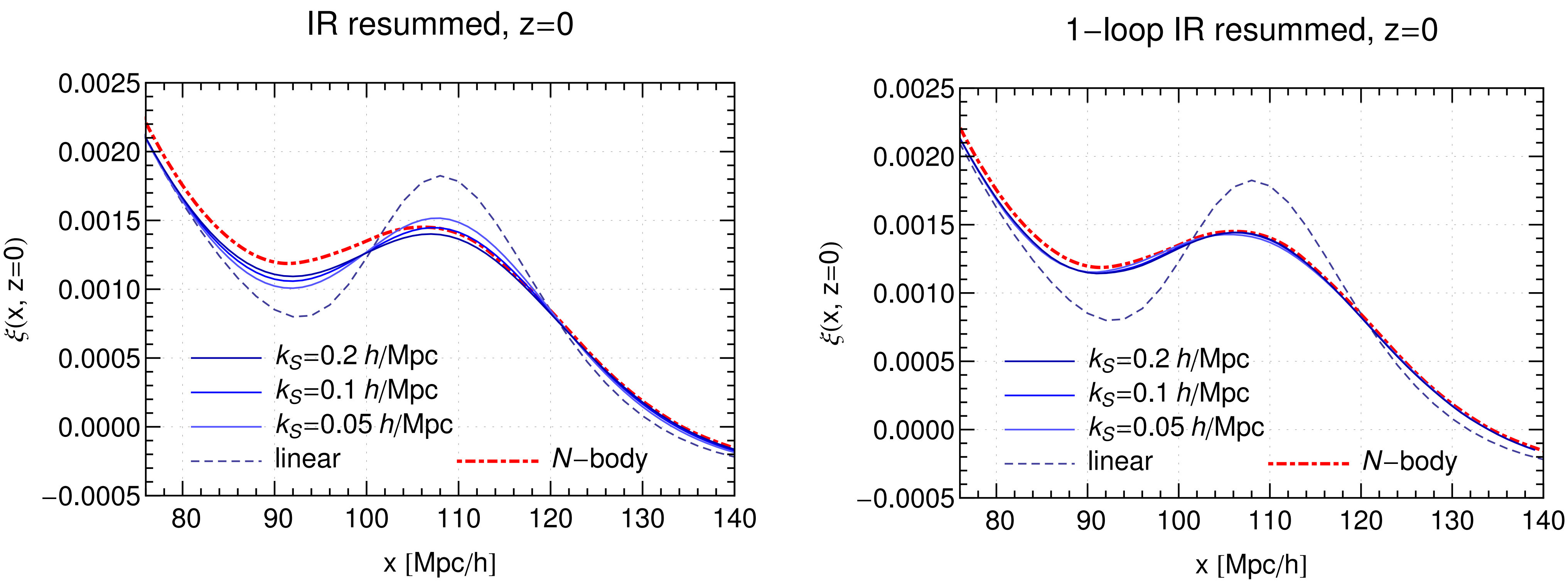
Further developments:

- NLO IR corrections. Important for the shift of BAO peak

Comparison with N-body



Sensitivity to the IR separation scale: LO vs NLO

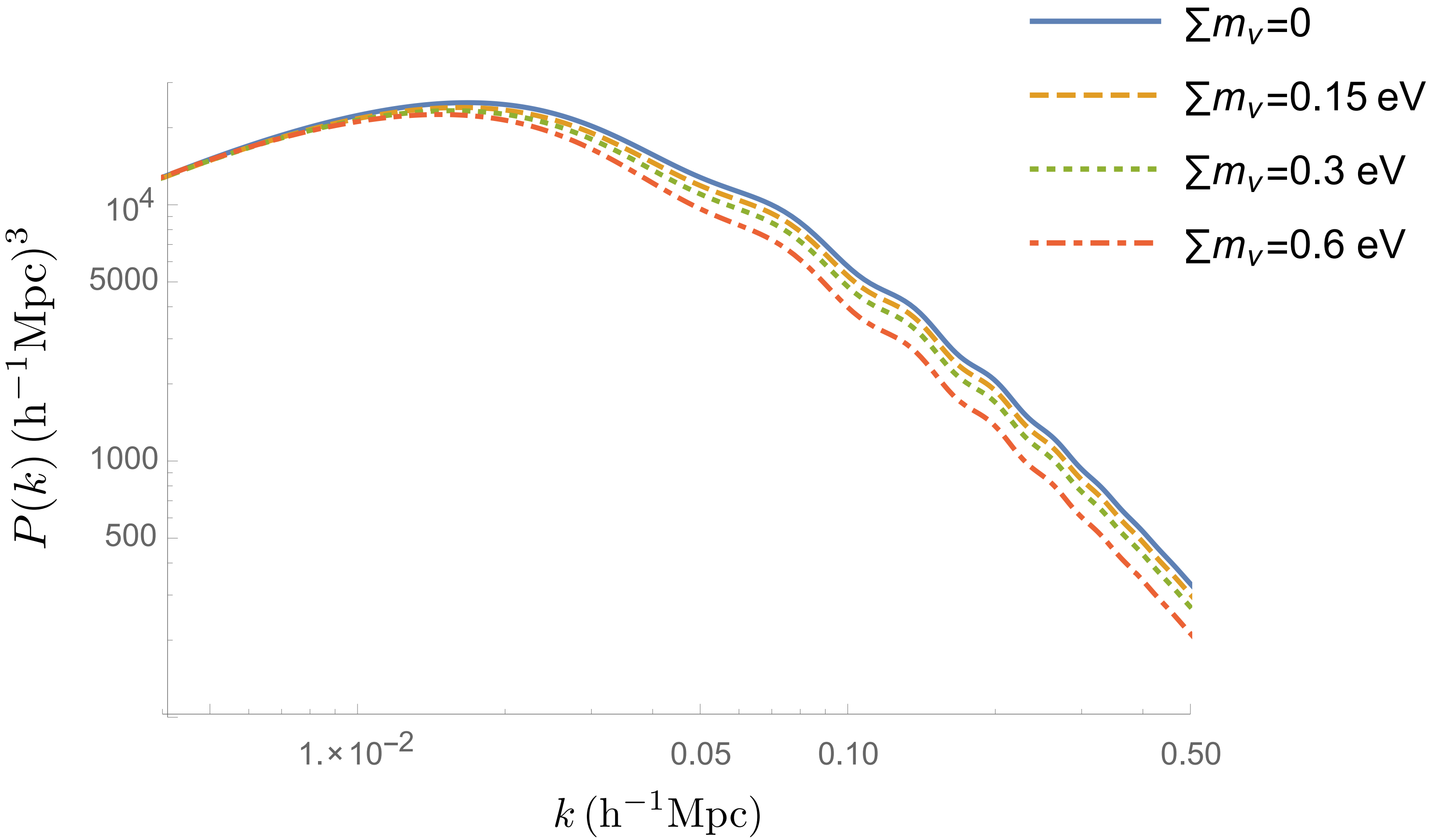


dependence on k_L decreases with the loop order

Residual dependence gives an estimate of the error ~ 2% in the BAO range

BAO and the neutrino mass

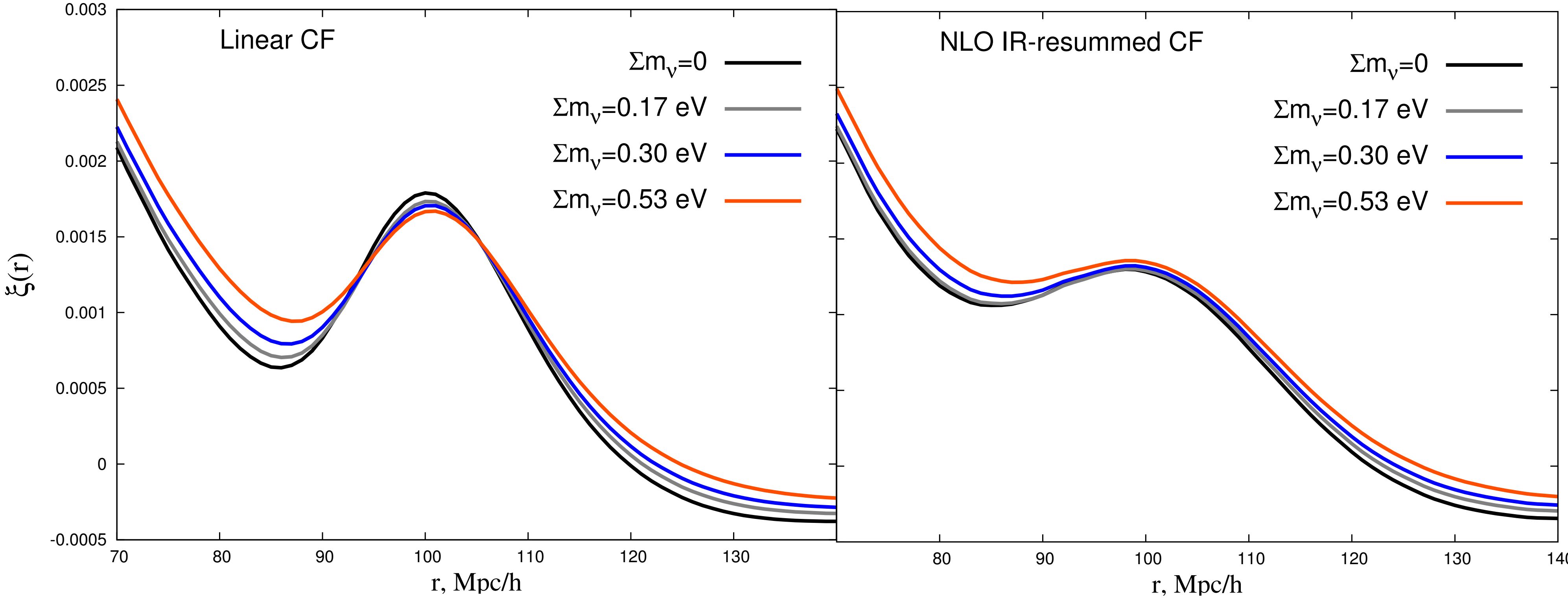
Effect on linear PS:



At $k > 0.05\ \mathrm{h^{-1}Mpc}$ degenerate with the overall normalization

BAO and the neutrino mass

Non-linear effects remove the degeneracy



A probe of m_ν alternative to CMB and Lyα ?

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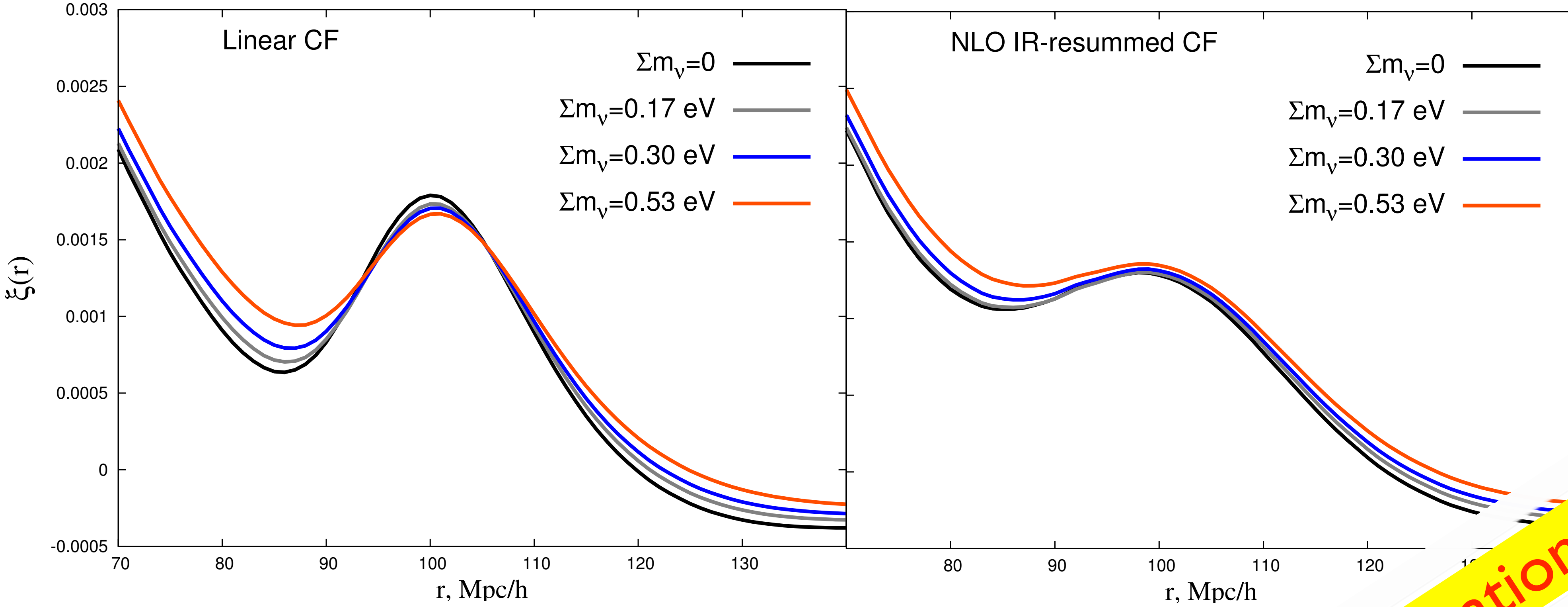

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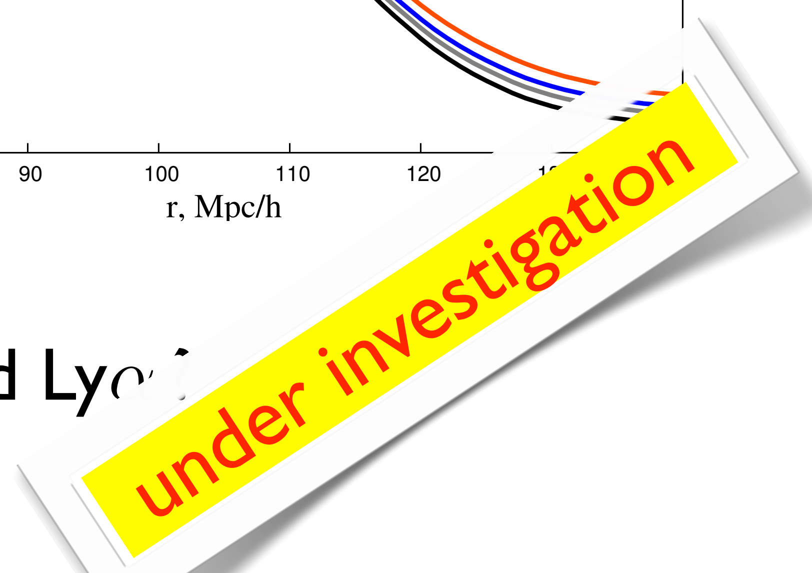

UV renormalization in TSPT

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Introduce a hard cutoff:

$$\hat{P}(k) \mapsto \hat{P}^{\Lambda}(k) = \begin{cases} \hat{P}(k), & k < \Lambda \\ 0, & k > \Lambda \end{cases}$$

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Boundary conditions = counterterms C_n encapsulating the effects of short modes

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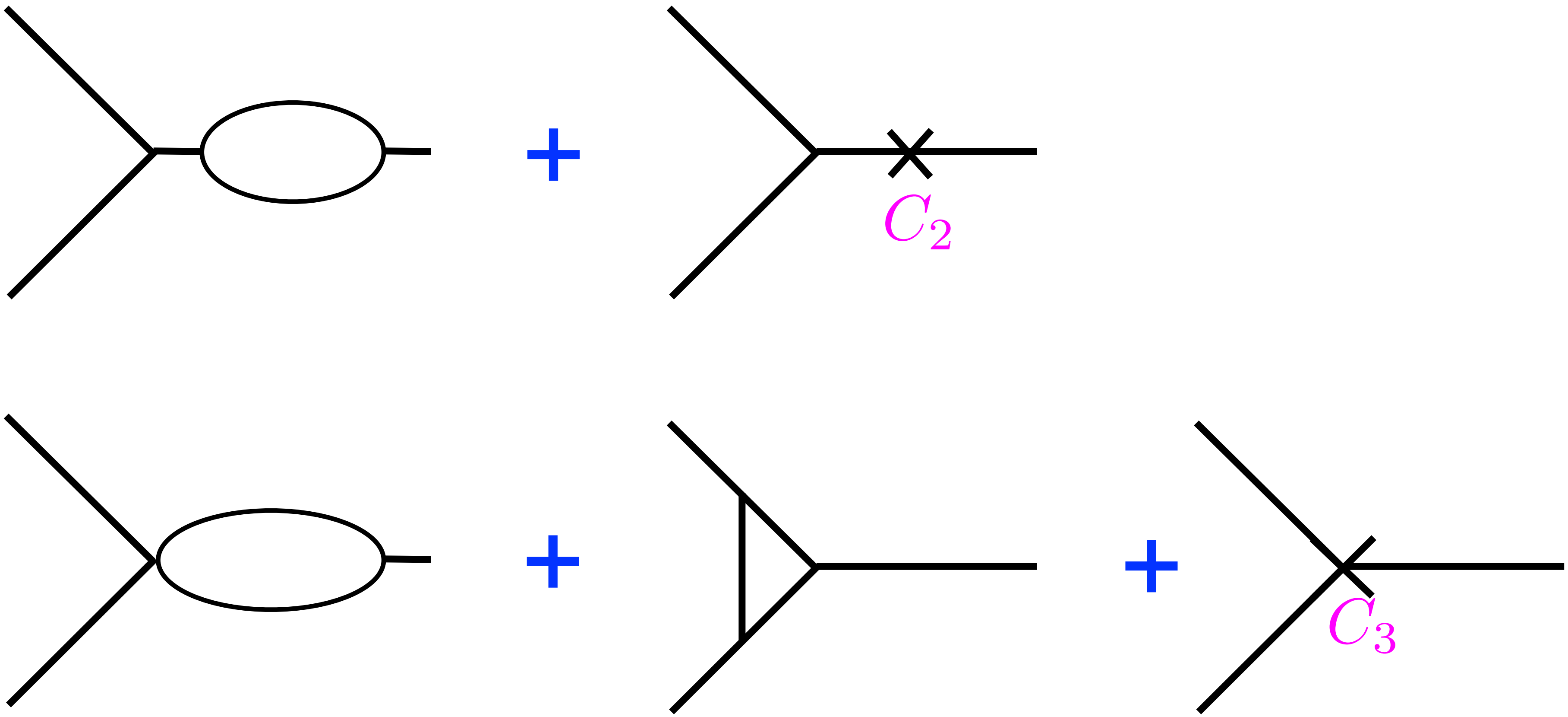
NB. Time is treated as an external parameter; different from

Matarrese, Pietroni (2007)
Floerchinger et al. (2015)

UV renormalization in TSPT

+ clear separation between PR and PI counterterms



+ stochastic contributions are at the same footing as viscous ones

Structure of counterterms

$C_n(\{k\}, \tau)$

local in time by construction

but non-polynomial in momenta

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⇒ an Ansatz $C_n = \mathcal{F}_n(\{k\})\, C_n^{(0)}(\tau)$ is stable under RG

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in progress
w. D'Amico, Scoccimarro

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Outlook

- large deviation statistics as semiclassical approximation
  in progress with Blas, Garny, Ivanov and Uhlemann
- inclusion of "astrophysical" effects (biases, redshift space distortion, baryons)