

# THE CRITICAL STRAIN ANGLE IN THE NEUTRON STAR CRUST

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# Layers of the NS Crust:

$$\rho_{\text{drip}} = 4 \times 10^{11} \text{ gcm}^{-3}$$

**Outer Crust:** Ions arrayed in (probably bcc) Coulomb lattice + gas of degenerate electrons

**Inner Crust** ( $\rho > \rho_{\text{drip}}$ ): Neutron rich, Lattice of nuclei+electrons + free neutrons

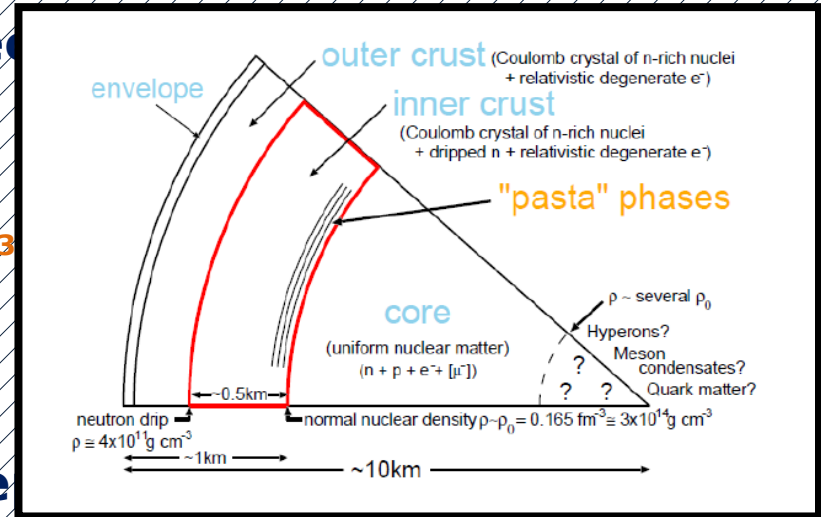
Screening effect

spacing)

At the highest densities ( $\rho \sim 10^{14} \text{ gcm}^{-3}$ )

➤ 'Nuclei' start to 'touch'

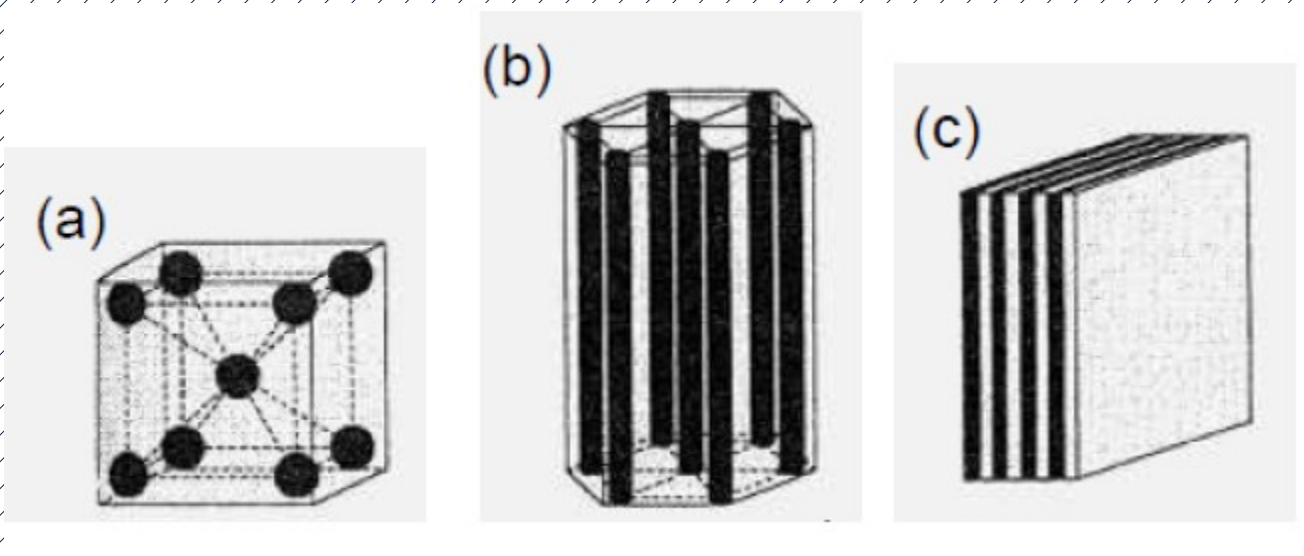
➤ Competition between Coulomb repulsion and Nuclear attraction



**Nuclear Pasta: Spherical, spaghetti-like rods, lasagna-like slabs**

*(Ravenhall, Pethick & Wilson-1983)*

*(Hashimoto, Seki, Yamada-1984)*



**tion modes, gravitational wave emission, magnetar giant flares, glitch**

# Glitch and Crustquakes:

- Sudden change in rotation rate and spindown rate with no correlated change in pulsar's electromagnetic signature  
(exceptions: i.e. PSR J1119-6127, 1E 2259+586, J1846-0258)

- **internal changes of the neutron star:**

- structural(crustquakes)

- angular momentum exchange between star's components (vortex unpinning )

$$\frac{\Delta\Omega}{\Omega} \sim 10^{-9} - 10^{-6}$$

$$\frac{\Delta\dot{\Omega}_c}{\dot{\Omega}_c} \sim 10^{-5} - 10^{-3}$$

- Stresses in the crust induced by spinning down of the star
- Readjusting to more spherical form that a fluid star would follow
- Reduction in crust's moment of inertia
- Increase in angular velocity by conservation of angular momentum

## Glitch!

- explains small glitches (i.e. Crab, PSR J0537-6910), but inadequate for larger glitches (Vela)

Time between glitches does not fit pure superfluid unpinning model.

Glitches are probably not pure superfluid events.

Crustquakes trigger, superfluid amplifies.

## critical (breaking) strain angle:

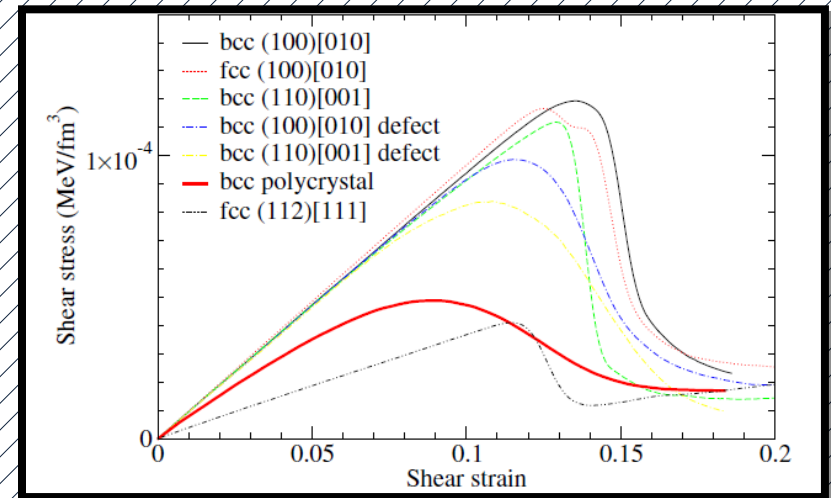
**Chowdhury & Welch (1970)** :  $10^{-5} < \theta_{cr} < 10^{-3}$  (doing analogy with terrestrial crystal)

**Pines (1985)** :  $\theta_{cr} \sim 10^{-2} - 10^{-1}$  (pure Coulomb lattice, fine structure)

**Wright & Kadau (2009)** :  $\theta_{cr} \sim 10^{-1}$  (based on molecular dynamics simulation  
-for outer crust,  $\rho \sim 10^{13} \text{ g cm}^{-3}$ )

➤ It is too hard for the crust to fracture along faults, but rather it fails collectively

➤ Crust is very strong and can support large mountains so that their gravitational wave radiation might be detectable



# Estimating $\theta_{cr}$ for all layers of NS crust (Akbal&Alpar 2016-to be submitted)

Hypothesis –  $\theta_{cr} \sim |E_C|/E_K$

Dimensionless value of the ratio total Coulomb energy to the relativistic electrons' kinetic energy is relevant to this critical value:

$$|E_C| = \int_{cell} (E^2/8\pi) d\tau. \quad E_K = Z(3/4)\hbar ck_F$$

Wigner-Seitz cell approximation  
Z electrons and Z protons

$$\theta_{cr}^{sph} \sim \frac{2}{5} \left( \frac{4}{9\pi} \right)^{1/3} Z^{2/3} \alpha \left( \frac{2r_c}{R} + \frac{R^2}{r_c^2} - 3 \right),$$

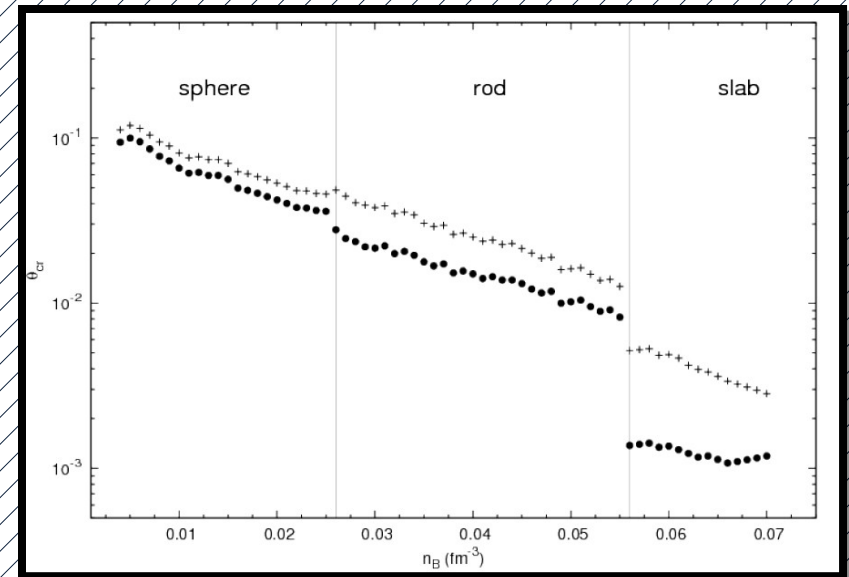
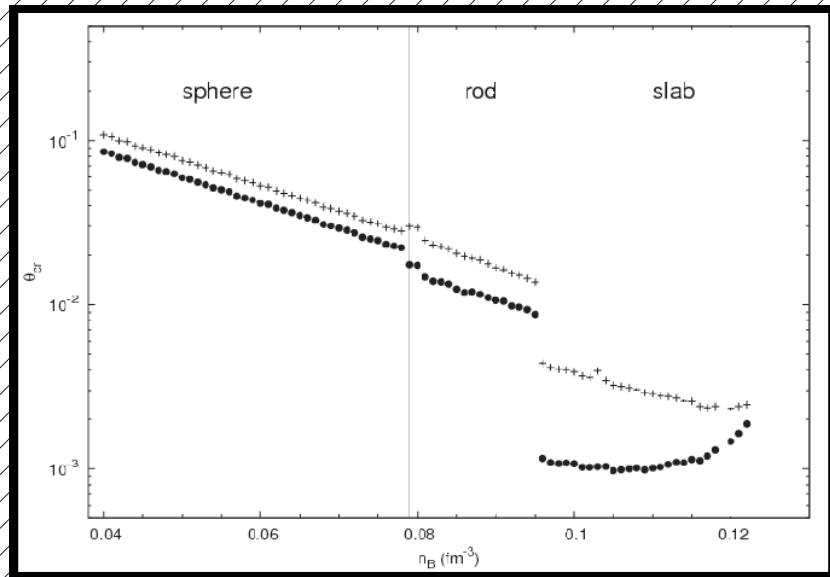
$$\theta_{cr}^{rod} \sim \frac{4}{3(3\pi)^{1/3}} \lambda^{2/3} r_c^{2/3} \alpha \left( \ln \left( \frac{r_c}{R} \right) - \frac{1}{2} + \frac{R^2}{2r_c^2} \right),$$

$$\theta_{cr}^{slab} \sim \frac{2}{9} \left( \frac{\pi}{3} \right)^{1/3} \sigma^{2/3} r_c^{1/3} \alpha \left( r_c - \frac{11R}{8} + \frac{R^2}{2r_c} \right).$$

$$\lambda = Z/r_c$$

$$\sigma = Z/r_c^2$$

# R and $r_c$ values obtained by Iida et al. 2001 and Maruyama et al. 2005



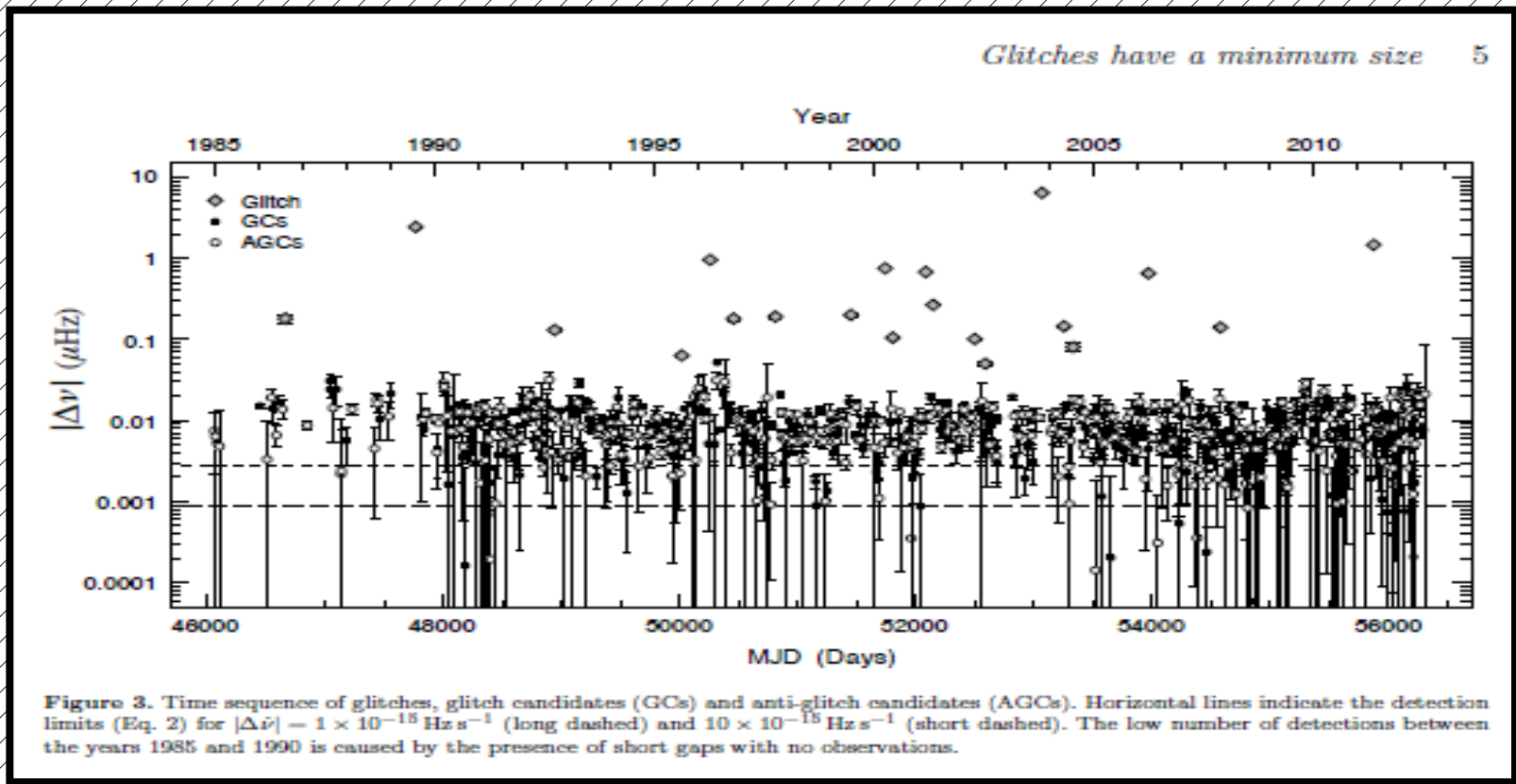
- Our estimation ( $\theta_{cr} = 4.4 \times 10^{-2}$ ) is also in agreement with Horowitz&Kadau(2009) that obtains  $\theta_{cr} = 8 \times 10^{-2}$  for bcc polycrystal orientation in outer layer.



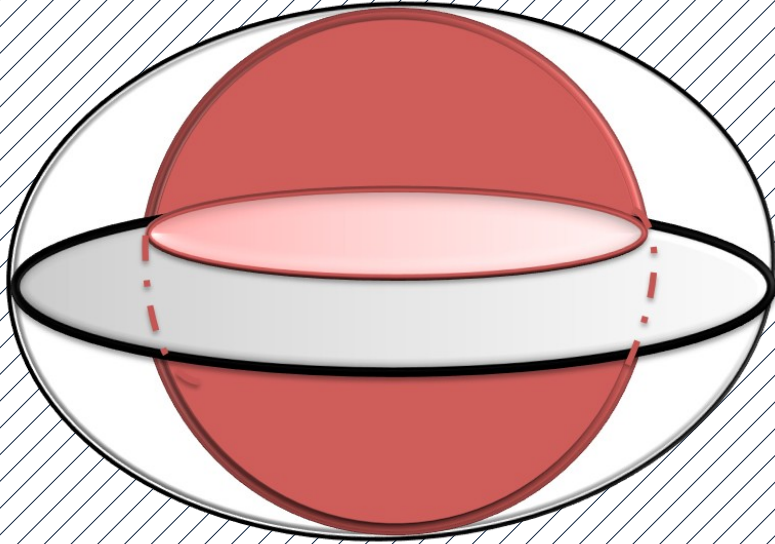
# Minimum Glitch Size in Crab:

Neutron star glitches have a substantial minimum size

C.M. Espinoza,<sup>1,2\*</sup> D. Antonopoulou,<sup>3</sup> B.W. Stappers,<sup>1</sup> A. Watts<sup>3</sup> and A.G. Lyne<sup>1</sup>



## Preliminary results (Akbal&Alpar 2016 -ongoing)



A ring including many plates moves inward

$D$ : plate size

$h$ : crust thickness

$$\Delta\nu = 0.05 \mu\text{Hz}, \Delta\Omega/\Omega = 1.7 \times 10^{-9}$$

$$\frac{\Delta\Omega}{\Omega}|_{\min} = \frac{\Delta I}{I} \propto f \left( \frac{D}{h} \right) \propto f(\theta_{cr})$$

$$\theta_{cr} \sim \frac{D_4 \times 10^4}{h_5 \times 10^5} \sim 3.3 \times 10^{-2} \frac{1}{h_5} \left( \frac{(M/M_S)}{\rho_{c,13} h_5 f} \right)^{1/2}$$

**ANY QUESTIONS?**

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Artist's impression of a Neutron star