Transport phenomena in neutron stars

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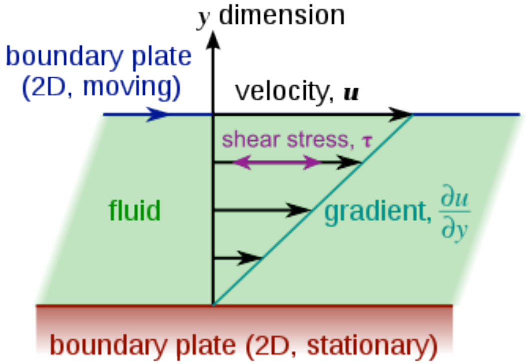
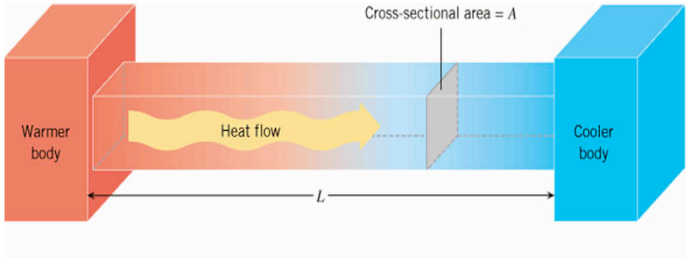
Transport coefficients

describe the response of the system to some external perturbation

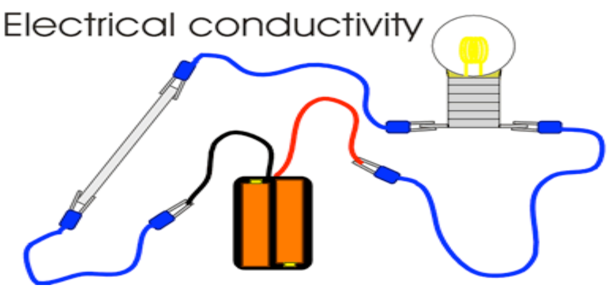
(Shear and Bulk) viscosity:
resistance to gradual deformation by shear stress or tensile stress



Thermal conductivity:
property of a material to conduct heat



Electrical conductivity:
measures a material's ability to conduct an electric current



CRUST

Chamel & Haensel '08
Page & Reddy '12

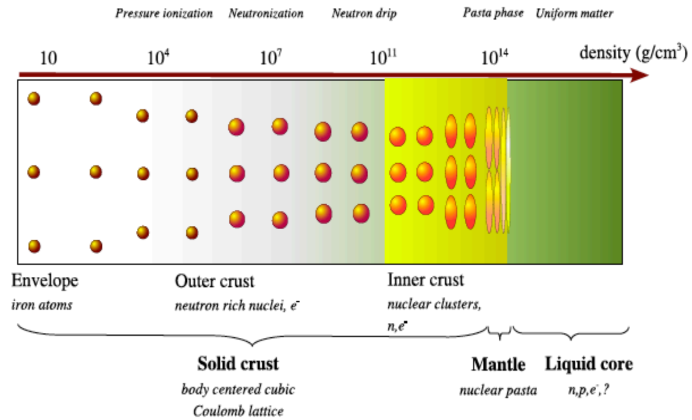
Composed of matter at sub-nuclear density ($\rho \leq 10^{14}$ g/cm^3) occupying the outermost 1-2 km region of the star

Outer crust: ionized nuclei in a degenerate electron gas

Inner crust: neutrons drip out from nucleus.

In shallower regions, spherical neutron-rich nuclei are embedded in a neutron superfluid. In deeper regions, non-spherical ("pasta") phases appear

cold non-accreting neutron star: ground state structure



Generalities:

-Electrons are strongly degenerate forming a dilute quasi-ideal gas of "electron excitations"

-Electrons interact with nuclei (equivalent to the scattering of electrons with a dilute gas of phonons), electrons and impurities in the crystal lattice

-The "screened" Coulomb interaction can be treated as small perturbation

-Analysis via the Boltzmann equation

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f}{\partial \boldsymbol{r}} + \boldsymbol{F} \cdot \frac{\partial f}{\partial \boldsymbol{p}} = I_e[f]$$

where $I_e[f]=I_{eN}[f]+I_{ee}[f]+I_{imp}[f]$, being the scatterers uncorrelated

- First approximation: relaxation time approximation

$$I_{eN}[f] \approx -\frac{\delta f}{\tau_0(\epsilon)}$$

- Second approximation: Matthiessen rule (sum of frequencies of different scattering processes)

Thermal (κ) and electrical (σ) conductivities

Chamel and Haensel ’08

$$\boldsymbol{j}_T = -Q_T T \boldsymbol{j}_e - \kappa \boldsymbol{\nabla} T$$

$$\boldsymbol{j}_e = \sigma \boldsymbol{E}^* + \sigma Q_T \boldsymbol{\nabla} T\,,$$

$$\sigma = \frac{e^2 n_e}{m_e^* \nu_\sigma} \qquad \nu_\sigma = \nu_{e\mathrm{N}}^\sigma + \nu_{\mathrm{imp}}^\sigma$$

$$\kappa = \frac{\pi^2 k_{\mathrm{B}}^2 T n_e}{3 m_e^* \nu_\kappa} \qquad \nu_\kappa = \nu_{e\mathrm{N}}^\kappa + \nu_{ee}^\kappa + \nu_{\mathrm{imp}}^\kappa$$

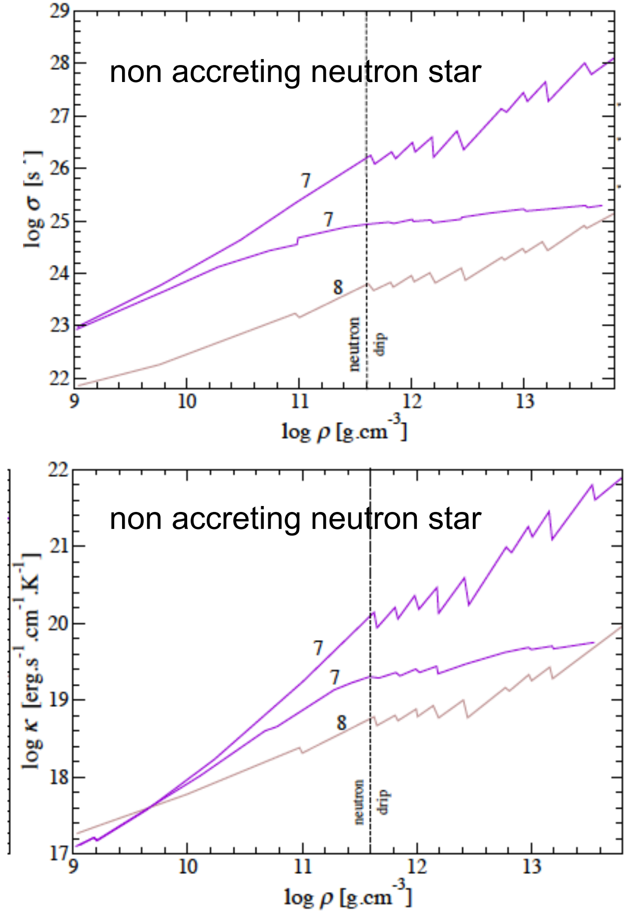


Figure 54: Electrical conductivity σ and thermal conductivity κ of the outer and inner crust, calculated for the ground-state model of Negele & Vautherin [303]. Labels 7 and 8 refer to $\log_{10} T[\mathrm{K}] = 7$ and 8, respectively. The thin line with label 7 corresponds to an impure crust, which contains in the lattice sites 5% impurities – nuclei with $|Z_{\mathrm{imp}} - Z| = 4$. Based on a figure made by A.Y. Potekhin.

Outer crust: main carriers of transport are electrons scattering with ions

$$\nu_\kappa = \nu_\sigma \simeq \nu_{e\mathrm{N}}(\mu_e)$$

For low temperatures, impurities dominate

Inner crust: the neutron gas plays a role

$$\kappa = \kappa_e + \kappa_n$$

with κ_n (κ_n^{diff},κ_n^{conv} in superfluids)

$$\kappa_n^{\mathrm{diff}} = \frac{\pi^2 k_{\mathrm{B}}^2 T n_n}{3 m_n^* \nu_n} \qquad \nu_n = \nu_{n\mathrm{N}} + \nu_{nn} \qquad \nu_n \approx \nu_{n\mathrm{N}}$$

Electron-electron might also be more important than electron-ion interactions (Shternin & Yakovlev ‘06)

Viscosities

$$\Pi_{ij}^{\mathrm{vis}} = \eta\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\boldsymbol{\nabla}\cdot\boldsymbol{U}\right) + \zeta\delta_{ij}\boldsymbol{\nabla}\cdot\boldsymbol{U}$$

Chamel and Haensel '08

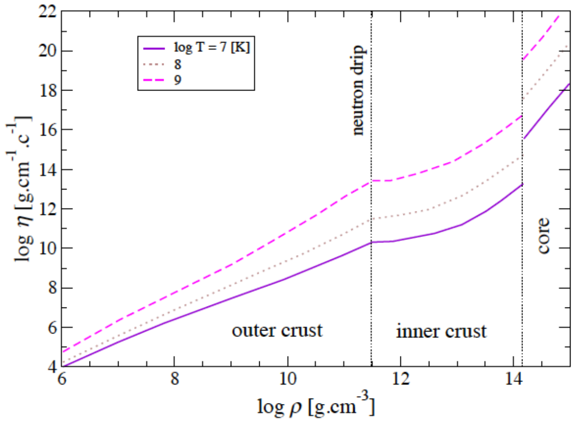


Figure 56: Electron shear viscosity of the crust and the upper layer of the core for $\log_{10} T[\mathrm{K}] = 7, 8, 9$. Calculated by Chugunov & Yakovlev [101] with a smooth composition model of the ground-state (Appendix B of Haensel, Potekhin, and Yakovlev [184]).

Shear viscosity η

For strongly non-ideal and solid plasma, the transport is mediated by electrons

Outer crust:

$$\eta = \eta_e + \eta_{\mathrm{N}}$$

$$\rho > 10^5 \text{ g cm}^{-3} \quad \eta_e \gg \eta_{\mathrm{N}} \text{ and } \eta \approx \eta_e$$

Electrons scatter on nuclei, on impurity nuclei and themselves

$$\nu_e^\eta = \nu_{e\mathrm{N}}^\eta + \nu_{\mathrm{imp}}^\eta + \nu_{ee}^\eta$$

For not too low T, $\nu_e \approx \nu_{e\mathrm{N}}$

At low temperatures, impurities dominate

Inner crust: extra contribution of gas of neutrons

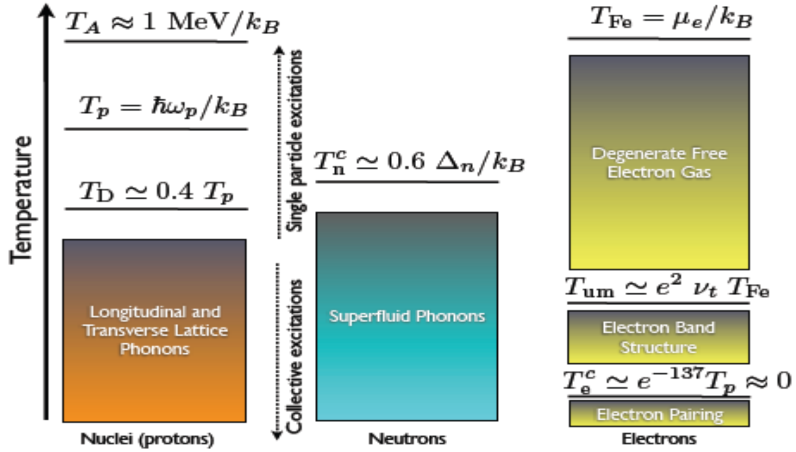
Bulk viscosity ξ

calculations? It is assumed that $\zeta \ll \eta$.

Remarks on inner crust

Page & Reddy ‘12

dynamics is dominated by electrons and lattice phonons (coherent motion of the clusters of protons located in neutron-rich nuclei). Another ingredient is superfluid phonons.

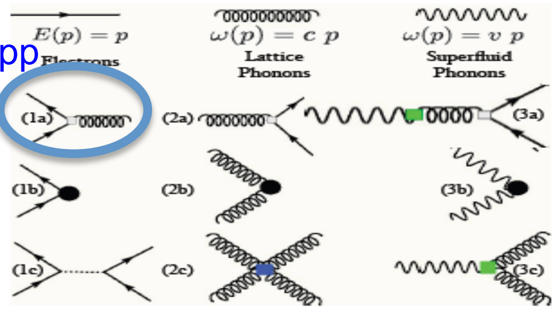


Conductivity and viscosity

- electron Umklapp scattering off the ion lattice is efficient for typical temperatures of interest

-at low temperature when Umklapp scattering is suppressed, electron-impurity scattering will likely dominate

Umklapp



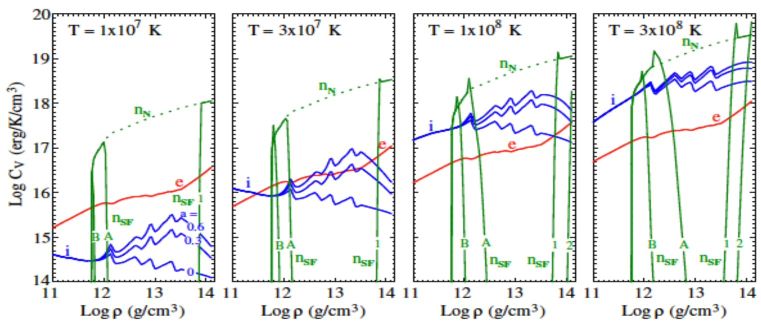


Figure 5. Specific heat of ions, electrons, and for neutrons with (labelled n_{SF}) and without the effects of the superfluid gap (labelled n_N) are shown for four representative temperatures.

Specific heat (thermal property)

ion, electron and neutron contributions

Strong magnetic fields

Overview of Chamel & Haensel '08
based on Potekhin '99; Ventura & Potekhin '01

strong surface magnetic field: $B \gg 10^9$ G
electron transport processes are affected

transport properties become anisotropic:
magnetic field bends electron trajectories in the (x,y) plane and suppresses the electron transport across B

Chamel and Haensel '08

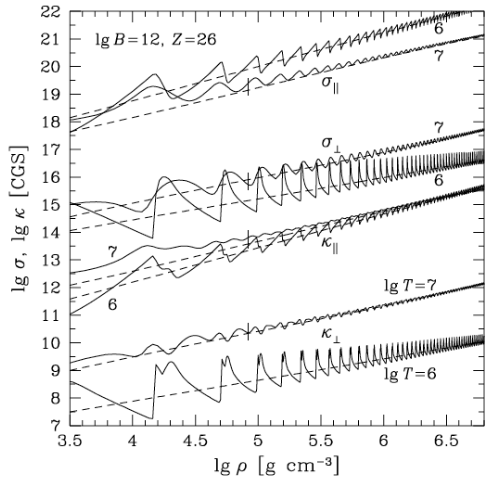


Figure 59: Longitudinal (∥) and transverse (⊥) electrical and thermal conductivities in the outer envelope composed of ^{56}Fe for $B = 10^{12}$ G and $\log_{10} T[\mathrm{K}] = 6, 7$. Quantum calculations (solid lines) are compared with classical ones (dash lines). Vertical bars: liquid-solid transition at $T = 10^7$ K. Based on Figure 5 from [338].

Different scenarios:

Nonquantizing magnetic fields:
many Landau orbitals are populated and quantum effects smeared by thermal effects

Weakly-quantizing magnetic fields
- many Landau orbitals are populated and quantum effects are well pronounced (oscillations due to filling of a new Landau level)
-two relaxation times, parallel and perpendicular to B field

Strongly-quantizing magnetic fields
quantization effects are well pronounced and electrons populate the ground Landau level

Some conclusions:
along B, thermal electron conduction is bigger than thermal ion conduction
across B, thermal electron conduction is suppressed

Possible observations to constrain transport properties of the crust

- supernova observations:
the constituents of the collapsing stellar cores and neutron star crusts are the same

- cooling of a young neutron star is very sensitive to its crust physics:
thermal relaxation of the crust depends on specific heat and thermal conductivity

- r-mode damping due to viscous crust-core Ekman layer

- relaxation of the crust after deposit of heat in
• accretion in a binary system,
such as Quasi-Persistent Sources of SXRTs
• giant flares

Chamel and Haensel '08

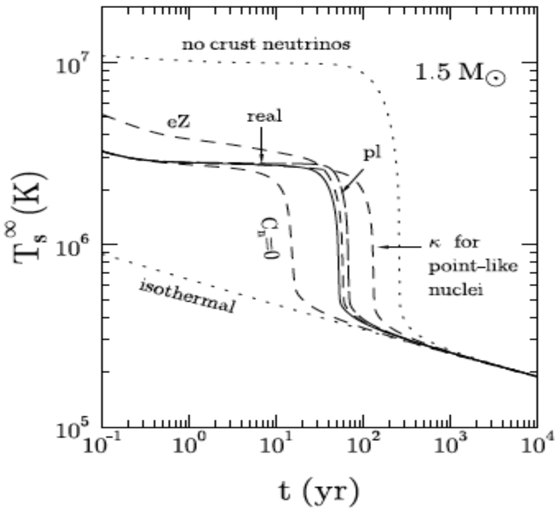


Figure 70: Effective surface temperature (as seen by an observer at infinity) of a $1.5\,M_\odot$ neutron star during the first hundred years for different crust models. Dotted lines: cooling without neutrino emission from the crust (upper line), infinite κ at $\rho > 10^{10}$ g cm^{-3}. Solid line: cooling curve for the best values of κ, C_v, and Q_ν. Dashed line $C_n = 0$: dripped neutrons heat capacity removed. Dashed curve κ: thermal conductivity calculated assuming point-like nuclei. Two other dashed lines: neutrino emission processes removed except for plasmon decay (pl) or electron-nucleus Bremsstrahlung (eZ). See also line $1.5\,M_\odot$ in Table 2 of [167].

CORE

Outer core extends in a density range $0.5\ \rho_0 \leq \rho \leq 2\rho_0$ and can be several kilometers deep. It is mainly composed of neutrons with some admixture of protons, electrons and muons. Neutron superfluidity and proton superconductivity are expected.

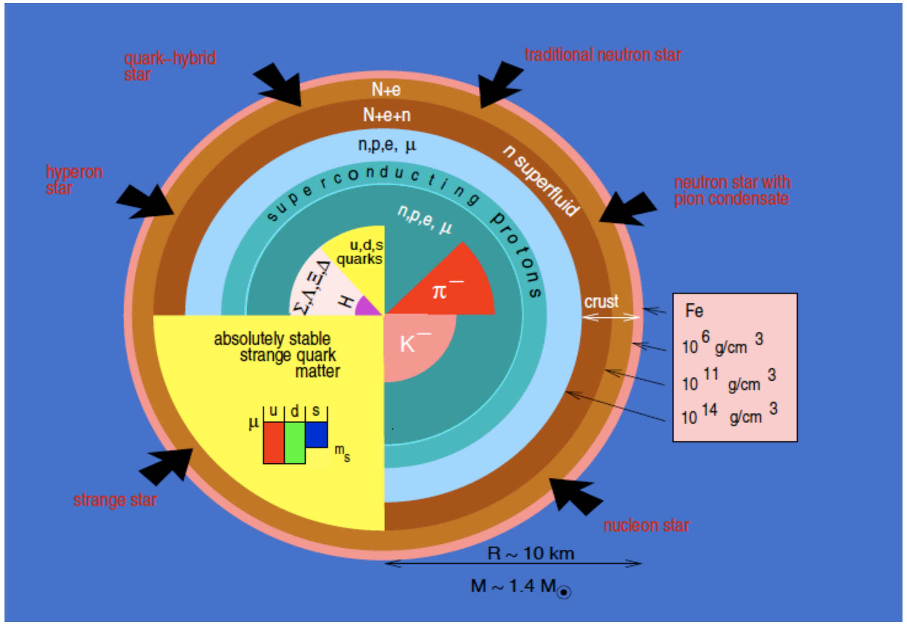
Inner core can be several kilometers in radius and have a central density as high as 10-15 ρ_0. The composition and equation of state of the inner core are poorly known.

Studied scenarios:

Matter made of n,p,e,μ

Matter made of n,p,e,μ,Y (=hyperons)

Matter made of quarks



Fridolin Weber

n,p,e,μ matter

electrons and muons (lightest and most mobile particles) and neutrons (most abundant)

Shear viscosity $\eta = \eta_{e\mu} + \eta_{n}$.

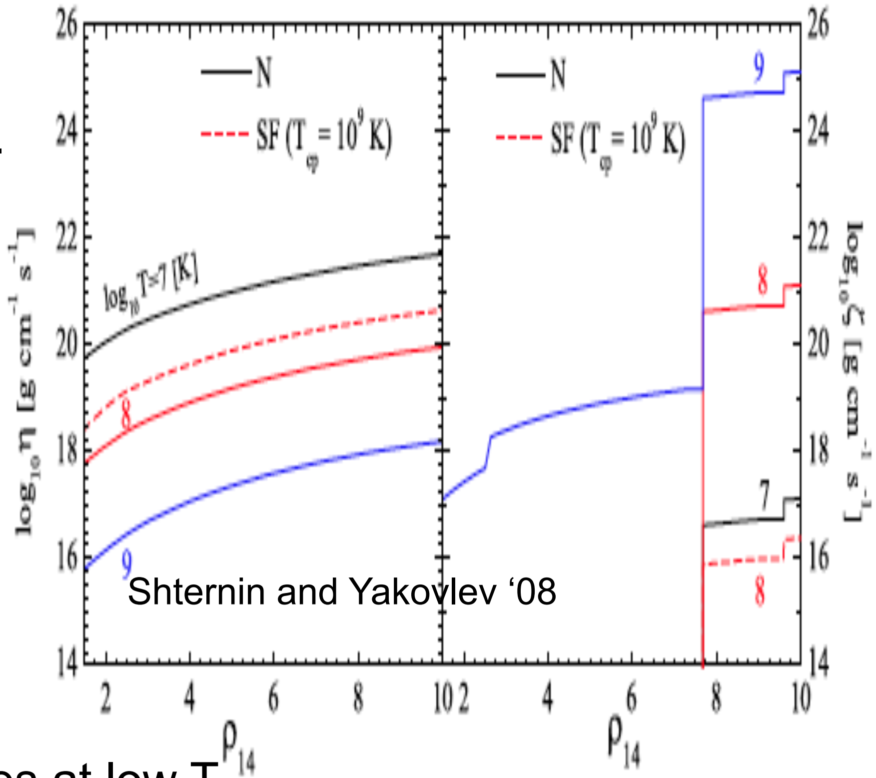
eμ collisions with protons:
Flowers and Itoh '76 '79; Cutler & Lindblom '87
Shternin and Yakovlev '08

n-n and n-p collisions:
Flowers and Itoh '79, Cutler and Lindblom '87,
Benhar and Valli '07, Shternin and Yakovlev '08..

superfluidity:
-proton superconductivity affects eμp interaction
-neutron superfluidity, phonon-phonon contribution (Manuel and LT '11,'13) and/or electron-phonon contribution (Bedaque & Reddy '13, Reddy et al'14)

Bulk viscosity

direct and modified Urca processes in the core of a vibrating neutron star by Sawyer'89; Haensel & Schaeffer'92; Haensel, Levenfish & Yakovlev '00 '01



Conclusions:

- $\eta_{e\mu}$ generally dominates over η_n
- η is comparable with ξ at $T\sim10^8$ K and dominates at low T
- proton superconductivity increases the importance of η in comparison with ξ
- neutron superfluidity: phonon-phonon/electron contributions might become important

n,p,e,μ matter

Thermal conductivity

$$\kappa \approx \kappa_b + \kappa_{e\mu},$$

κ_b: Baiko, Hensel & Yakovlev '01, …
$\kappa_{e\mu}$: Flowers & Itoh '76 '79; Gnedin & Yakovlev '95; Shternin & Yakovlev '07

Conclusions:

- $\kappa_{e\mu} \geq \kappa_n$ for $T \geq 2\text{x}\ 10^9$ K in normal matter and for any T in superconducting matter with proton critical temperatures $T_{cp} \geq 3 \text{ x } 10^9$ K
- in neutron superfluid matter:
phonon contribution might be important (Manuel & LT '14) and/or electron-phonon (Bedaque & Reddy '13, Reddy et al'14)

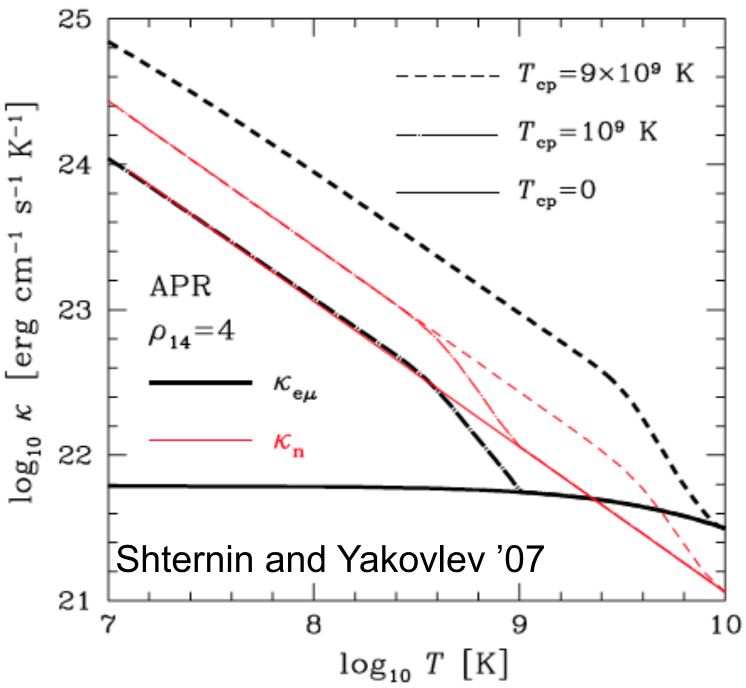


FIG. 5 (color online). Temperature dependence of the electron-muon thermal conductivity $\kappa_{e\mu}$ (thick lines) and the neutron conductivity κ_n (thin lines) in a neutron star core with the APR EOS at $\rho = 4 \times 10^{14}$ g cm^{-3}. Solid lines refer to nonsuperconducting matter, while dot-dashed and dashed lines are for matter with proton superconductivity (with $T_{cp} = 10^9$ and 9×10^9 K, respectively).

n,p,e,μ,Y (=hyperon) matter

Bulk viscosity Jones ‘71 ‘01, Lindblom & Owen ’02, Haensel, Levenfish & Yakovlev ’02, Chatterjee & Bandyopadhyay ‘07,…

Hyperons: direct non-leptonic hyperon collisions which go via weak interaction such as

$$\boxed{\begin{array}{c} n + n \rightleftharpoons p + \Sigma^-, \\ \Lambda + N \rightleftharpoons n + N, \end{array}}$$

are the most efficient mechanism involving hyperons

Only with Σ-

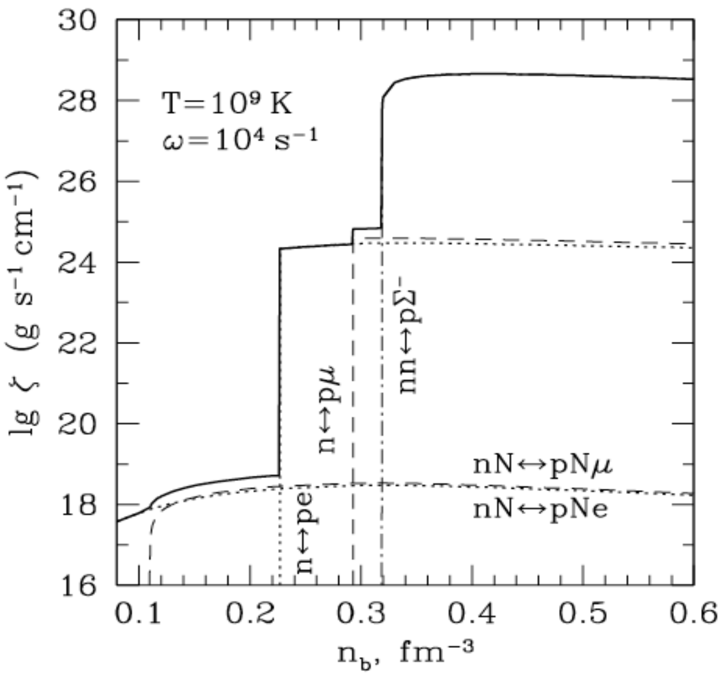


Fig. 1. Density dependence of partial bulk viscosities associated with various processes (indicated near the curves) at $T = 10^9$ K and $\omega = 10^4\ \mathrm{s}^{-1}$ in non-superfluid matter. Dotted and dashed lines refer to Urca processes involving electrons and muons, respectively; dot-and-dashed line refers to hyperon process (3). Thick solid line is the total bulk viscosity.

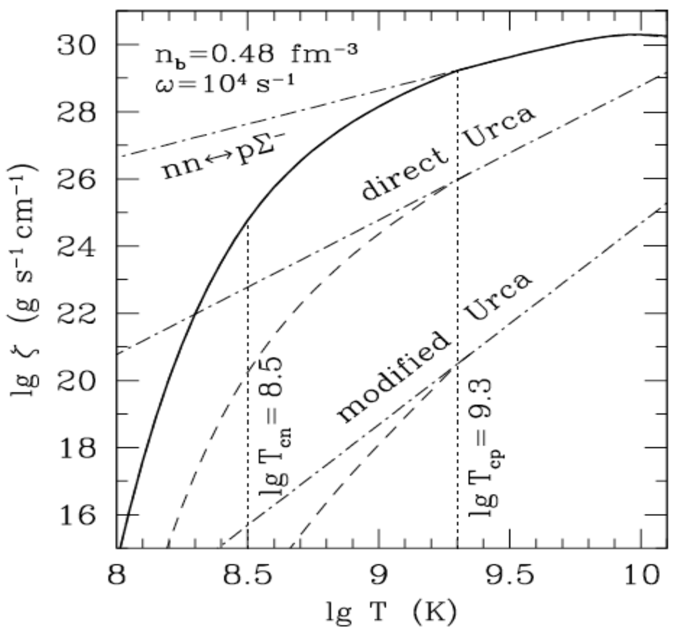
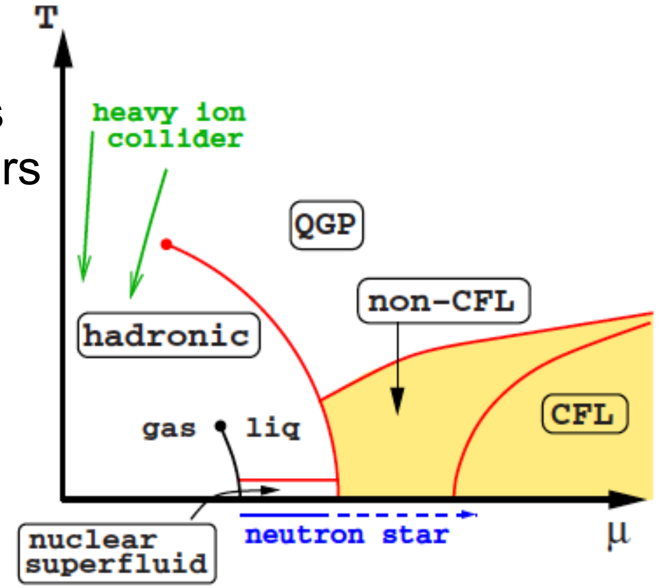


Fig. 3. Temperature dependence of bulk viscosity at $n_b = 0.48\ \mathrm{fm}^{-3}$ and $\omega = 10^4\ \mathrm{s}^{-1}$ in the presence of proton superfluidity with $\lg T_{cp} = 9.3$ and neutron superfluidity with $\lg T_{cn} = 8.5$. Dot-and-dashed lines (from up to down): partial bulk viscosities due to hyperonic, direct Urca and modified Urca processes, respectively, in non-superfluid matter. Associated solid and dashed lines: the same bulk viscosities in superfluid matter. Vertical dotted lines show $\lg T_{cn}$ and $\lg T_{cp}$.

Haensel, Levenfish and Yakovlev ’02

Quark matter Alford, Schmitt, Rajagopal and Schaefer '08

there are different phases, such as CFL phase,
where all three colors and up, down and strange flavors
form conventional zero-momentum spinless Cooper pairs



CFL phase

-free-parameter predictions of QCD at
very large densities and rigorous consequences of
QCD in terms of few phenomenological parameters
at low densities

-all quark modes are gapped and relevant excitations are
Goldstone modes (phonons) for
thermal conductivity and viscosities
(φ<->φφ φφ<->φφ processes)

$$\kappa = \frac{2\pi^2 T^3}{45 v^2} l_\varphi$$

$$l_\varphi \propto \mu^8 / T^9$$

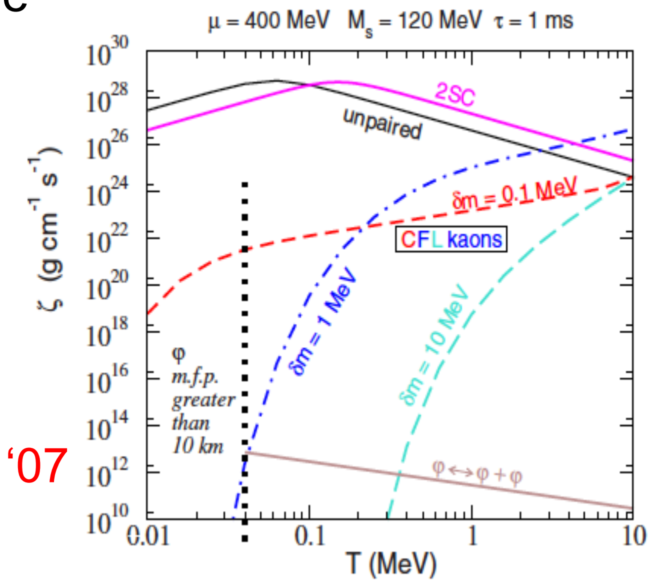
Shovkovy & Ellis'02

$$\eta = 1.3 \times 10^{-4} \frac{\mu^8}{T^5}$$

Manuel et al '05

$$\zeta = 0.011 \frac{M_s^4}{T}$$

Manuel &
Llanes-Estrada '07

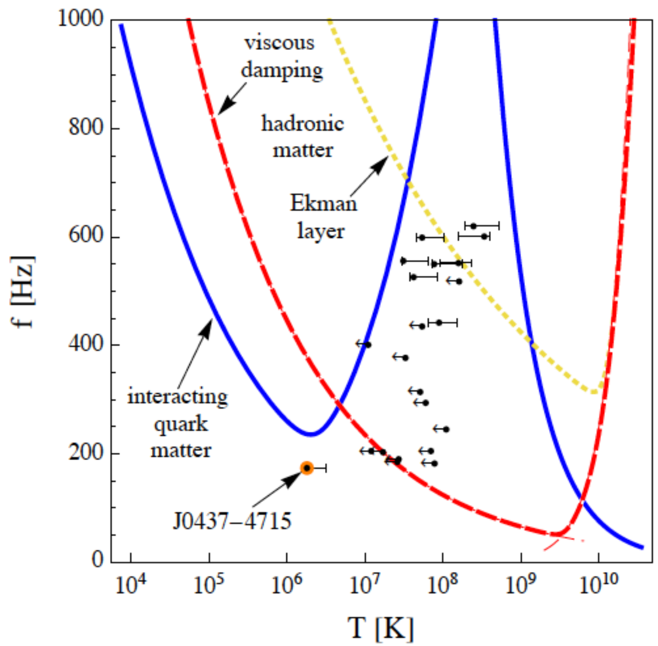


comparison of bulk viscosity for CFL
with other phases

- electrical conductivity in quark matter is dominated by
electrons and positrons (Shovkovy & Ellis '03)

Possible observations to constrain transport properties of the core

- shear and bulk viscosities inside the neutron star core can help to understand r-mode instability and emission of gravitational waves



Data for accreting pulsars in binary systems (LMXBs) vs instability curves for nuclear and hybrid stars.

Alford & Schwenzer ‘13

- heat conduction problem in neutron star cores is needed to model cooling of neutron stars (especially in the first 100 years of their life), thermal relaxation of pulsars after glitches,… (taken from Shternin & Yakovlev ‘07 and references herein)

DISCUSSION: Some open questions..

Transport phenomena in the CRUST

inner crust (Page & Reddy '12)

- existence and extent of pasta phase and transport properties
- impurities in the inner crust (need of nuclear structure studies)
- nuclear excitations relevant in the crust for $T \leq 10^{10}$ K?

…..

Transport phenomena in the CORE

- superfluidity/superconductivity affecting transport coefficients
- effect of strong magnetic fields

…..

WG2:
Physics of the strong interaction, theory and experiment

Transport coefficients in superfluid neutron star matter

Laura Tolós
Cristina Manuel, Sreemoyee Sarkar and Jaume Tarrús

- ✧ EFT and superfluid phonons
- ✧ EoS for superfluid neutron star matter
- ✧ Shear viscosity and the r-mode instability window
- ✧ Bulk viscosity
- ✧ Thermal conductivity
- ✧ Summary

Manuel and Tolos, Physical Review D 84 (2011) 123007
Manuel and Tolos, Physical Review D 88 (2013) 043001
Manuel, Tarrus and Tolos, JCAP 1307 (2013) 003
Manuel, Sarkar and Tolos, Physical Review C 90 (2014) 055803

EFT and superfluid phonon

Exploit the universal character of EFT at leading order by obtaining the effective Lagrangian associated to a superfluid phonon and implement the particular features of the system, associated to the coefficients of the Lagrangian, via the EoS

Son '02
Son and Wingate '06

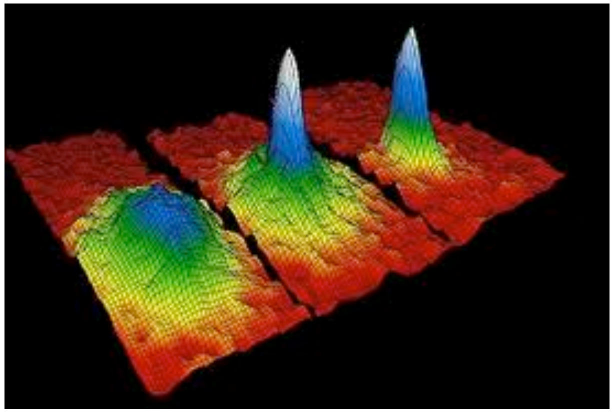
non-relativistic case

$$\mathcal{L}_{\rm LO} = P(X)$$

$$X = \mu - \partial_t \varphi - \frac{(\nabla \varphi)^2}{2m}$$

$P(\mu)$	pressure
μ	chemical potential
φ	phonon field
m	mass condense particles

Applicable in superfluid systems such as cold Fermi gas at unitary, ^{4}He or neutron stars



EoS for superfluid neutron star matter

In order to obtain the speed of sound at T=0 and the different phonon self-couplings one has to determine the EoS for neutron matter in neutron stars.

A common benchmark for nucleonic EoS is APR98
Akmal, Pandharipande and Ravenhall '98

which was later parameterized in a causal form Heiselberg and Hjorth-Jensen '00

$$E/A = \mathcal{E}_0 y \frac{y-2-\delta}{1+\delta y} + S_0 y^\beta (1-2x_p)^2$$

$$y = n/n_0 \qquad x_p = n/n_0$$

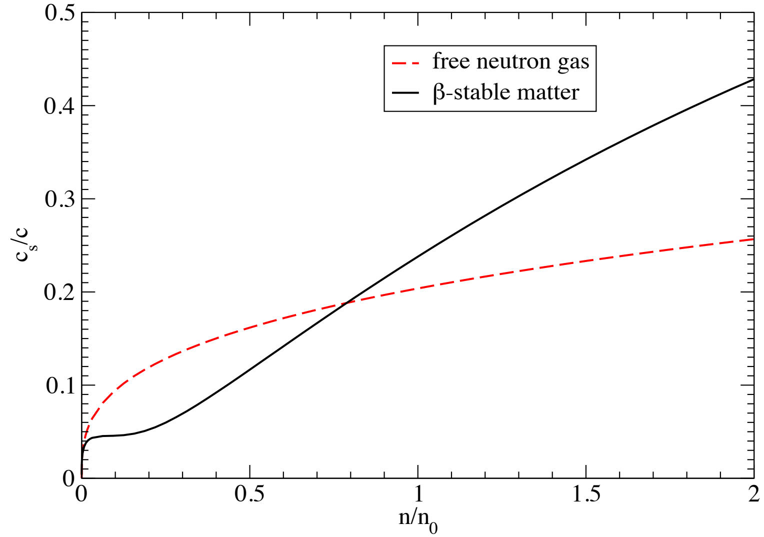
$n_0 = 0.16\,\mathrm{fm}^{-3}$

$\mathcal{E}_0 = 15.8\,\mathrm{MeV}$ $\delta = 0.2$

$S_0 = 32\,\mathrm{MeV}$ $\beta = 0.6$

For β-stable matter made up of neutrons, protons and electrons, the speed of sound at T=0 is

$$\sqrt{\frac{\partial P}{\partial \rho}} \equiv c_s$$



Effective Lagrangian for superfluid phonon at LO

$$\mathcal{L}_{\rm LO} = \frac{1}{2}((\partial_t\phi)^2 - v_{\rm ph}^2(\nabla\phi)^2) - g((\partial_t\phi)^3 - 3\eta_g\partial_t\phi(\nabla\phi)^2) + \lambda((\partial_t\phi)^4 - \eta_{\lambda,1}(\partial_t\phi)^2(\nabla\phi)^2 + \eta_{\lambda,2}(\nabla\phi)^4) + \cdots.$$

with Φ the rescaled phonon field, and where the different phonon self-couplings can be expressed in terms of the speed of sound at T=0

$$v_{ph} = \sqrt{\frac{\frac{\partial P}{\partial \mu}}{m\frac{\partial^2 P}{\partial \mu^2}}} = \sqrt{\frac{\partial P}{\partial \rho}} \equiv c_s$$

and derivatives with respect to mass density:

Escobedo and Manuel '10

$$u = \frac{\rho}{c_s}\frac{\partial c_s}{\partial \rho}\,, \quad w = \frac{\rho}{c_s}\frac{\partial^2 c_s}{\partial \rho^2}\,,$$

$$g = \frac{1-2u}{6c_s\sqrt{\rho}}\,, \qquad \eta_g = \frac{c_s^2}{1-2u}\,, \qquad \lambda = \frac{1-2u(4-5u)-2w\rho}{24c_s^2\rho}\,,$$

$$\eta_{\lambda,1} = \frac{6c_s^2(1-2u)}{1-2u(4-5u)-2w\rho}\,, \qquad \eta_{\lambda,2} = \frac{3c_s^4}{1-2u(4-5u)-2w\rho}$$

Results valid for neutrons pairing in $^1S_0$ channel and also valid for $^3P_2$ neutron pairing if corrections $\bar{\Delta}(^3P_2)^2/\mu_n^2$ are ignored Bedaque, Rupak and Savage '03

Including NLO corrections in the phonon dispersion law

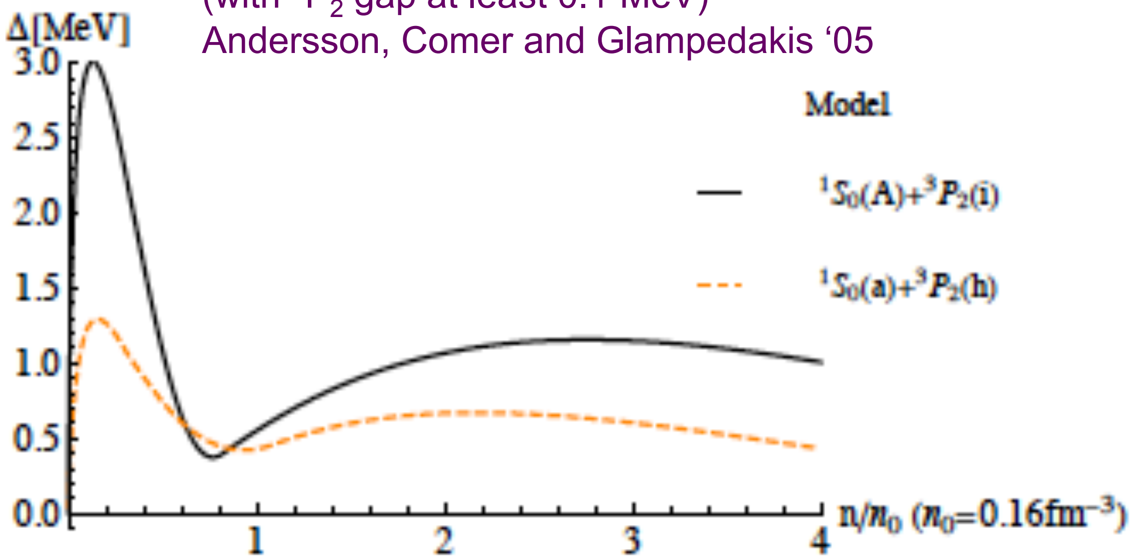
$$E_P = c_s p(1 + \gamma p^2)$$

$$\gamma = -\frac{v_F^2}{45\Delta^2}$$

v_F: Fermi velocity
Δ: gap function

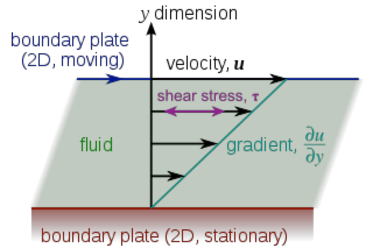
ϒ < 0: first allowed phonon scattering are binary collisions

taken from parameterizations of different models
(with $^3P_2$ gap at least 0.1 MeV)
Andersson, Comer and Glampedakis '05



Shear viscosity due to superfluid phonons

Shear viscosity

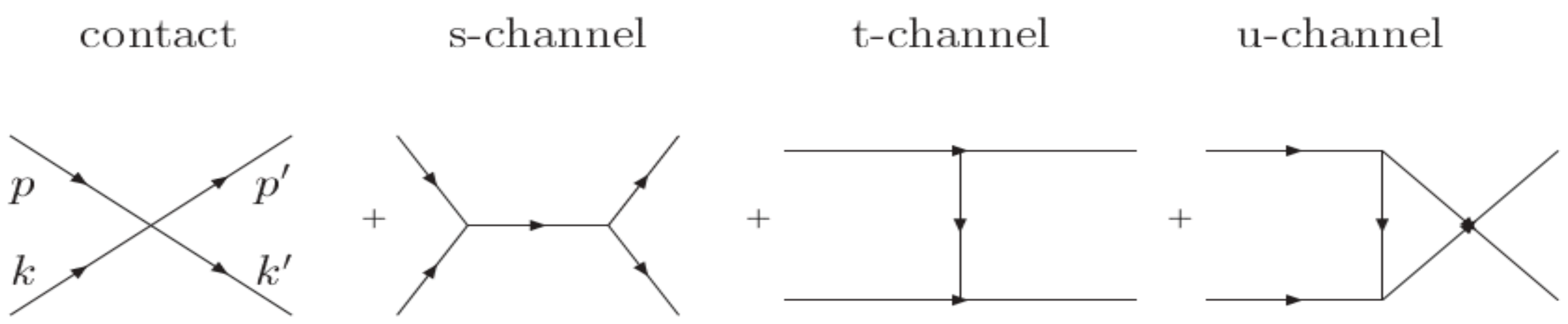


wikipedia

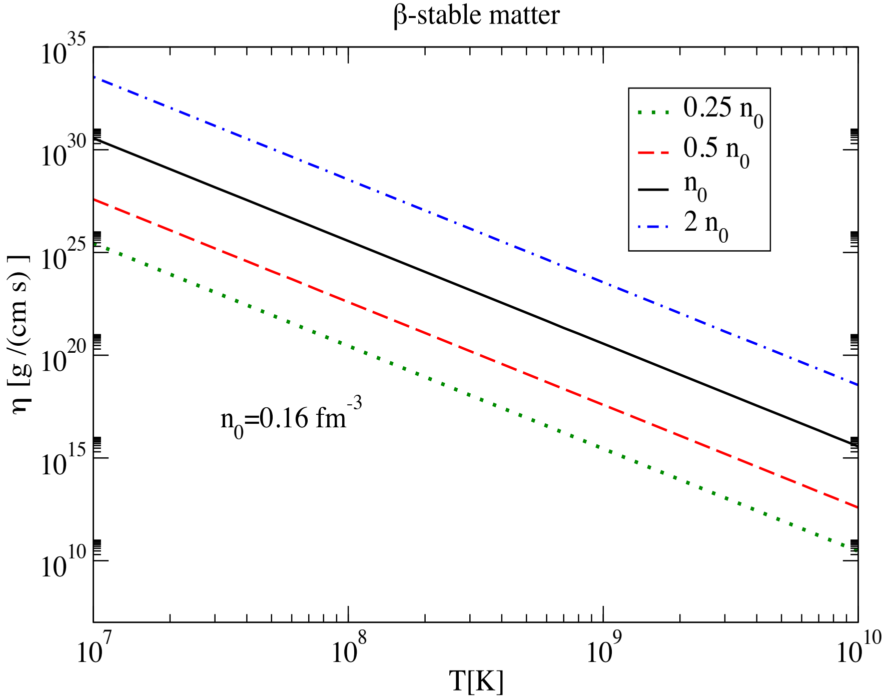
The shear viscosity is calculated using variational methods for solving the transport equation as

$$\eta = \left(\frac{2\pi}{15}\right)^4 \frac{T^8}{c_s^8} \frac{1}{M}$$

where M represents a multidimensional integral that contains the thermally weighted scattering matrix for phonons.



Shear viscosity due to binary collisions of phonons scales as $\eta \propto 1/T^5$ (also for ^{4}He and cold Fermi gas at unitary) while the coefficient depends on EoS.

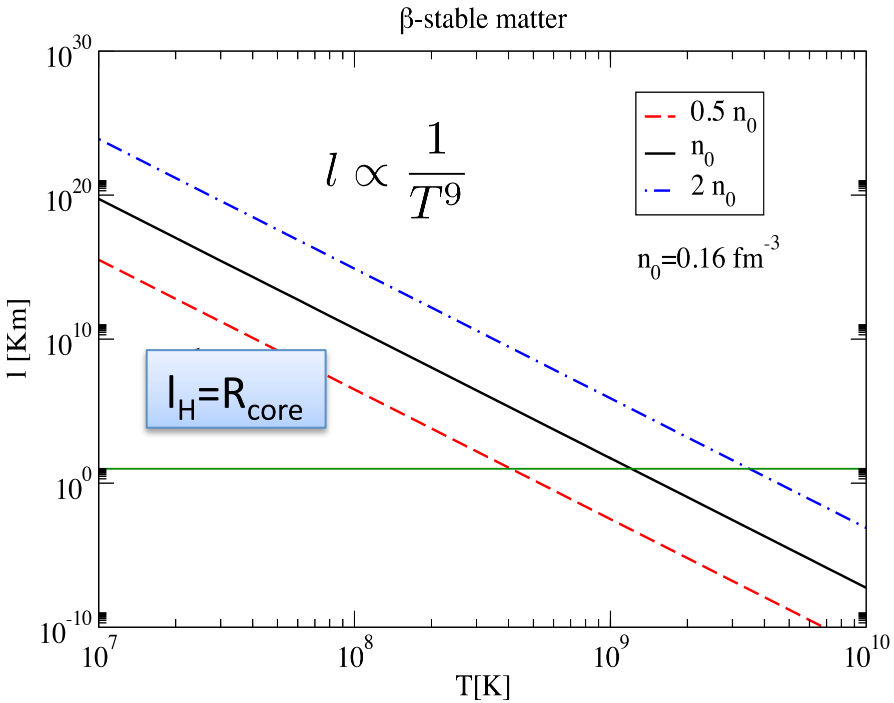


Mean free path of phonons: establish when phonons become hydrodynamic

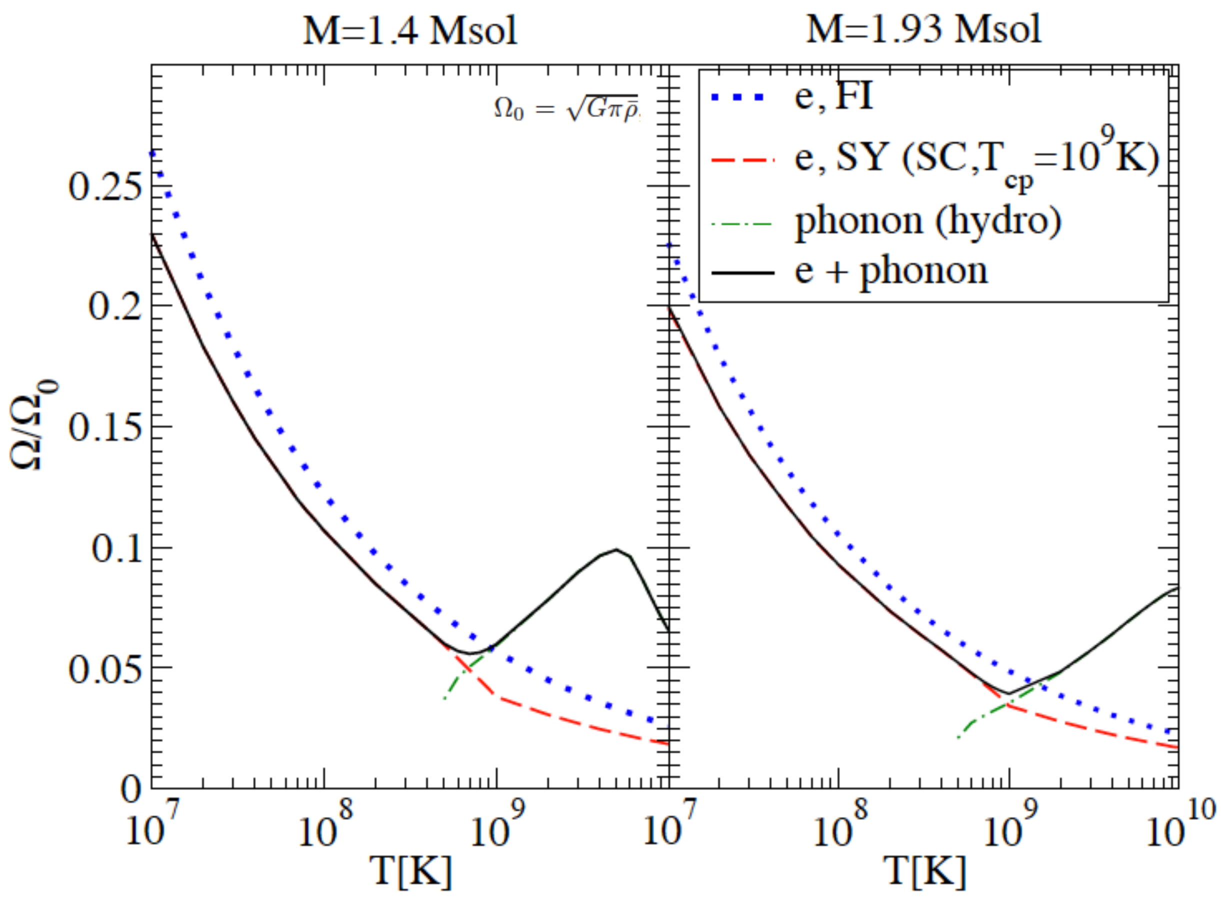
$$l = \frac{\eta}{n < p >}$$

$< p >$: thermal average

n : phonon density



Alford, Braby, and Mahmoodifar '10



r-mode instability window (only shear)

$$-\frac{1}{|\tau_{\rm GR}(\Omega)|} + \frac{1}{\tau_{\eta_{\rm hydro}}(T)} = 0$$

Dissipation due to superfluid phonons start to be relevant at T≈7 x 10^8 K for 1.4 $M_\odot$ and T≈10^9 K for 1.93 $M_\odot$

$$\frac{1}{|\tau_{\rm GR}(\Omega)|} = \frac{32\,\pi\,G\,\Omega^{2l+2}}{c^{2l+3}}\frac{(l-1)^{2l}}{((2l+1)!!)^2}\left(\frac{l+2}{l+1}\right)^{2l+2}\int_0^R \rho r^{2l+2}dr$$

$$\frac{1}{\tau_{\eta_{\rm hydro}}(T)} = (l-1)(2l+1)\int_{R_c}^R \eta_{\rm hydro} r^{2l}dr\left(\int_0^R \rho r^{2l+2}dr\right)^{-1}$$

l=2 (dominant)

Bulk viscosities due to superfluid phonons

The bulk viscosity coefficients are calculated from the dynamical evolution of the phonon number density* or, equivalently, by using the Boltzmann equation for phonons in the relaxation time approximation

$$\zeta_i(\omega) = \frac{1}{1+\left(\omega I_1^2 \frac{\partial\rho}{\partial n}\frac{\partial\rho}{\partial\mu}\frac{T}{\Gamma_{ph}}\right)^2}\frac{T}{\Gamma_{ph}}C_i \,, \qquad i=1,2,3,4$$

$$C_1 = C_4 = -I_1 I_2 \,, \qquad C_2 = I_2^2 \,, \qquad C_3 = I_1^2 \,,$$

$$I_1 = \frac{60T^5}{7c_s^7\pi^2}\left(\pi^2\zeta(3) - 7\zeta(5)\right)\left(c_s\frac{\partial B}{\partial\rho} - B\frac{\partial c_s}{\partial\rho}\right) \,, \qquad B = c_s\gamma$$

NLO corrections in phonon dispersion law

$$I_2 = -\frac{20T^5}{7c_s^7\pi^2}\left(\pi^2\zeta(3) - 7\zeta(5)\right)\left(2Bc_s + 3\rho\left(c_s\frac{\partial B}{\partial\rho} - B\frac{\partial c_s}{\partial\rho}\right)\right) \,,$$

Three independent coefficients: $\zeta_1 = \zeta_4 \quad \zeta_1^2 \leq \zeta_2\zeta_3 \quad \zeta_2, \zeta_3 \geq 0$

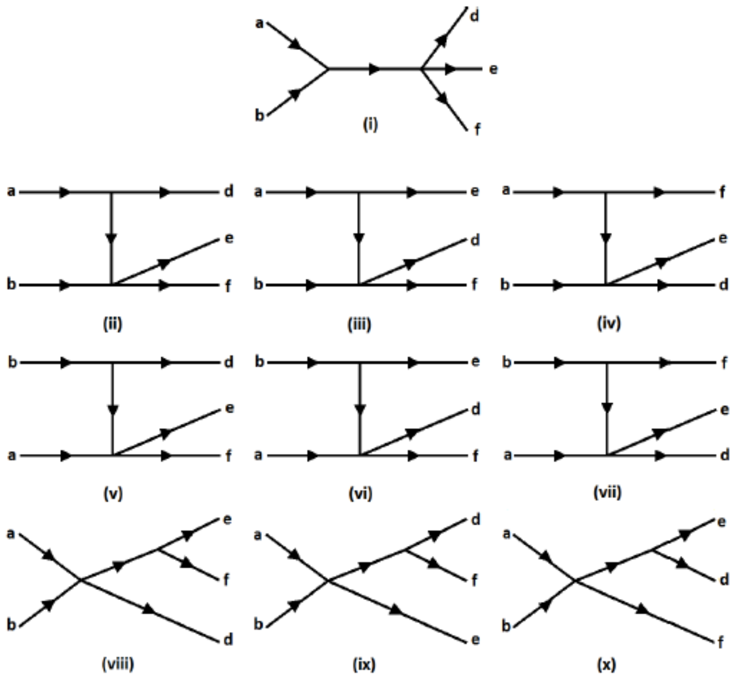
In the static limit

$$\zeta_i = \frac{T}{\Gamma_{ph}}C_i \,, \qquad i=1,2,3,4 \,,$$

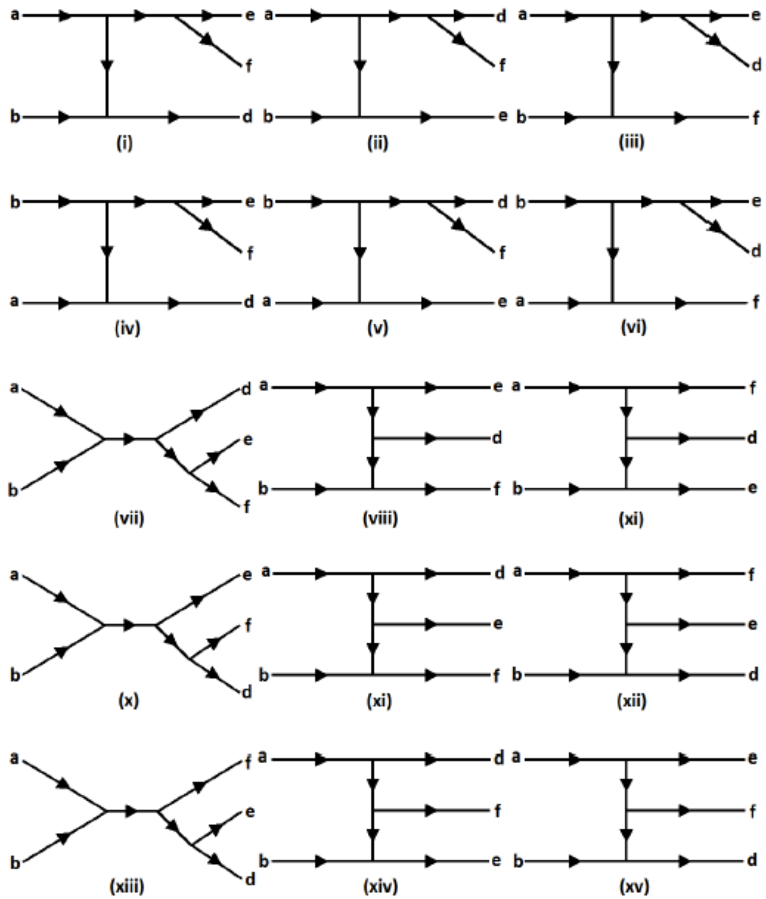
* Khalatnikov

Phonon decay rate for phonon number changing processes: 2 <-> 3

$$\Gamma_{ph} = \int d\Phi_5(p_a, p_b; p_d, p_e, p_f)\|\mathcal{A}\|^2 f(E_a) f(E_b)\,(1 + f(E_d))\,(1 + f(E_e))\,(1 + f(E_f))$$

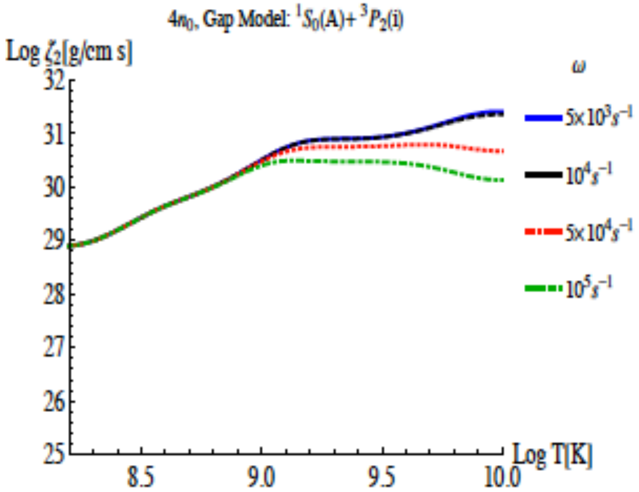


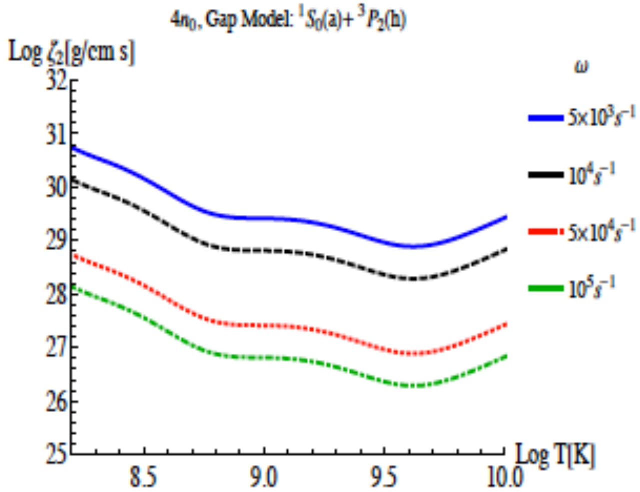
with one 4-phonon vertex
and one 3-phonon vertex



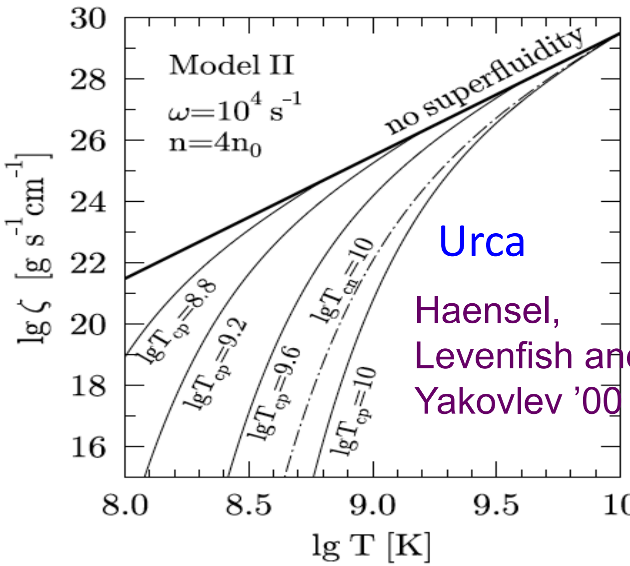
with only 3-phonon vertices

ξ_2 at $n \geq 4n_0$ is within 10% of the static value for $T \leq 10^9$ K and for the case of maximum values of the $^3P_2$ gap > 1 MeV, while, otherwise, the static solution is not valid. Bulk viscosity coefficients strongly depend on the gap.





Compared to the contribution of Urca (also modified Urca) processes to the bulk viscosities in neutron stars, those are dominated by phonon-phonon processes



Urca

Haensel, Levenfish and Yakovlev '00 '01

Thermal conductivity due to superfluid phonons

The thermal conductivity relates the heat flux with the temperature gradient

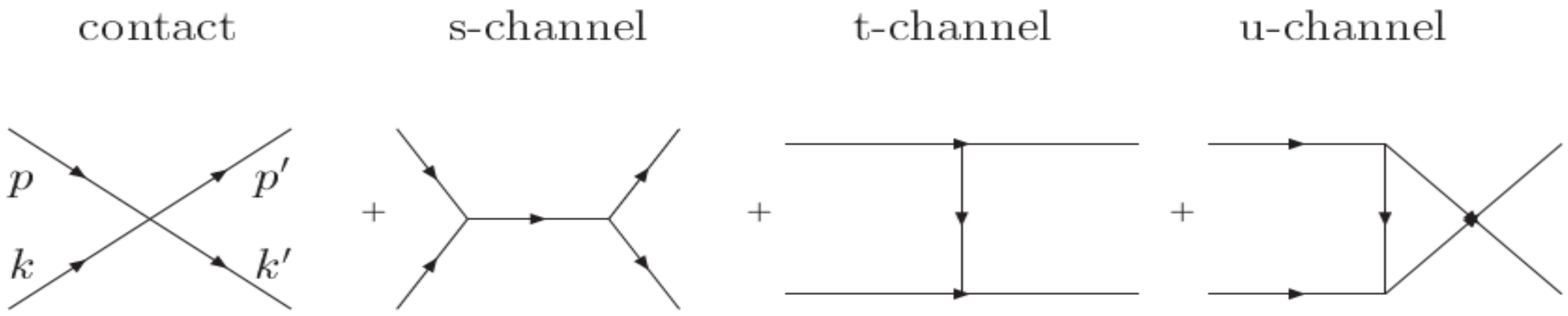
$$\mathbf{q} = -\kappa \nabla T$$

and is calculated using variational methods* for solving the transport equation as

$$\kappa \geq \left(\frac{4a_1^2}{3T^2}\right) A_1^2 M_{11}^{-1}$$

$$a_1 = \frac{4c_s^4}{15\Delta^2}\,, \qquad A_1 = \frac{256\pi^6}{245c_s^9}T^9$$

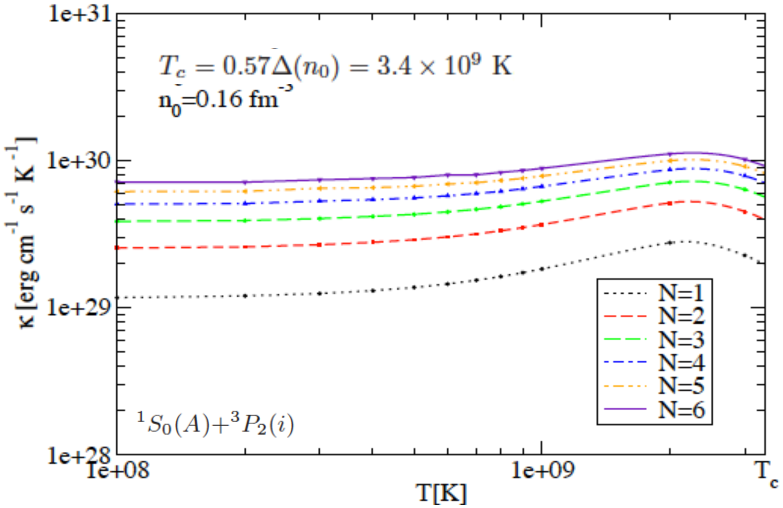
where M_{11} is the (1,1) element of a N x N matrix (N determined by convergence). Each element is a multidimensional integral that contains the thermally weighted scattering matrix for phonons:



* Braby, Chao and Schafer '10

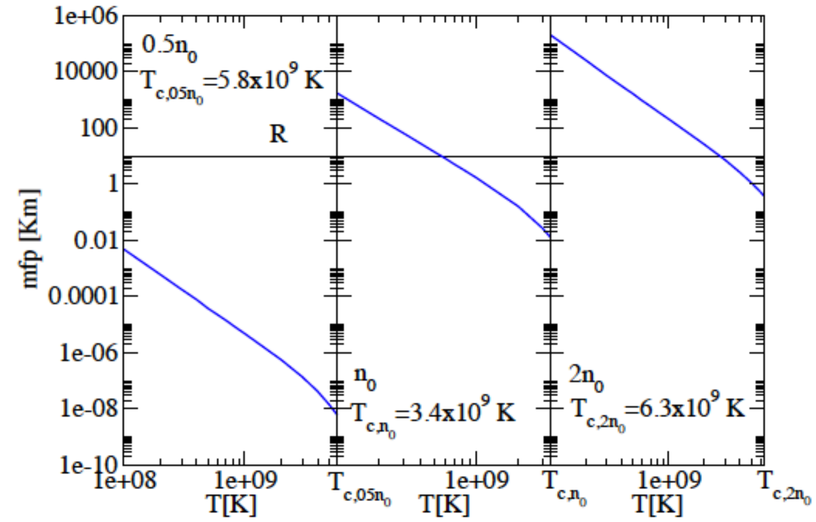
Need of NLO corrections in phonon dispersion law (seen for He^4 and CFL) Perform a variational calculation up to N=6 (deviation from N=5 ≤ 10%). We find that as in CFL

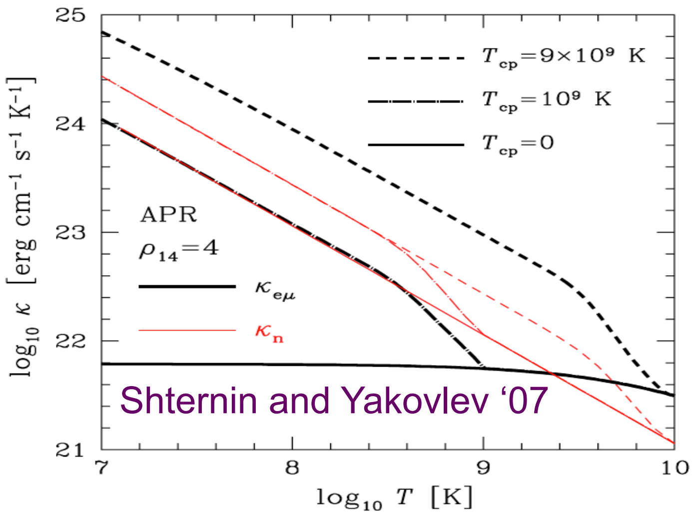
$$\kappa \propto \frac{1}{\Delta^6} \qquad T \lesssim 10^9 \text{ K}$$



Mean free path of phonons (different from shear mean free path)

$$l = \frac{\kappa}{\frac{1}{3}c_v c_s} \qquad l \propto 1/T^3$$





$$10^{25} \lesssim \kappa_{ph} \lesssim 10^{32} \text{ erg cm}^{-1}\text{ s}^{-1}\text{ K}^{-1}$$

Thermal conductivity in neutron stars is dominated by phonon-phonon collisions

Summary for transport coefficients in superfluid neutron matter

Starting from a general formulation for the collisions of superfluid phonons using EFT techniques, we compute the shear and bulk viscosities as well as the thermal conductivity in terms of the EoS of the system (and the gap function)

- Binary collisions of phonons produce a shear viscosity that scales with $1/T^5$ (universal feature seen for ^{4}He and cold Fermi gas at unitary) while the coefficient depends on EoS (microscopic theory)

r-mode window modified for $T \geq (10^8\text{-}10^9)$ K due to phonon shear viscosity

- Bulk viscosity coefficients strongly depend on the gap and they are dominated by phonon-phonon collisions as compared to Urca (modified Urca) processes

- Thermal conductivity due to phonons scales as $1/\Delta^6$, the constant of proportionality depending on the EoS. As compared to electron-muon collisions, phonon-phonon collisions dominate the thermal conductivity

Need of an accurate analysis of electron-phonon collisions Bertoni, Reddy and Rrapaj '15

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