Relativistic Models for Electron and Neutrino-Nucleus Scattering

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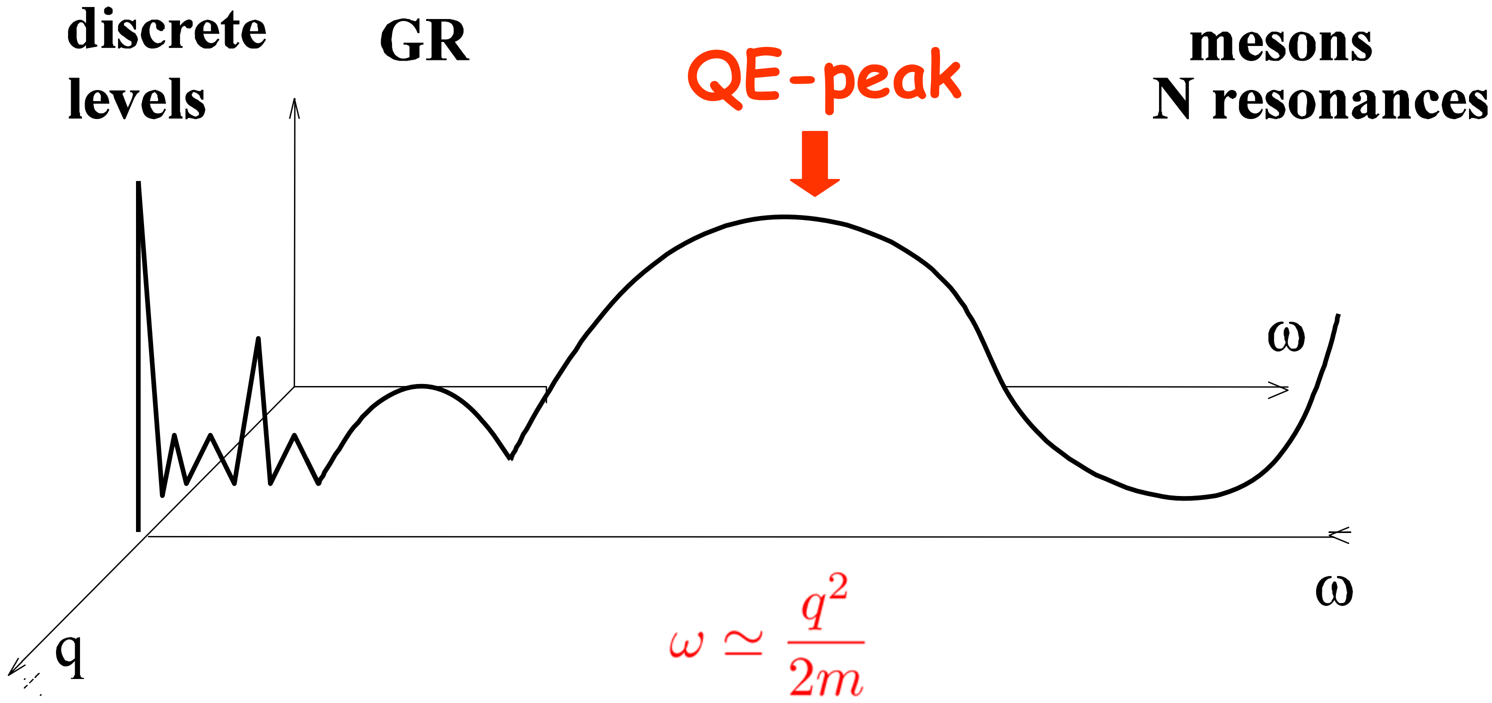
J.M. Udías (Madrid)

NuInt09 Sitges May 18^{th}-22^{nd} 2009

nuclear response to the electroweak probe



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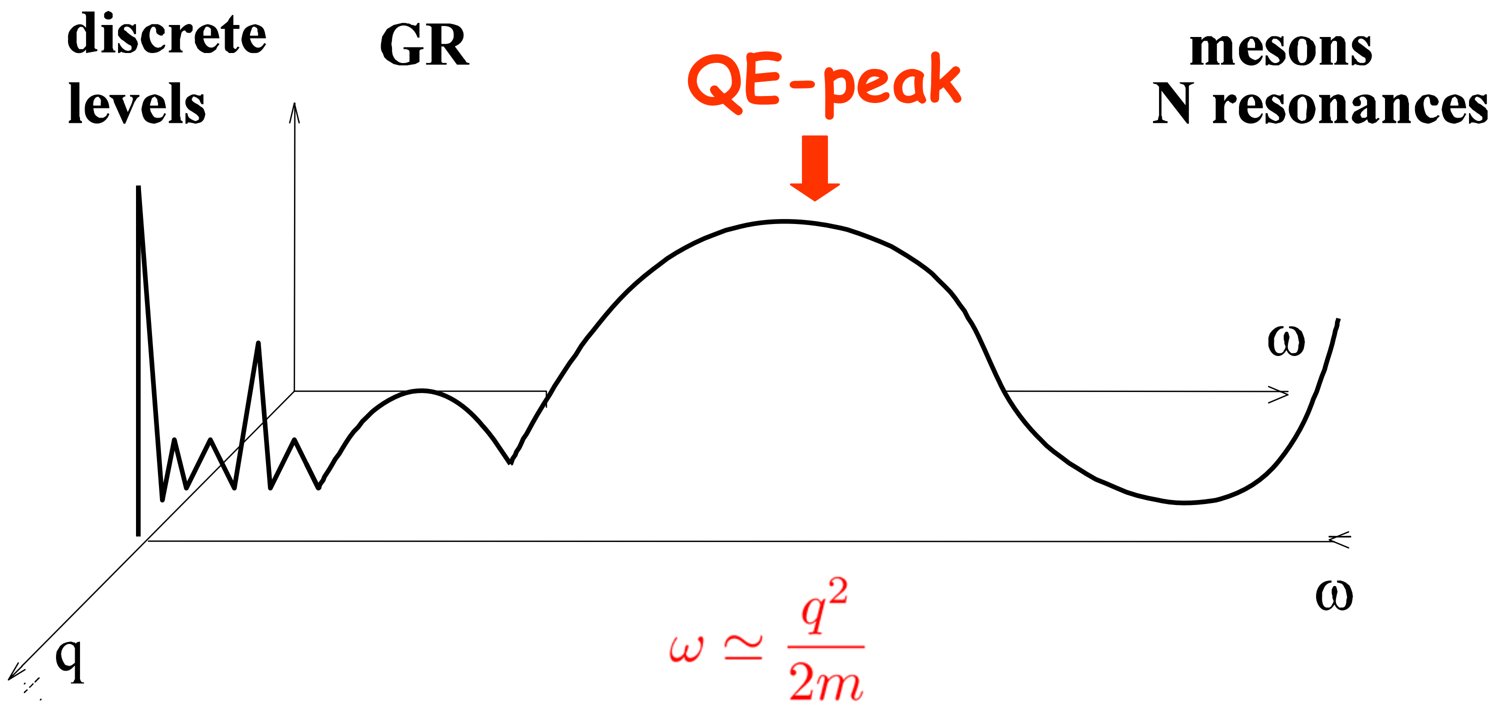

QE-peak dominated by one-nucleon knockout

QE e-nucleus scattering

$$e + A \Longrightarrow e' + N + (A - 1)$$

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$$e + A \Longrightarrow e' + N + (A-1)$$

- both e' and N detected one-nucleon-knockout (e,e'p)

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QE ν-nucleus scattering

$$\nu_l(\bar{\nu}_l) + A \Longrightarrow \nu_l(\bar{\nu}_l) + N + (A-1) \qquad \text{NC}$$

$$\nu_l(\bar{\nu}_l) + A \Longrightarrow l^-(l^+) + N + (A-1) \qquad \text{CC}$$

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- only N detected semi-inclusive NC and CC

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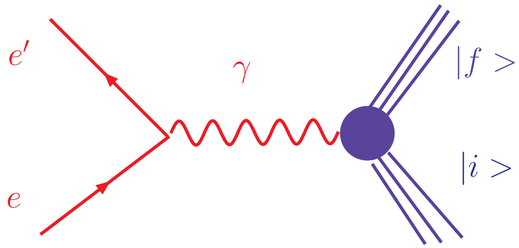
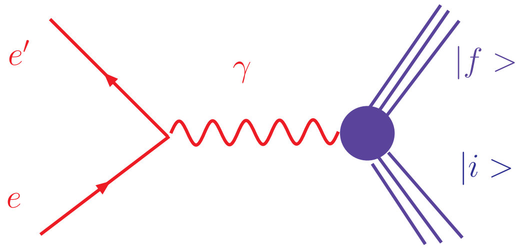
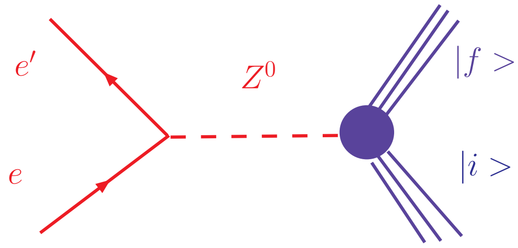
- only N detected semi-inclusive NC and CC
- only final lepton detected inclusive CC



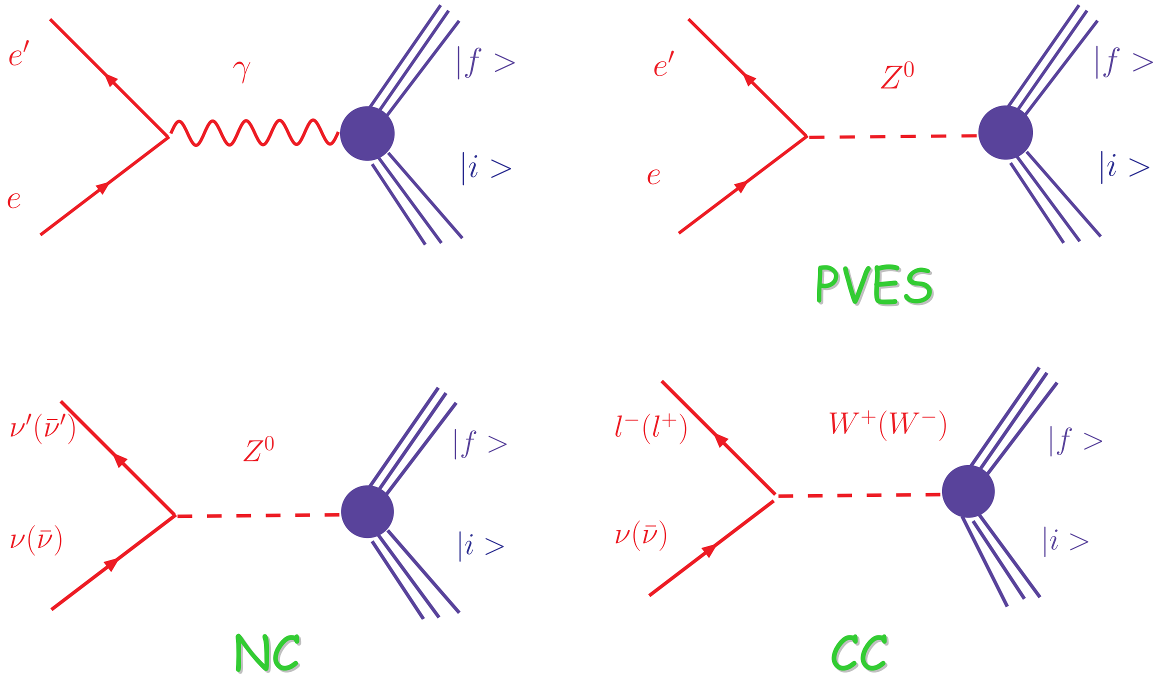
electron scattering





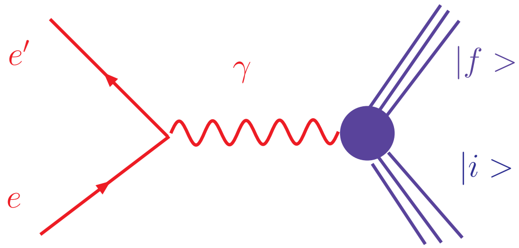
PVES

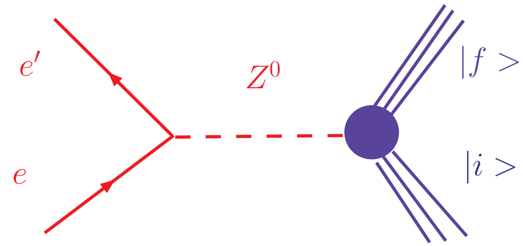
electron scattering



electron scattering

neutrino scattering





PVES

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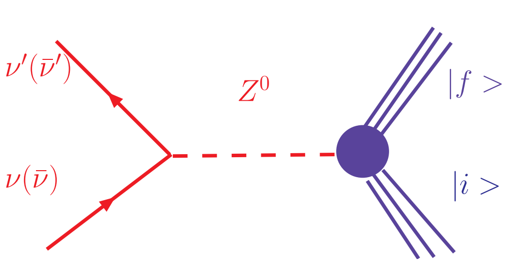

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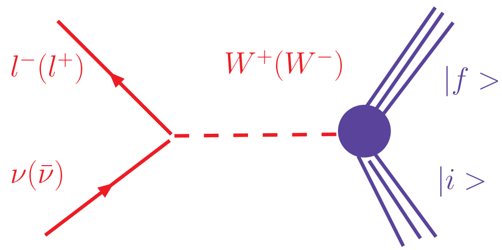
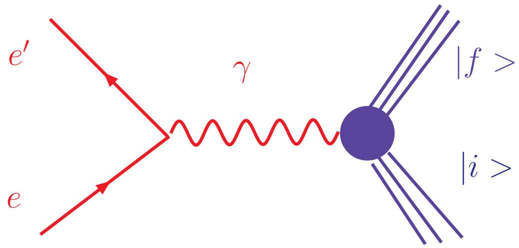
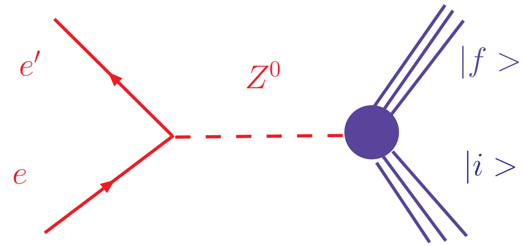


CC

neutrino scattering

$$\sigma = K L^{\mu\nu} \; W_{\mu\nu}$$





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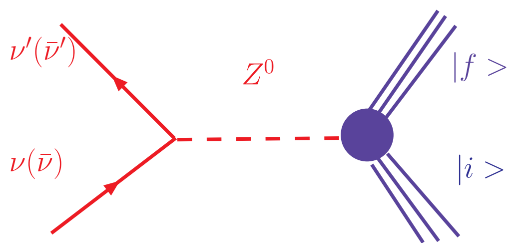
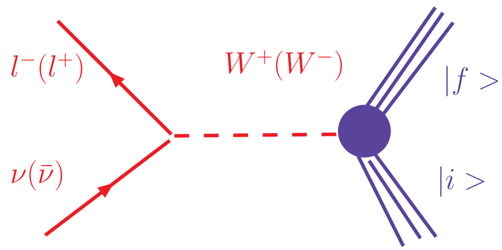


NC



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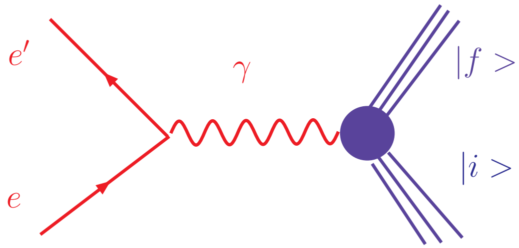
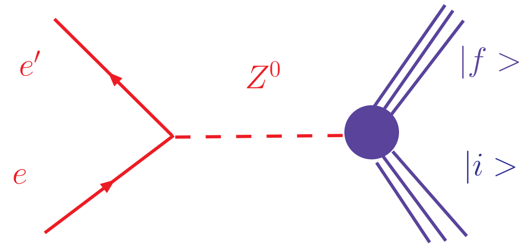
kin factor





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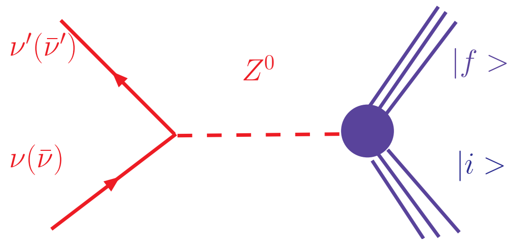

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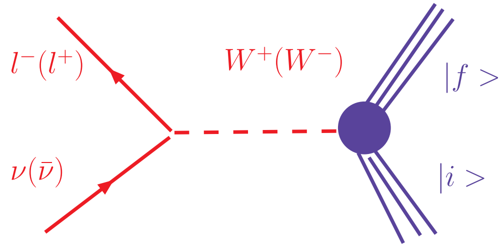

CC

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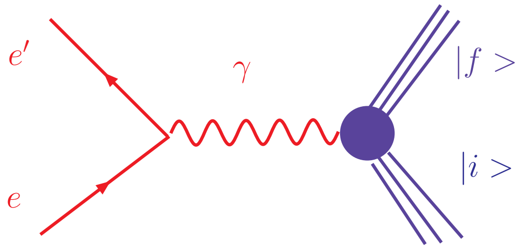
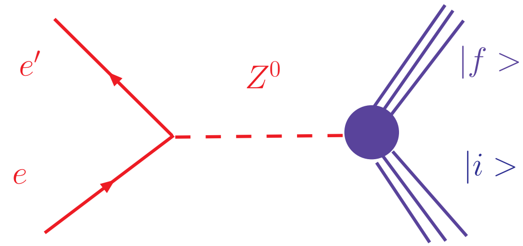
$$\sigma = K L^{\mu\nu} W_{\mu\nu}$$

lepton tensor contains lepton kinematics





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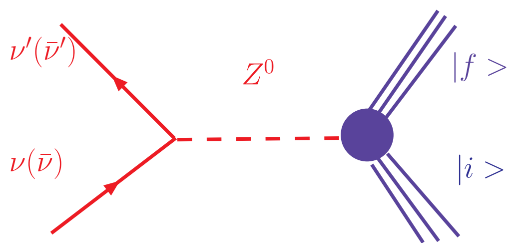

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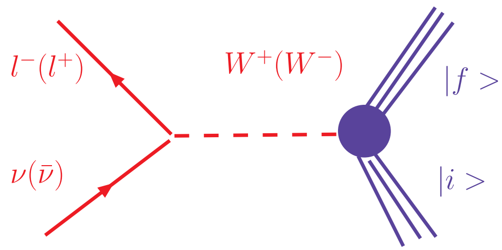

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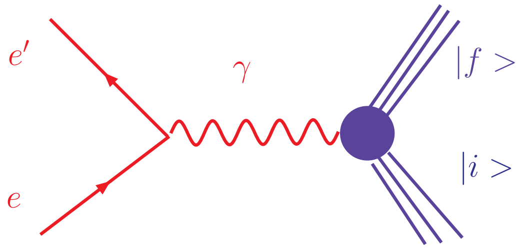
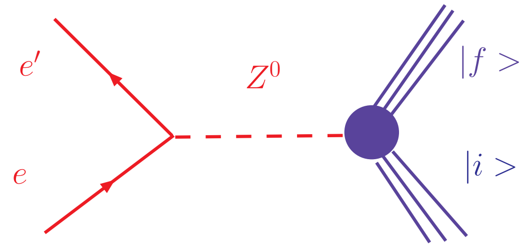
hadron tensor

$$W^{\mu\nu} = \overline{\sum_{\mathrm{i,f}}} J^{\mu}(\boldsymbol{q})\, J^{\nu *}(\boldsymbol{q})\, \delta(E_{\mathrm{i}} + \omega - E_{\mathrm{f}})$$

$$J^{\mu}(\boldsymbol{q}) = \int \mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}} \langle f \mid \hat{J}^{\mu}(\boldsymbol{r}) \mid i \rangle \mathrm{d}\boldsymbol{r}$$





PVES

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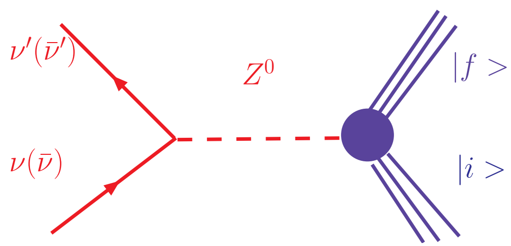

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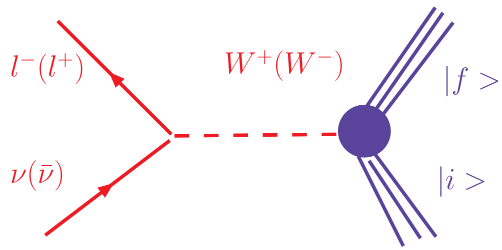

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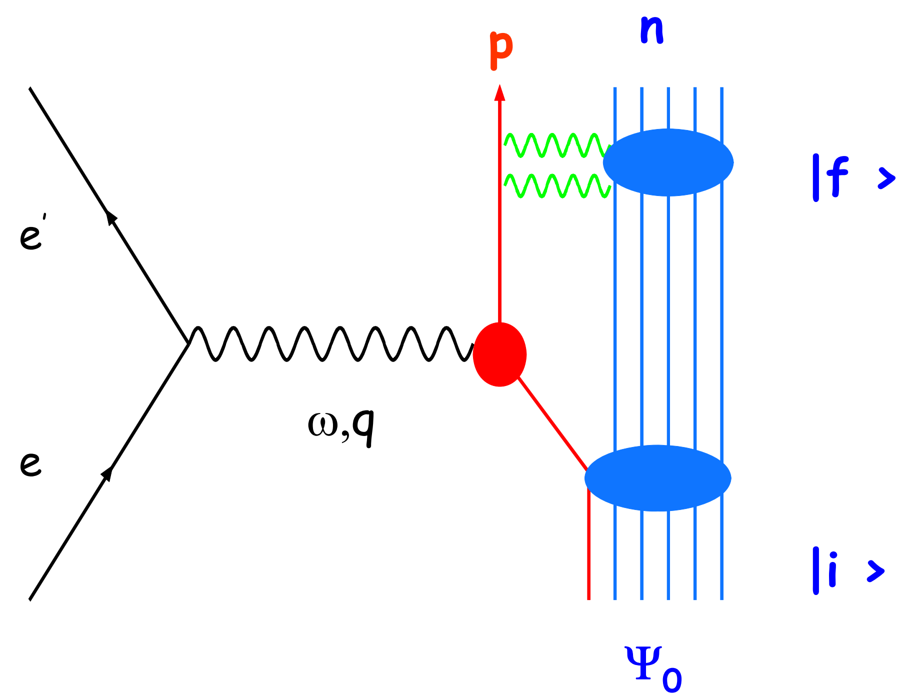
Direct knockout DWIA (e,e′p)

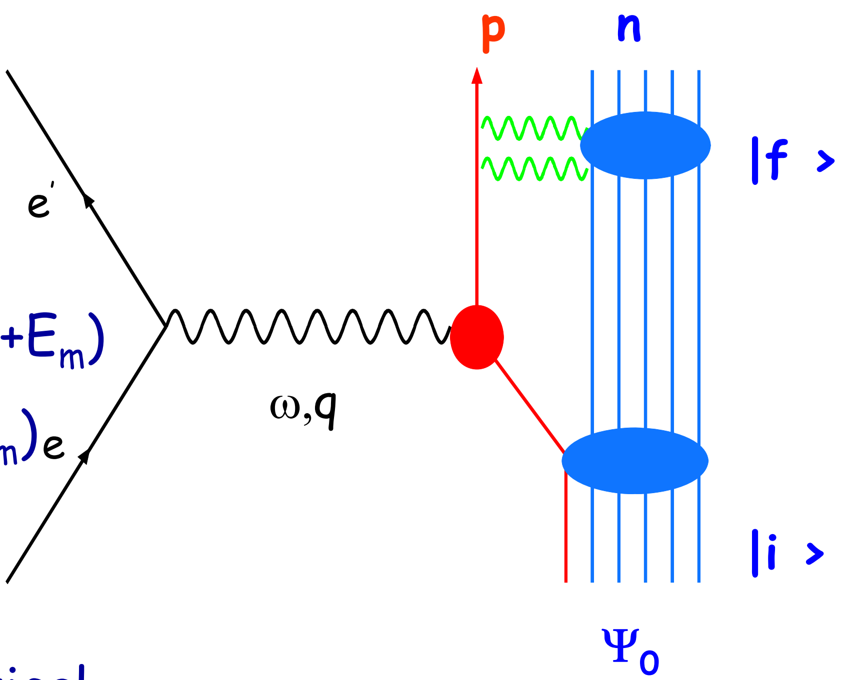
- exclusive reaction: n
- DKO mechanism: the probe interacts through a one-body current with one nucleon which is then emitted the remaining nucleons are spectators



$$\langle f \mid J^\mu(\boldsymbol{q}) \mid i \rangle \Rightarrow \lambda_n^{1/2} \langle \chi_{\boldsymbol{p}}^{(-)} \mid j^\mu(\boldsymbol{q}) \mid \phi_n \rangle$$

Direct knockout DWIA (e,e′p)

$$\lambda_n^{1/2}\ \langle \chi_{\boldsymbol{p}}^{(-)} \mid j^{\mu}(\boldsymbol{q}) \mid \phi_n \rangle$$

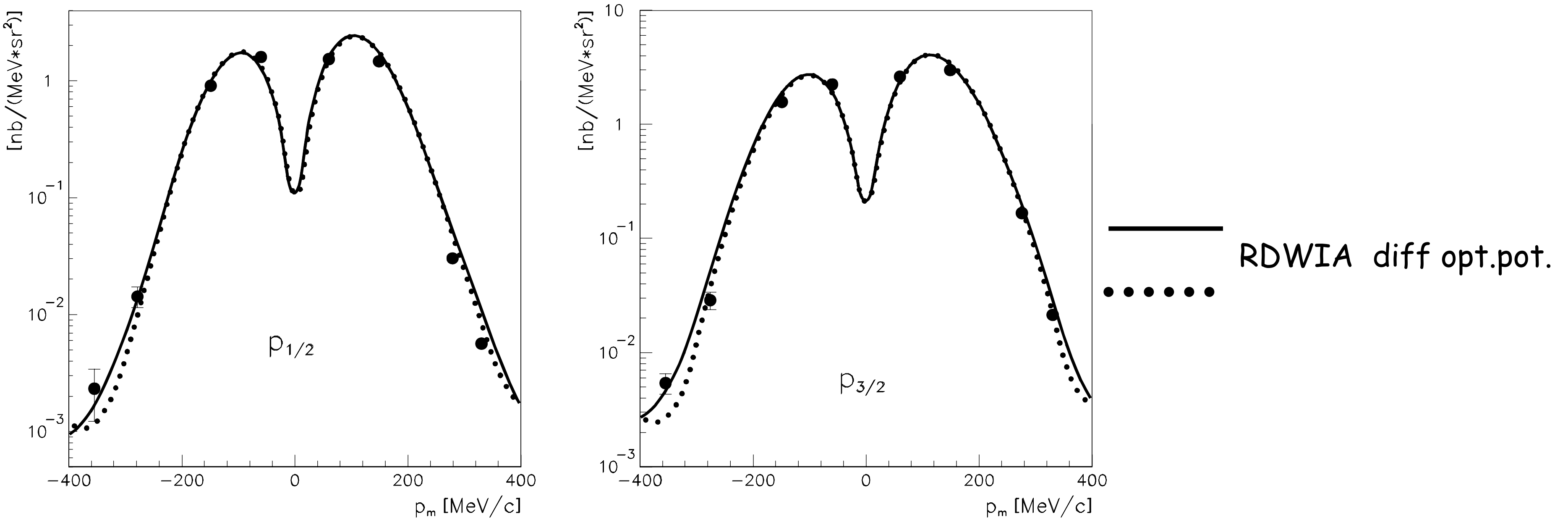


- j^{μ} one-body nuclear current
- $\chi^{(-)}$ = < n|f > s.p. scattering w.f. $H^{+}(\omega+E_m)$
- ϕ_n = <n|Ψ_0> one-nucleon overlap $H(-E_m)$
- λ_n spectroscopic factor
- $\chi^{(-)}$ and ϕ consistently derived as eigenfunctions of a Feshbach-type optical model Hamiltonian
- phenomenological ingredients used in the calculations for $\chi^{(-)}$ and ϕ

RDWIA: (e,e′p) comparison to data

$^{16}O(e,e'p)$

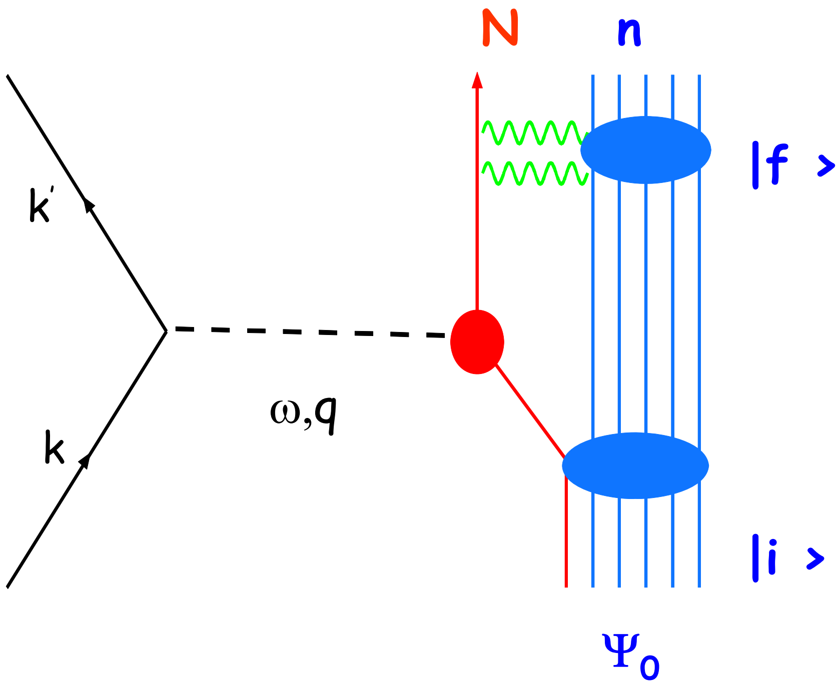
JLab (ω,q) const kin e=2445 MeV ω =439 MeV T_p= 435 MeV



A. Meucci, C. Giusti, F.D. Pacati, PRC 64 (2001) 014604

RDWIA: NC and CC ν -nucleus scattering

$$\lambda_n^{1/2} \, \langle \chi_{\boldsymbol{p}}^{(-)} \mid j^\mu(\boldsymbol{q}) \mid \phi_n \rangle$$



- transition amplitudes calculated with the same model used for (e,e′p)
- the same phenomenological ingredients are used for $\chi^{(-)}$ and ϕ
- j^μ one-body nuclear weak current

ν -nucleus scattering

- only the outgoing nucleon is detected: semi inclusive process
- cross section integrated over the energy and angle of the outgoing lepton
- integration over the angle of the outgoing nucleon
- final nuclear state is not determined : sum over n

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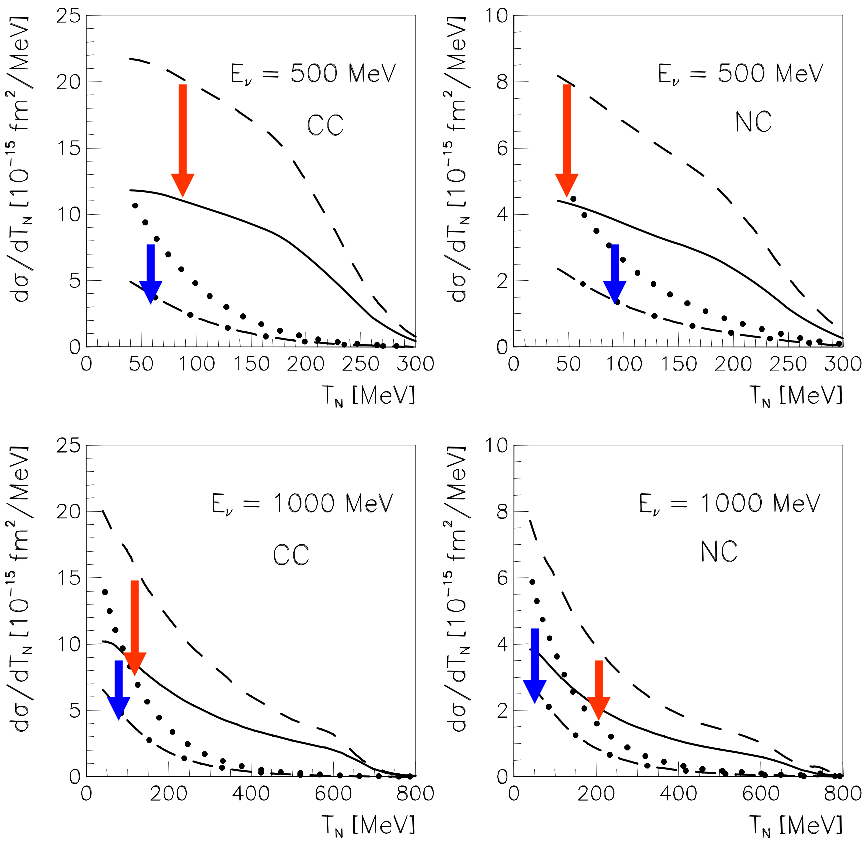
$$W^{\mu\nu}(\omega, q) = \sum_n \langle n; \chi^{(-)}_{\boldsymbol{p}_{\mathrm{N}}} \mid J^{\mu}(\boldsymbol{q}) \mid \Psi_0 \rangle \langle \Psi_0 \mid J^{\nu\dagger}(\boldsymbol{q}) \mid n; \chi^{(-)}_{\boldsymbol{p}_{\mathrm{N}}} \rangle \ \delta(E_0 + \omega - E_{\mathrm{f}})$$

calculations

- pure Shell Model description: ϕ_n one-hole states in the target with an unitary spectral strength
- $\sum_n$ over all occupied states in the SM: all the nucleons are included but correlations are neglected
- the cross section for the ν-nucleus scattering where one nucleon is detected is obtained from the sum of all the integrated one-nucleon knockout channels
- FSI are described by a complex optical potential with an imaginary absorptive part

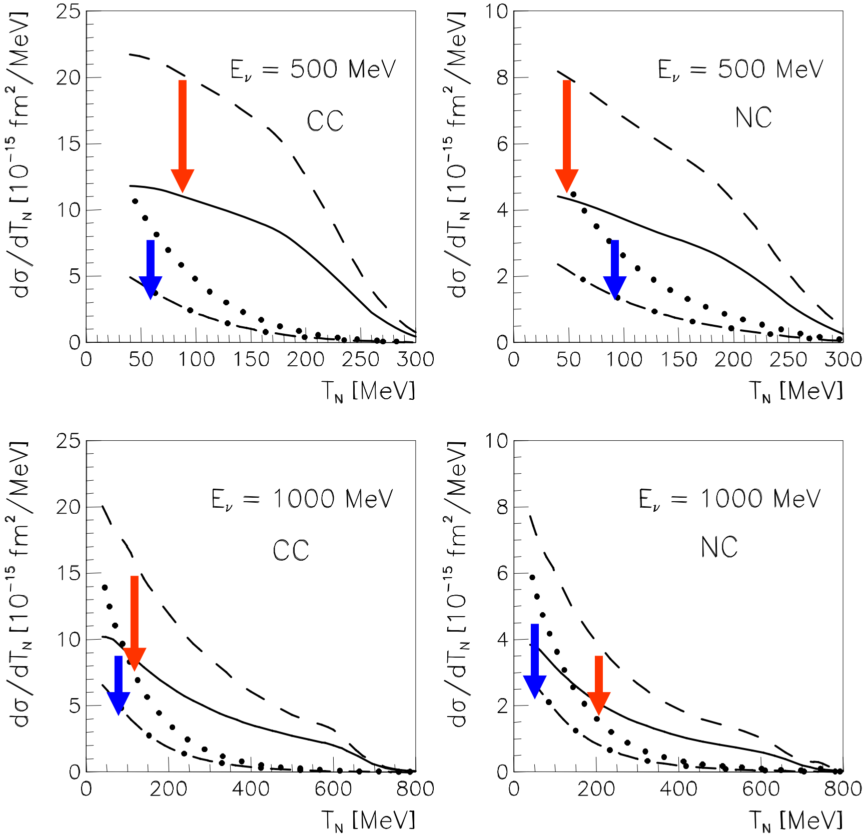
CC **NC**



$^{12}\mathrm{C}(\nu_\mu, \mu^- p)$ RPWIA / RDWIA $^{12}\mathrm{C}(\nu, \nu' p)$ FSI

$^{12}\mathrm{C}(\bar{\nu}_\mu, \mu^+ n)$ RPWIA / RDWIA $^{12}\mathrm{C}(\bar{\nu}, \bar{\nu}' p)$ FSI

CC NC



FSI

the imaginary part of the optical potential gives an absorption that reduces the calculated cross sections

$^{12}\mathrm{C}(\nu_\mu, \mu^- p)$ RPWIA RDWIA $^{12}\mathrm{C}(\nu, \nu' p)$ **FSI**

$^{12}\mathrm{C}(\bar{\nu}_\mu, \mu^+ n)$ RPWIA RDWIA $^{12}\mathrm{C}(\bar{\nu}, \bar{\nu}' p)$ **FSI**

FSI for the inclusive scattering : Green's Function Approach

(e,e') nonrelativistic

F. Capuzzi, C. Giusti, F.D. Pacati, Nucl. Phys. A 524 (1991) 281

F. Capuzzi, C. Giusti, F.D. Pacati, D.N. Kadrev Ann. Phys. 317 (2005) 492 (AS CORR)

(e,e') relativistic

A. Meucci, F. Capuzzi, C. Giusti, F.D. Pacati, PRC (2003) 67 054601

A. Meucci, C. Giusti, F.D. Pacati Nucl. Phys. A 756 (2005) 359 (PVES)

CC relativistic

A. Meucci, C. Giusti, F.D. Pacati Nucl. Phys. A739 (2004) 277

FSI for the inclusive scattering : Green's Function Approach

- the components of the inclusive response are expressed in terms of the Green's operators
- under suitable approximations can be written in terms of the s.p. optical model Green's function
- the explicit calculation of the s.p. Green's function can be avoided by its spectral representation which is based on a biorthogonal expansion in terms of a non Herm optical potential V and V^+
- matrix elements similar to RDWIA
- scattering states eigenfunctions of V and V^+ (absorption and gain of flux): the imaginary part redistributes the flux and the total flux is conserved
- consistent treatment of FSI in the exclusive and in the inclusive scattering

FSI for the inclusive scattering : Green's Function Approach

$$W^{\mu\mu}(\omega,q)=\sum_n\left[\mathbf{Re}T_n^{\mu\mu}(E_{\mathbf{f}}-\varepsilon_n,E_{\mathbf{f}}-\varepsilon_n)-\frac{1}{\pi}\mathcal{P}\int_M^{\infty}\mathbf{d}\mathcal{E}\,\frac{1}{E_{\mathbf{f}}-\varepsilon_n-\mathcal{E}}\mathbf{Im}T_n^{\mu\mu}(\mathcal{E},E_{\mathbf{f}}-\varepsilon_n)\right]$$

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$$T_n^{\mu\mu}(\mathcal{E},E)=\lambda_n\langle\varphi_n\mid j^{\mu\dagger}(\boldsymbol{q})\sqrt{1-\mathcal{V}'(E)}\mid\tilde{\chi}_{\mathcal{E}}^{(-)}(E)\rangle\langle\chi_{\mathcal{E}}^{(-)}(E)\mid\sqrt{1-\mathcal{V}'(E)}j^{\mu}(\boldsymbol{q})\mid\varphi_n\rangle$$

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interference between different channels

FSI for the inclusive scattering : Green's Function Approach

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eigenfunctions of V
and V⁺

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FSI for the inclusive scattering : Green's Function Approach

$$W^{\mu\mu}(\omega,q) = \sum_n \left[\mathbf{Re} T_n^{\mu\mu}(E_{\mathbf{f}} - \varepsilon_n, E_{\mathbf{f}} - \varepsilon_n) - \frac{1}{\pi}\mathcal{P}\int_M^{\infty} \mathbf{d}\mathcal{E} \frac{1}{E_{\mathbf{f}} - \varepsilon_n - \mathcal{E}} \mathbf{Im} T_n^{\mu\mu}(\mathcal{E}, E_{\mathbf{f}} - \varepsilon_n) \right]$$

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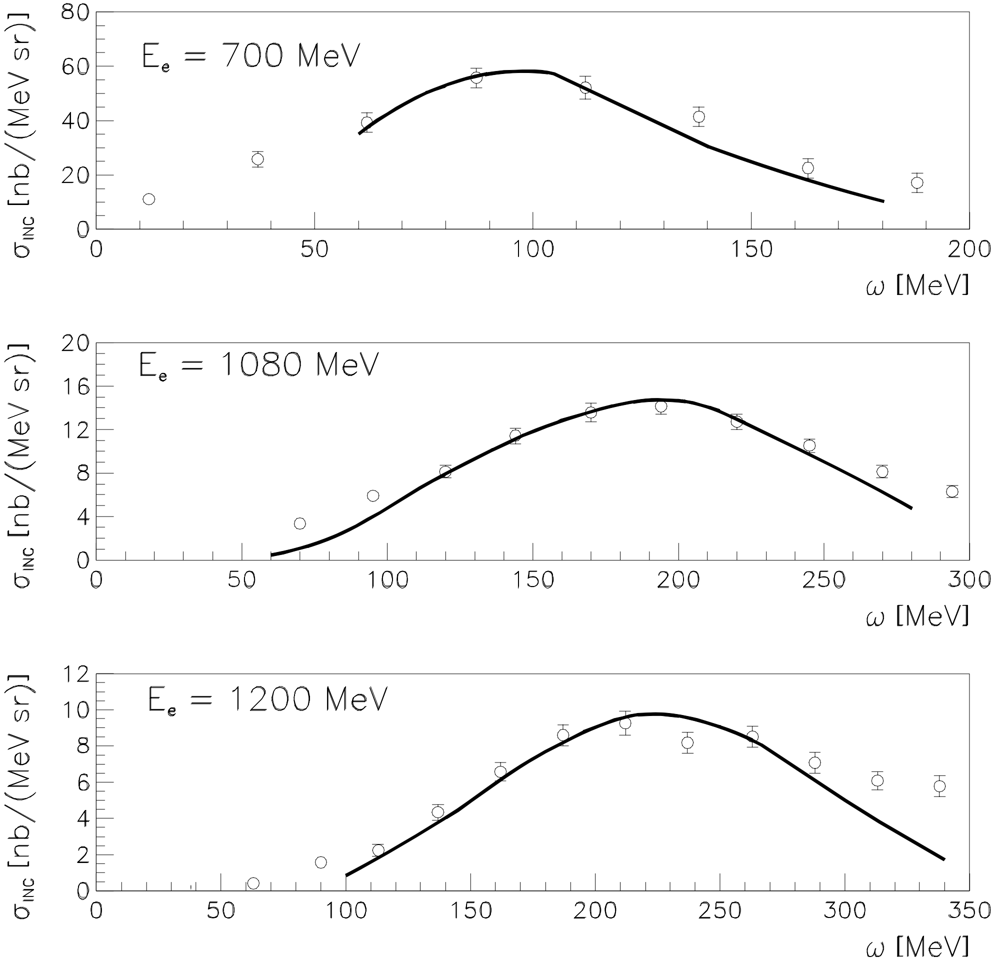
Flux redistributed and conserved

The imaginary part of the optical potential is responsible for the redistribution of the flux among the different channels

$^{16}O(e,e')$

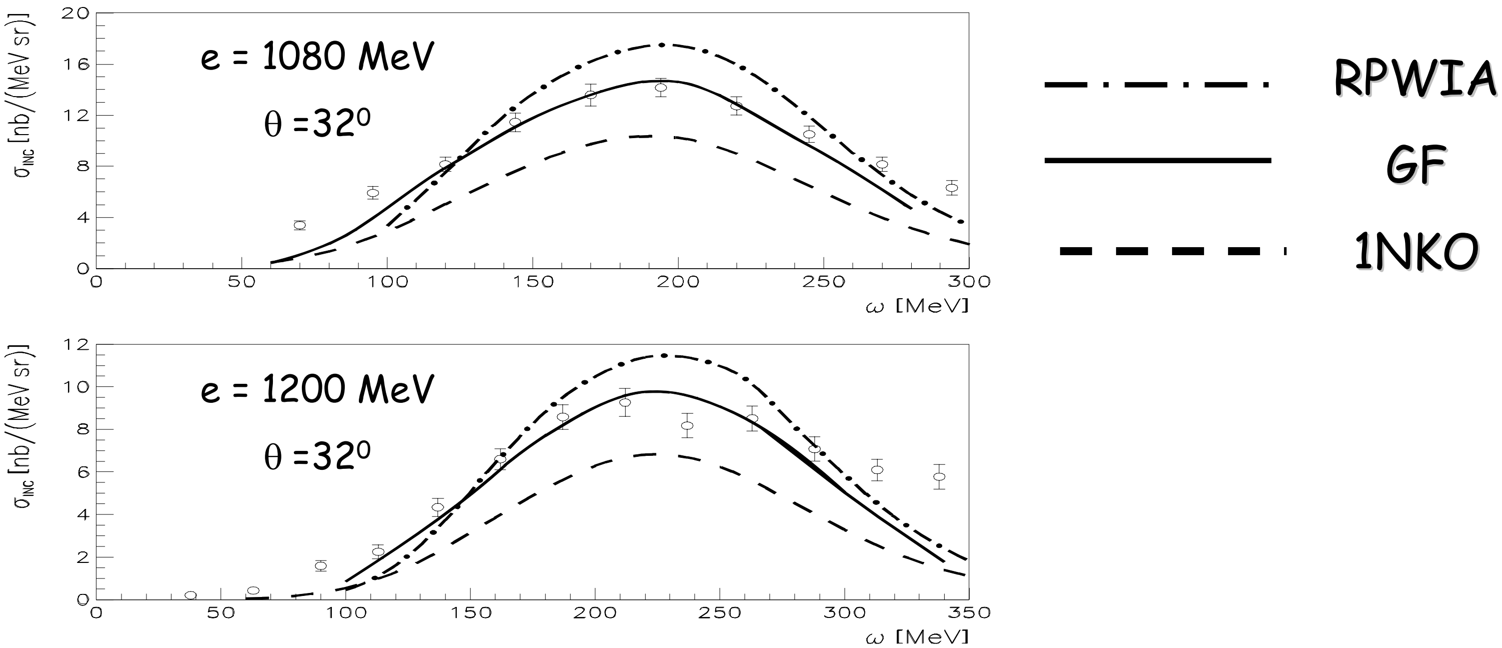
Green's function approach GF



data from Frascati NPA 602 405 (1996)

A. Meucci, F. Capuzzi, C. Giusti, F.D. Pacati, PRC 67 (2003) 054601

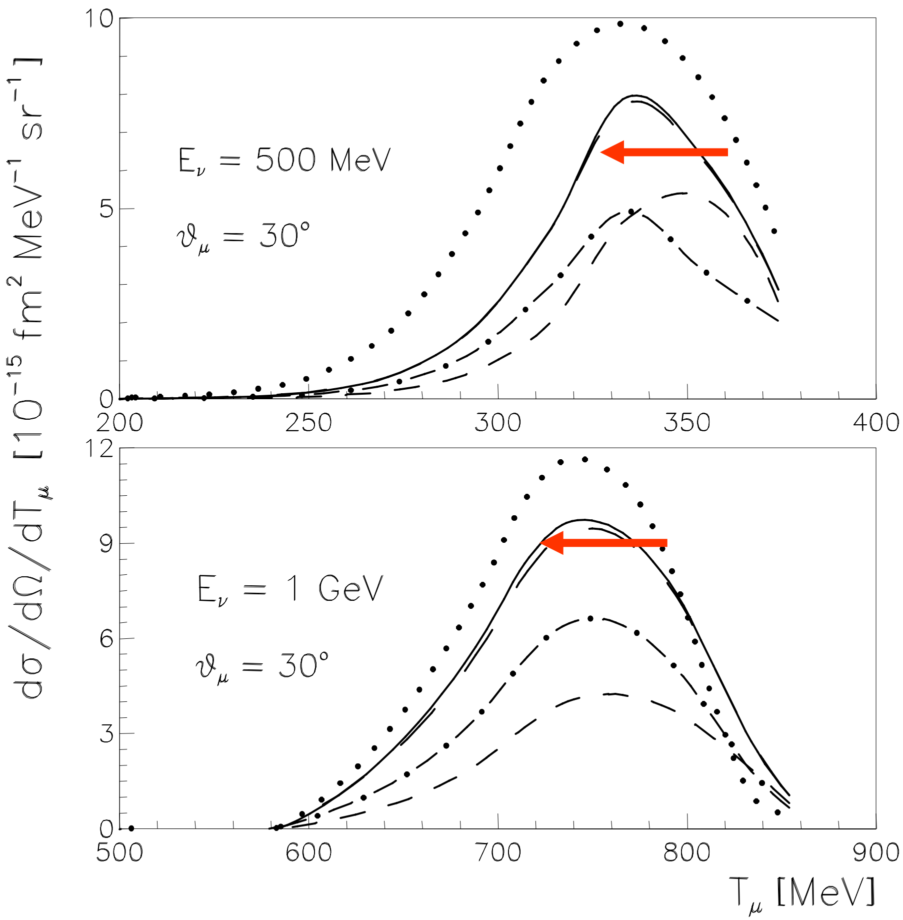
$^{16}O(e,e')$



FSI

data from Frascati NPA 602 405 (1996)

$$^{16}O(\nu_\mu, \mu^-)$$



FSI

RPWIA

GF

rROP

1NKO

$^{16}O(\bar{\nu}_\mu, \mu^+)$ GF

COMPARISON OF RELATIVISTIC MODELS

PAVIA ⟷ MADRID-SEVILLA

COMPARISON OF RELATIVISTIC MODELS

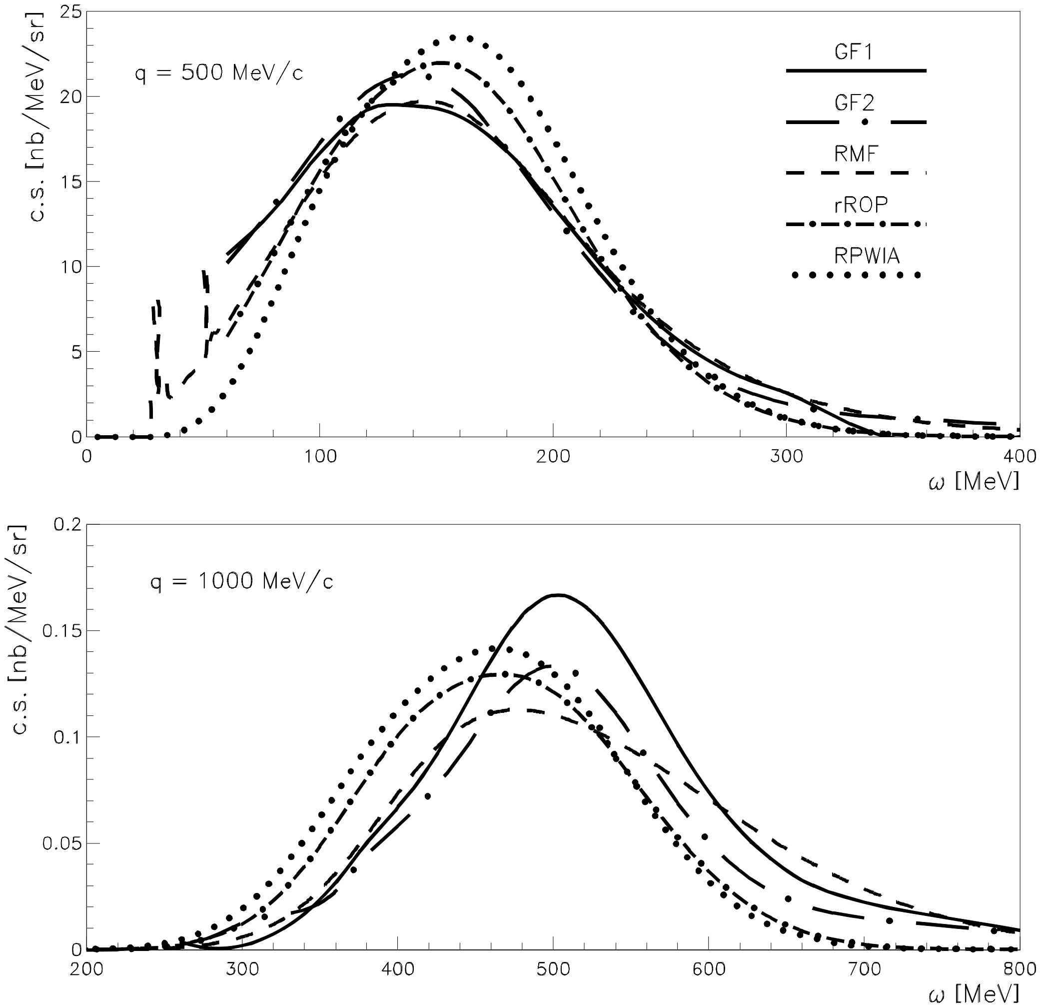
PAVIA ⟺ MADRID-SEVILLA

- consistency of numerical results
- comparison of different descriptions of FSI

$^{12}C(e,e')$

relativistic models

$e = 1$ GeV



FSI

RPWIA

rROP

GF1

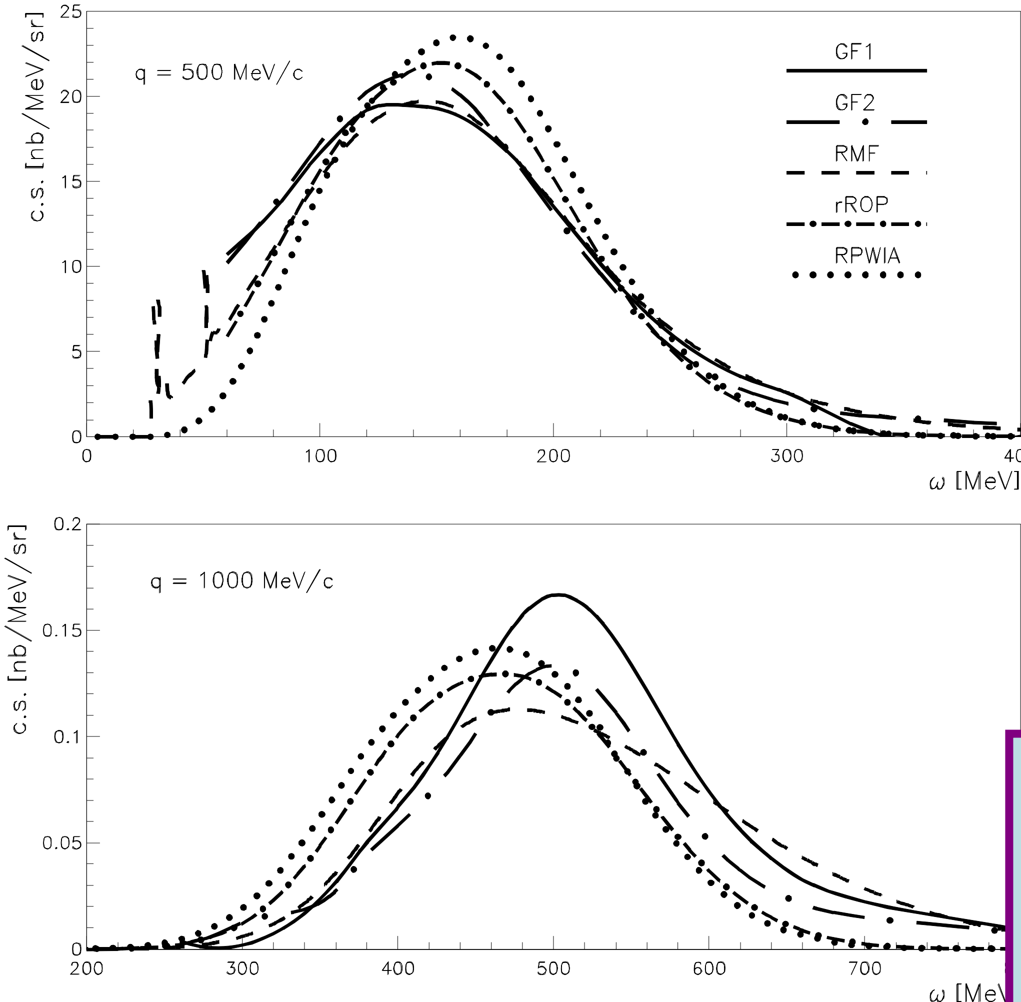
GF2

RMF

$^{12}C(e,e')$

relativistic models

e = 1 GeV



FSI

RPWIA

rROP

GF1

GF2

RMF

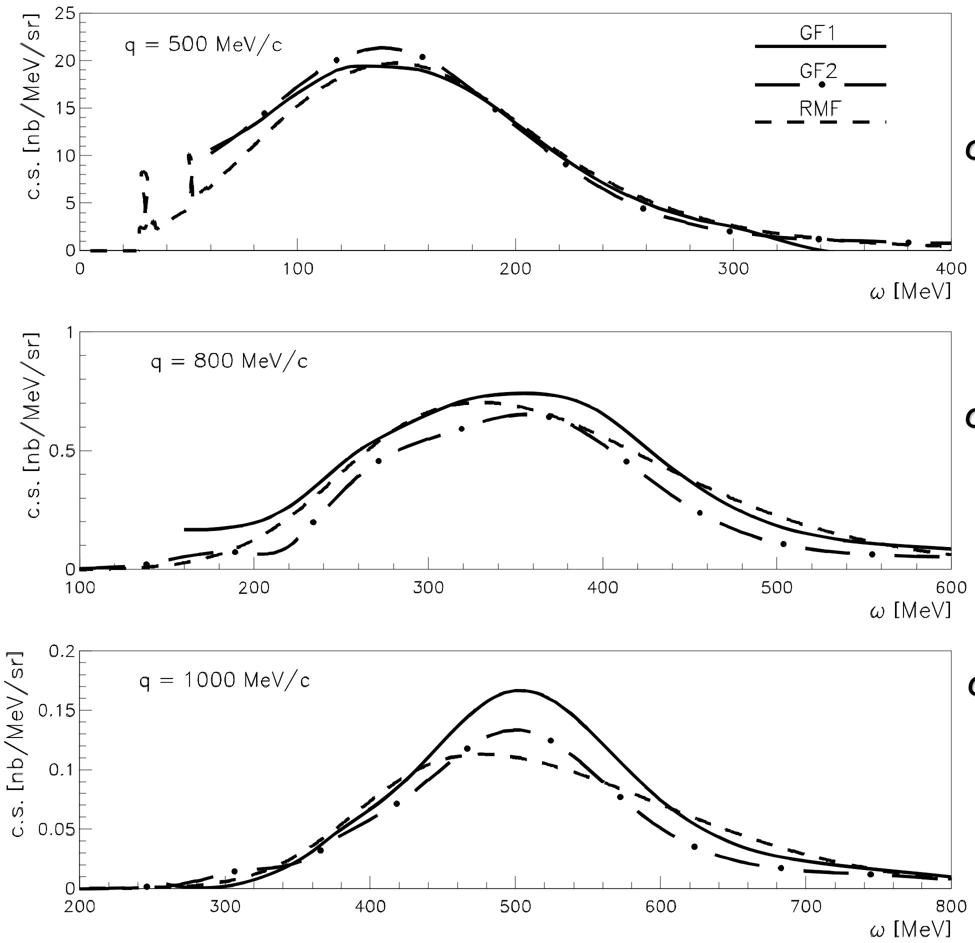
Relativistic Mean Field: same real energy-independent potential of bound states

Orthogonalization, fulfills dispersion relation and maintains the continuity equation

$^{12}C(e,e')$

relativistic models

e = 1 GeV



q=500 MeV/c

q=800 MeV/c

q=1000 MeV/c

FSI

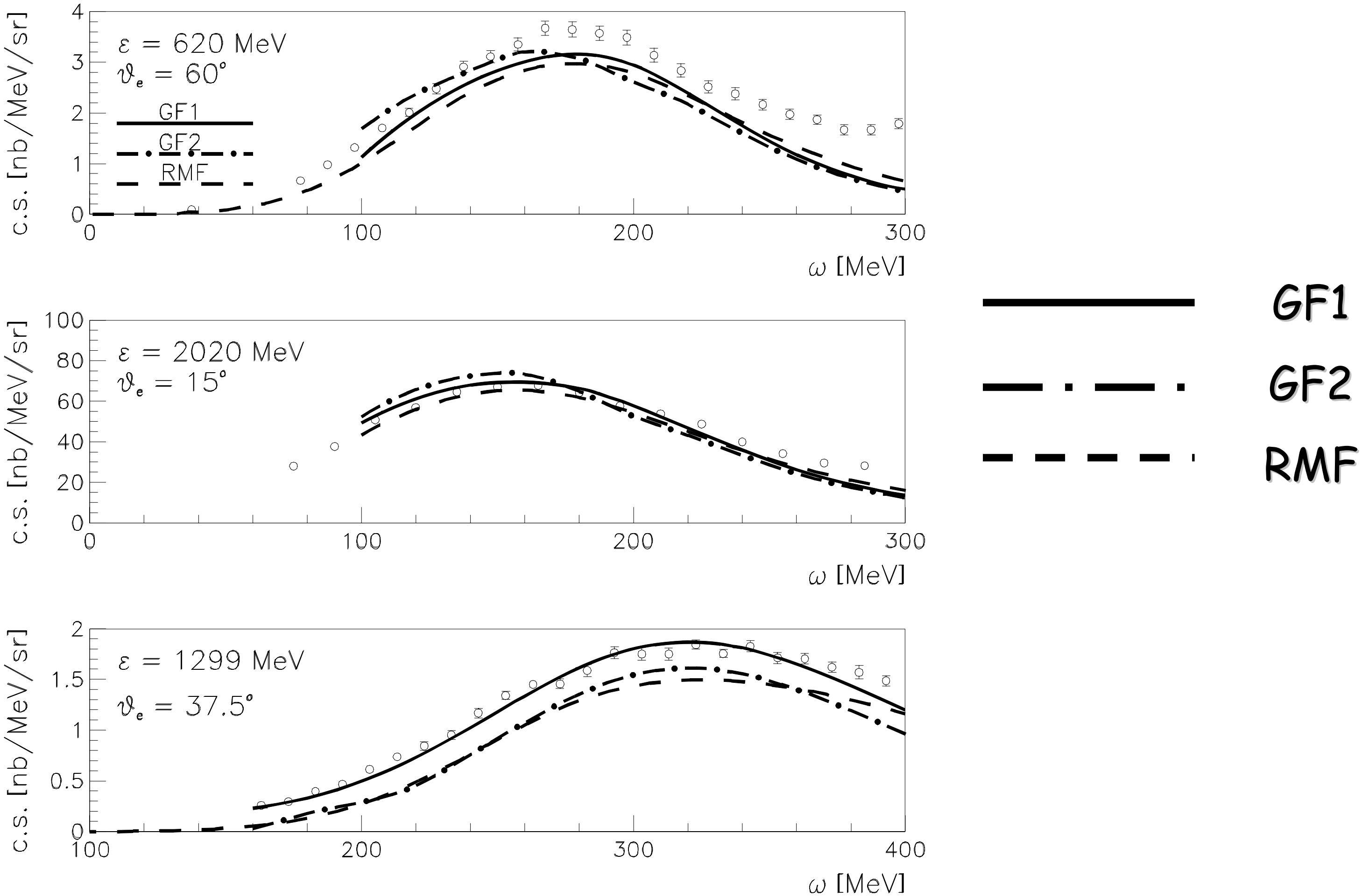
GF1

GF2

RMF

$^{12}C(e,e')$

relativistic models



SCALING PROPERTIES

GF ⟺ RMF

SCALING FUNCTION

Scaling properties of the electron scattering data

At sufficiently high q the scaling function $f = \dfrac{d^2\sigma(q,\omega)/d\Omega dk'}{S^{s.n.}(q,\omega)}$

depends only upon one kinematical variable (scaling variable)

(SCALING OF I KIND)

is the same for all nuclei

(SCALING OF II KIND)

I+II SUPERSCALING

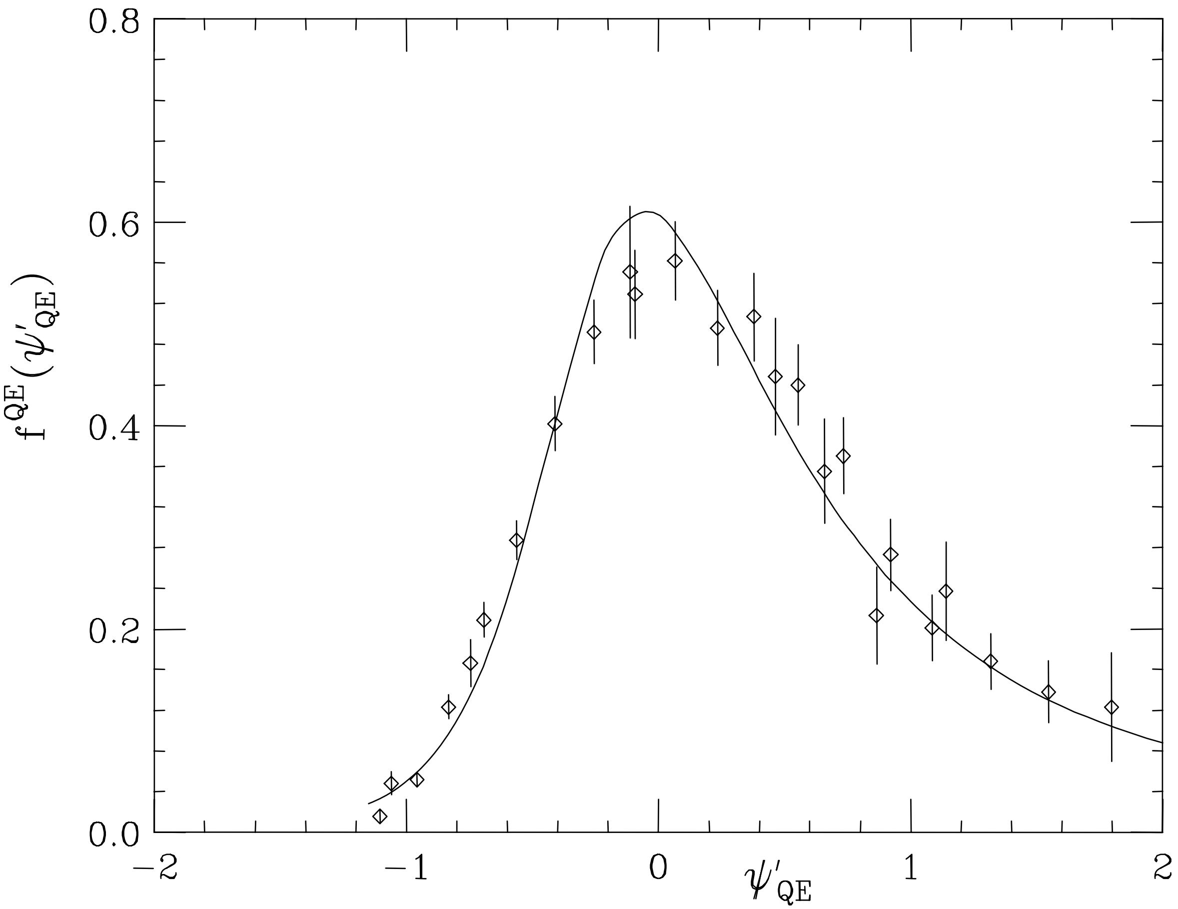
Scaling variable (QE) $\psi_{\rm QE} = \pm\sqrt{1/(2T_F)\left(q\sqrt{1+1/\tau}-\omega-1\right)}$

+ (-) for ω lower (higher) than the QEP, where ψ=0

- Reasonable scaling of I kind at the left of QEP
- Excellent scaling of II kind in the same region
- Breaking of scaling particularly of I kind at the right of QEP (effects beyond IA)
- The longitudinal contribution superscales

f^{QE} extracted from the data

Experimental QE superscaling function



M.B. Barbaro, J.E. Amaro, J.A. Caballero, T.W. Donnelly, A. Molinari, and I. Sick, Nucl. Phys Proc. Suppl 155 (2006) 257

SCALING FUNCTION

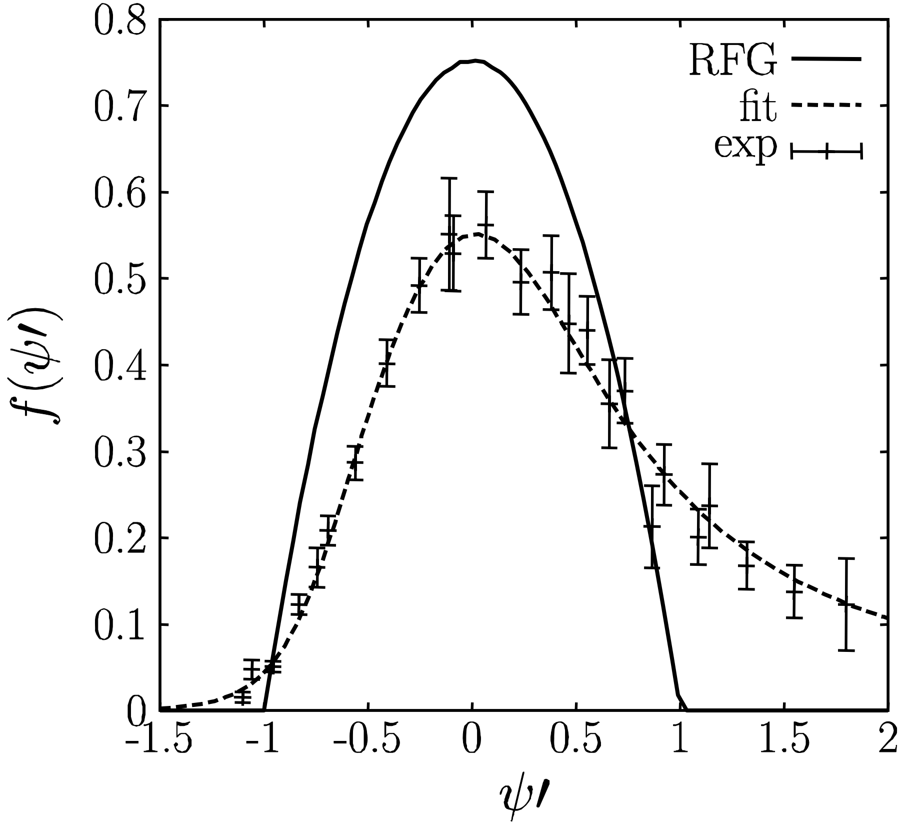
The properties of the experimental scaling function should be accounted for by microscopic calculations

The asymmetric shape of f^{QE} should be explained

The scaling properties of different models can be verified

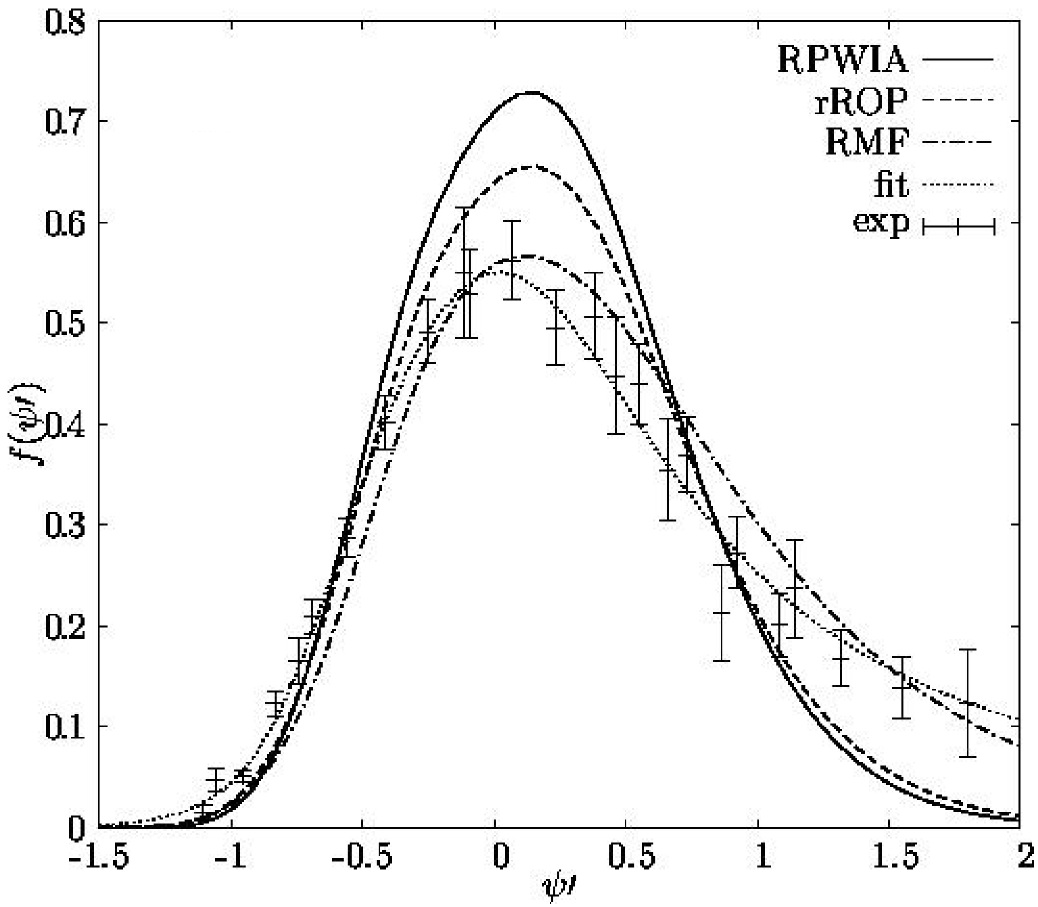
The associated scaling functions compared with the experimental f^{QE}

QE SUPERSCALING FUNCTION: RFG



Relativistic Fermi Gas

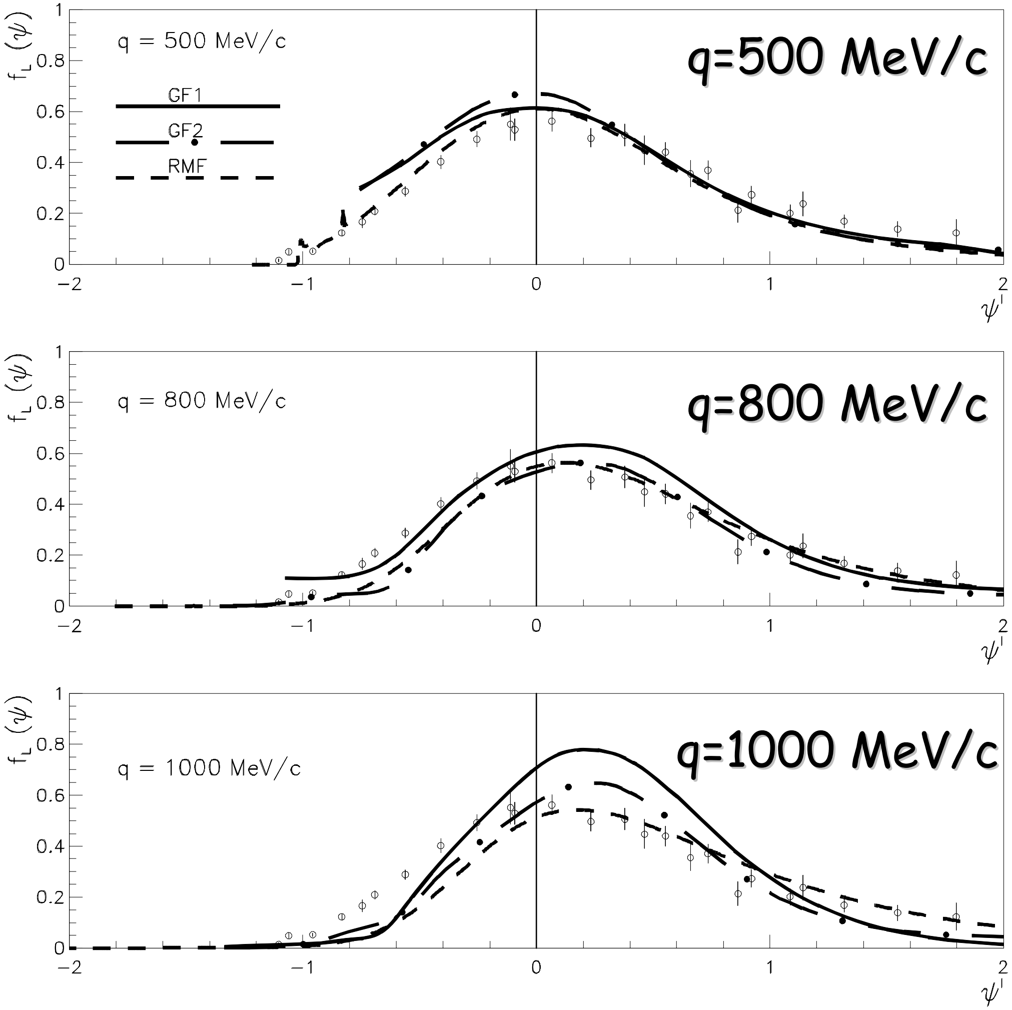
QE SUPERSCALING FUNCTION: RPWIA, rROP, RMF



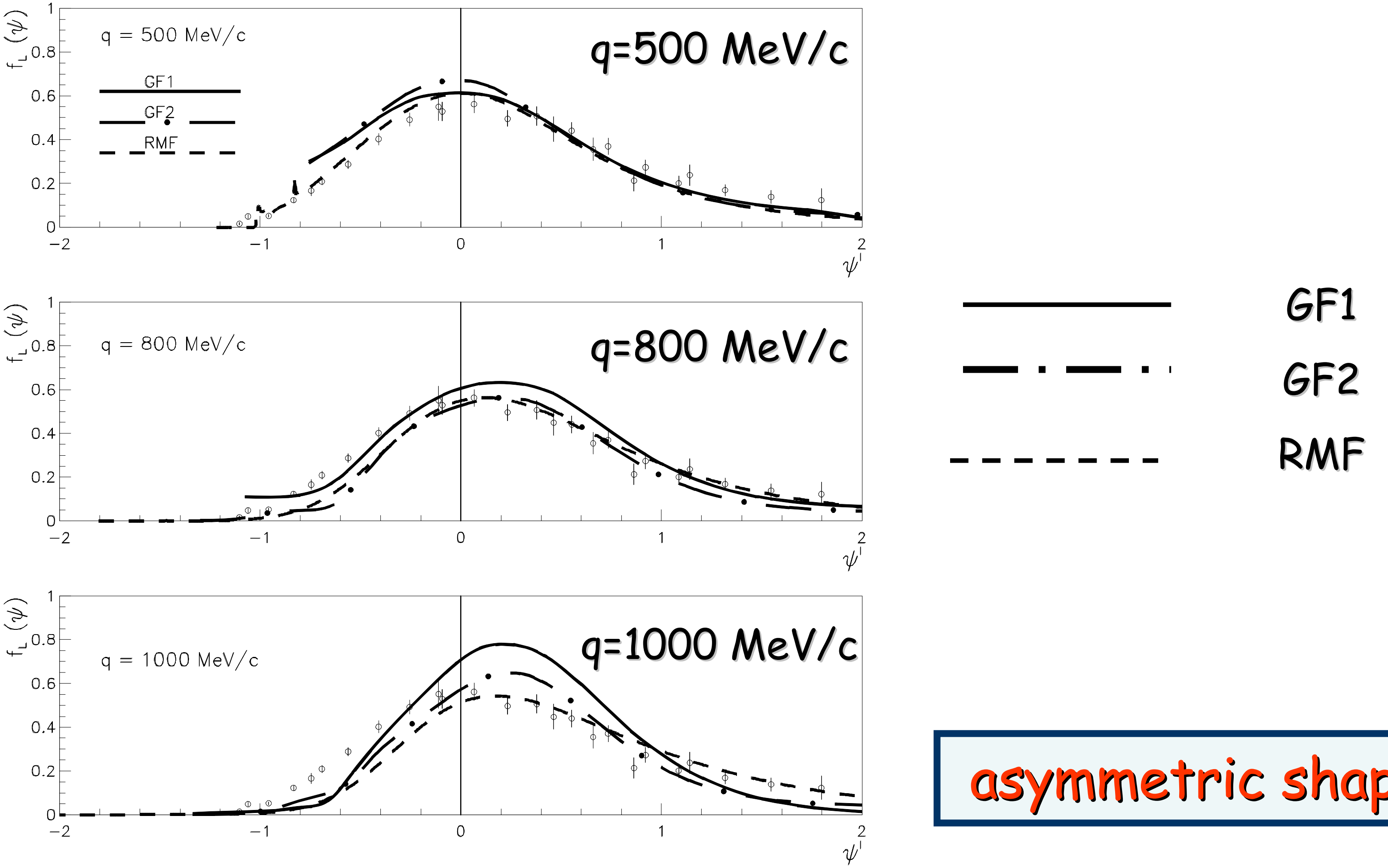
only RMF gives an asymmetric shape

J.A. Caballero J.E. Amaro, M.B. Barbaro, T.W. Donnelly, C. Maieron, and J.M. Udias
PRL 95 (2005) 252502

QE SCALING FUNCTION: GF, RMF

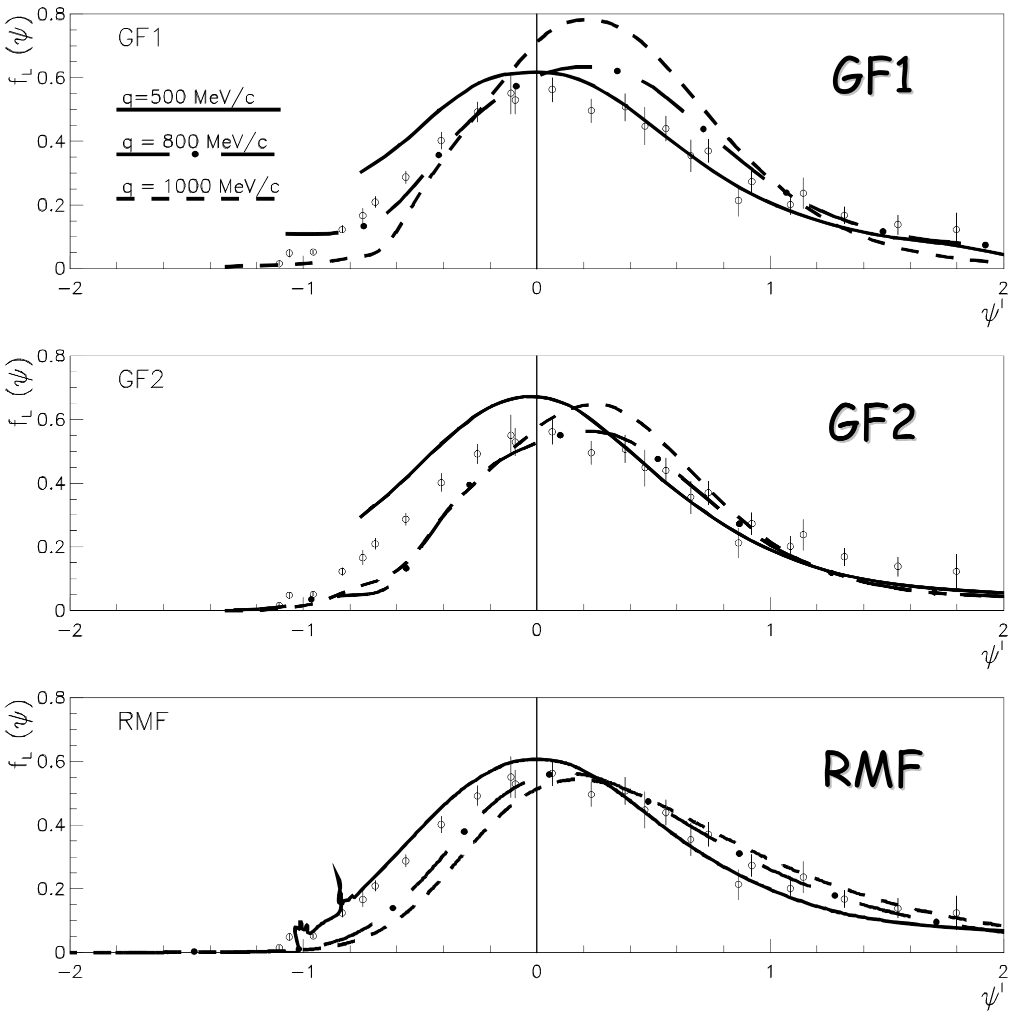


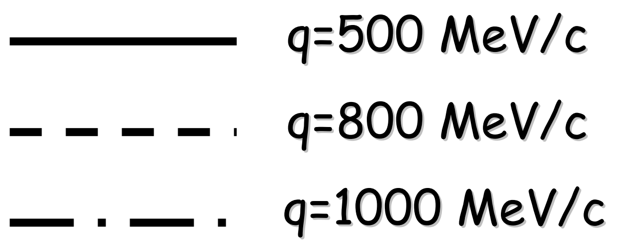
QE SCALING FUNCTION: GF, RMF



asymmetric shape

Analysis first-kind scaling : GF RMF





CONCLUSIONS

- relativistic models developed for QE electron-nucleus scattering and tested in comparison with electron-scattering data have been extended to neutrino-nucleus scattering
- consistent models for exclusive, semi-inclusive, inclusive processes with CC and NC
- numerical predictions can be given for different nuclei and kinematics
- comparison of the results of different models important to reduce theoretical uncertainties on nuclear effects
- comparison Pavia Madrid-Sevilla: consistency of numerical results (RPWIA, rROP)
- comparison Pavia Madrid-Sevilla: GF and RMF for inclusive (e,e') cross sections, scaling properties, similar results for $q \simeq$ 500-700 MeV/c, visible discrepancies for q = 1000 MeV/c

SCALING FUNCTION: RPWIA rROP GF RMF