Duality and neutrinos

Olga Lalakulich

+ E.Paschos (Dortmund Uni), W.Melnitchouk (JLab)

+ N. Jachowicz, Ch.Praet, J.Ryckebusch (Gent Uni)

+ U. Mosel, T.Leitner, O. Buss (Giessen Uni)

thanks to German Academic Exchange Service (DAAD) for financial support

Outline

Duality is a general feature of strongly interacting landscape

Quark–hadron duality in different phenomena:

- in $e^- e^+$ scattering establish relations between $R = \frac{\sigma(e^- e^+ \to h\bar{h})}{\sigma(e^- e^+ \to \mu^- \mu^+)}$ and quark counting
- in πp scattering helps to establish the Froissart bound on the x-section
- in hadronic decays of heavy mesons (extraction of V_{cb} and V_{ub} relies crucially on duality)
- in semileptonic decays of heavy mesons
- in electron-nucleon and neutrino-nucleon scattering
  an average of the structure functions in the resonance region should be close to predictions from parton distribution functions determined in DIS
- in πN scattering establish relations between resonances and Regge pole exchange, background and Pomeron exchange

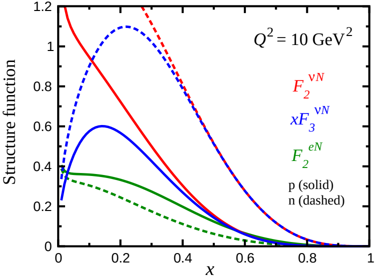
see W. Melnitchouk, R. Ent, C. Keppel, Phys Rept 406 for the review

What to compare

DIS part:

scaling curve from parton distributions

$$\frac{1}{x}F_2^{ep} = \frac{4}{9}(u+\bar{u}) + \frac{1}{9}(d+\bar{d}+s+\bar{s})...$$
$$\frac{1}{x}F_2^{en} = \frac{4}{9}(d+\bar{d}) + \frac{1}{9}(u+\bar{u}+s+\bar{s})...$$
$$xF_3^{\nu p} = 2x(d-\bar{u}+s),$$
$$xF_3^{\nu n} = 2x(u-\bar{d}+s)$$

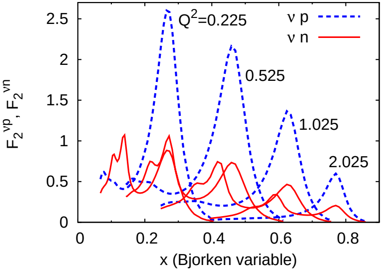


different parametrizations (GRV, MRST, CTEQ) give nearly the same results

RES part:

from the xsec or directly via form factors

$P_{33}(1232)$
$P_{11}(1440)$ $D_{13}(1520)$ $S_{11}(1535)$
$P_{33}(1600)$
$S_{11}(1650)$ $D_{15}(1675)$ $F_{15}(1680)$
$D_{33}(1700)$
$D_{13}(1700)$ $P_{11}(1710)$ $P_{13}(1720)$ $\ldots$



Giessen BUU

Scaling variables for duality

The most general scaling variable includes target mass correstion and finite quark mass

$$\xi_B = \frac{Q^2 + \sqrt{Q^4 + 4m_q^2 Q^2}}{2m_N\nu(1+\sqrt{1+Q^2/\nu^2})}$$

Barbieri, Ellis, Gaillard, Ross

Nachtmann scaling variable ξ

$$\xi = \lim_{m_q\to 0} \xi_B = \frac{2Q^2/2m_N\nu}{(1+\sqrt{1+Q^2/\nu^2})} = \frac{2x}{(1+\sqrt{1+4m_N^2x^2/Q^2})}$$

Expanding ξ in powers of $1/Q^2$ at high Q^2 gives the variable $\frac{2m_N\nu+m_N^2}{Q^2}$, found empirically in 1970 by Bloom and Gilman and used in their pioneering work on duality

$$\frac{1}{\xi} \approx \frac{1}{x}\left(1+\frac{m_N^2x^2}{Q^2}\right) = \frac{2m_N\nu + m_N^2}{Q^2}$$

At very high Q^2, neglecting m_N^2/Q^2, we get $\xi \approx \frac{2x}{1+1} = x$ - Bjorken variable (see Melnitchouk, Ent, Keppel, Phys.Rep. 406)

$F_2^{\nu p,\nu n}$: In neutrino–nucleon scattering duality does NOT hold for proton and neutron targets separately

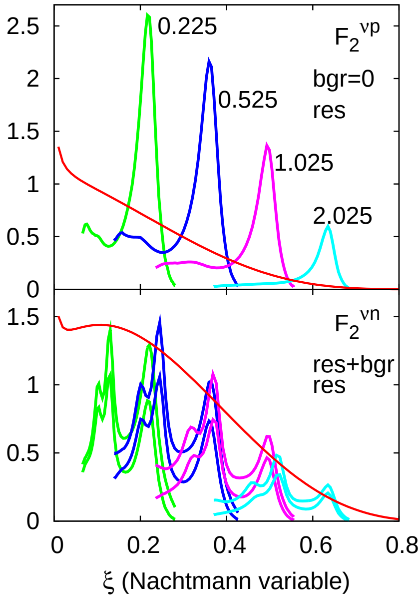
Low-lying resonances: $F_2^{\nu n(res)} < F_2^{\nu p(res)}$
neutron<proton

DIS: $F_2^{\nu n(DIS)} > F_2^{\nu p(DIS)}$
neutron>proton

$F_2^{\nu p(res-3/2)} = 3F_2^{\nu n(res-3/2)}$
$F_2^{\nu p(res-1/2)} \equiv 0$

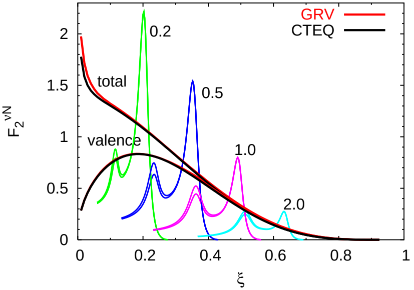
$F_2^{\nu n(res)}$: finite contributions from isospin-3/2 and -1/2 resonances

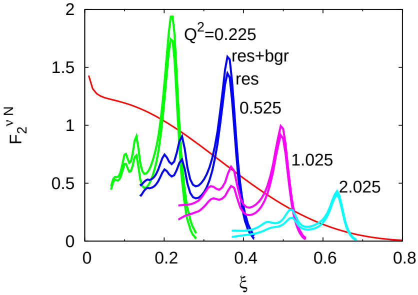
Interplay between the resonances with different isospins: isospin-3/2 resonances give strength to the proton structure functions, while isospin-1/2 resonances contribute to the neutron structure function only



$F_2^{\nu p, \nu n}$: Duality HOLDS for the averaged structure functions

Duality: on average the resonances appear to oscillate around and slide down the leading twist function





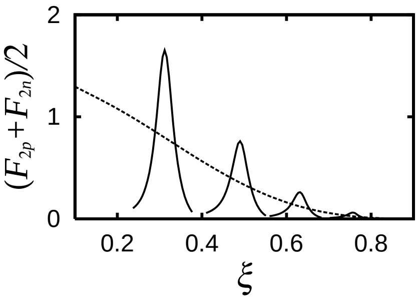
OL, Melnitchouk, Paschos, PRC 75

included: 4 resonances
F_2 calculated analytically
investigation of F_3 and $2xF_1$ is also done

Giessen BUU

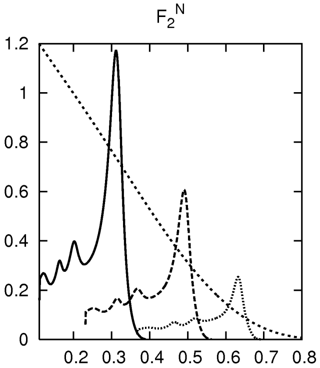
included: 12 resonances + phenomenological 1-pion background
F_2 is restored from xsec

Similar results in Sato-Lee model
Matsui,Sato,Lee, PRC 72
($P_{33}(1232)$) resonance considered so far)



$Q^2 = 0.4,\ 1.0,\ 2,\ 4\ \mathrm{GeV}^2$

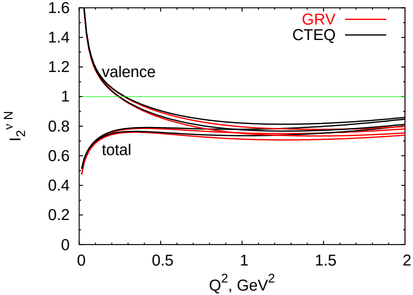
and Rein–Sehgal model
Graczyk, Juszczak, Sobczyk, Nucl Phys A781
(19 resonances included in the model)



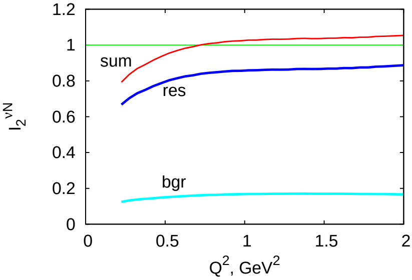
$Q^2 = 0.4,\ 1.0,\ 2\ \mathrm{GeV}^2$

Local duality: ratio of the integrals over the finite interval of ξ

OL,Melnitchouk,Paschos, PRC 75



Giessen BUU



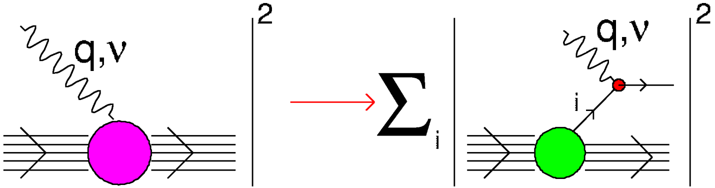
quantitative measure for validity of Bloom–Gilman duality

$$I_2(Q^2) = \frac{\int_{\xi_{\min}}^{\xi_{\max}} d\xi \, \mathcal{F}_2^{(\mathrm{res})}(\xi, Q^2)}{\int_{\xi_{\min}}^{\xi_{\max}} d\xi \, \mathcal{F}_2^{(\mathrm{LeadingTwist})}(\xi, Q^2_{\mathrm{DIS}})} \to 1 \quad \text{if perfect}$$

integration limits $\xi_{\min} = \xi(Q^2, W = 1.6\ \mathrm{GeV}), \quad \xi_{\max} = \xi(Q^2, W = 1.1\ \mathrm{GeV})$

Impulse approximation and nuclear shell model

picture from Benhar et al, PRD 72



struck protons and neutrons in the nucleus are off-mass-shell with the binding (removal) energy of the corresponding nuclear shell

$$\frac{d\sigma^A}{dQ^2 d\nu} = \sum_{occupied} \int d^3p \, \frac{d\sigma^N}{dQ^2 d\nu} n^{(N)}(p)$$

$n^{(N)}(p)$ — nucleon momentum distribution for each nuclear shell

Definition of the nuclear structure functions

- Standard expansion of the nuclear hadronic tensor is valid

$$W^A_{\mu\nu} = -g_{\mu\nu}\mathcal{W}^A_1 + \frac{p^A_\mu p^A_\nu}{M_A^2}\mathcal{W}^A_2 - i\varepsilon_{\mu\nu\lambda\sigma}\frac{p^\lambda_A q^\sigma}{2M_A^2} + \frac{q_\mu q_\nu}{M_A^2}\mathcal{W}^A_4 + \frac{p^A_\mu q_\nu + p^A_\nu q_\mu}{M_A^2}\mathcal{W}^A_5$$

  One-nucleon nucleon structure functions — for bound nucleon now
  Assumption: standard definition, BUT $\mathcal{W}_i(Q^2,\nu,|\vec{p}|)$

- Prescription of Atwood PRD 7, Ferree Koltun PRC 55 and express the nuclear structure functions via the nucleon ones as

$$W^A_{\mu\nu} = \sum_{nucleons}\int d^3p\, n(p)\left(W^p_{\mu\nu}(Q^2,\nu,|\vec{p}|) + W^n_{\mu\nu}(Q^2,\nu,|\vec{p}|)\right)$$

- Nuclear structure functions

$$\mathcal{W}^A_1(Q^2,\nu) = \sum_{nucleons}\int d^3p\, n(p)\left[\mathcal{W}_1(Q^2,\nu,p) + \mathcal{W}_2(Q^2,\nu,p)\frac{|\vec{p}|^2 - p_z^2}{m_N^2}\right] ,$$

$$\mathcal{W}^A_2(Q^2,\nu) = \sum_{nucleons}\int d^3p\, n(p)\mathcal{W}_2(Q^2,\nu,p)\left[\frac{|\vec{p}|^2 - p_z^2}{m_N^2}\frac{Q^2}{q_z^2} + \left(\frac{(p\cdot q)}{m_N\nu}\right)^2\left(1 + \frac{p_z}{q_z}\frac{Q^2}{(p\cdot q)}\right)^2\right]$$

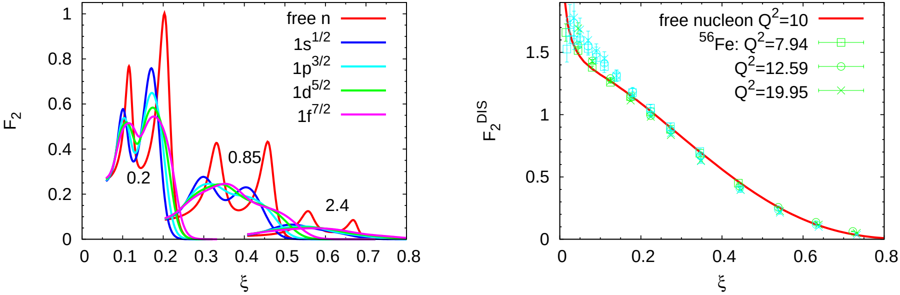
$$x_A F^A_3 = \sum_{nucleons}\int d^3p\, n(p) x F_3(Q^2,\nu,p)\frac{1}{m_N}\frac{p^0 q^z - \nu p^z}{q_z} .$$

- Checking the asymptotic value

$$\lim_{Q^2\to 0}\left[\frac{\nu^2}{Q^2}\mathcal{W}^A_2(Q^2,\nu) - \mathcal{W}^A_1(Q^2,\nu)\right] = 0$$

Nuclear effects for structure functions

For nuclei, the Fermi motion broaden resonances, thus performing averaging



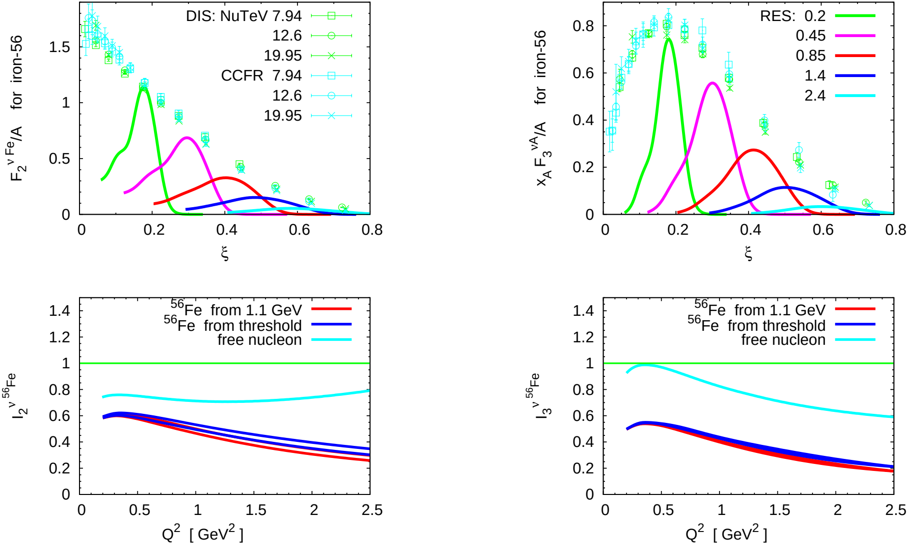
OL, Praet, Jachowicz, Ryckebusch (Ghent group), PRC 79

Resonance structure functions: isobar model with phenominological form factors
OL,Paschos, PRD 71, 74 includes the first four low-lying baryon resonances
$P_{33}(1232)$, $P_{11}(1440)$, $D_{13}(1520)$, $S_{11}(1535)$

DIS structure functions: experimental data by CCFR (Seligman el at, PRL 79) and NuTeV (Tzanov et al, PRD 74) collaborations

Duality in neutrino–iron reaction

For nuclei, the Fermi motion and other medium effects broaden resonances, thus performing averaging OL et al, (Ghent group), PRC 79



Conclusions

- Intermediate W region in neutrino–nucleon and –nucleus scattering: both hadronic and partonic description should give the same result — if quark–hadron duality holds
- Several groups investigated duality for lepton scattering on nucleonsand came to the same conclusion, that duality holds with reasonable accuracy for the average of proton and neutron structure functions
- We begin to investigate the role of background (Giessen) and the role of nuclear effects (Ghent). Problem for carbon/iron — lower resonance contribution than expected. Possible solution — background ?

Backup slides

x-sec via structure functions

The inclusive x-sec can be presented in a form close to DIS

$$\frac{d\sigma}{dQ^2 dW} = \frac{G^2}{4\pi}\cos^2\theta_C \frac{W}{m_N E^2}\left\{\mathcal{W}_1(Q^2+m_\mu^2) + \frac{\mathcal{W}_2}{m_N^2}\left[2(pk)(pk') - \frac{1}{2}m_N^2(Q^2+m_\mu^2)\right]\right.$$
$$\left. -\frac{\mathcal{W}_3}{m_N^2}\left[Q^2(pk) - \frac{1}{2}(pq)(Q^2+m_\mu^2)\right] + \frac{\mathcal{W}_4}{m_N^2}m_\mu^2\frac{(Q^2+m_\mu^2)}{2} - 2\frac{\mathcal{W}_5}{m_N^2}m_\mu^2(pk)\right\}$$

and the hadronic structure functions are defined as usual

$$\mathcal{W}^{\mu\nu} = -g^{\mu\nu}\,\mathcal{W}_1 + p^\mu p^\nu\,\frac{\mathcal{W}_2}{m_N^2} - i\varepsilon^{\mu\nu\sigma\lambda}p_\sigma q_\lambda\,\frac{\mathcal{W}_3}{2m_N^2} + \frac{\mathcal{W}_4}{m_N^2}q^\mu q^\nu + \frac{\mathcal{W}_5}{m_N^2}(p^\mu q^\nu + p^\nu q^\mu)$$

These structure functions are ”low–energy“ part of our duality study

- The functional dependence of the structure functions on the form factors vary with resonance
- In addition to $\mathcal{W}_1$ and $\mathcal{W}_2$ in neutrino scattering one has $\mathcal{W}_3$, which describes the vector-axial interference

Structure functions via form factors

$$\mathcal{W}_i(Q^2,\nu) = \frac{1}{m_N} V_i(Q^2,\nu) R(W, M_R)$$

$$\begin{aligned} V_1 = & \frac{(g_1^V)^2}{\mu^4} Q^4[(pq + m_N^2 \mp m_N M_R)] + \frac{(g_2^V)^2}{\mu^2}[2(pq)^2 \\ & + Q^2(m_N^2 \pm m_N M_R - q\cdot p)] \\ & + \frac{g_1^V g_2^V}{\mu^3} 2Q^2[(pq)(M_R \mp m_N) \pm m_N Q^2] \\ & + (g_1^A)^2 (m_N^2 \pm m_N M_R + q\cdot p) \end{aligned} \quad (4.25)$$

$$V_2 = 2m_N^2 \left[\frac{(g_1^V)^2}{\mu^4} Q^4 + \frac{(g_2^V)^2}{\mu^2} Q^2 + (g_1^A)^2 \right] \quad (4.26)$$

$$V_3 = 4m_N^2 \left[\frac{g_1^V g_1^A}{\mu^2} Q^2 + \frac{g_2^V g_1^A}{\mu} (M_R \pm m_N) \right] \quad (4.27)$$

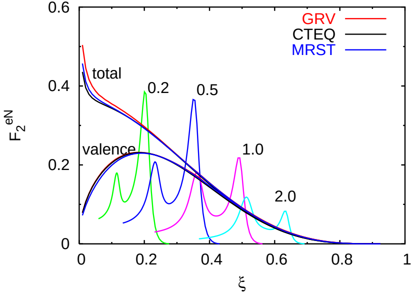
These are the formulas for the spin-1/2 resonances

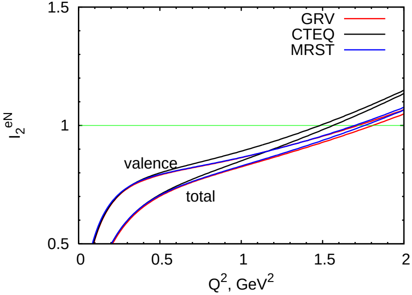
$\pm$ signs here correspond to P_{11} and S_{11} resonances, respectively

Formulas for spin-3/2 resonances are more cumbersome

Analytical form is useful for comparison with other models, for studing quark–hadron duality

Local duality: ratio of the integrals over the finite interval of ξ





$Q^2 = 0.2,\ 0.5,\ 1.0,\ 2\ \mathrm{GeV}^2$

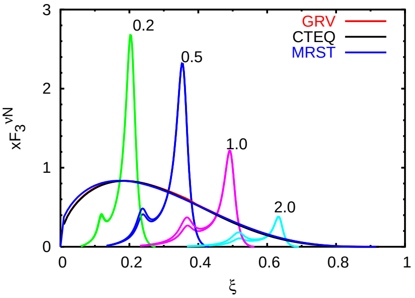
$$I_2(Q^2) = \frac{\int_{\xi_{\min}}^{\xi_{\max}} d\xi\ \mathcal{F}_2^{(\mathrm{res})}(\xi, Q^2)}{\int_{\xi_{\min}}^{\xi_{\max}} d\xi\ \mathcal{F}_2^{(\mathrm{LeadingTwist})}(\xi, Q^2)},$$

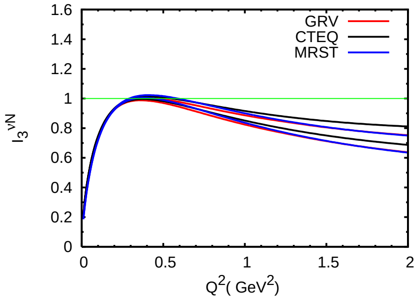
OL,Melnitchouk,Paschos, PRC 75

$$\xi_{\min} = \xi(Q^2, W = 1.6\ \mathrm{GeV}), \qquad \xi_{\max} = \xi(Q^2, W = 1.1\ \mathrm{GeV})$$

Two component duality: resonance curve agrees better with the valence–only structure function. The resonances are dual to the valence quarks, background (not shown here) to the sea quarks

Duality for $xF_3^{\nu N}$ structure function





$$I_3^{\nu N}(Q^2) = \frac{\int_{\xi_{\min}}^{\xi_{\max}} d\xi \, xF_3^{(\mathrm{res})}(\xi, Q^2)}{\int_{\xi_{\min}}^{\xi_{\max}} d\xi \, xF_3^{(\mathrm{LeadingTwist})}(\xi, Q^2)},$$

$F_3^{\nu N}$ is generally more sensitive to the choice of the axial form factors

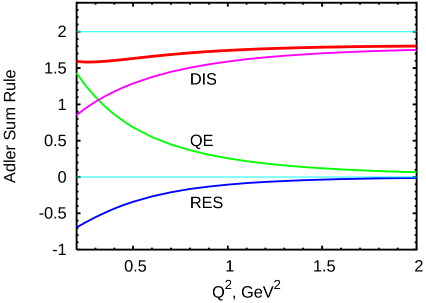
The accuracy of local duality about 30% is consistent with the estimated uncertainly of the axial form factors

Adler sum rule

$$\underbrace{\left[g_{1V}^{(QE)}\right]^2 + \left[g_{1A}^{(QE)}\right]^2 + \left[g_{2V}^{(QE)}\right]^2 \frac{Q^2}{2M^2}}_{\text{QE contribution}} + \int d\nu \underbrace{\left[W_2^{\nu n}(Q^2,\nu) - W_2^{\nu p}(Q^2,\nu)\right]}_{\text{RES+DIS}} = 2$$

RES: integration of resonance W_2 over ν region, corresponding to the $\xi_{min}(Q^2) < \xi < \xi_{max}(Q^2)$

DIS: integration of the leading twist F_2 over the remaining ξ interval $0 < \xi < \xi_{min}$, corresponding to $W > 1.6\ \mathrm{GeV}$

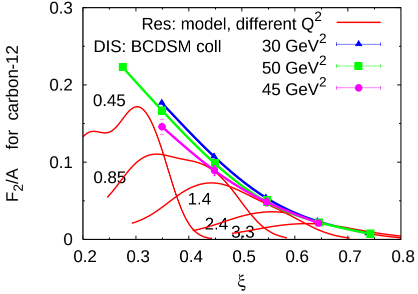


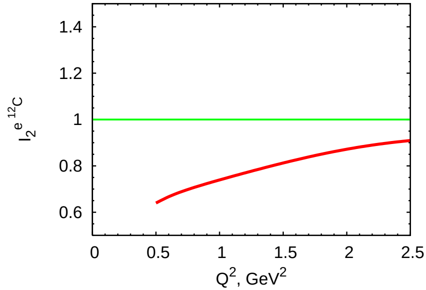
High-Q^2 limit (DIS)

$$\int d\nu \left[W_2^{\nu n}(Q^2,\nu) - W_2^{\nu p}(Q^2,\nu)\right] \equiv \int dx \frac{F_2^{\nu n}(Q^2,\nu) - F_2^{\nu p}(Q^2,\nu)}{x} = 2$$

Duality in electron–carbon reaction

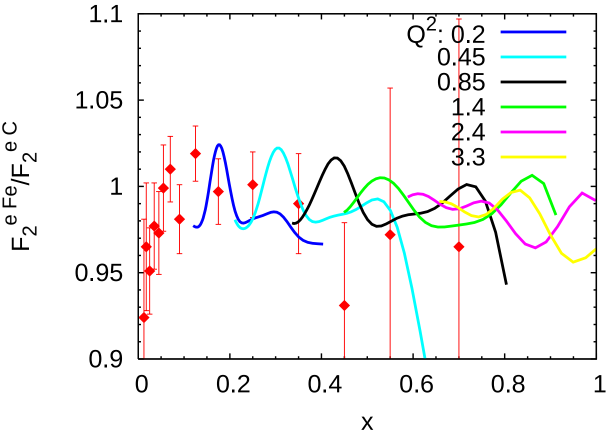
For nuclei, the Fermi motion and other medium effects broaden resonances, thus performing averaging





Resonance structure functions: isobar model with phenominological form factors OL,Paschos, PRD 71, 74 includes the first four low-lying baryon resonances $P_{33}(1232)$, $P_{11}(1440)$, $D_{13}(1520)$, $S_{11}(1535)$

Duality in iron/carbon ratio



Ratio $F_2^{e^{56}Fe}/F_2^{e^{12}C}$ for $Q^2 = 0.2, 0.45, 0.85, 1.4, 2.4$ and $3.3\ \mathrm{GeV}^2$ compared with the experimental data in DIS region