A study of nuclear effects in F_2 and F_3 structure functions in the deep inelastic $\nu(\bar{\nu})$ reactions in nuclei

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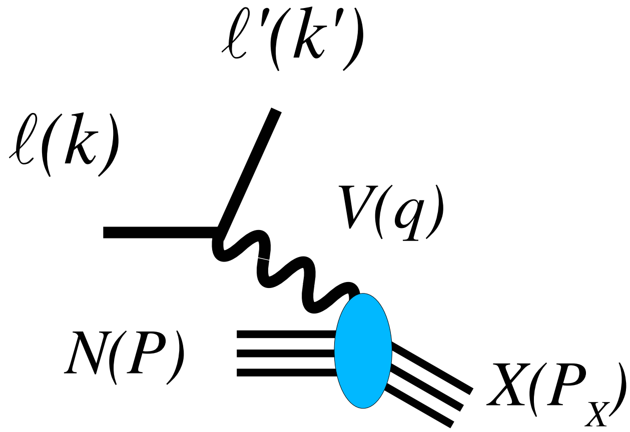
References:

Marco, Oset and Fernandez de Cordoba, NPA 611 (1996) 484

Athar, Singh and Vicente-Vacas PLB 668(2008) 133

Ruiz Simo and Vicente Vacas J.Phys.G36:015104,2009.

Athar, Ruiz Simo and Vicente Vacas, (in Preparation)



For the basic lepton–nucleon inelastic scattering process,

$$\ell(k) + N(P) \to \ell'(k') + X(P_X)$$

the matrix element is given by

$$-i\mathcal{M} = \bar{u}(k')\left(-\frac{ie}{2\sqrt{2}sin\theta_W}\right)\gamma^\mu(1-\gamma_5)u(k)$$
$$-i\frac{\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2}\right)}{q^2 - M_W^2} < X|J^\nu|N > \left(-\frac{ie}{2\sqrt{2}sin\theta_W}\right)$$

The general expression of the cross section for the reaction $\nu_l + N \to l^- + X$ is

$$\sigma = \frac{1}{v_{rel}}\frac{2m_\nu}{2E_\nu}\frac{2M}{2E(\mathbf{p})}\int\frac{d\mathbf{k}'}{(2\pi)^3}\times$$
$$\frac{2m_l}{2E_l}\prod_{i=1}^{N}\int\frac{d\mathbf{p}'_\mathbf{i}}{(2\pi)^3}\prod_{l\epsilon f}\frac{2M_l}{2E'_l}\times$$
$$\prod_{j\epsilon b}\frac{1}{2E_j}\bar{\Sigma}\Sigma|\mathcal{M}|^2(2\pi)^4\delta^4(p+k-k'-\sum_{i=1}^{N}p'_i)$$

The differential cross section in the rest frame of the nucleon is given by

$$\frac{d^2\sigma^N_{\nu\bar{\nu}}}{d\Omega' dE'} = \frac{G_F^2}{(2\pi)^2}\frac{|\mathbf{k}'|}{|\mathbf{k}|}\left(\frac{m_W^2}{q^2 - m_W^2}\right)^2 L^{\alpha\beta}_{\nu\bar{\nu}} W^N_{\alpha\beta}$$

The most general form of the hadronic tensor $W^N_{\alpha\beta}$ is expressed as:

$$\begin{aligned} W^N_{\alpha\beta} = \quad & \left(\frac{q_\alpha q_\beta}{q^2} - g_{\alpha\beta}\right) W_1^{\nu(\bar{\nu})} + \frac{1}{M^2}\left(p_\alpha - \frac{p.q}{q^2} q_\alpha\right) \\ & \left(p_\beta - \frac{p.q}{q^2} q_\beta\right) W_2^{\nu(\bar{\nu})} - \frac{i}{2M^2}\epsilon_{\alpha\beta\rho\sigma} p^\rho q^\sigma W_3^{\nu(\bar{\nu})} \\ & + \frac{1}{M^2} q_\alpha q_\beta W_4^{\nu(\bar{\nu})} + \frac{1}{M^2}(p_\alpha q_\beta + q_\alpha p_\beta) \\ & W_5^{\nu(\bar{\nu})} + \frac{i}{M^2}(p_\alpha q_\beta - q_\alpha p_\beta) W_6^{\nu(\bar{\nu})} \end{aligned}$$

W_i^N: structure functions which depend on the scalars q^2 and $p.q$

Bjorken variables x and y are defined as

$$x = \frac{Q^2}{2M\nu}, \quad y = \frac{\nu}{E_\nu}, \quad Q^2 = -q^2, \quad \nu = \frac{p.q}{M}$$

The differential scattering cross section (in the limit of lepton mass $m_l \to 0$) as

$$\frac{d^2\sigma^{\nu(\bar{\nu})}}{dxdy} = \frac{{G_F}^2 M E_\nu}{\pi}\Big[xy^2 F_1^{\nu(\bar{\nu})}(x,Q^2) + (1 - y - \frac{xyM}{2E_\nu})F_2^{\nu(\bar{\nu})}(x,Q^2) \pm xy(1-y/2)F_3^{\nu(\bar{\nu})}(x,Q^2)\Big]$$

$+$ (-) sign stands for the neutrino (antineutrino). $F_i^{\nu,\bar{\nu}}(x,Q^2)$ are dimensionless structure functions defined as

$$F_1^{\nu(\bar{\nu})}(x) = MW_1^{\nu(\bar{\nu})}(\nu,Q^2)$$
$$F_2^{\nu(\bar{\nu})}(x) = \nu W_2^{\nu(\bar{\nu})}(\nu,Q^2)$$
$$F_3^{\nu(\bar{\nu})}(x) = \nu W_3^{\nu(\bar{\nu})}(\nu,Q^2)$$

- In the Bjorken limit ($Q^2 \to \infty$, $\nu \to \infty$, x finite),

$F_i^{\nu(\bar{\nu})}(x)$ are independent of Q^2 and depend only on the single dimensionless variable x. In the quark parton model these structure functions are determined in terms of PDFs for quarks and antiquarks.

Nuclear effects in neutrino scattering

There are two main nuclear effects:

I. A kinematic effect which arises as the struck nucleon is not at rest but is moving with a Fermi momentum in the rest frame of the nucleus, leading to a Lorentz contraction of the incident flux.

II. The dynamic effects which arise due to Fermi motion, Pauli blocking and strong interaction of the initial nucleon in the nuclear medium.

The expression for the cross section in nuclear medium is given by:

$$\frac{d^2\sigma^A_{\nu,\bar{\nu}}}{d\Omega' dE'} = \frac{G_F^2}{(2\pi)^2} \frac{|\mathbf{k}'|}{|\mathbf{k}|} \left(\frac{m_W^2}{q^2 - m_W^2}\right)^2 L^{\alpha\beta}_{\nu,\bar{\nu}} \, W^A_{\alpha\beta}$$

$W^A_{\alpha\beta}$: nuclear hadronic tensor defined in terms of nuclear hadronic structure functions $W_{iA}(x, Q^2)$

In the rest frame of the nucleus the expression of the differential cross section modifies from

$$\frac{d^2\sigma^N_{\nu,\bar\nu}}{d\Omega' dE'} = \frac{{G_F}^2}{(2\pi)^2}\frac{|\mathbf{k}'|}{|\mathbf{k}|}\left(\frac{m_W^2}{q^2-m_W^2}\right)^2 L^{\alpha\beta}_{\nu,\bar\nu} W^N_{\alpha\beta}$$ to

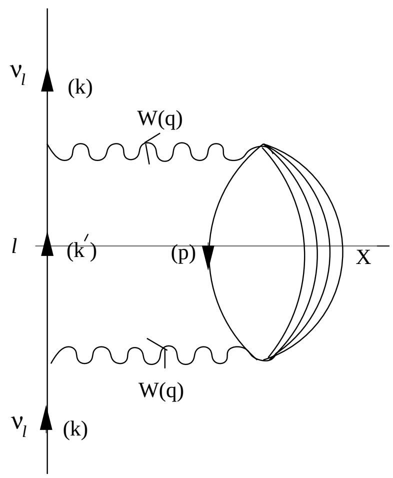
$$\frac{d^2\sigma^A_{\nu,\bar\nu}}{d\Omega' dE'} = \frac{{G_F}^2}{(2\pi)^2}\frac{|\mathbf{k}'|}{|\mathbf{k}|}\left(\frac{m_W^2}{q^2-m_W^2}\right)^2 L^{\alpha\beta}_{\nu,\bar\nu} W^A_{\alpha\beta}$$

$W^A_{\alpha\beta}$ is the nuclear hadronic tensor:

$$\begin{aligned} W^A_{\alpha\beta} &= \left(\frac{q_\alpha q_\beta}{q^2} - g_{\alpha\beta}\right) W_1^A + \frac{1}{M_A^2}\left(p_\alpha - \frac{p.q}{q^2}\, q_\alpha\right) \\ &\quad \left(p_\beta - \frac{p.q}{q^2}\, q_\beta\right) W_2^A - \frac{i}{2M_A^2}\epsilon_{\alpha\beta\rho\sigma} p^\rho q^\sigma W_3^A \end{aligned}$$

$$\begin{aligned} F_1^A(x_A) &= M_A W_1^A(\nu, Q^2) \\ F_2^A(x_A) &= \nu W_2^A(\nu, Q^2) \\ F_3^A(x_A) &= \nu W_3^{\nu(\bar\nu)}(\nu, Q^2) \end{aligned}$$

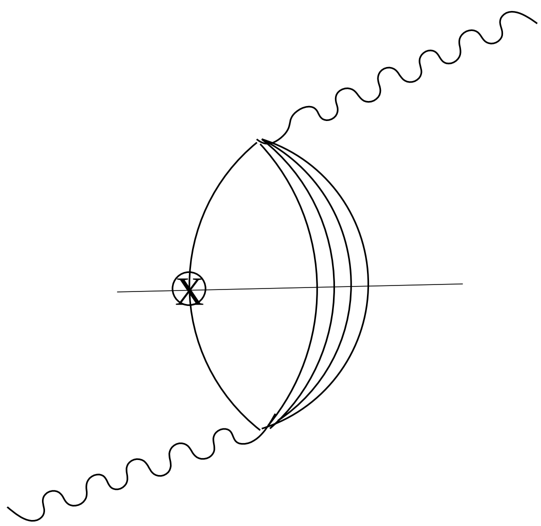
In the present formalism the neutrino nuclear cross sections are obtained in terms of neutrino self energy $\Sigma(k)$ in the nuclear medium which also defines the dimensionless nuclear structure functions $F_i^A(x, Q^2)$.



The neutrino self-energy in nuclear matter is given by

$$-i\sum(k) = \int \frac{d^4q}{(2\pi)^4} \bar{u}_\nu(k) \left(-\frac{ie}{2\sqrt{2}sin\theta_W}\gamma^\mu(1-\gamma_5)\right)$$
$$\left(\frac{i(\not{k}' + m_l)}{k'^{\ 2} - m_l^2 + i\epsilon}\right)\left(-\frac{ie}{2\sqrt{2}sin\theta_W}\gamma^\nu(1-\gamma_5)\right)$$
$$u_\nu(k)\left(-i\frac{\left(g_{\mu\rho} - \frac{q_\mu q_\rho}{M_W^2}\right)}{q^2 - M_W^2}\right)(-i)\Pi^{\rho\sigma}(q)i\frac{\left(g_{\sigma\nu} - \frac{q_\sigma q_\nu}{M_W^2}\right)}{q^2 - M_W^2}$$

$\Pi^{\alpha\beta}(q)$ is the W self-energy in the nuclear medium and is written as:



$$\Pi^{\alpha\beta}(q) = (-i) \int \frac{d^4p}{(2\pi)^4} \frac{1}{k'^2 - m_\mu^2 + i\epsilon}$$

$$\sum_{s_p, s_i} \prod_{i=1}^{n} \int \frac{d^4p'_i}{(2\pi)^4} \prod_l iG_l(p'_l) \prod_j iD_j(p'_j)$$

$$\left(\frac{-G_F m_W^2}{\sqrt{2}}\right) \times \langle X|J^\alpha|N\rangle\langle X|J^\beta|N\rangle^*$$

$$(2\pi)^4 \delta^4(q + p - \Sigma_{i=1}^{n} p'_i)$$

This relativistic nucleon propagator after summing over the ladder approximation give

$$G(p_0, \mathbf{p}) = \frac{M}{E(\mathbf{p})} \sum_r \frac{u_r(p)\bar{u}_r(p)}{p^0 - E(\mathbf{p}) - \bar{u}_r(p) \sum(p^0, \mathbf{p}) u_r(p) \frac{M}{\mathbf{E(p)}}}$$

This allows us to write the relativistic nucleon propagator in a nuclear medium in terms of the Spectral functions of holes and particles as

$$G(p^0, \mathbf{p}) = \frac{M}{E(\mathbf{p})} \sum_r u_r(\mathbf{p})\bar{u}_r(\mathbf{p}) \left[\int_{-\infty}^{\mu} d\omega \frac{S_h(\omega, p)}{p^0 - \omega - i\eta} + \int_{\mu}^{\infty} d\omega \frac{S_p(\omega, p)}{p^0 - \omega + i\eta} \right]$$

$S_h(\omega, p)$ and $S_p(\omega, p)$ being the hole and particle spectral functions given by

$$S_h(\omega, \mathbf{p}) = \frac{1}{\pi} \frac{\frac{M}{E(\mathbf{p})} Im\Sigma^N(p^0, p)}{(p^0 - E(\mathbf{p}) - \frac{M}{E(\mathbf{p})} Re\Sigma^N(p^0, p))^2 + (\frac{M}{E(\mathbf{p})} Im\Sigma^N(p^0, p))^2}$$

for $p^0 \leq \mu$

$$S_p(\omega, \mathbf{p}) = -\frac{1}{\pi} \frac{\frac{M}{E(\mathbf{p})} Im\Sigma^N(p^0, p)}{(p^0 - E(\mathbf{p}) - \frac{M}{E(\mathbf{p})} Re\Sigma^N(p^0, p))^2 + (\frac{M}{E(\mathbf{p})} Im\Sigma^N(p^0, p))^2}$$

for $p^0 > \mu$.

We use local density approximation (LDA) and for a symmetric nuclear matter of density $\rho(\mathbf{r})$,

$$4 \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\mu} S_h(\omega, p, k_F(\mathbf{r})) d\omega = \rho(\mathbf{r})$$

leading to the normalization condition given by

$$4 \int d^3r \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\mu} S_h(\omega, p, \rho(r))\, d\omega = A$$

where $\rho(r)$ is the baryon density for the nucleus which is normalized to A.

For an isospin symmetric nucleus

$$W_A^{\alpha\beta} = 4 \int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(\mathbf{p})} \int_{-\infty}^{\mu} dp^0 S_h(p^0, \mathbf{p}, \rho(r)) W_N^{\alpha\beta}(p,q)$$

where $W_N^{\alpha\beta}(p,q)$

$$W_{\alpha\beta}^N = \left(\frac{q_\alpha q_\beta}{q^2} - g_{\alpha\beta}\right) W_1^{\nu(\bar{\nu})} + \frac{1}{M^2}\left(p_\alpha - \frac{p.q}{q^2}\, q_\alpha\right) \left(p_\beta - \frac{p.q}{q^2}\, q_\beta\right) W_2^{\nu(\bar{\nu})} - \frac{i}{2M^2} \epsilon_{\alpha\beta\rho\sigma} p^\rho q^\sigma W_3^{\nu(\bar{\nu})}$$

Chosing $\mathbf{q}$ along the z-axis and if the nucleus is at rest

$$W^{A}_{xx}(x_A,Q^2) = W^{\nu(\bar{\nu})}_{1A}(x_A,Q^2)$$

$$\frac{F_{1A}(x_A,Q^2)}{AM} = 4\int d^3r \int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\mathbf{p})}\int_{-\infty}^{\mu} dp^0 S_h(p^0,\mathbf{p},\rho(r)) \times \left[\frac{F_{1N}(x_N,Q^2)}{M} + \frac{1}{M^2}{p_x}^2\frac{F_{2N}(x_N,Q^2)}{\nu}\right]$$

Using Callan Gross relation $F_2(x) = 2xF_1(x)$

$$F_{2A}(x_A,Q^2) = 4\int d^3r \int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\mathbf{p})}\int_{-\infty}^{\mu} dp^0 S_h(p^0,\mathbf{p},\rho(r)) \times \left[\frac{x}{x_N}[1 + \frac{2x_N {p_x}^2}{\nu}]F_{2N}(x_N,Q^2)\right]$$

Defining γ as $\gamma = \frac{q_z}{q^0} = \left(1 + \frac{4M^2x^2}{Q^2}\right)^{1/2}$

we get

$$F_3^A(x,Q^2) = 4\int d^3r \int \frac{d^3p}{(2\pi)^3}\frac{M}{E(p)}$$

$$\int_{-\infty}^{\mu} dp^0 S_h(p^0,p,\rho(r))\left(\frac{p_0\gamma - p_z}{(p_0 - p_z\gamma)\gamma}\right)F_3^N(x_N,Q^2)$$

Target Mass Corrections

Ingo Schienbein et al. J. of Phys. G 35:053101,2008

$$F_2^{\mathrm{TMC}}(x,Q^2) \simeq \frac{x^2}{\xi^2 r^3} F_2^{(0)}(\xi)[1 + \frac{6\mu x \xi}{r}(1-\xi)^2]$$

$F_3^{\mathrm{TMC}}(x,Q^2)$ is approximated by

$$F_3^{\mathrm{TMC}}(x,Q^2) \simeq \frac{x}{\xi r^2} F_3^{(0)}(\xi)[1 - \frac{\mu x \xi}{r}(1-\xi)\ln\xi]\,.$$

$$r = \sqrt{1 + \frac{4x^2M^2}{Q^2}};\, \xi = \frac{2x}{1+r};\, \mu = \frac{M^2}{Q^2}$$

Effect of Target Mass Correction

Q2	WITH	Without	Difference in %
x= 0.45			
1.2590 5.0120 **x= 0.65**	0.594435543 0.463959802	0.77166 0.50072	30 7
1.259 5.0120 19.950 **x=0.75**	0.241981271 0.134176562 0.0853798181	0.41732 0.16212 0.089995	72 21 5
1.259 5.0120 19.950	0.148675201 0.0608069109 0.0312409955	0.29576 0.78358E-01 0.33593E-01	100 28 8

Off shell correction

S. A. Kulagin and R. Petti PHYSICAL REVIEW D 76, 094023 (2007) & Nucl. Phys. A765 (2006) 126.

$$\delta f_2 = C_N(x - x_1)(x - x_0)(1 + x0 - x)$$

with the parameters extracted from the fit $C_N = 8.10 \pm 0 : 30(stat) \pm 0 : 53(syst)$, $x0 = 0.448 \pm 0.005(stat) \pm 0 : 007(syst)$ and x_1=0.05.

No off shell effects at low x(<0.55)

This is independent of Q^2

1. **x=0.55 Around 4-5%**

2. **x=0.65** Around 8-9%

3. **x=0.75 Around 10-12%**

This is independent of the Structure Function

Shadowing and Anti-shadowing effects

S. A. Kulagin and R. Petti PHYSICAL REVIEW D 76, 094023 (2007) & Nucl. Phys. A765 (2006) 126.

Effect of Shadowing in F_2 and F_3

1. **x=0.0001 to x=0.015**

I. Q^2=1.259GeV^2 $\approx$ 25% reduction in F_2 and 50% reduction in F_3

II. Q^2=3.162eV^2 $\approx$ 15% reduction in F_2 and 35% reduction in F_3

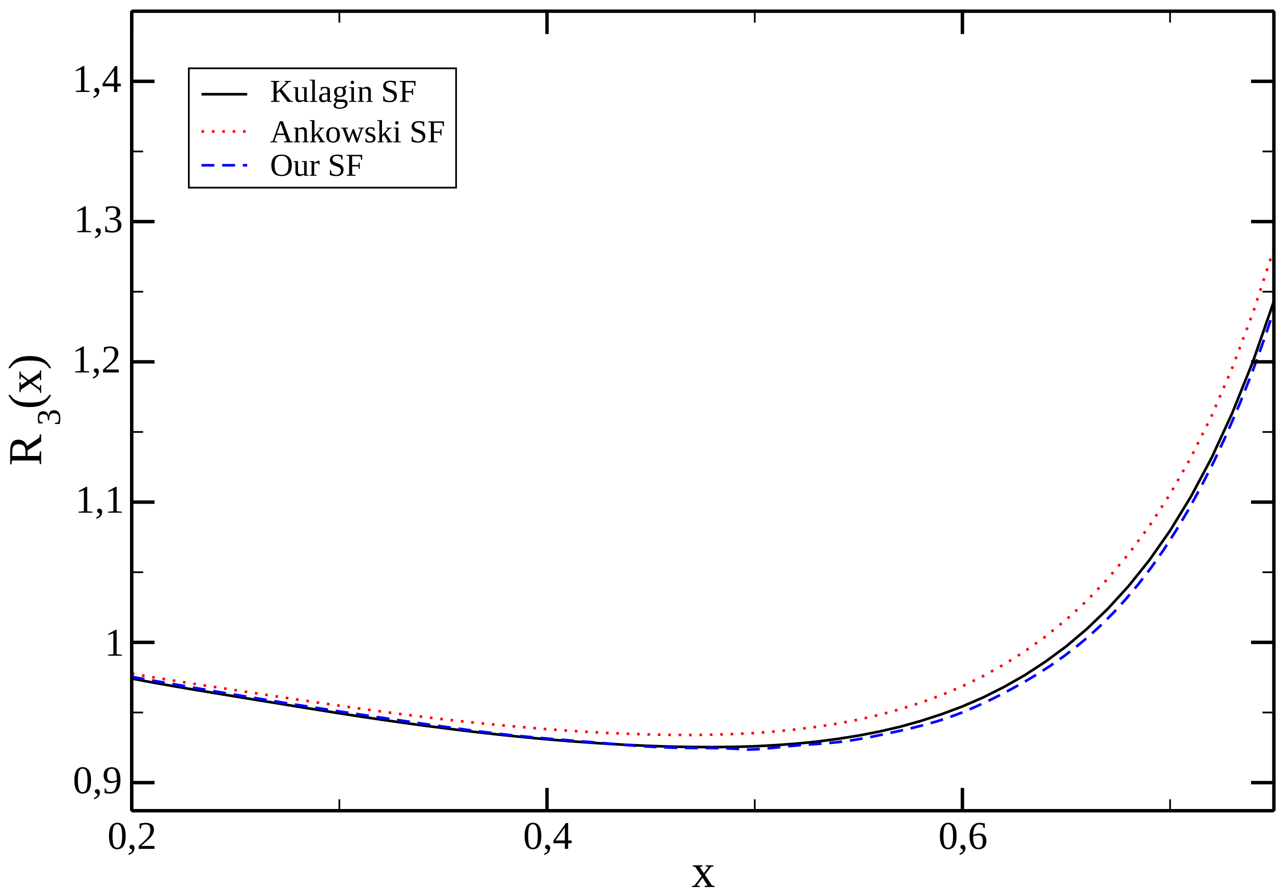
III. Q^2=20GeV^2 Around 4% reduction in F_2 and 10% reduction in F_3

2. **x=0.045**

I. Q^2=1.259GeV^2 $\approx$ 20% reduction in F_2

II. Q^2=3.162eV^2 $\approx$ 14% reduction in F_2

$R = \frac{F_{2A}}{AF_{2N}}$ for ^{40}Ca at $Q^2 = 20\ GeV^2$ with the nuclear spectral functions

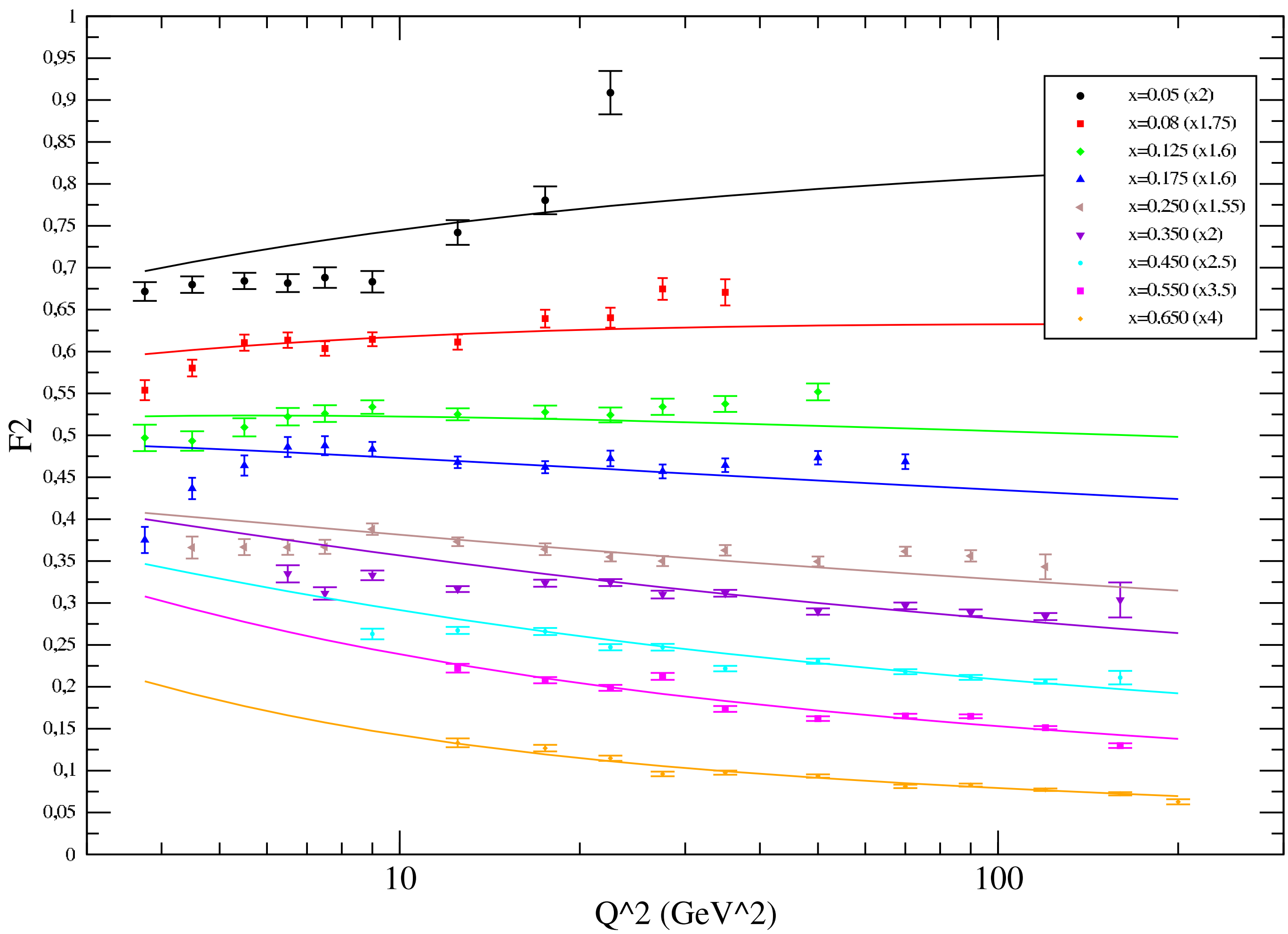


Ruiz Simo and Vicente Vacas J.Phys.G36:015104,2009.

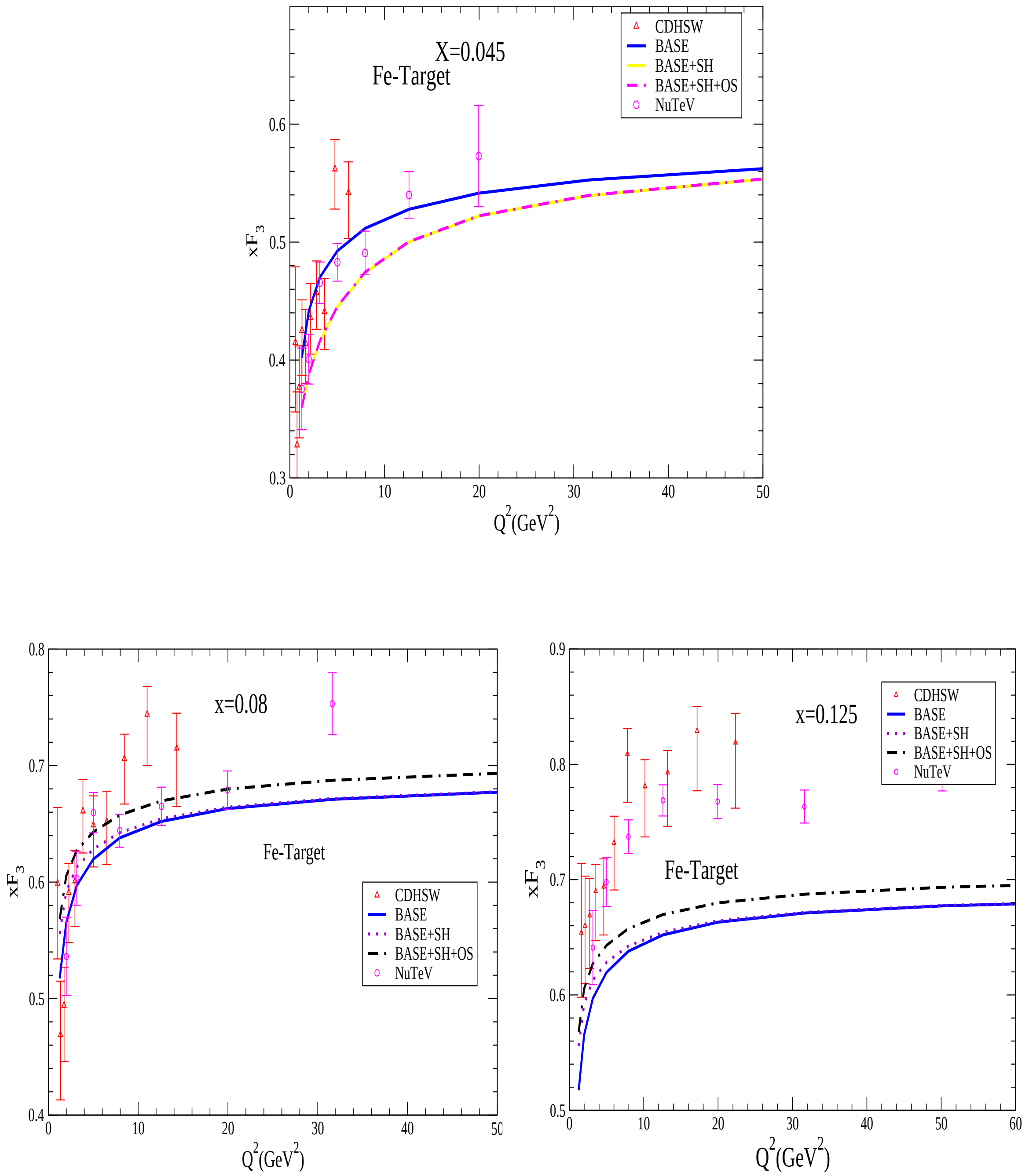
Kulagin and Petti, Nucl. Phys. A 765 (2006) 126

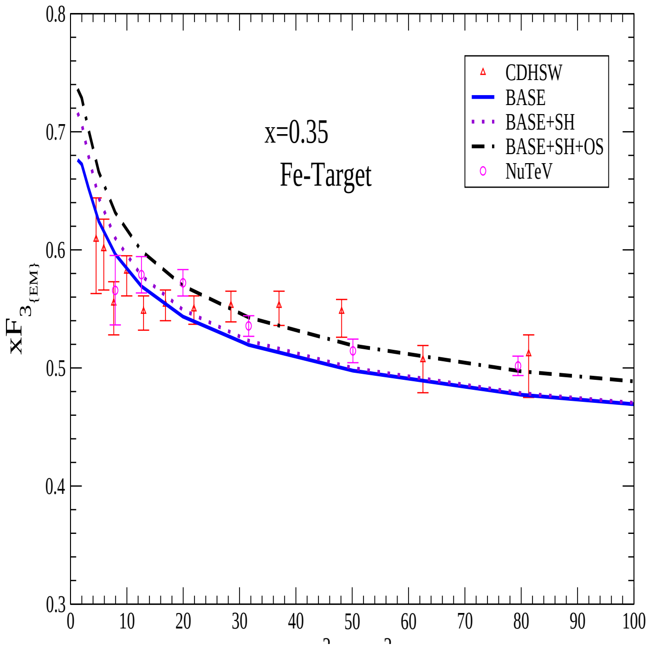
Ankowski and Sobczyk, Phys. Rev. C 77 (2008) 044311

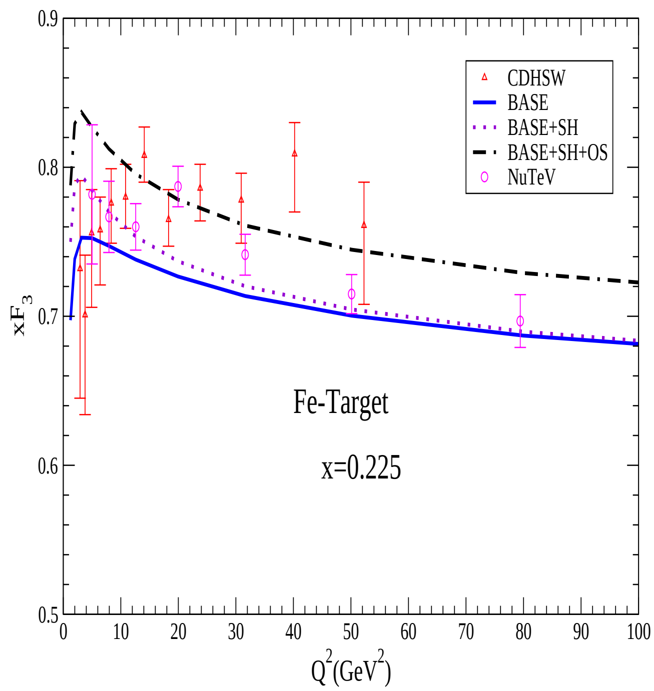
With nuclear medium effects only (No Shadowing and No Off shell correction)

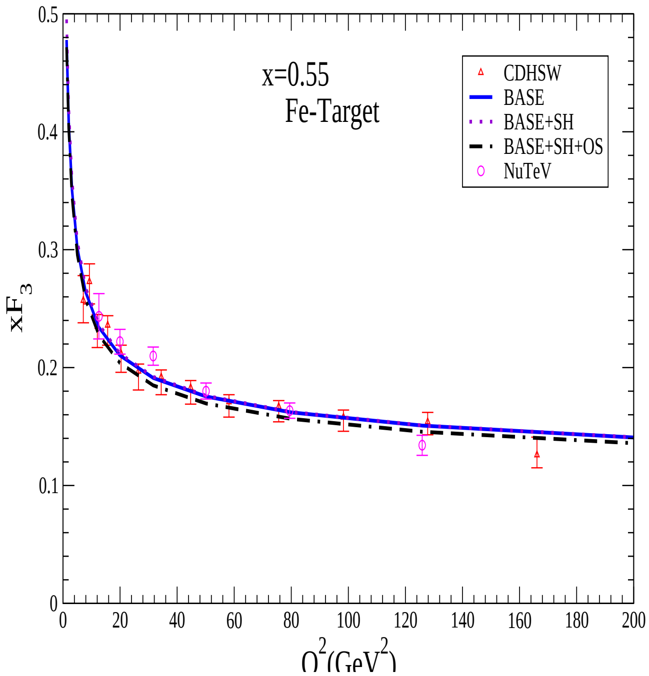


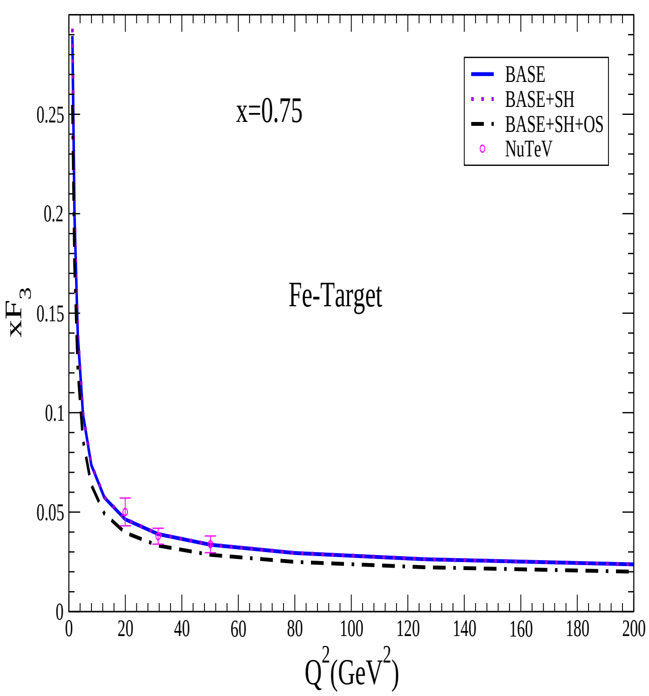
F_{2EM} in Iron: :Experimental Results from Aubert et al.NPB 272 (1986) 15

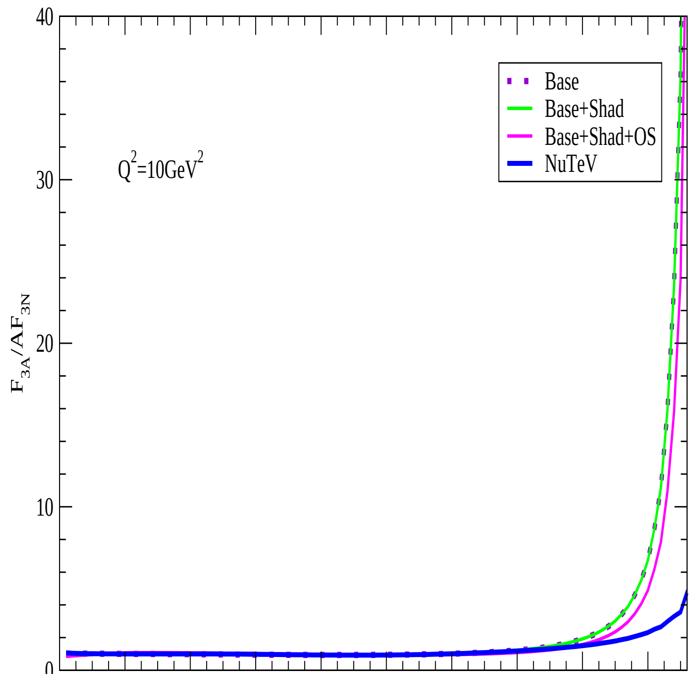

X=0.045
Fe-Target
CDHSW
BASE
BASE+SH
BASE+SH+OS
NuTeV
xF3
Q2(GeV2)
x=0.08
Fe-Target
CDHSW
BASE
BASE+SH
BASE+SH+OS
NuTeV
xF3
Q2(GeV2)
x=0.125
Fe-Target
CDHSW
BASE
BASE+SH
BASE+SH+OS
NuTeV
xF3
Q2(GeV2)


CDHSW
BASE
BASE+SH
BASE+SH+OS
NuTeV
x=0.35
Fe-Target
$xF_{3(EM)}$

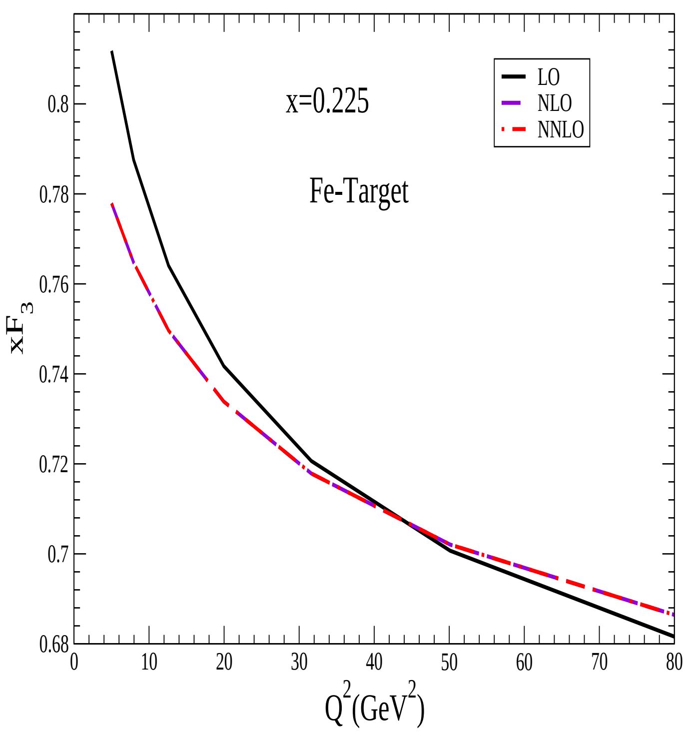

CDHSW
BASE
BASE+SH
BASE+SH+OS
NuTeV
Fe-Target
x=0.225
xF_3
$Q^2(GeV^2)$


x=0.55
Fe-Target
CDHSW
BASE
BASE+SH
BASE+SH+OS
NuTeV
xF3
Q2(GeV2)


x=0.75
Fe-Target
BASE
BASE+SH
BASE+SH+OS
NuTeV
xF3
Q2(GeV2)



Base
Base+Shad
Base+Shad+OS
NuTeV
Q^2=10GeV^2
F_{3A}/AF_{3N}



LO
NLO
NNLO
x=0.225
Fe-Target
xF_3
Q^2(GeV^2)

Conclusions

1. NME on $F_2(x,Q^2)$ and $F_3(x,Q^2)$ lead to reduction at large Q^2. Our results without shadowing and off shell effects give a χ^2=2.8 for $F_3^A(x,Q^2)$ when compared with NuTeV experimental results in Iron.

2. NME results on $F_2(x,Q^2)$ and $F_3(x,Q^2)$ are in qualitative agreement with phenomenological analysis of NuTeV collaboration but not with Hirai et al.

3. NME lead to better agreement with NuTeV and CCFR results.

4. The target mass correction is important at high x(x>0.45) and at very low Q^2. It dies out rapidly with increase in Q^2.

5. The off shell effects are important at high x and they are independent of Q^2

6. Effect of shadowing is more in the case of F_3 than in the case of F_2. It is important at low Q^2 and low x.