Effect of Final state interaction and Coulomb distortion of charged current neutrino-nucleus scattering in quasi-elastic region

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Introduction

Motivation :

- Is there loss of flux in the inclusive neutrino scattering ? (no detection of the knocked-out nucleon)
- Is Coulomb effect of charged current neutrino-nucleus scattering different from electron scattering ?

Goal :

- Investigate the contribution of the final state interaction (FSI) on the total cross section.
- Study Coulomb effect in charged current neutrino-nucleus scattering.

Ingredients :

- Calculate total cross section for the neutral and charged current reactions in quasi-elastic region for ^{12}C nucleus.

- Use a relativistic single particle model ($\sigma - \omega$ model).

- Compare a relativistic optical potential generated by OSU group and a same potential of the bound nucleon for the FSI of knocked out nucleons.

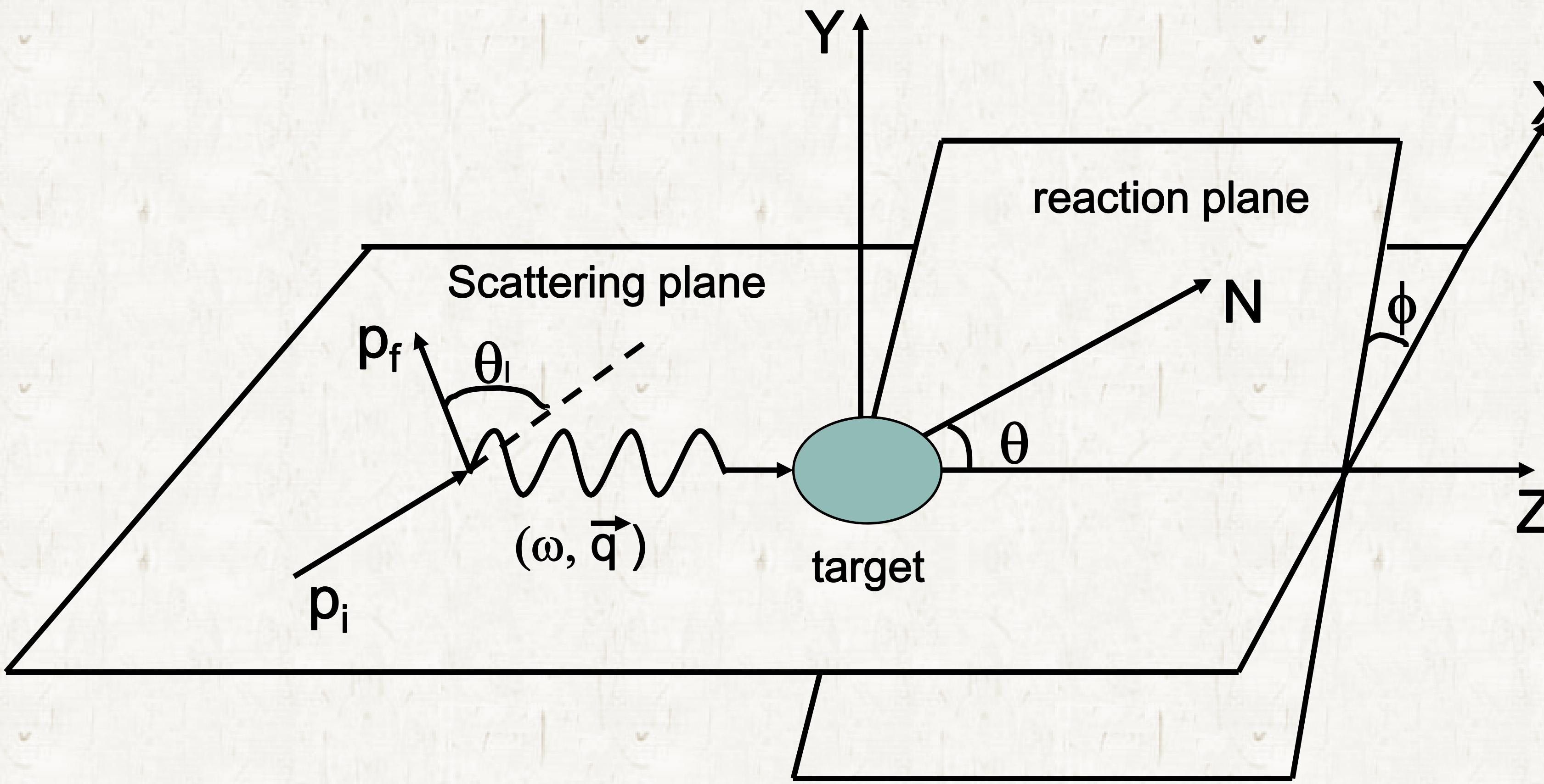
- Compare our results with experimental data by scaling the number of nucleons.

- Include the Coulomb distortion of final leptons for charged current reaction using approximate method developed by Ohio University group.

Formalism

$\nu(\bar{\nu}) + A \longrightarrow \nu(\bar{\nu}) + N + (A-1)$ neutral-current (NC) reaction

$\nu(\bar{\nu}) + A \longrightarrow \ell(\bar{\ell}) + N + (A-1)$ charged-current (CC) reaction



The relativistic nucleon weak current operator

$$J^\mu = F_1(Q^2)\gamma^\mu + i\frac{\kappa}{2M_N}F_2(Q^2)\sigma^{\mu\nu}q_\nu + G_A(Q^2)\gamma^\mu\gamma_5 + \frac{1}{2M_N}G_P(Q^2)q^\mu\gamma_5$$

weak vector form factors

$$F_i^{V,\ p(n)} = (\frac{1}{2} - 2\sin^2\theta_W)F_i^{p(n)} - \frac{1}{2}F_i^{n(p)} - \frac{1}{2}F_i^s$$

$$F_1^s(Q^2) = \frac{F_1^s Q^2}{(1+\tau)(1+Q^2/M_V^2)^2}$$

$$\tau = Q^2/(4M_N^2)$$

$$F_2^s(Q^2) = \frac{F_2^s(0)}{(1+\tau)(1+Q^2/M_V^2)^2}$$

$$M_V = 0.843 \text{ GeV}$$

$$F_1^s = dG_E^s(Q^2)/dQ^2|_{Q^2=0} = 0.53 \text{ GeV}^{-2}$$

$F_2^s(0) = \mu_s = -0.4$: strange magnetic moment

$\sin^2\theta_W = 0.2224$: Weinberg angle

The axial form factors are given by

$$G_A^{NC}(Q^2) = \frac{1}{2}(\mp g_A + g_A^s)/(1+Q^2/M_A^2)^2$$ for NC

$$G_A^{CC}(Q^2) = -g_A/(1+Q^2/M_A^2)^2$$ for CC

$M_A = 1.032$ GeV : axial cut-off mass

$$G_P(Q^2) = \frac{2M_N}{Q^2+m_\pi^2}G_A(Q^2)$$: pseudoscalar form factor disappears for NC reaction

m_π : pion mass

$g_A = 1.262$ $g_A^s = -0.19$

Neutrino-nucleus (^{12}C) scattering

The differential cross section is given by

$$\frac{d\sigma}{dT_p} = 4\pi^2 \frac{M_N M_{A-1}}{(2\pi)^3 M_A} \int \sin\theta_l d\theta_l \int \sin\theta_p d\theta_p$$
$$p f_{rec}^{-1} \sigma_M^{Z,W^\pm} [v_L R_L + v_T R_T + h v'_T R'_T]$$

M_N : mass of nucleon

M_{A-1} : mass of residual nucleus

M_A : mass of target nucleus

h : helicity for neutrino (antineutrino)

The recoil factor is given by

$$f_{rec} = \frac{E_{A-1}}{M_A} \left| 1 + \frac{E}{E_{A-1}} \left[1 - \frac{\mathbf{q}\cdot\mathbf{p}}{p^2} \right] \right|$$

The kinematic factor $\sigma_M^{Z,W^\pm}$ is defined by

$$\sigma_M^Z = \left(\frac{G_F \cos(\theta_l/2) E_f M_Z^2}{\sqrt{2}\pi(Q^2 + M_Z^2)} \right)^2$$

NC reaction

$$\sigma_M^{W^\pm} = \sqrt{1 - \frac{M_l^2}{E_f}} \left(\frac{G_F \cos(\theta_C) E_f M_W^2}{2\pi(Q^2 + M_W^2)} \right)^2$$

CC reaction

M_Z and M_W : mass of Z- and W-boson

θ_l : scattering angle $\cos^2\theta_C \simeq 0.9749$: Cabibbo angle

Nucleon current represents the Fourier transform given by

$$J^\mu = \int \bar{\psi}_p \hat{\mathbf{J}}^\mu \psi_b e^{i\mathbf{q}\cdot\mathbf{r}} d^3 r$$

For the NC reaction, the kinematic factors are given by

$$v_L = 1 \qquad v_T = \tan^2\frac{\theta_l}{2} + \frac{Q^2}{2q^2} \qquad v_T' = \tan\frac{\theta_l}{2}\left[\tan^2\frac{\theta_l}{2} + \frac{Q^2}{q^2}\right]^{1/2}$$

The response functions are given by

$$R_L = \left| J^0 - \frac{\omega}{q} J^z \right|^2 \qquad R_T = |J^x|^2 + |J^y|^2 \qquad R_T' = 2\mathrm{Im}(J^{x*} J^y)$$

For the CC reaction, the kinematics factors are given by

$$v_L^0 = 1 + \sqrt{1 - \frac{M_l^2}{E_f^2}} \cos\theta_l \quad v_L^z = 1 + \sqrt{1 - \frac{M_l^2}{E_f^2}} \cos\theta_l - \frac{2E_i E_f}{q^2}\left(1 - \frac{M_l^2}{E_f^2}\right)\sin^2\theta_l$$

$$v_L^{0z} = \frac{\omega}{q}\left(1 + \sqrt{1 - \frac{M_l^2}{E_f^2}} \cos\theta_l\right) + \frac{M_l^2}{E_f q}$$

$$v_T = 1 - \sqrt{1 - \frac{M_l^2}{E_f^2}} \cos\theta_l + \frac{E_i E_f}{q^2}\left(1 - \frac{M_l^2}{E_f^2}\right)\sin^2\theta_l \quad v_T' = \frac{E_i + E_f}{q}\left(1 - \sqrt{1 - \frac{M_l^2}{E_f^2}} \cos\theta_l\right) - \frac{M_l^2}{E_f q}$$

and corresponding response functions are given by

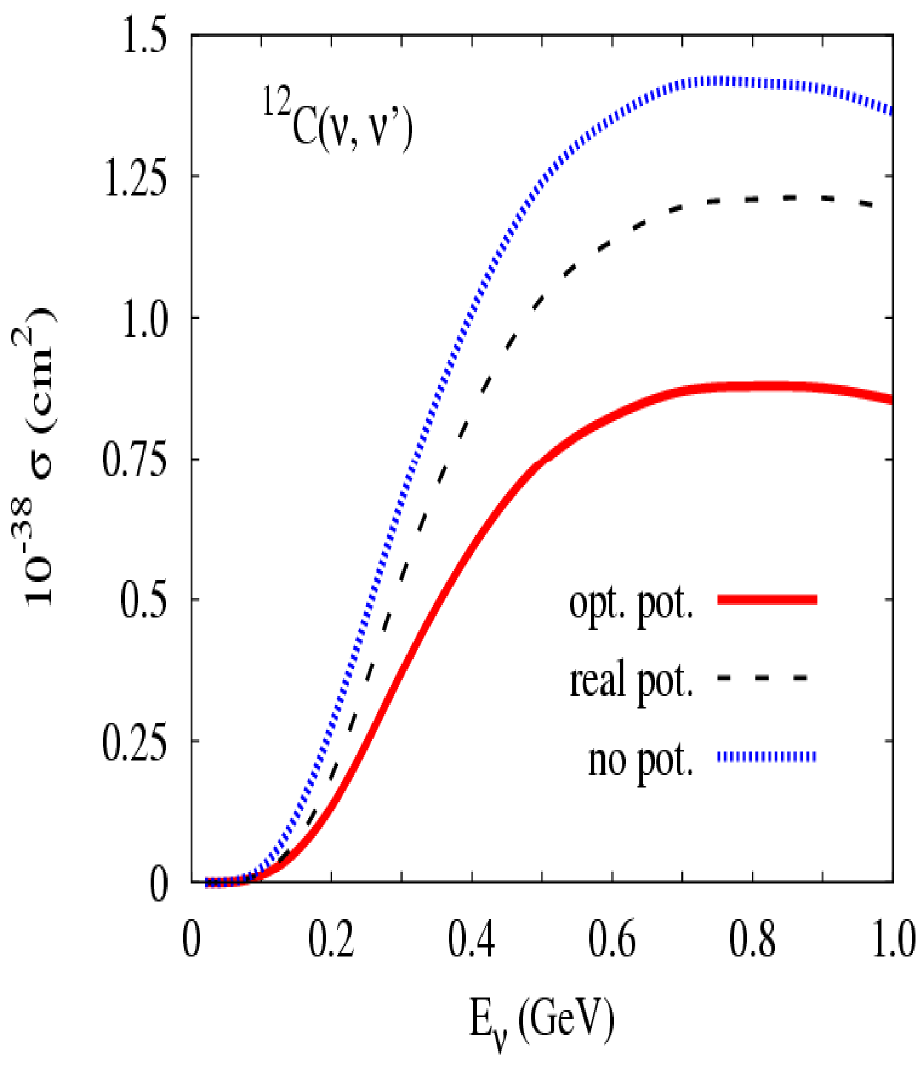
$$R_L^0 = |J^0|^2 \qquad R_L^z = |J^z|^2 \qquad R_L^{0z} = -2\mathrm{Re}(J^0 J^{z*})$$

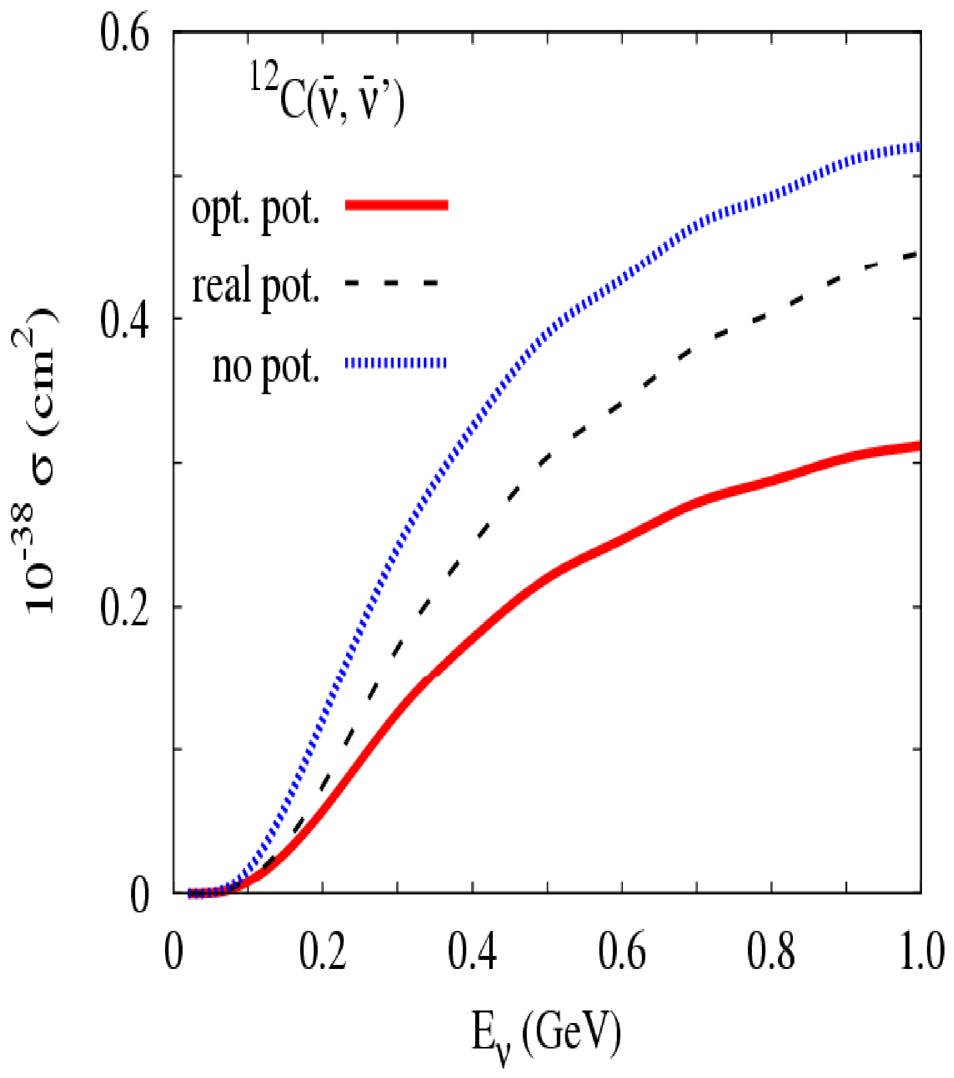
$$R_T = |J^x|^2 + |J^y|^2 \qquad R_T' = 2\mathrm{Im}(J^x J^{y*})$$

with $v_L R_L = v_L^0 R_L^0 + v_L^z R_L^z + v_L^{0z} R_L^{0z}$

The total cross section $\sigma = \int \frac{d\sigma}{dT_p} dT_p$

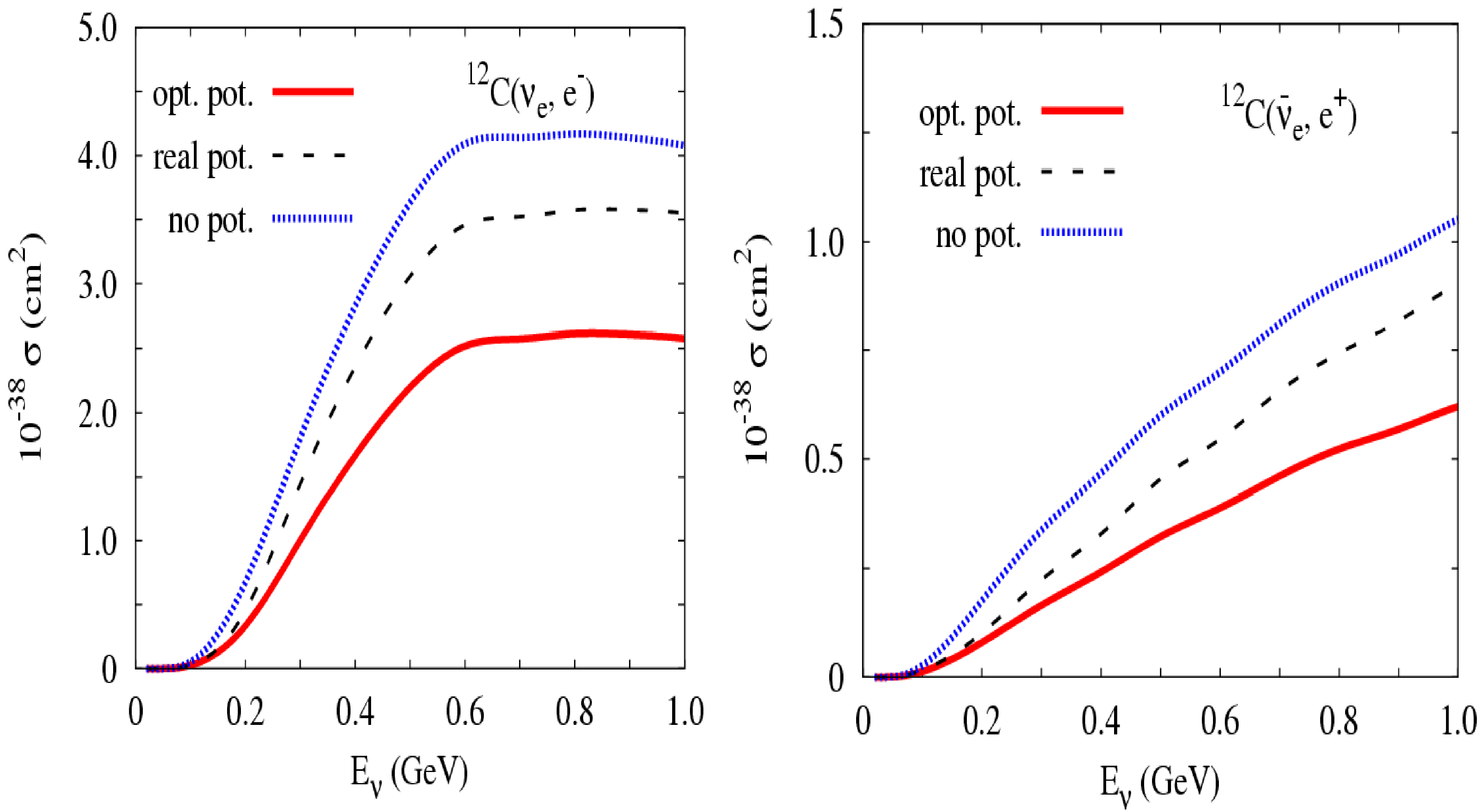
Total cross section (NC reaction)





Total cross section (CC reaction)

without the Coulomb effect of final leptons

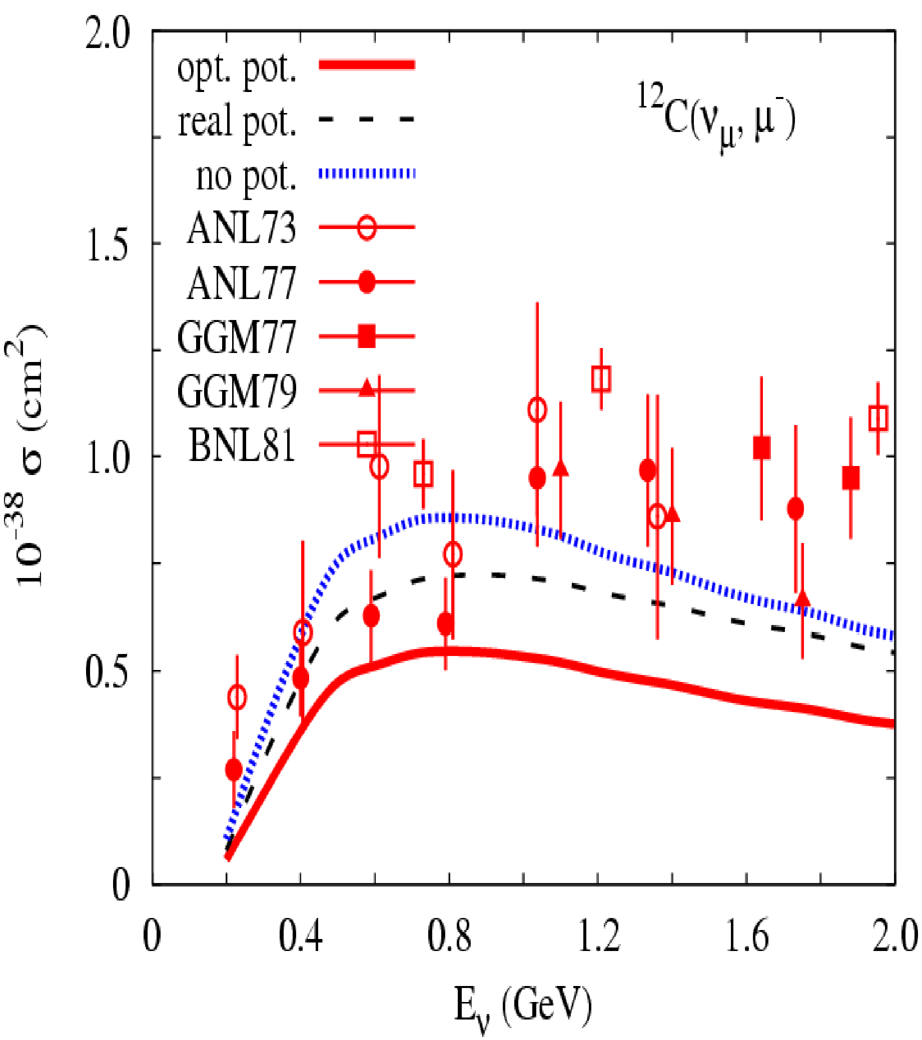


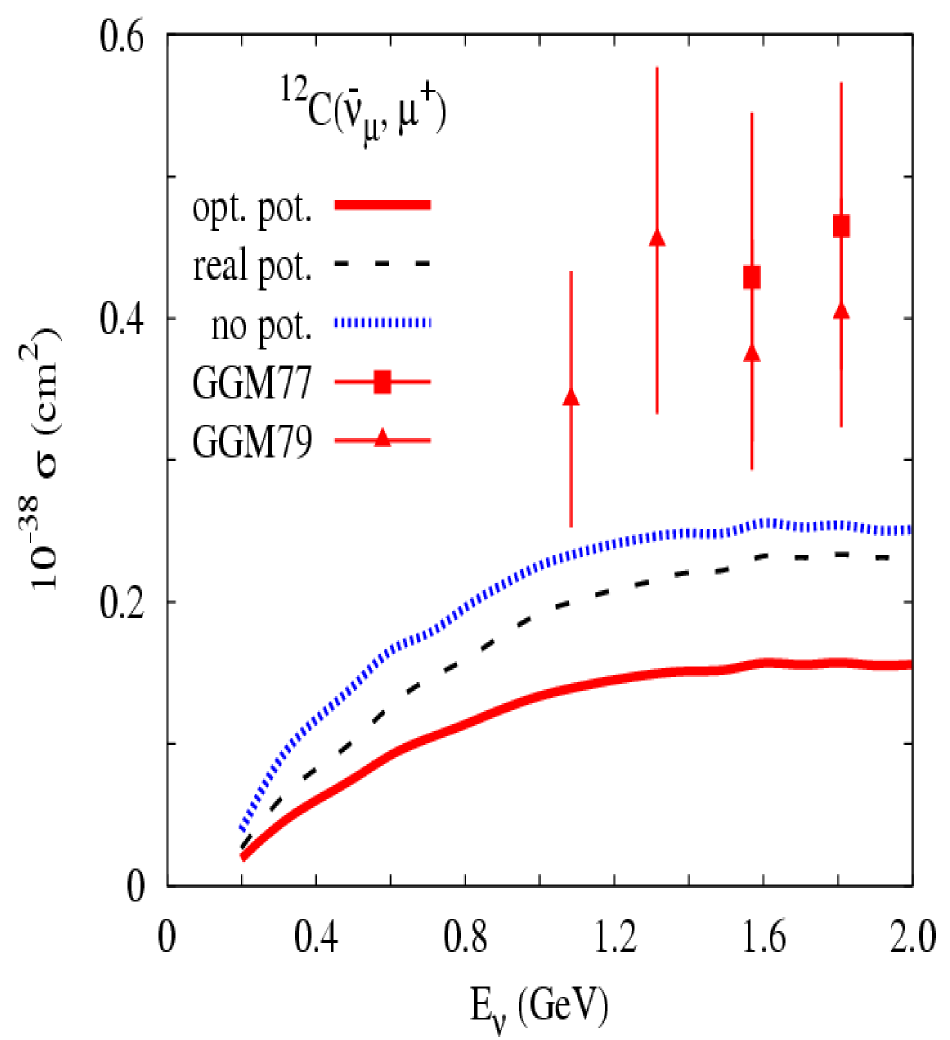
Total cross section (CC reaction)

without the Coulomb effect of final leptons

$\nu_\mu + n \longrightarrow p + \mu^-$

$\bar{\nu}_\mu + p \longrightarrow n + \mu^+$





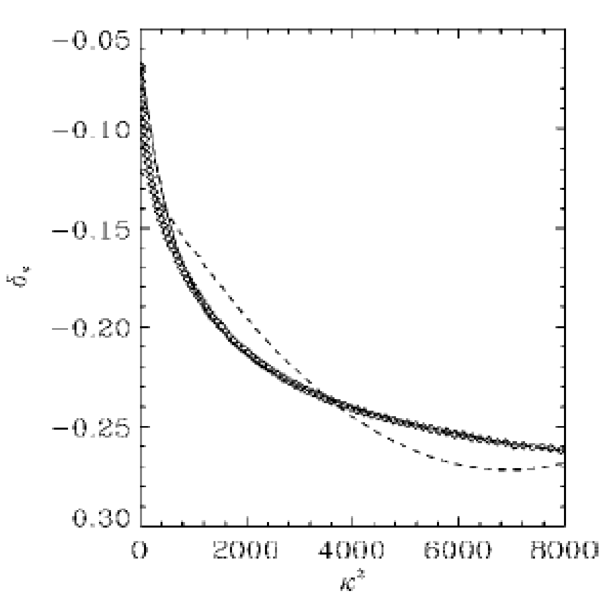
Coulomb Effect

Approximate electron wave functions are given by

$$\Psi^{\pm}(\mathbf{r}) = \frac{p'(r)}{p} \; e^{\pm i\delta(\mathbf{L}^2)} \; e^{i\Delta} \; e^{i\mathbf{p}'(r)\cdot\mathbf{r}} \; u_p$$

$$\mathbf{p}'(\mathbf{r}) = \left(\; p - \frac{1}{r}\int_0^r V(r)\,dr \right)\hat{\mathbf{p}}$$

local effective momentum approximation (LEMA)



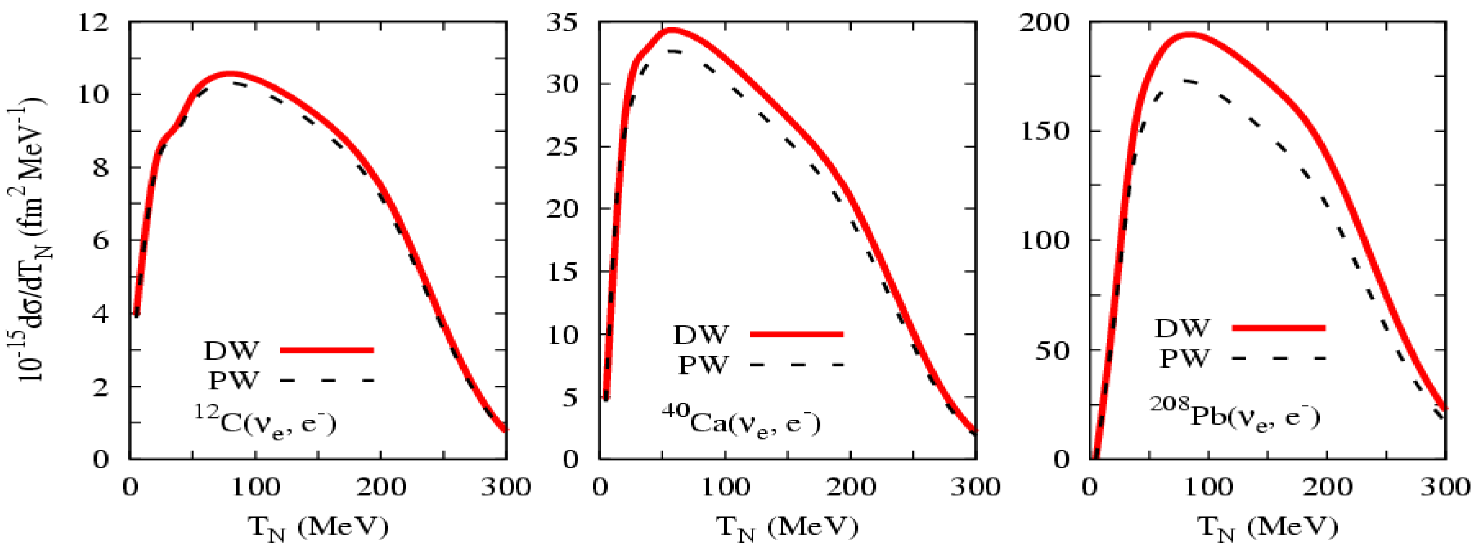
$$\Delta = a[\hat{\mathbf{p}}'(r)\cdot\hat{r}]\mathbf{L}^2$$

$$a = -\alpha Z[(16 \;\; \mathrm{MeV}/c)/p]^2$$

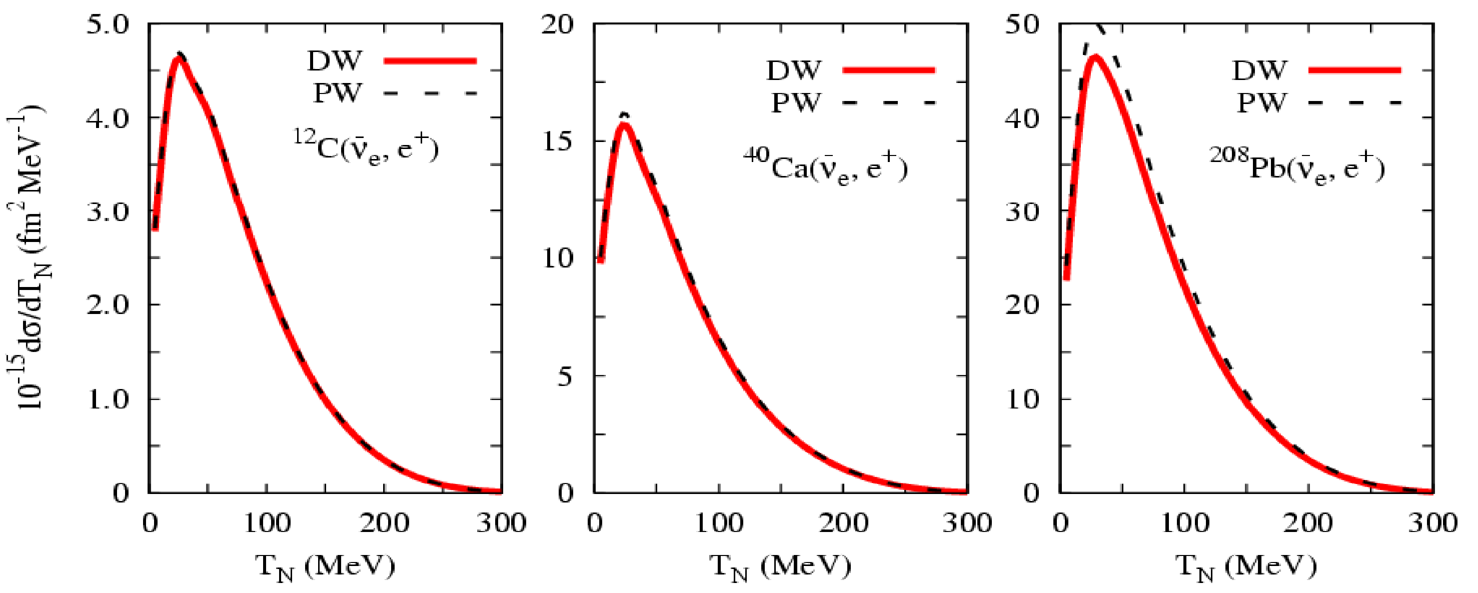
$$\delta(\kappa) = \left[a_0 + a_2 \frac{\kappa^2}{(pR)^2} \right] e^{-1.4\kappa^2/(pR)^2} - \frac{\alpha Z}{2}(1 - e^{-\kappa^2/(pR)^2})\ln(1+\kappa^2)$$

charged current neutrino-nucleus scattering

incoming neutrino (antineutrino) energy 500 MeV including the FSI



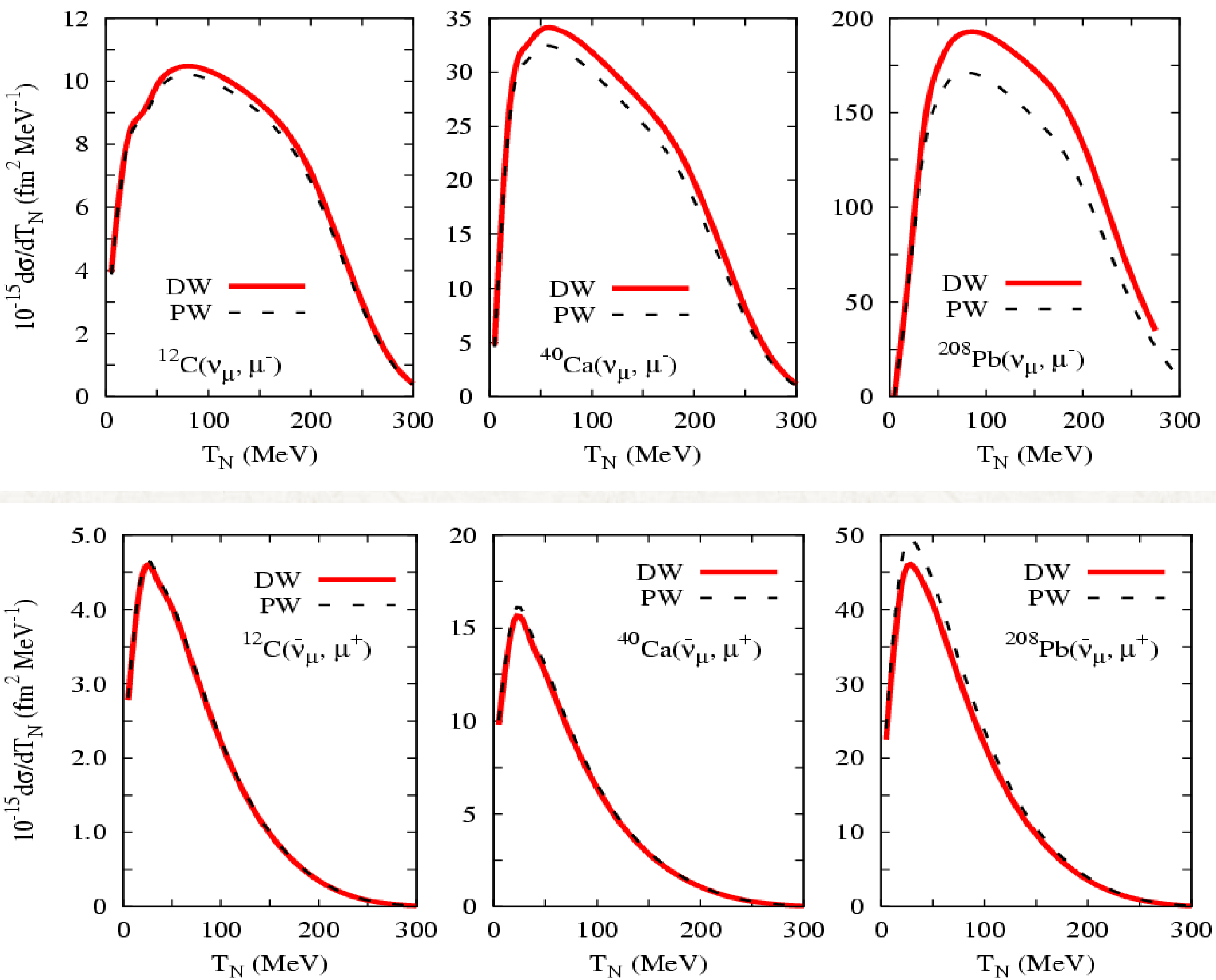
A (ν_e, e$^-$)



A ($\bar{\nu}_e$, e$^+$)

charged current neutrino-nucleus scattering

incoming neutrino (antineutrino) energy 500 MeV including the FSI



$A (\nu_\mu, \mu^-)$

$A (\bar{\nu}_\mu, \mu^+)$

Summary

- The effect of the FSI on the NC total cross section is about 50% for the optical potential and about 15% for the real potential for both incident neutrino and antineutrino.

- For the CC reaction, the effect reduces the total cross section about 50% for the optical potential and about 15% for the real potential for the incident neutrino and antineutrino without the Coulomb distortion of the final leptons.

- At low energies, the real potential describes the experimental data better than the optical potential for the neutrino-muon reaction.

- In (e,e') reaction, the electron Coulomb distortion effect is of the order of 3 % for ^{12}C, 7 % for ^{40}Ca, and 30 % for ^{208}Pb at intermediate electron energy.

- In (ν_e, e^-) reaction and (ν_μ, μ^-), the Coulomb distortion effect is about 2 % for ^{12}C, 4 % for ^{40}Ca, and 13 % for ^{208}Pb at incident neutrino energy 500 MeV.

- In ($\bar{\nu}_e$, e^+) reaction and ($\bar{\nu}_\mu$, μ^+), the Coulomb distortion effect is about 1 % for ^{12}C, 3 % for ^{40}Ca, and 8 % for ^{208}Pb at incident antineutrino energy 500 MeV.

- As for the case of positron, the Coulomb effect of (ν_e, $\bar{e}^+$) and (ν_μ, $\bar{\mu}^+$) reactions tends to saturate.

- The effect of the Coulomb distortion is about half of the electron scattering.

- In conclusion, it is difficult to say whether there is loss of flux, or not because of no enough experimental data even with inclusion of the Coulomb effect.