Neutrino-induced one-pion coherent production in nuclei. A (brief) theoretical status report

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General formulae

For NC processes

$$\nu_l(k) + \mathcal{N}_{gs} \rightarrow \nu_l(k') + \mathcal{N}_{gs} + \pi^0(k_\pi)\,.$$

Defining $q = k - k'$, $y = q^0/E$, $t = (q - k_\pi)^2$, and $\phi_{k_\pi q}$ the pion azimuthal angle, and taking $\vec{q}$ along the positive $z-$axis and $\vec{k} \times \vec{k}\,'$ along the positive $y-$axis, one has in the LAB frame

$$\frac{d\sigma}{dq^2 dy dt\, d\phi_{k_\pi q}} = \frac{G^2}{16\pi^2} E\,\kappa \left(-\frac{q^2}{|\vec{q}\,|^2}\right) \left(u^2 \frac{d\sigma_L}{dt\, d\phi_{k_\pi q}} + v^2 \frac{d\sigma_R}{dt\, d\phi_{k_\pi q}} + 2uv \frac{d\sigma_S}{dt\, d\phi_{k_\pi q}} + \frac{d\mathcal{A}}{dt\, d\phi_{k_\pi q}}\right),$$

where

$$\frac{d\sigma_S}{dt\, d\phi_{k_\pi q}} = \frac{1}{2\pi}\frac{d\sigma_S}{dt}\;,\quad \frac{d\sigma_R}{dt\, d\phi_{k_\pi q}} = \frac{1}{2\pi}\frac{d\sigma_R}{dt}\;,\quad \frac{d\sigma_L}{dt\, d\phi_{k_\pi q}} = \frac{1}{2\pi}\frac{d\sigma_L}{dt}\;,\quad \int \frac{d\mathcal{A}}{dt\, d\phi_{k_\pi q}}\, d\phi_{k_\pi q} = 0$$

Thus (T. D. Lee and C. N. Yang, Phys. Rev. **126**, 2239 (1962))

$$\frac{d\sigma}{dq^2 dy dt} = \frac{G^2}{16\pi^2} E\,\kappa \left(-\frac{q^2}{|\vec{q}\,|^2}\right) \left(u^2 \frac{d\sigma_L}{dt} + v^2 \frac{d\sigma_R}{dt} + 2uv \frac{d\sigma_S}{dt}\right),$$

This is the starting point for PCAC-based models

PCAC models

At $q^2 = 0$ only σ_S contributes

$$q^2 \, \frac{d\sigma_S}{dt} = -\frac{\pi}{\kappa} \left(|\vec{q}\,|^2 H_{00} + q^0 |\vec{q}\,| \left(H_{0z} + H_{z0} \right) + (q^0)^2 \, H_{zz} \right)$$

For $q^2 = 0$ one has $q^0 = |\vec{q}\,|$ and then

$$\left(|\vec{q}\,|^2 H_{00} + q^0 |\vec{q}\,| \left(H_{0z} + H_{z0} \right) + (q^0)^2 \, H_{zz} \right) = q_\mu q_\nu H^{\mu\nu} = q_\mu \mathcal{J}^\mu_{NC} (q_\nu \mathcal{J}^\nu_{NC})^*$$

Since the vector NC is conserved we are left with the divergence of the axial part. Using PCAC

$$\langle \mathcal{N}_{gs} \pi^0(k_\pi) | q_\mu \mathcal{A}^\mu_{NC} | \mathcal{N}_{gs} \rangle_{q^2=0} = 2 f_\pi \, \langle \mathcal{N}_{gs} \pi^0(k_\pi) | -\mathrm{i}T | \pi^0(q) \mathcal{N}_{gs} \rangle_{q^2=0} \, ,$$

Thus

$$q^2 \, \frac{d\sigma_S}{dt} \bigg|_{q^2=0} = -4 \frac{E_\pi}{\kappa} \, f_\pi^2 \, \frac{d\sigma(\pi^0 \mathcal{N}_{gs} \to \pi^0 \mathcal{N}_{gs})}{dt} \bigg|_{q^2=0}$$

and then, neglecting the nucleus recoil $(q^0 = E_\pi)$, one can further write

$$\frac{d\sigma}{dq^2 dy \, dt} \bigg|_{q^2=0} = \frac{G^2 f_\pi^2}{2\pi^2} \frac{E \, u \, v}{|\vec{q}\,|} \, \frac{d\sigma(\pi^0 \mathcal{N}_{gs} \to \pi^0 \mathcal{N}_{gs})}{dt} \bigg|_{q^2=0, \, E_\pi = q^0} \, .$$

PCAC models

For $q^2 \neq 0$ a form factor is added

$$\frac{d\sigma}{dq^2 dy dt} = \frac{G^2 f_\pi^2}{2\pi^2} \frac{E\, u\, v}{|\vec{q}|} G_A^2 \left. \frac{d\sigma(\pi^0 \mathcal{N}_{gs} \to \pi^0 \mathcal{N}_{gs})}{dt} \right|_{E_\pi = q^0}$$

with

$$G_A = 1/(1 - q^2/m_A^2)$$

This is the Berger-Sehgal model for NC coherent π^0 production [Phys. Rev. D79,053003 (2009)]. (See dedicated talk on friday)

Kartavtsev, Paschos and Gounaris [Phys. Rev. D74, 054007 (2006)] keep also the non-PCAC σ_L, σ_R contributions, while in Paschos and Schalla [arXiv:0903.0451] the latter have been neglected. (See dedicated talk on friday)

In the Rein-Sehgal model [Nuc. Phys. B223, 29 (1983)] one further uses

$$\frac{Euv}{|\vec{q}|} \quad \to \quad \frac{1-y}{y} \quad (\text{exact for } \; q^2 = 0)$$

$$\frac{d\sigma(\pi^0 \mathcal{N}_{gs} \to \pi^0 \mathcal{N}_{gs})}{dt} \quad = \quad |F_A(t)|^2 \; F_{\text{abs}} \left. \frac{d\sigma(\pi^0 N \to \pi^0 N)}{dt} \right|_{t=0}$$

PCAC models, caveats

One would have to eliminate the initial pion distortion from the physical $\pi - \mathcal{N}$ cross sections

For $q^2 \neq 0$, $d\sigma/dq^2 dy\, dt$ is not enough to determine $d\sigma/dq^2 dy\, d\cos\theta_\pi$.
(q^2, y, t and the incoming neutrino energy are not enough to determine $\cos\theta_\pi$. For that one also needs the value of $\phi_{k_\pi q}$)
Thus to get differential cross sections with respect to $cos\theta_\pi$ one needs to start from

$$\frac{d\sigma}{dq^2 dy\, dt\, d\phi_{k_\pi q}}$$

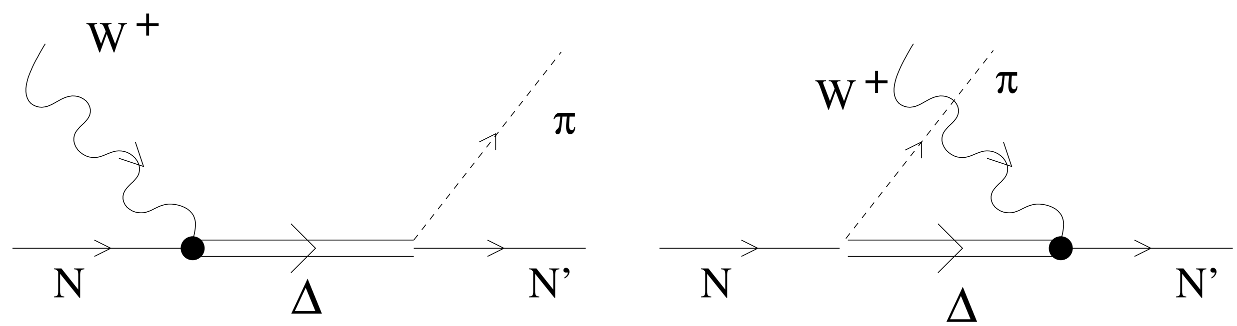
In PCAC models that information is lost and they have to assume

$$\left.\frac{d\sigma}{dq^2 dy\, dt\, d\phi_{k_\pi q}}\right|_{PCAC} = \frac{1}{2\pi}\frac{d\sigma}{dq^2 dy\, dt}$$

This is not correct for $q^2 \neq 0$

Microscopic models

The dominant contribution to the elementary amplitude at low energies is given by the Δ pole mechanism



Dominated by the C_5^A axial contribution

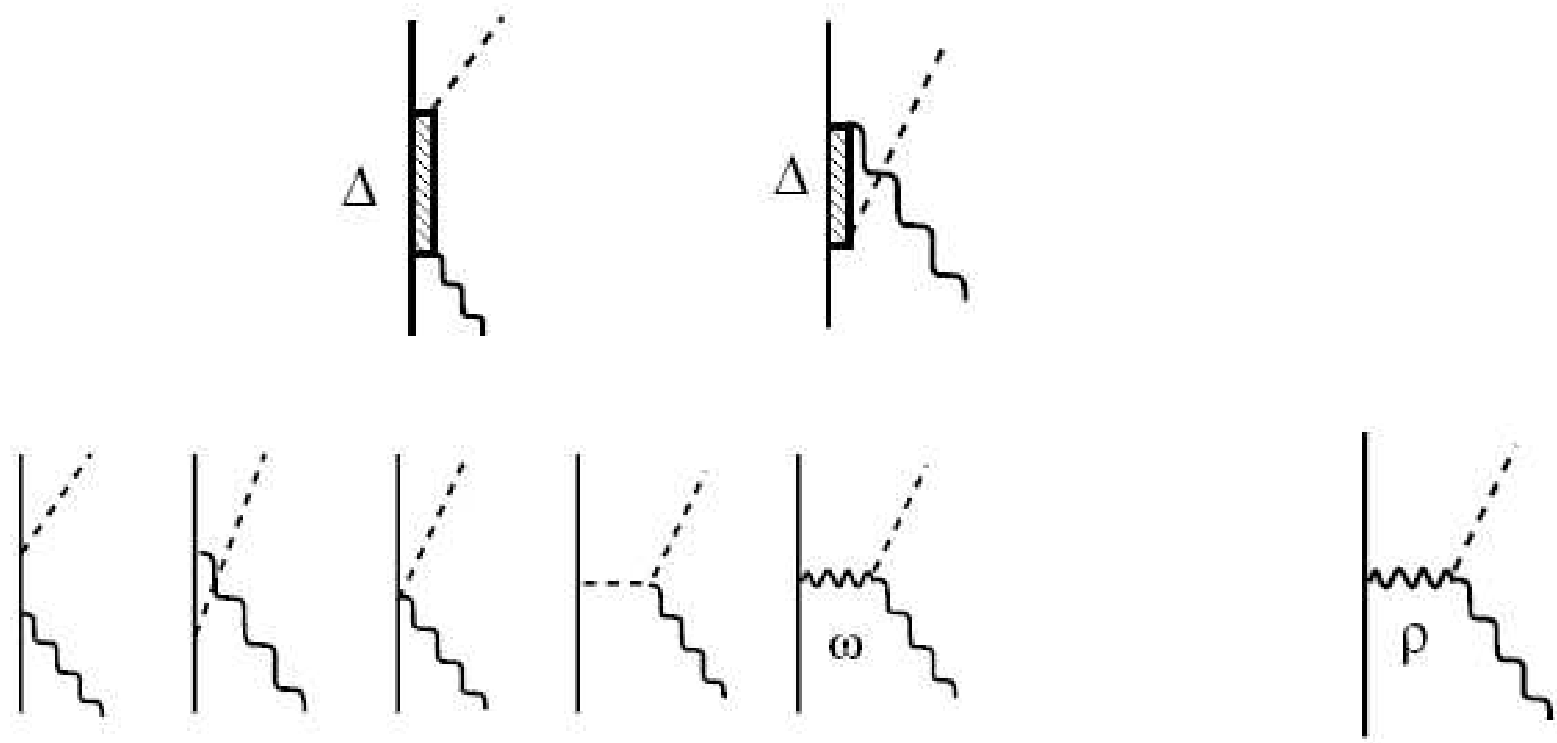
$$C_5^A(0) = 1.2$$

(Off-diagonal Goldberger-Treiman relation)

This is the case for the calculations of coherent pion production by L. Alvárez-Ruso et al. [Phys. Rev. D75, 055501 (2007); Phys. Rev. D76, 068501 (2007)]

Microscopic models

Background terms in the elementary amplitude were included by Sato, Uno and Lee [Phys. Rev. C67, 065201 (2003)]

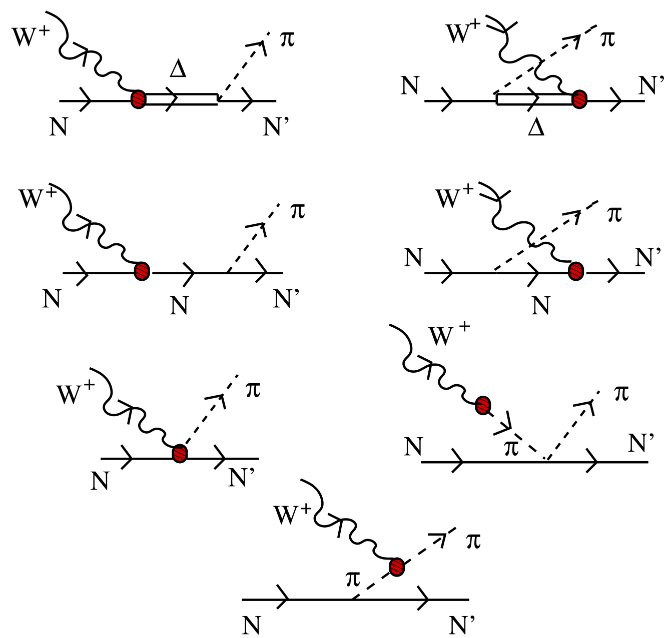


Bare form factors extracted from quark models ($C_5^A(0)|_{bare} \approx 0.8$) further dressed by dynamical pion cloud effects

This is the elementary model used in the recent calculation of the coherent pion production process by Nakamura et al. (arXiv:0901.2366) (See dedicated talk on friday)

Microscopic models

Background terms have been also included in the works of Hernández et al. [Phys. Rev. D76, 033005 (2007); Phys. Rev. D79, 013002 (2009)] by means of a SU(2) chiral lagrangian



$$C_5^A(0) = 0.867 \ \ (Fit\ to\ ANL\ data\ on\ the\ nucleon)$$

Background terms turn out to be irrelevant for coherent production in symmetric nuclei $\Longrightarrow$ Reduction of the coherent cross section

Microscopic models

Medium modifications, pion distortion, nonlocalities

Δ properties are strongly modified inside the nuclear medium.

$$M_\Delta \rightarrow M_\Delta + \mathrm{Re}\Sigma_\Delta$$
$$\Gamma_\Delta/2 \rightarrow \Gamma_\Delta^{\mathrm{Pauli}}/2 - \mathrm{Im}\Sigma_\Delta$$

Pion distortion effects are also very important, specially for $|\vec{k}_\pi| < 0.5$ GeV

$$e^{-\mathrm{i}\vec{k}_\pi \cdot \vec{r}} \rightarrow \widetilde{\varphi}_\pi^*(\vec{r}; \vec{k}_\pi) \quad (Klein - Gordon\ equation)$$

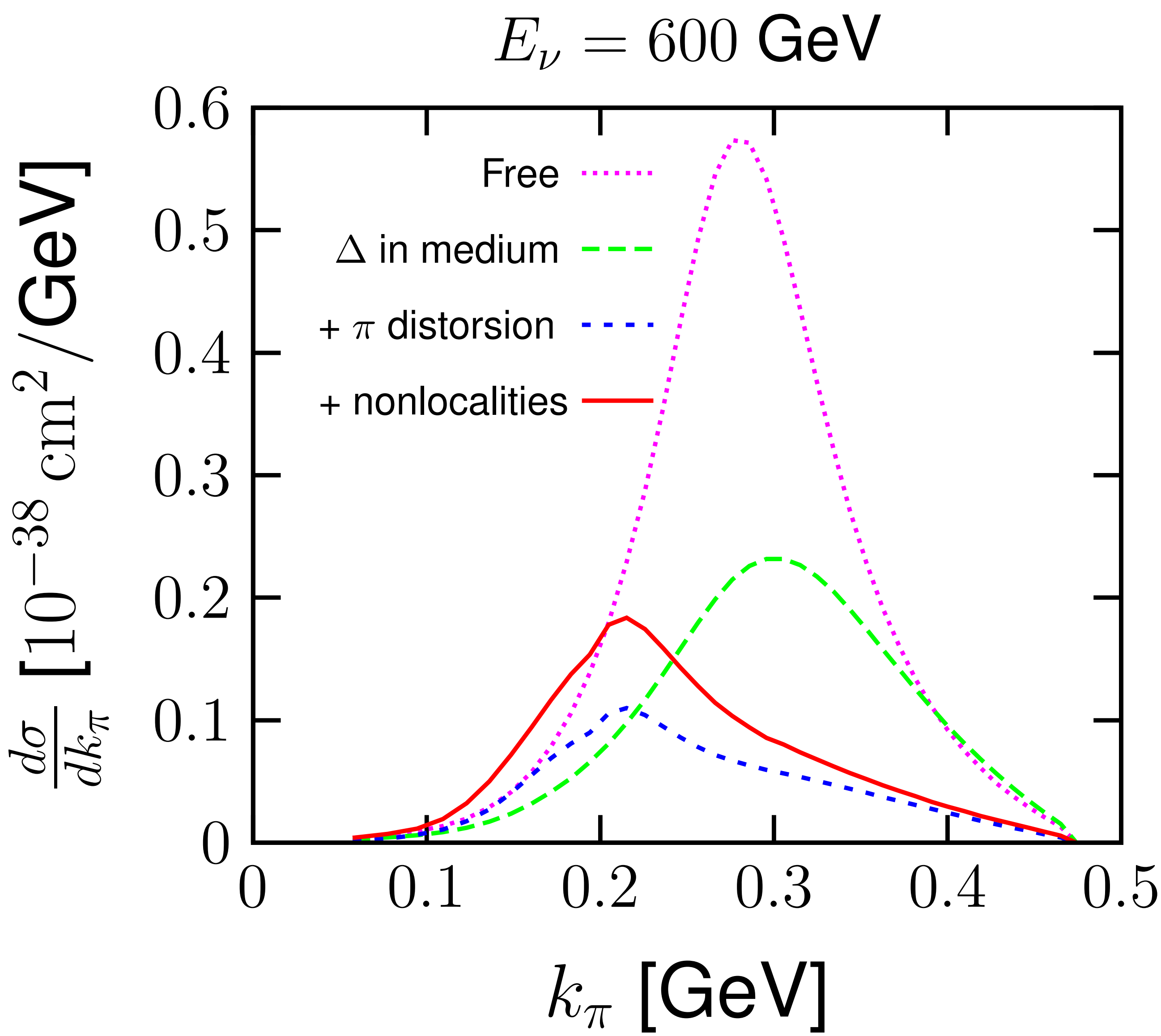
Nonlocalities in the pion momenta

$$\vec{k}_\pi e^{-\mathrm{i}\vec{k}_\pi \cdot \vec{r}} \quad \rightarrow \quad \mathrm{i}\vec{\nabla}\widetilde{\varphi}_\pi^*(\vec{r}; \vec{k}_\pi) \quad (first\ order\ terms\ in\ k_\pi)$$

Alvarez-Ruso et al., Hernández et al.

Microscopic models

Medium modifications, pion distortion, nonlocalities



$\nu\ ^{16}O \rightarrow \mu^-\ ^{16}O\ \pi^+$

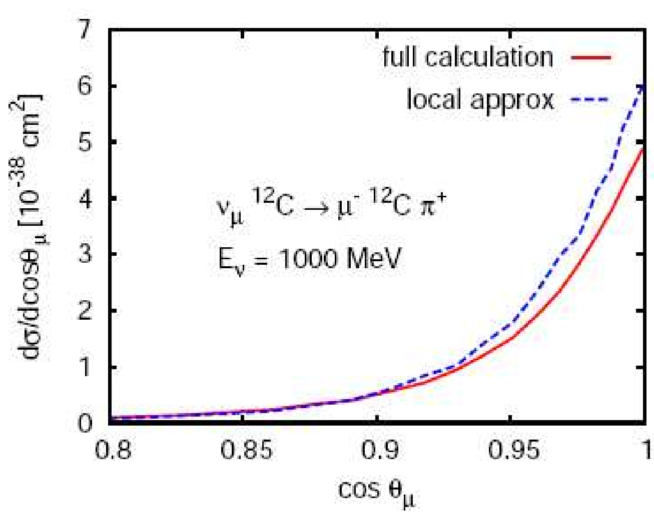
Microscopic models, caveats

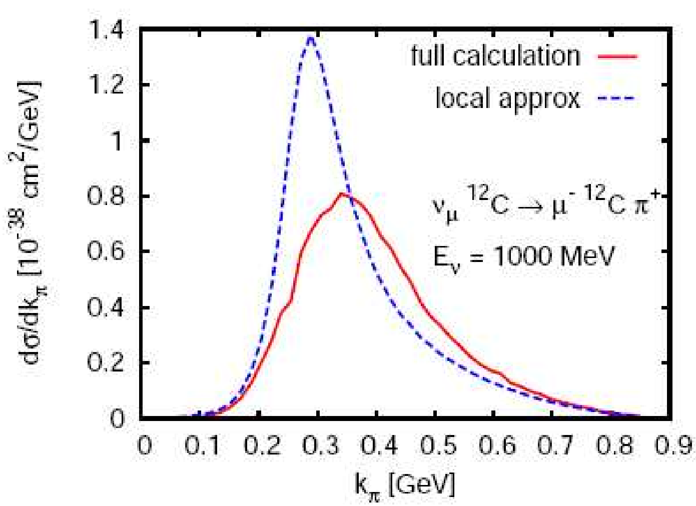
C_5^A value

- Goldberger-Treiman relation gives $C_5^A(0) = 1.2$
- Quark models favor a smaller $C_5^A(0)$ value (≈ 0.9)
- Lattice determination by C. Alexandrou et al. [Phys. Rev. Lett. 98, 052003 (2007)] not conclusive (It is not clear what the extrapolation to the chiral limit would give)

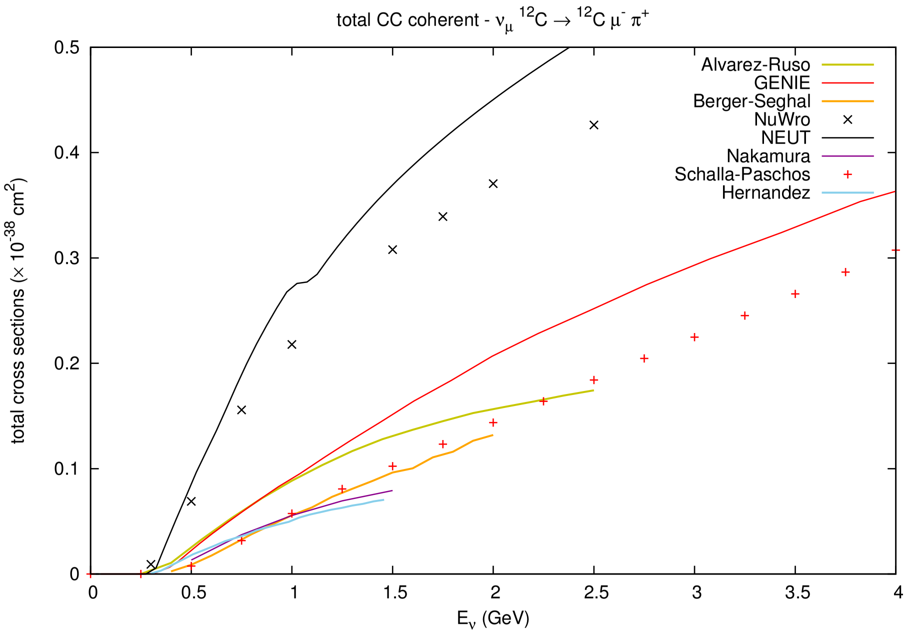
Role of higher energy resonances

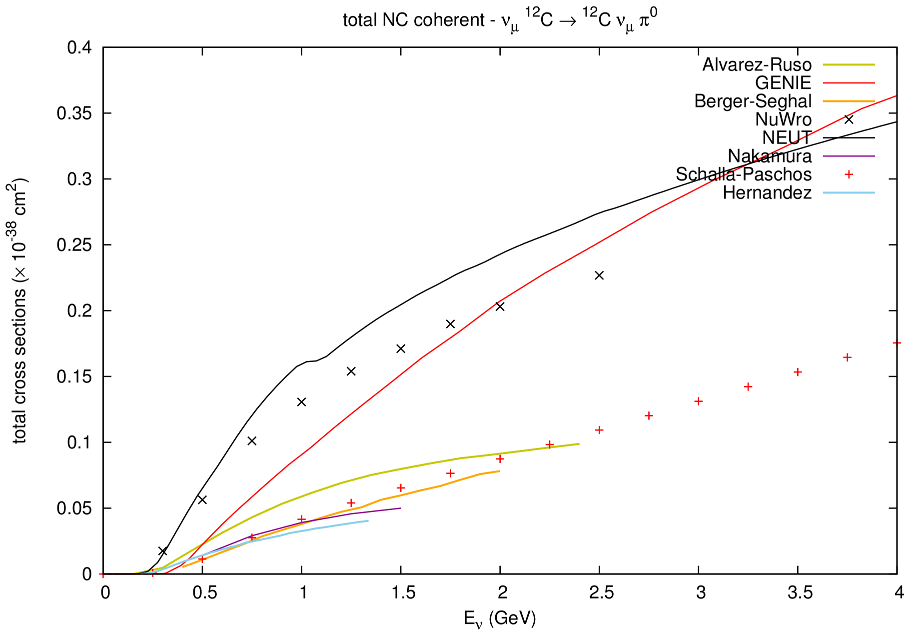
Nonlocalities in nucleon momenta could be important (Leitner, Mosel, Winkelmann, arXiv:0901.2837). (See poster this afternoon)





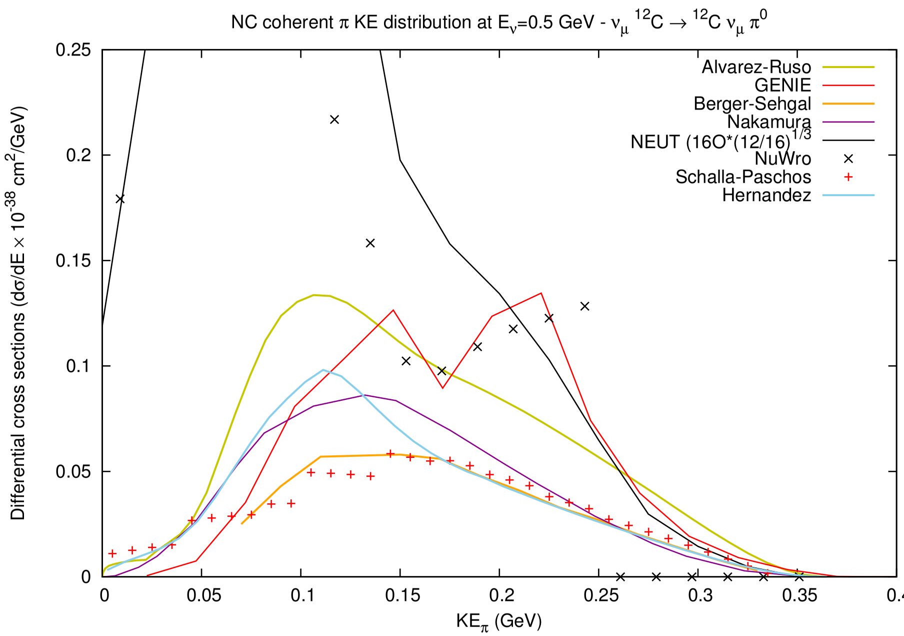
Results I

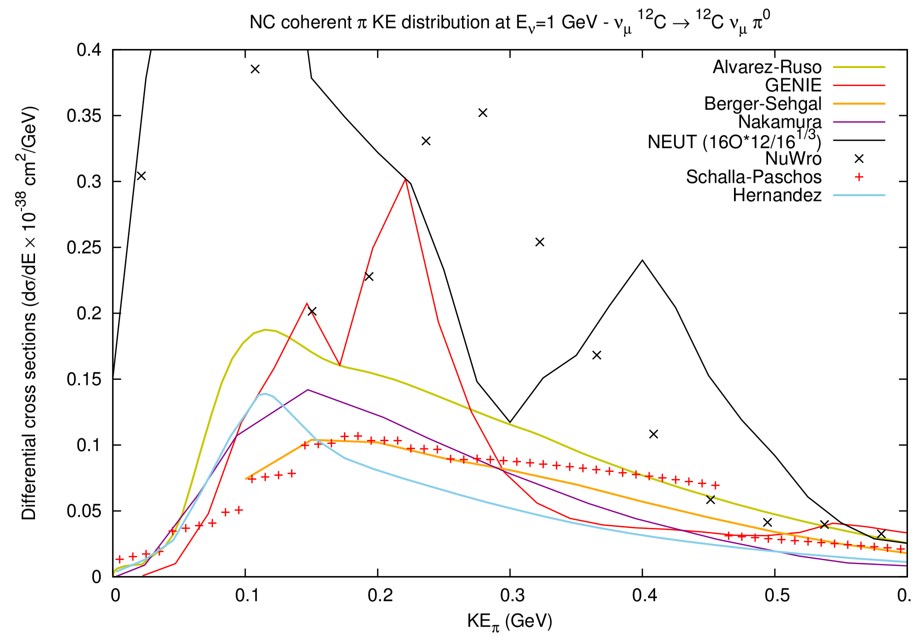


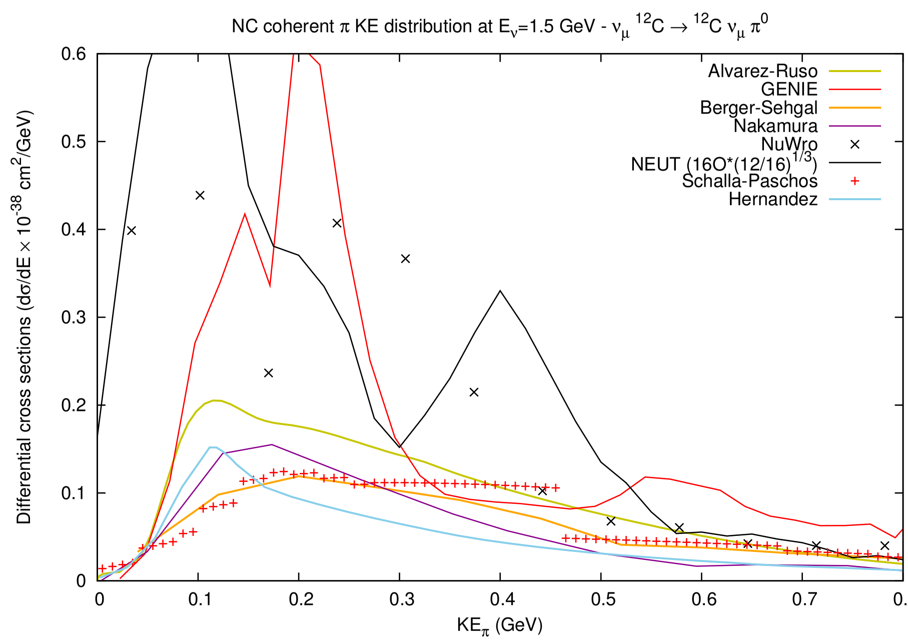


Total σ for CC and NC coherent pion production on Carbon as a function of E_ν

Results II

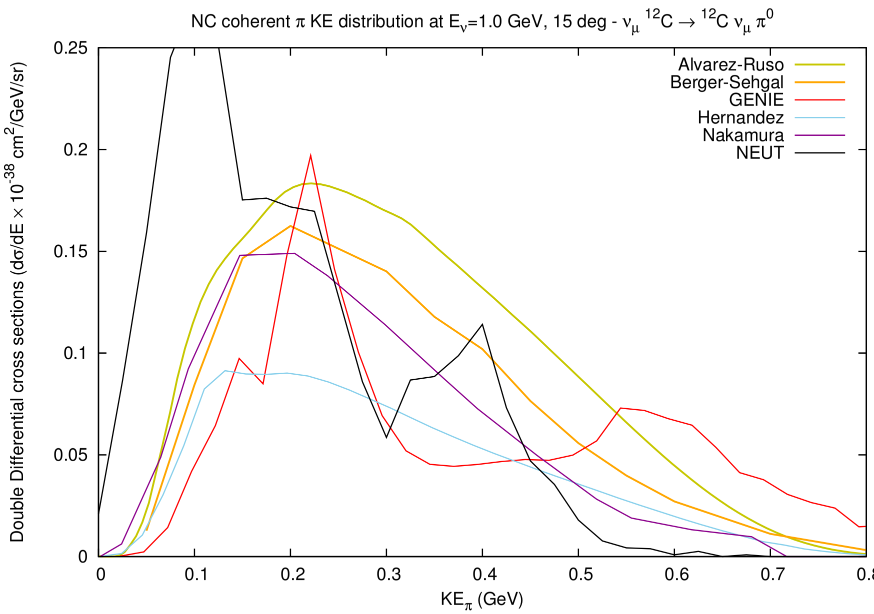


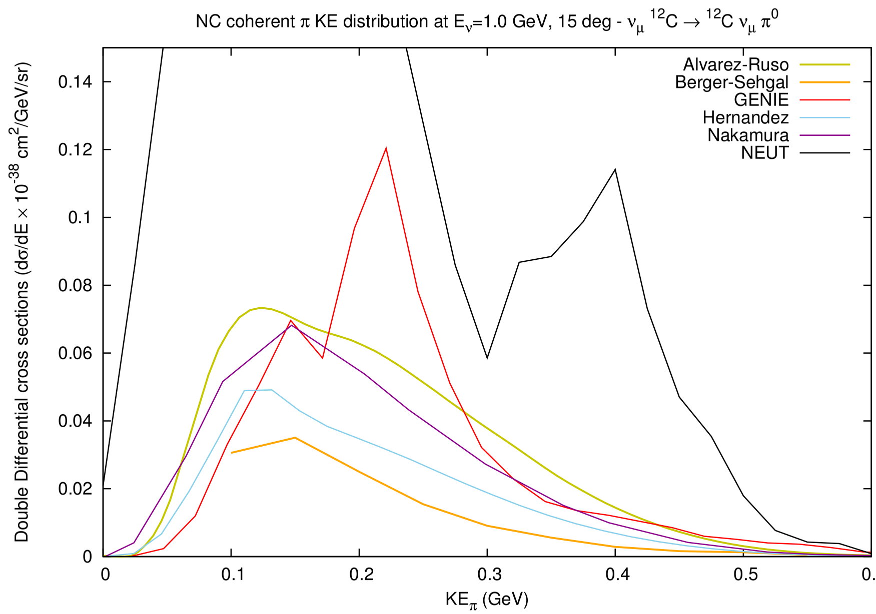


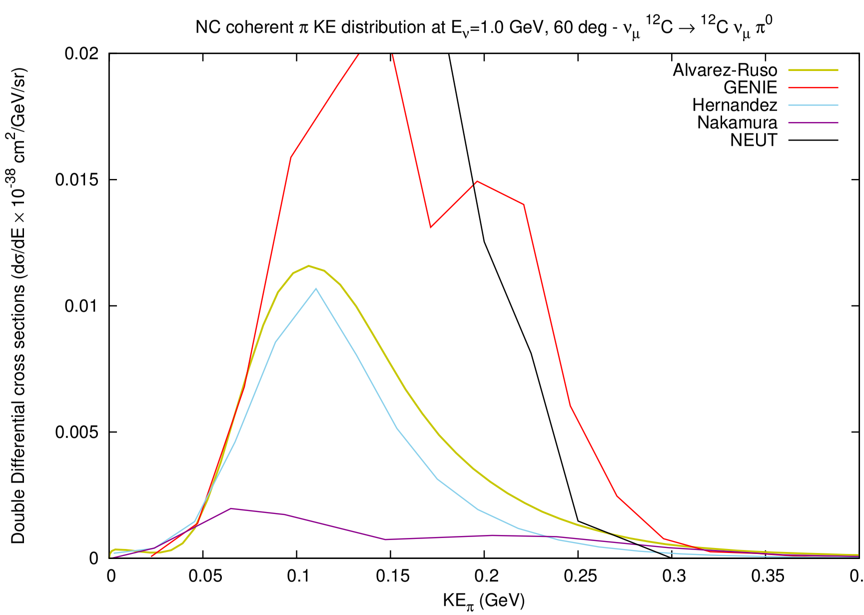


$d\sigma/dT_\pi$ for NC π^0 coherent production on Carbon and $E_\nu = 0.5, 1.0, 1.5$ GeV

Results III







$\frac{1}{2\pi}\frac{d\sigma}{dT_\pi\, d\cos\theta_\pi}$ for NC π^0 coherent production on Carbon for $E_\nu = 1$ GeV