PCAC and coherent pion production by neutrinos

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$$CC : \nu_\mu\, ^{12}C \rightarrow \mu^-\, ^{12}C\pi^+$$

$$NC : \nu_\mu\, ^{12}C \rightarrow \nu_\mu\, ^{12}C\pi^\circ$$

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Our starting point is the general formula for neutrino scattering off a nucleus or nucleon at rest

$$\frac{d\sigma^{CC}}{dQ^2dy} = \frac{G_F^2\cos^2\theta_C}{4\pi^2}\kappa E\frac{Q^2}{|\boldsymbol{q}|^2}\left[u^2\sigma_L + v^2\sigma_R + 2uv\sigma_S\right] \tag{1}$$

already derived by Lee and Yang[b] for zero mass of the outgoing lepton[c]. For $Q^2 \to 0$ only the term $\sim \sigma_S$ survives.

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[b] Phys.Rev. 126,2239 (1962)

[c] $y = (E - E')/E$, $\kappa = (W^2 - M_N^2)/2M_N$, $u, v = (E + E' \pm |\boldsymbol{q}|)/2E$

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Here Adlers forward scattering theorem[d] based on PCAC predicts

$$\sigma_{S,\nu N \to l' F}(W) = \frac{|\boldsymbol{q}|}{\kappa Q^2} f_\pi^2 \sigma_{\pi N \to F}(W) \ . \qquad (2)$$

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[b] Phys.Rev. 126,2239 (1962)

[c] $y = (E - E')/E$, $\kappa = (W^2 - M_N^2)/2M_N$, $u, v = (E + E' \pm |\boldsymbol{q}|)/2E$

[d] Phys.Rev. 135,B963 (1964)

resulting in

$$\left.\frac{d\boldsymbol{\sigma}^{CC}}{dQ^2dy}\right|_{Q^2\to 0} = \frac{G_F^2\cos^2\theta_C f_\pi^2}{2\pi^2}\frac{E}{|\boldsymbol{q}|}uv\boldsymbol{\sigma}_{\pi N}(W) \tag{3}$$

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and[e]

$$\left.\frac{d\boldsymbol{\sigma}^{NC}}{dQ^2dy}\right|_{Q^2\to 0} = \frac{G_F^2 f_\pi^2}{4\pi^2}\frac{E}{|\boldsymbol{q}|}uv\boldsymbol{\sigma}_{\pi N}(W) \quad . \tag{4}$$

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For CC the limit $Q^2 = 0$ cannot be reached. Therefore and for comparison with experiments we extrapolate to finite values of Q^2 by introducing a formfactor $G_A = m_A^2/(Q^2 + m_A^2)$. In addition we include a correction (already contained in Adlers paper) due to the nearby pion pole in the hadronic axial vector current[f]

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[e] $f_{\pi^\circ} = f_{\pi^+}/\sqrt{2}$

[f] Kopeliovich,Marage Int.J.Mod.Phys. A8,1513 (1993)

$$\frac{d\boldsymbol{\sigma}^{CC}}{dQ^2dy} = \frac{G_F^2 \cos^2\theta_C f_\pi^2}{2\pi^2}\frac{E}{|\boldsymbol{q}|}uv\left(G_A - \frac{1}{2}\frac{Q^2_{\min}}{Q^2+m_\pi^2}\right)^2 \sigma_{\pi N} \quad . \qquad (5)$$

With $Q^2_{\min} = m^2_{l'}y/(1-y)$ the pion pole term vanishes for $m_{l'} = 0$, it is a lepton mass correction.

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Coherent scattering Coherent πN scattering is strongly peaked in forward direction distinguishing it from incoherent background. We therefore expect coherent single pion production by neutrinos to be well described by the PCAC Ansatz. Like in the original Rein Sehgal (RS) paper[g] this approximation is assumed to hold also for the differential cross section

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[g] Nucl.Phys. B223,29(1983)

$$\frac{d\boldsymbol{\sigma}^{CC}}{dQ^2dydt} = \frac{G_F^2\cos^2\theta_C f_\pi^2}{2\pi^2}\frac{E}{|\boldsymbol{q}|}uv\left(G_A - \frac{1}{2}\frac{Q^2_{\min}}{Q^2+m_\pi^2}\right)^2 \times \frac{d\boldsymbol{\sigma}(\pi^+N\to\pi^+N)}{dt} \tag{6}$$

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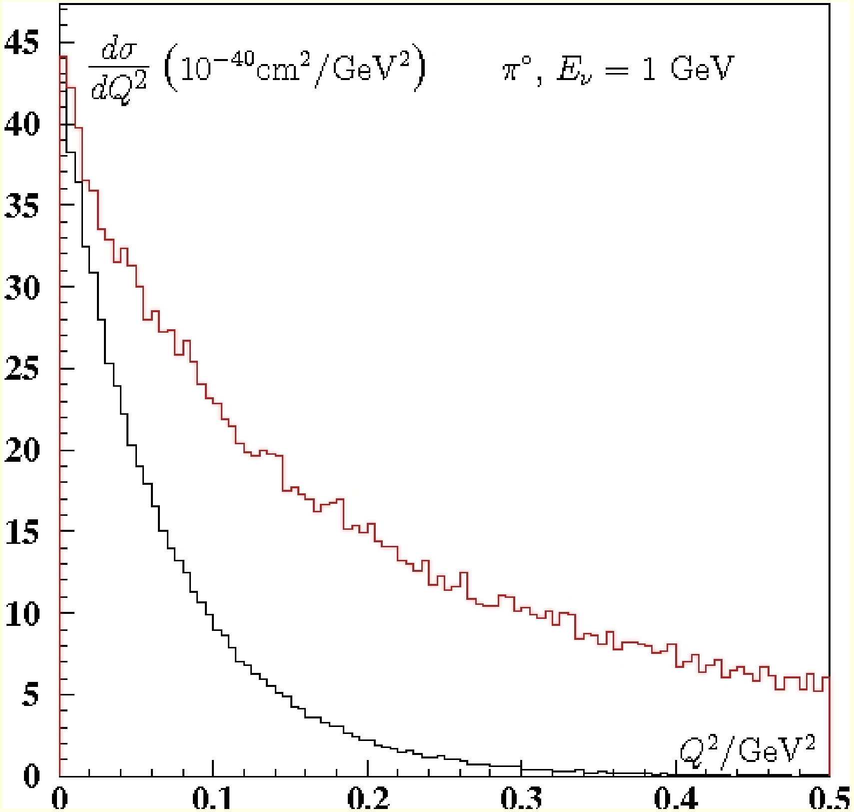
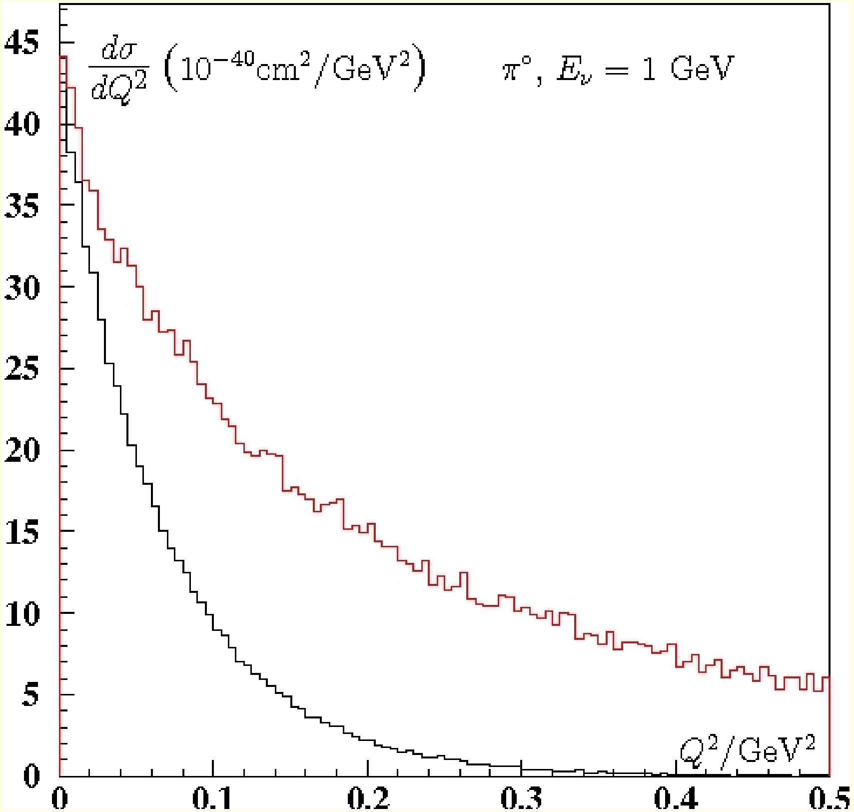
This extension is by no means trivial. t is the four momentum transfer squared between the incoming virtual boson and the outgoing pion. Therefore $t = 0$ cannot be reached. $t_{\min} = f(Q^2)$ results in a very effective Q^2 cutoff for cross sections $\sim A\exp(-bt)$. The t-integral of (6) approaches (5) only for $Q^2 \to 0$.



Black histogram calculated from integrating (6) over (t, y), red histogram from integrating (5) over y. Hadronic toy model: constant cross section ($80\,\mathrm{mb}$), constant slope ($40\,\mathrm{GeV}^{-2}$).



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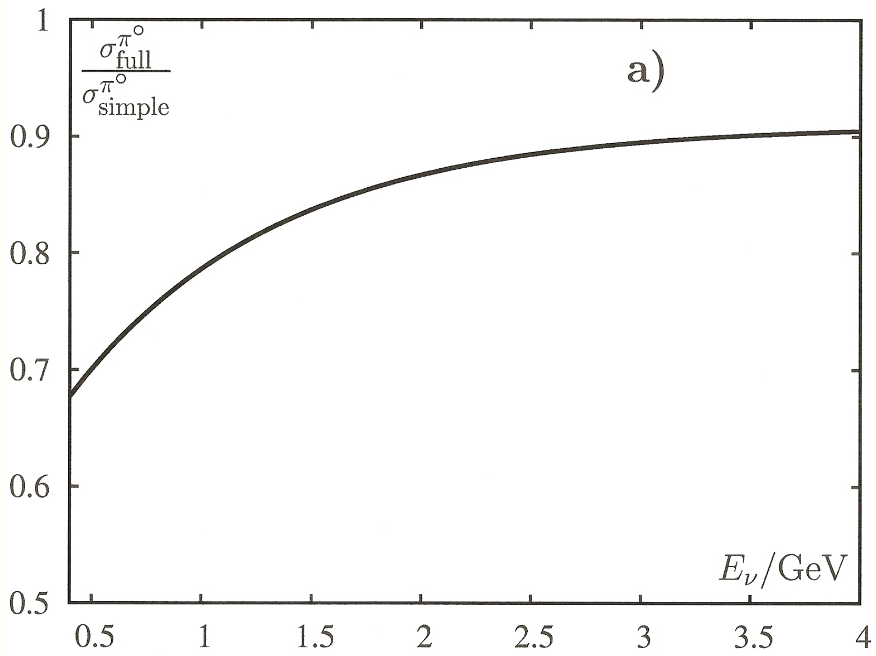
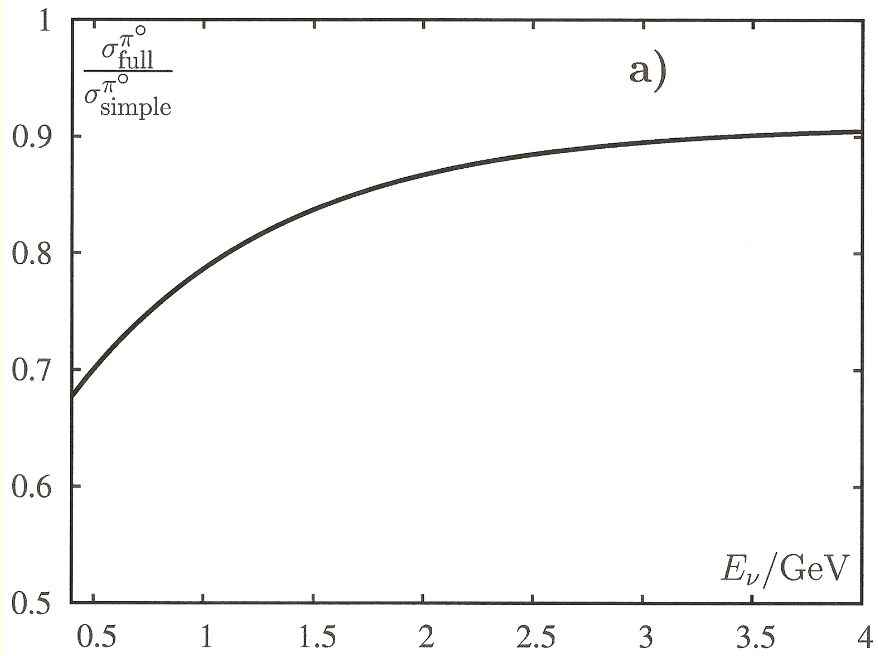
The RS paper evaluates the kinematical factor always at $Q^2 = 0$, i.e. $Euv/|\boldsymbol{q}| \to (1-y)/y$. At high energies the differences are negligible. At threshold they are very important (hadronic toy model). Note also the strong violation of isospin symmetry.



$\frac{\sigma^{\pi^\circ}_{\text{full}}}{\sigma^{\pi^\circ}_{\text{simple}}}$
a)
E_ν/GeV
1
0.9
0.8
0.7
0.6
0.5
0.5
1
1.5
2
2.5
3
3.5
4



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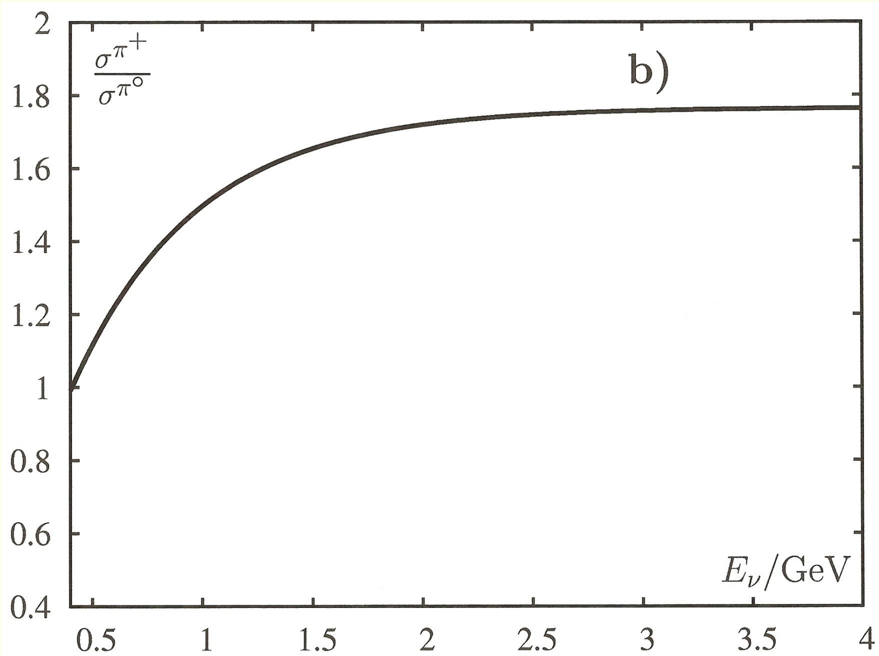
3.5
4



$\frac{\sigma^{\pi^+}}{\sigma^{\pi^\circ}}$
b)
E_ν/GeV
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.5
1
1.5
2
2.5
3
3.5
4

The elastic πN cross section The RS paper contained a simple model for elastic pion nucleus scattering, which can be easily implemented into MC generators. Starting from

$$\frac{d\sigma(\pi N \to \pi N)}{dt} = A^2 \frac{d\sigma_{\mathrm{el}}}{dt}\Big|_{t=0} e^{-bt} F_{\mathrm{abs}} \qquad (7)$$

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the elastic differential cross section at $t = 0$ is calculated via the optical theorem

$$\frac{d\sigma_{\rm el}}{dt}\Big|_{t=0} = \frac{1}{16\pi}\left(\frac{\sigma_{\rm tot}^{\pi^+ p} + \sigma_{\rm tot}^{\pi^- p}}{2}\right)^2 \tag{8}$$

where the total pion nucleon cross sections are taken from data. The slope b is determined via the optical model

$$b = \frac{1}{3} R_0^2 A^{2/3} \;\; \text{e.g. } R_0 = 1.057\,\text{fm} \quad . \tag{9}$$

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Finally using a simple geometrical picture the absorption factor

$$F_{\text{abs}} = \exp\left(-\frac{9A^{1/3}}{16\pi R_0^2}\sigma_{\text{inel}}\right) \tag{10}$$

is calculated from data for inelastic pion nucleon scattering.

$$\sigma_{\text{inel}} = \frac{\sigma_{\text{inel}}^{\pi^+ p} + \sigma_{\text{inel}}^{\pi^- p}}{2} \quad . \tag{11}$$

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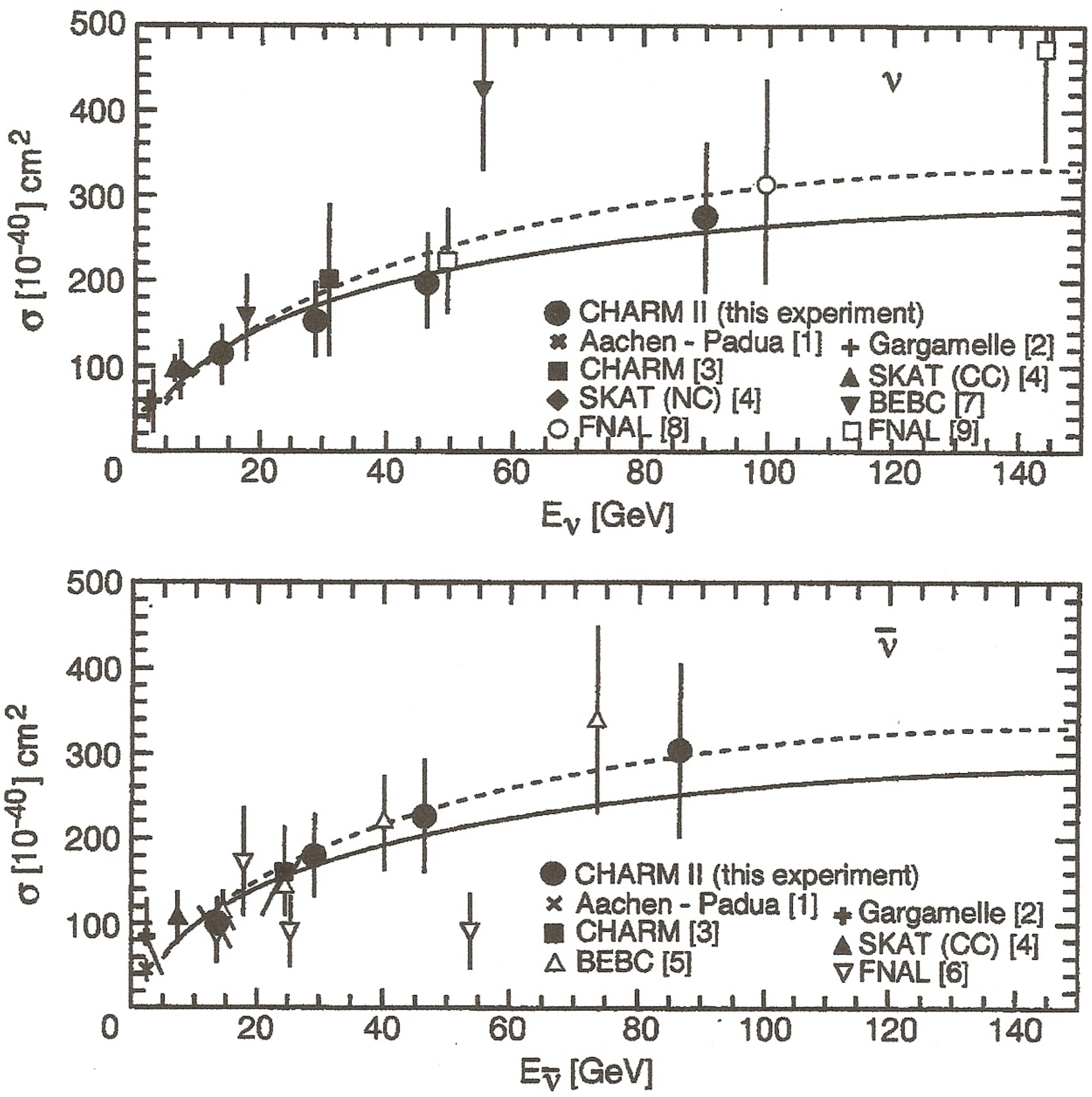
Although this model has its limitations (e.g. it predicts $\sigma \to 0$ for $A \to \infty$) it has been very successful in describing high energy coherent neutrino scattering.


ν
CHARM II (this experiment)
Aachen - Padua [1]
Gargamelle [2]
CHARM [3]
SKAT (CC) [4]
SKAT (NC) [4]
BEBC [7]
FNAL [8]
FNAL [9]
σ [10^-40] cm^2
E_ν [GeV]
ν̄
CHARM II (this experiment)
Aachen - Padua [1]
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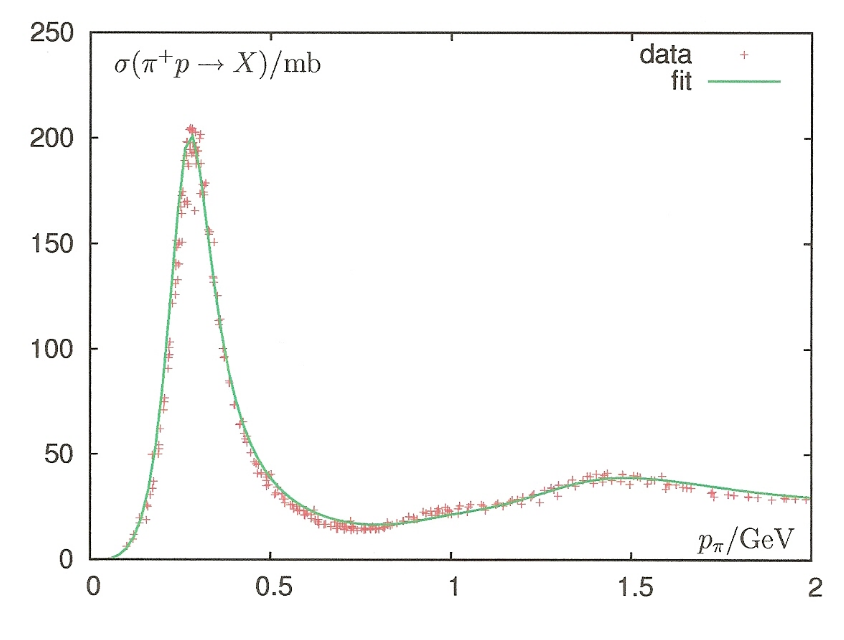
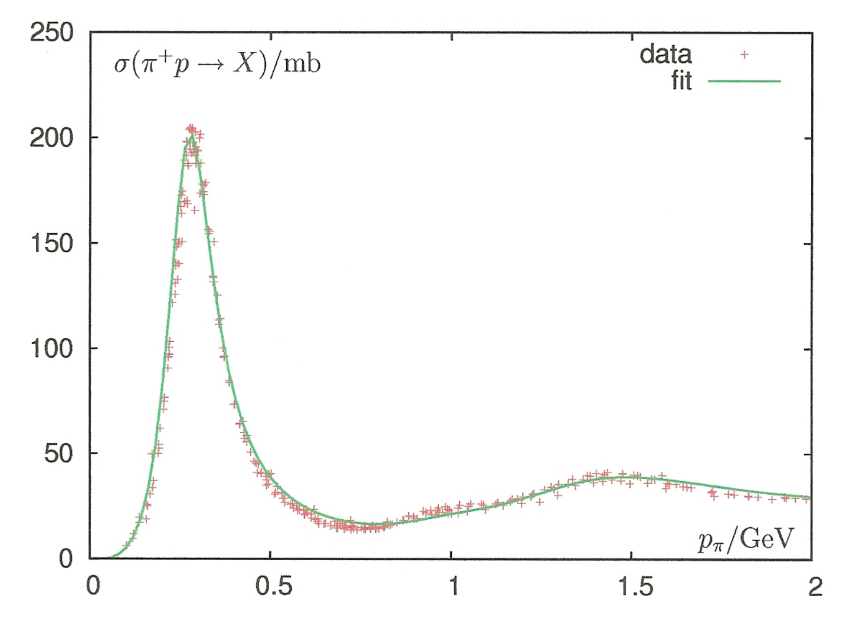
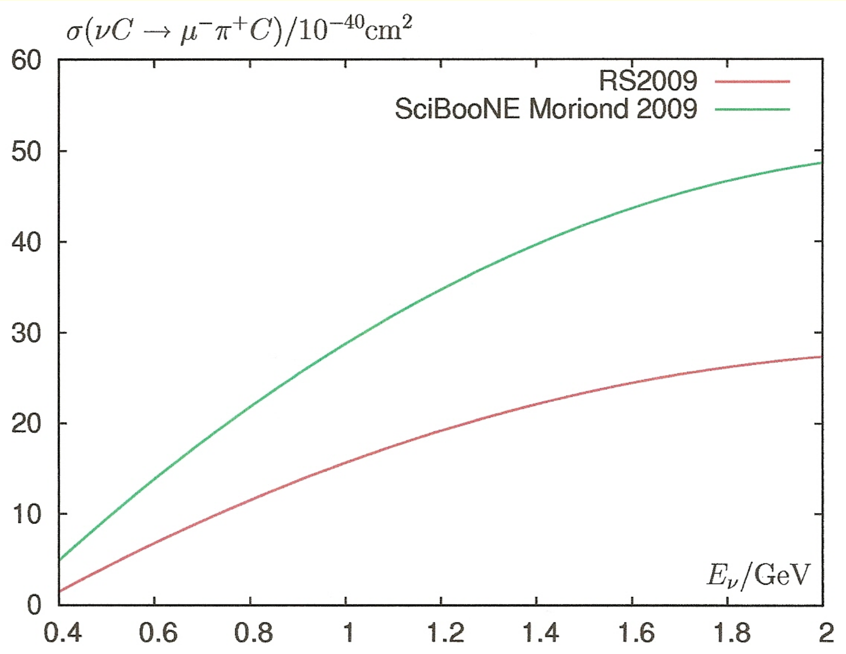
σ [10^-40] cm^2
E_ν̄ [GeV]



The updated hadronic RS 2009 model uses detailed fits to the πN cross sections which is important for low neutrino energies.





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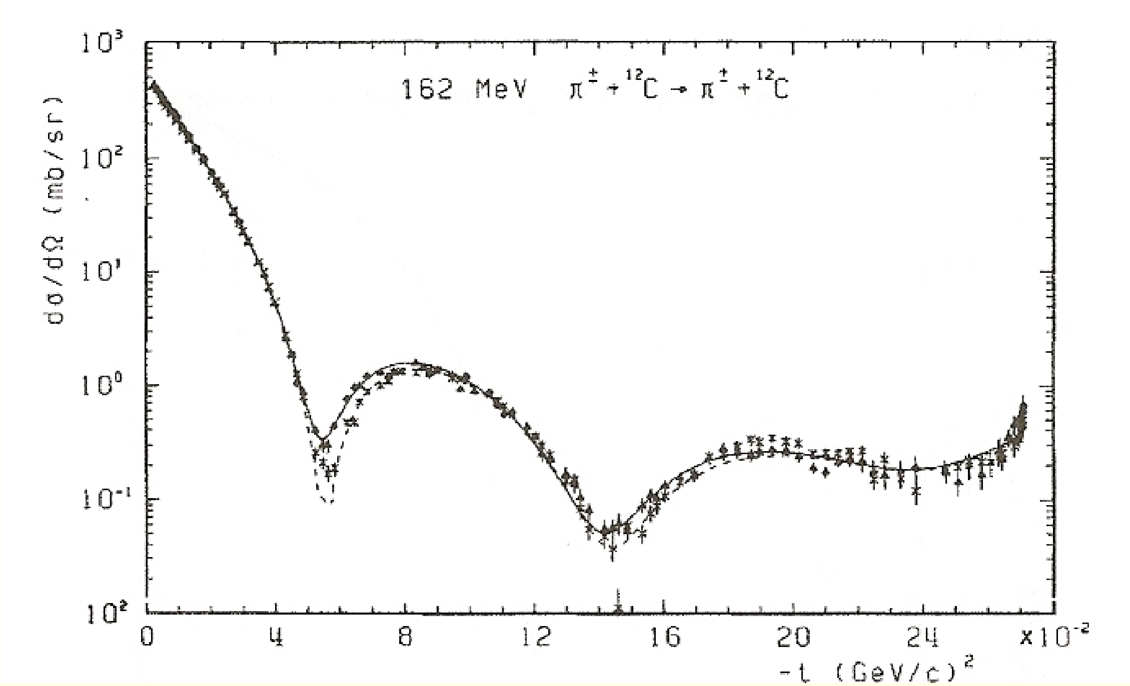
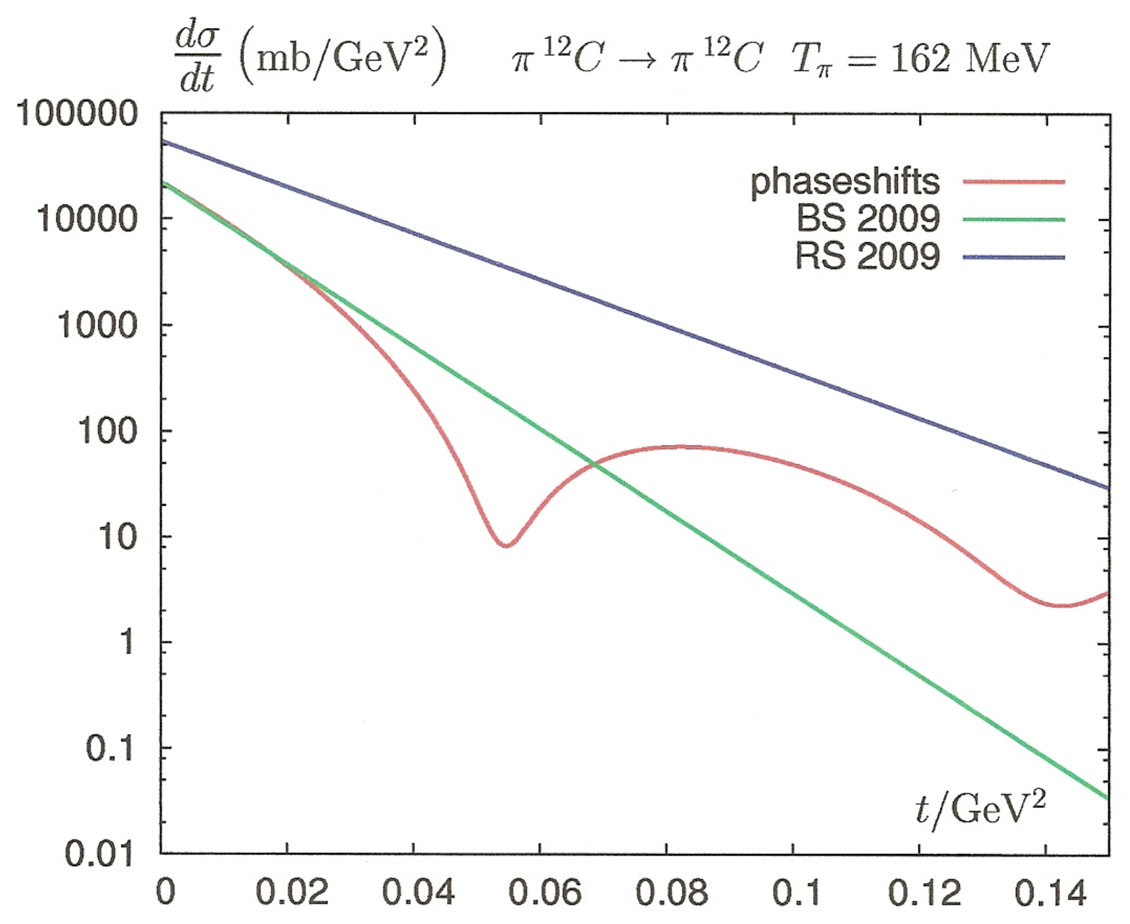
This can, however, not explain the differences between various implementations of the RS model.

For the resonance region with its rapidly varying cross sections and angular distributions the RS model is probably too simple. It describes badly the experimental data on elastic $\pi^{12}C$ scattering. Instead of refining it we – in the spirit of Adlers theorem – directly revert to the measured $\pi^{12}C$ cross sections.



We use data with $30 < T_\pi < 870\,\mathrm{MeV}$ which have been subjected to a phase shift analysis by the Karlsruhe group[h]. The forward scattering is fitted by a $A\exp(-bt)$ Ansatz resulting in energy dependent coefficients A, b.



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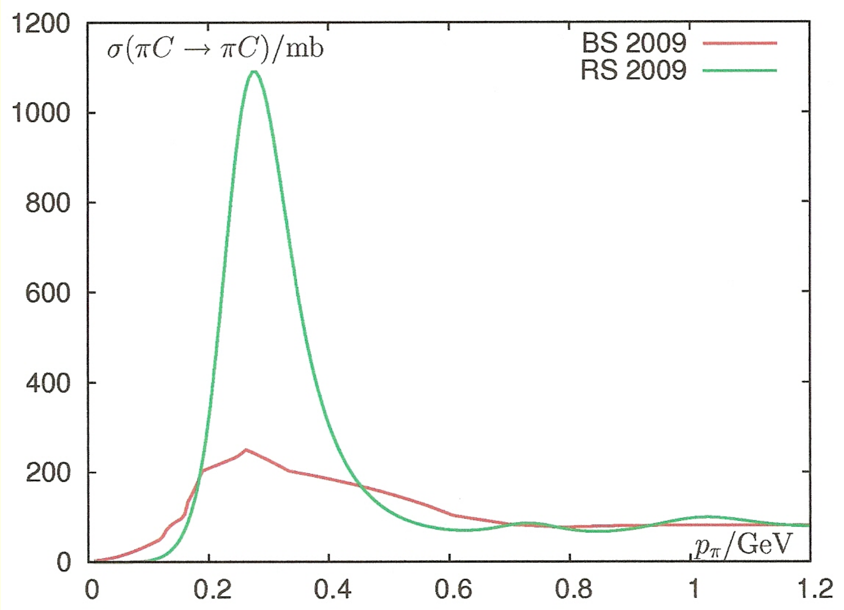
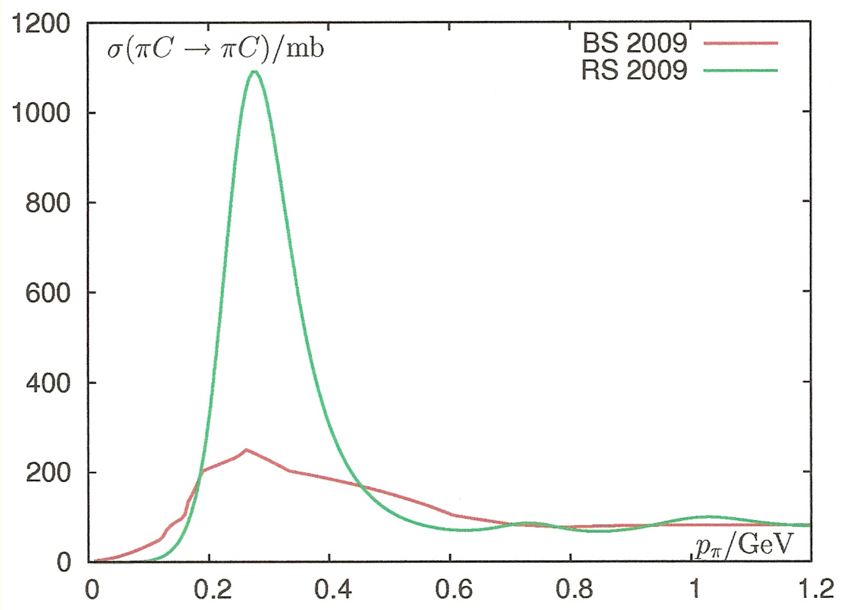
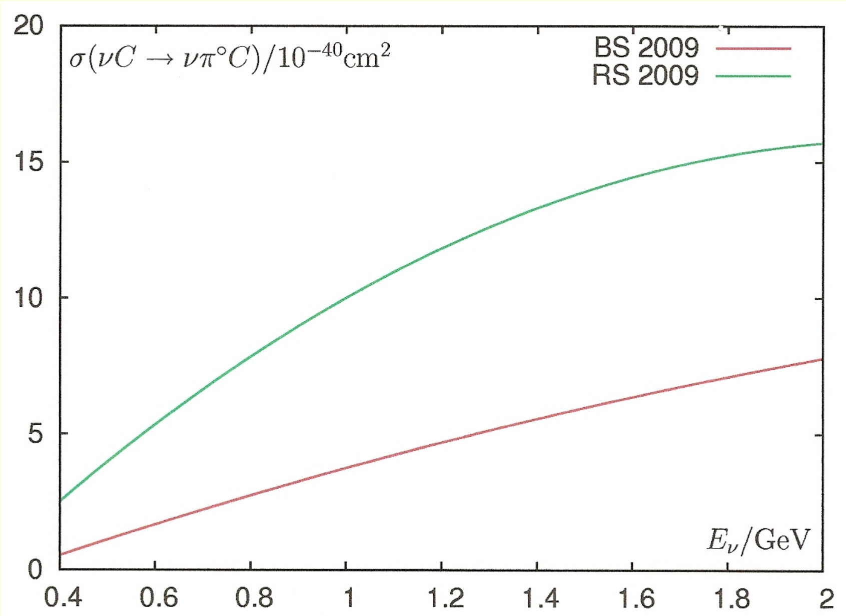
[h] Phys.Rev C29, 581 (1984); Phys.Rev. C55, 2584 (2007)



Using this parameterization the pion carbon elastic cross section is below the hadronic RS model but approaches it quickly at higher p_π





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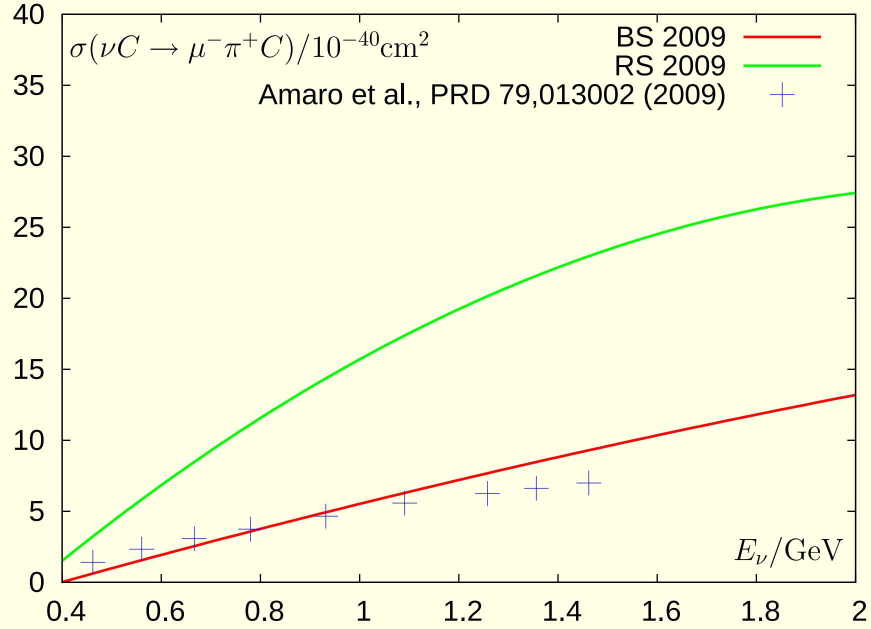
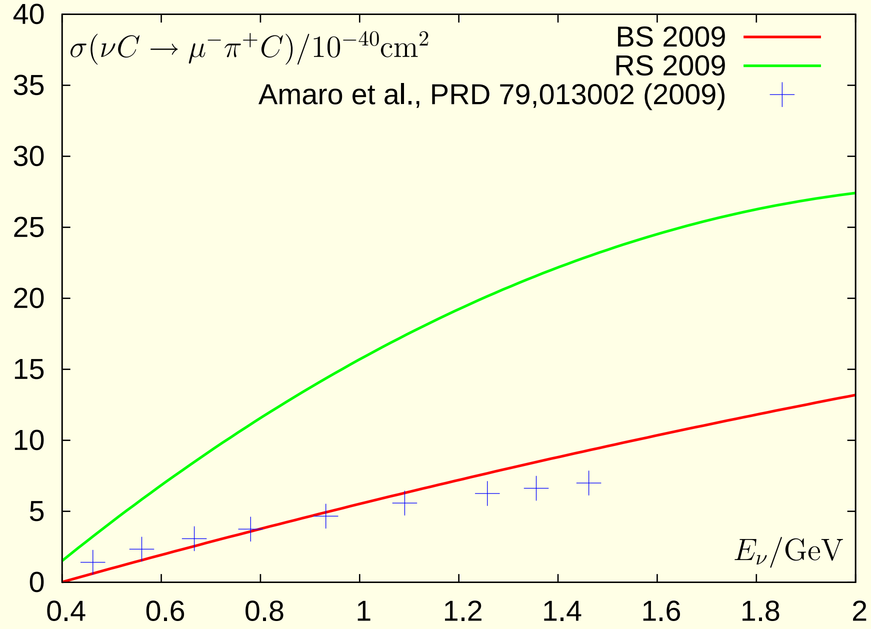
We thus get a substantial modification of the PCAC prediction for pion production off carbon nuclei for NC and CC reactions at low neutrino energies.



$\sigma(\nu C \to \mu^- \pi^+ C)/10^{-40}\mathrm{cm}^2$
BS 2009
RS 2009
Amaro et al., PRD 79,013002 (2009)
E_ν/GeV
0
5
10
15
20
25
30
35
40
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2



Our results are compatible with other PCAC based calculations[i] and remarkably close to some variants of microscopip nuclear physics models (Singh et al., Amaro et al.[j])

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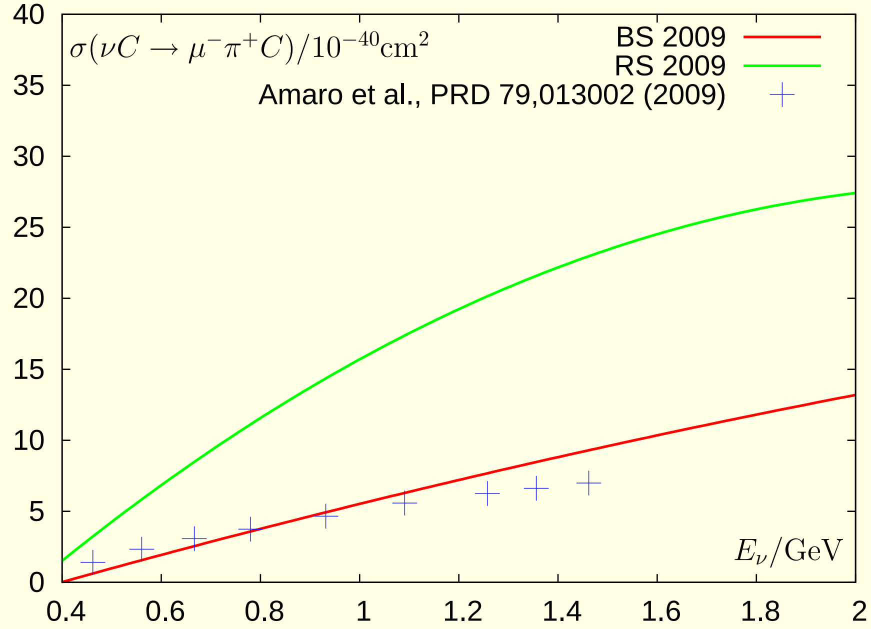
[i] Kartavtsev et al., Phys.Rev. D74, 054007 (2006)

[j] Phys.Rev.Lett. 96,24801,(2006); Phys.Rev. D79,013002 (2009)



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At this moment the model can be applied to other nuclei by using a $A^{2/3}$ scaling law.

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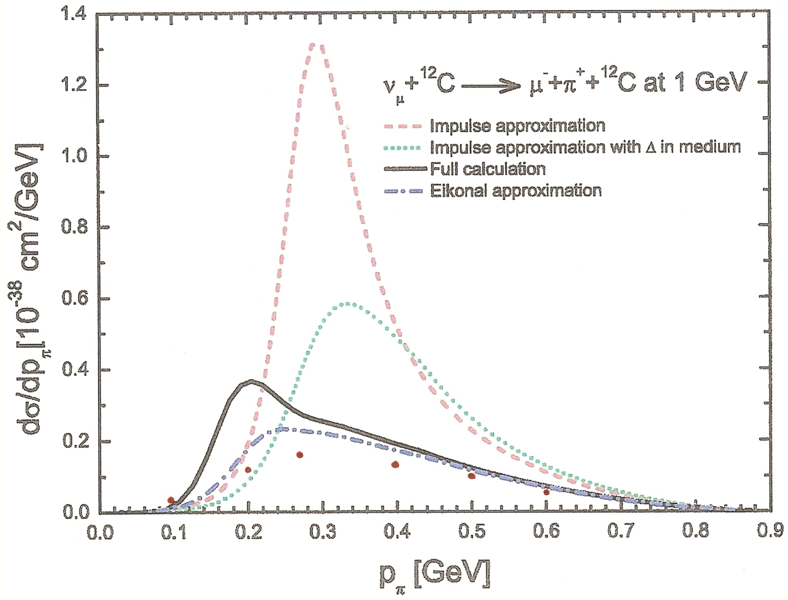
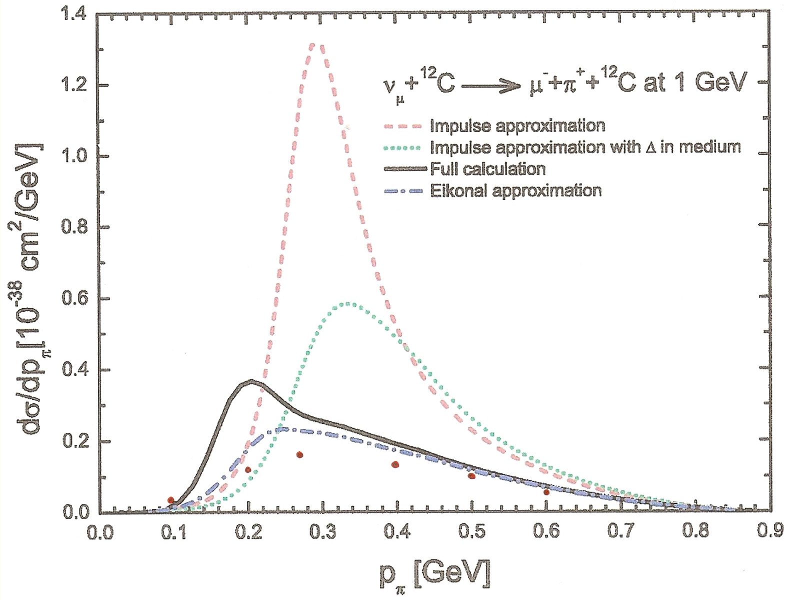
[j] Phys.Rev.Lett. 96,24801,(2006); Phys.Rev. D79,013002 (2009)



Differential distributions are more sensitive to model details (Alvarez-Ruso et al.[k]). The red points are BS 2009.

[k] Phys.Rev. C76, 068501 (2007)



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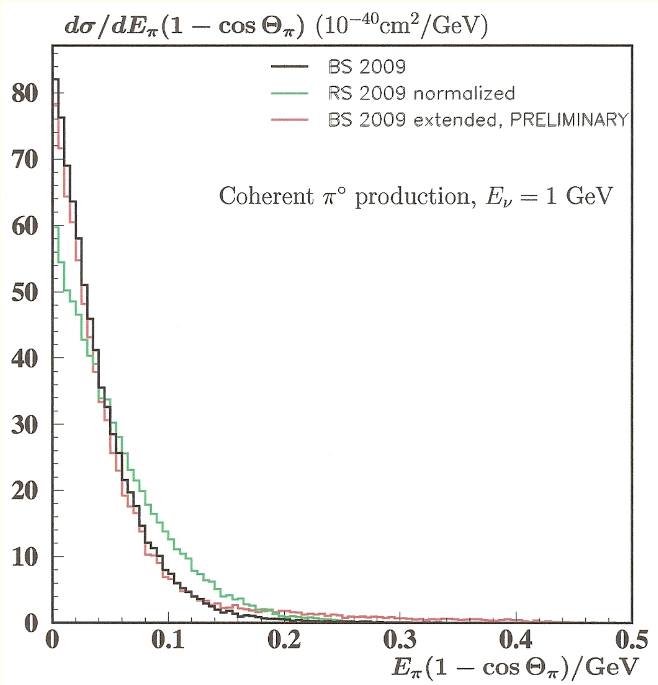
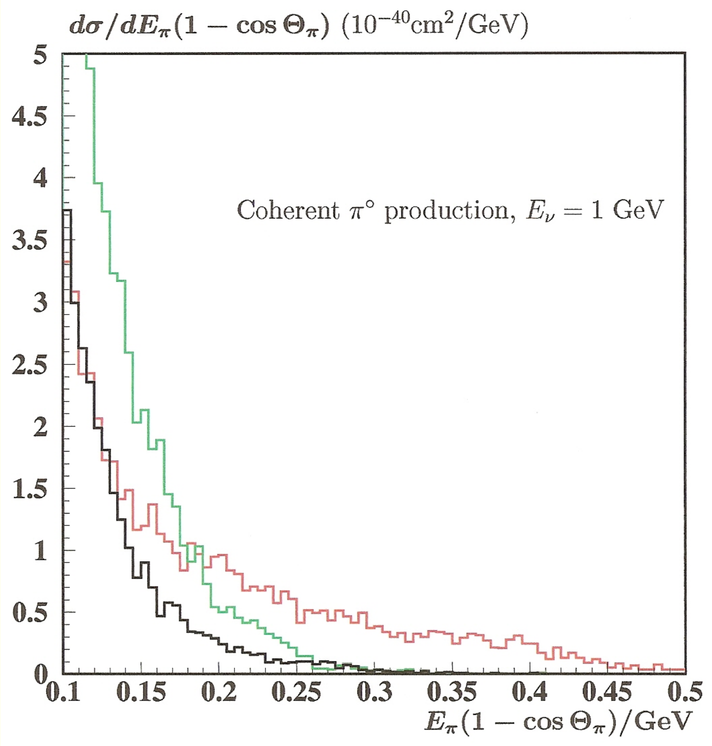
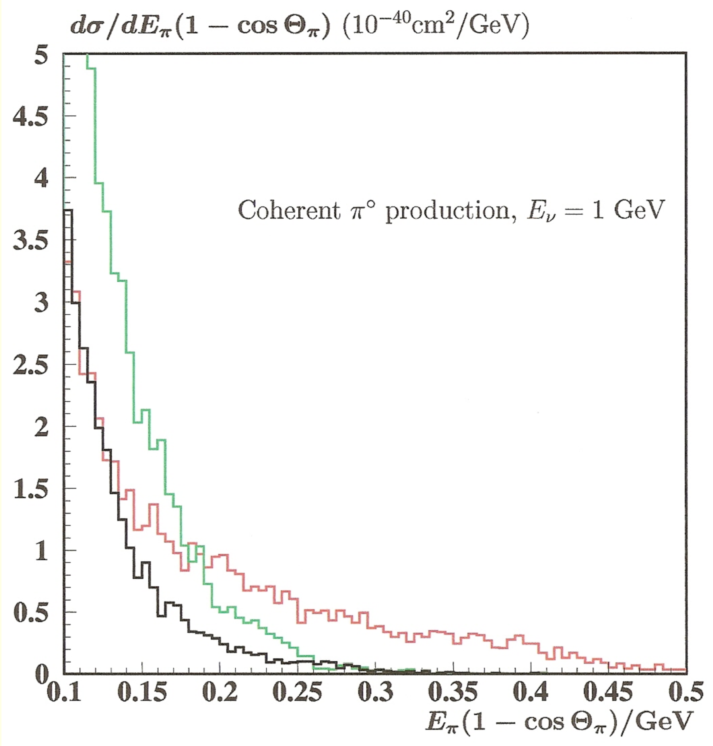


More on differential distributions: The MiniBooNE variable $E_\pi(1-\cos\Theta_\pi)$ and a first look at future results.



Inclusion of the full angular distribution measured in $\pi^{12}C$ scattering produces fat tail.



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Our results agree with all published experimental limits on coherent π^+ production. Like everybody else we have a problem with the MiniBooNE π° result. Before claiming that this falsifies the PCAC model one would like to clarify several questions, e.g. how the experiments ensure the coherence of the process.

Summary and conclusions

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- Better interpolation scheme
- More nuclei
- Improved transition to the high energy RS model