Empirical Fit to Inclusive Electron Scattering on Nuclei

P. Bosted (with Vahe Mamyan, Eric Christy)

NUINT09, May 2009

- **Motivation**
- **Proton fit for F_1 and F_2**
- **Deuteron fit for F_1 and n/p ratios**
- **A>2 fit**
- **Outlook: R for A>1 using new data**

Motivation

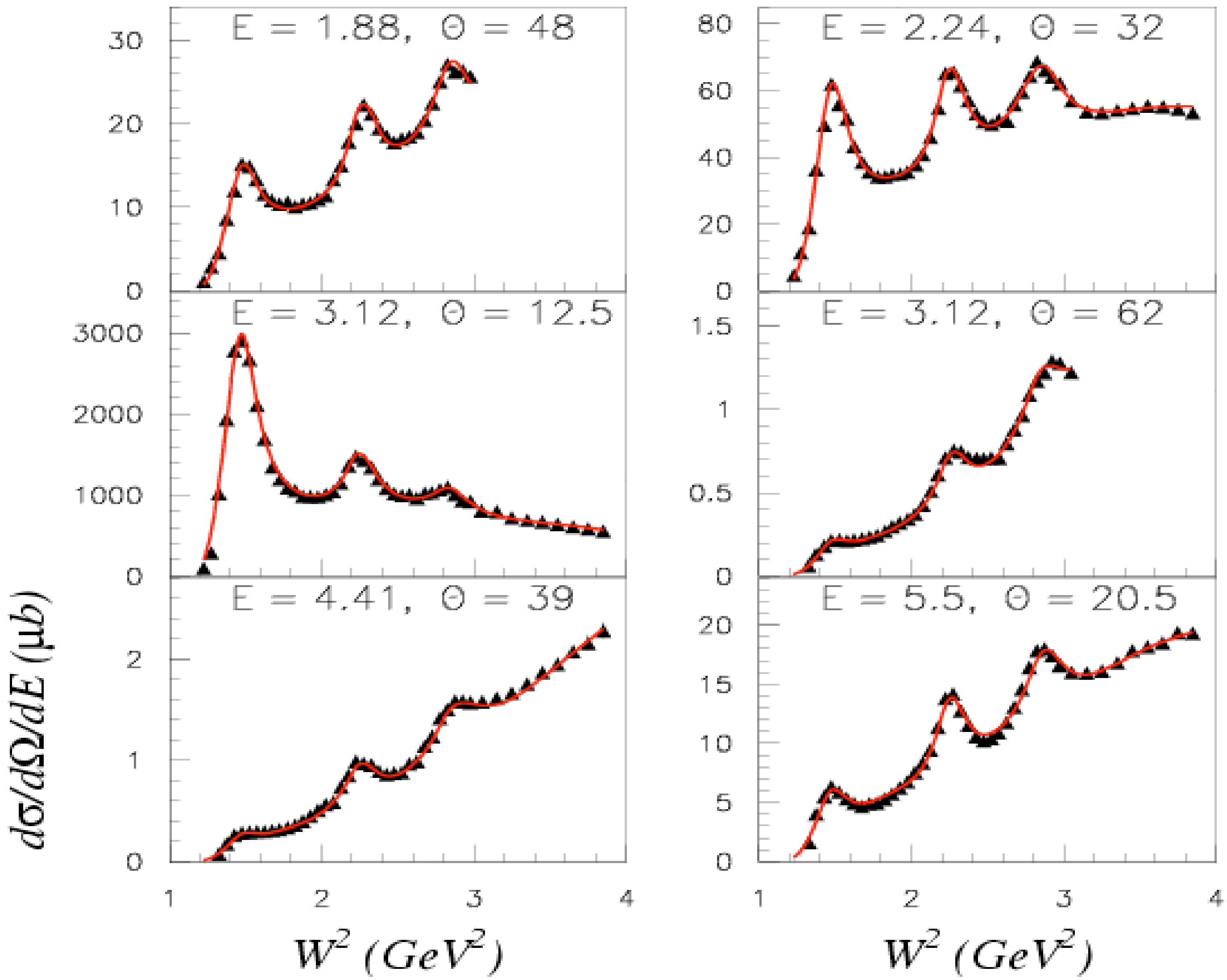
- Replace 1000's data points with smooth function
- Needed for reliable radiative corrections
- Dilution in (g_1, g_2) experiments using ammonia
- Predictions DIS PV at low Q^2
- Modeling for neutrino scattering experiments
- Needed to get g_1 from A_{1}.
- Helpful in sum rule evaluations.
- Study of higher twist, QCD evolution low Q^2

New fit to inclusive electron-proton scattering for $0<Q^2<8$ GeV^2, W<3 GeV

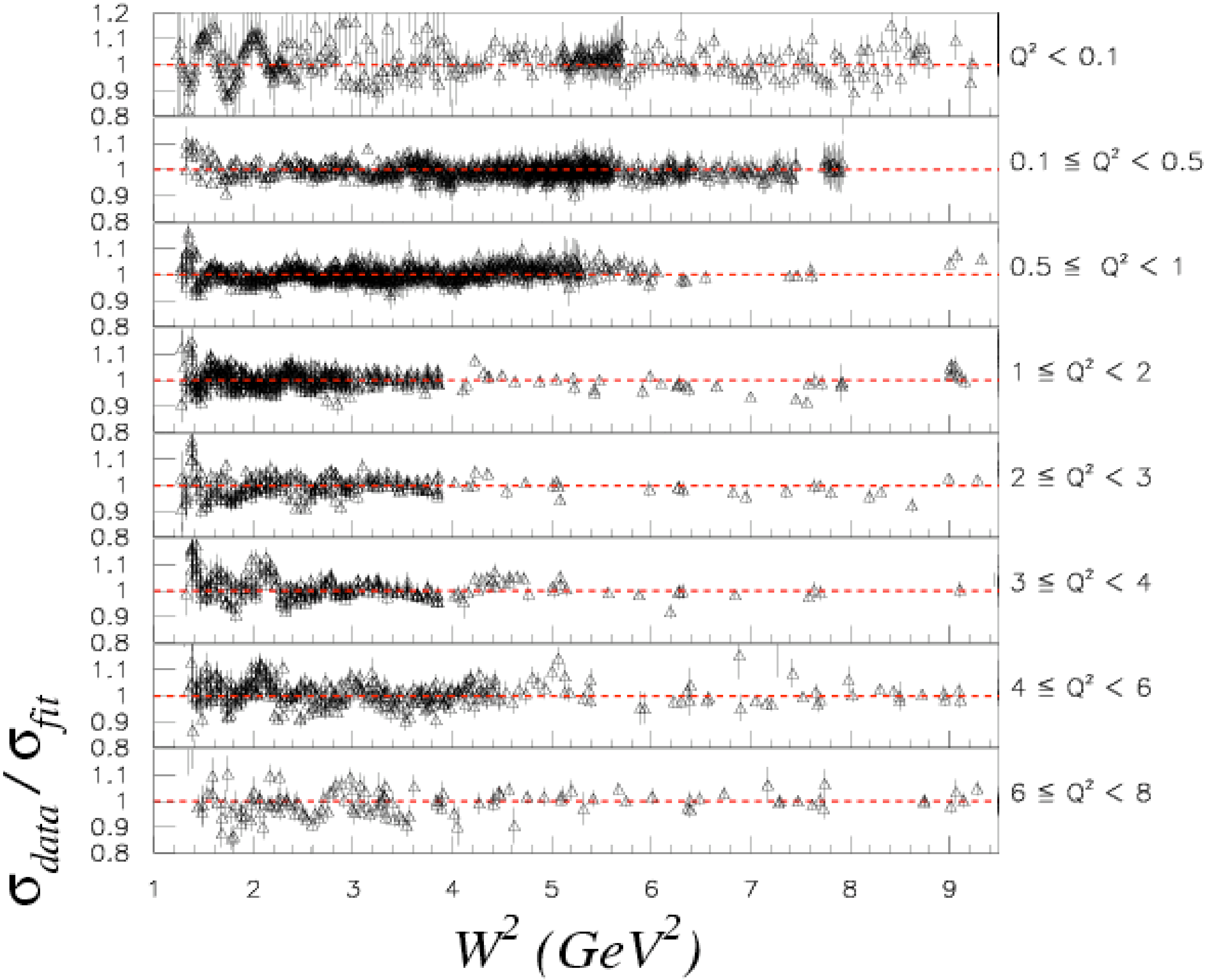
M.E. Christy and P.E. Bosted, arXiv:0711.0159)2007)

- Based on new Jefferson Lab Hall C data.
- Fit to both F_1, F_2 (or F_2, R)
- Includes $Q^2=0$ photo production data
- SLAC data added for smooth transition DIS region
- Fit variables are Q^2, $W^2 = M^2 + Q^2 (1/x - 1)$, and ε
- Polynomial non-resonant background
- Relativistic Breit-Wigner shapes for 6 resonances
- Total of 100 free parameters in fit

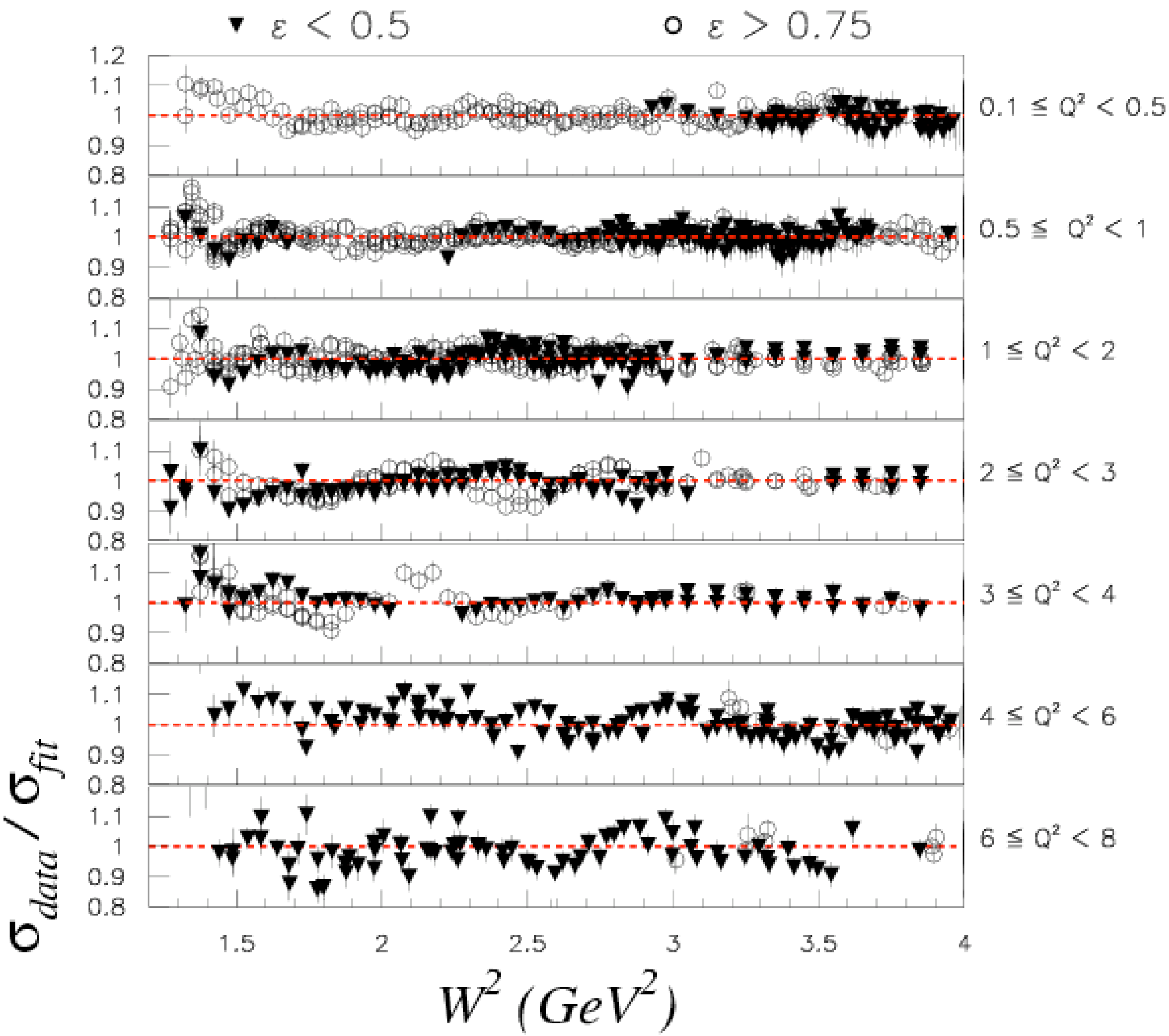
Proton Cross Sections: a few examples



Ratio of data to proton fit



Ratio of data to proton fit at low, high ε



World Data Set for A>1

- **Used data collected by Donal Day for inclusive electron scattering A>1 in quasi-elastic (q.e.) and resonance region. Mainly Bates, Saclay, SLAC. See:**
http://faculty.virginia.edu/qes-archive/index.html

- **Added new data from Jefferson Lab from several experiments, most of which are not completely finalized or published. Present empirical fit being used to iterate on the radiative corrections.**

New Jefferson Lab Data

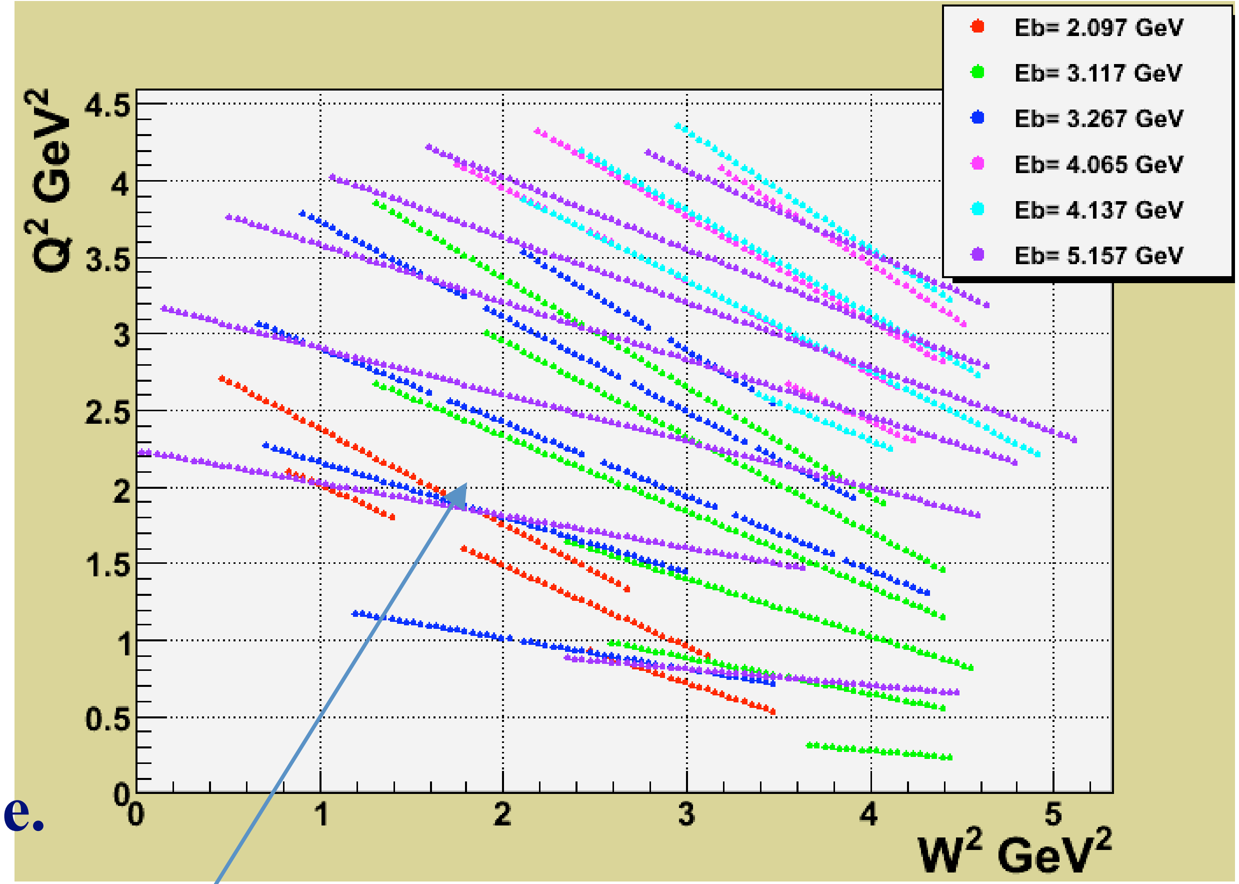
Most experiments cover $0.5<W<3$ GeV
Electron eam energies 1 to 6 GeV

- **Jlab E00-002. Target: D. $Q^2<1$ GeV^2. Unpuslished.**
- **Jlab CLAS. Target: D. $0.5<Q^2<5$ GeV$^{2.}$ Phys.Rev.C73:045205,2006**.
- **Jlab E00-116. Target: D. $3<Q^2<7$ GeV^2. S. Malace thesis (to be submitted P.R.C. soon).**
- **Jlab E02-109. Targets: D, C, Al, Fe. $Q^2<2$ GeV$^{2.}$ Wide range of ε (for R). Unpublished (resolution and radiative corrections still in progress).**
- **Jlab 03-103. Targets: D, ^{3}He, ^{4}He, Be, C, Al, Cu. $2<Q^2<5$ GeV$^{2.}$ Small ε range. Data for $A<12$ in J. Seely et al.,** arXiv:0904.4448 (2009). **Rest soon.**
- **Jlab 06-109. Targets: D, C, Al, Fe, Cu. $1.5< Q^2<5$ GeV$^{2.}$ Wide range of ε (for R). Unpublished (analysis almost done: iterating on rad. corr. With present fit).**
- **Jlab Hall A Coulomb Sum Rule experiment. Several nuclei. Data not available to be included in fitting.**
- **Hall A data on ^{3}He (unpublished) from spin experiments kindly provided by spokespersons to use in this fitting.**

Kinematic Range of E06-109

- ➢**Measurements for each energy are done at least for 3 different angles.**
- ➢**Wide epsilon range allows Rosenbluth separation.**

Targets : D, C, Al, Cu, Fe.

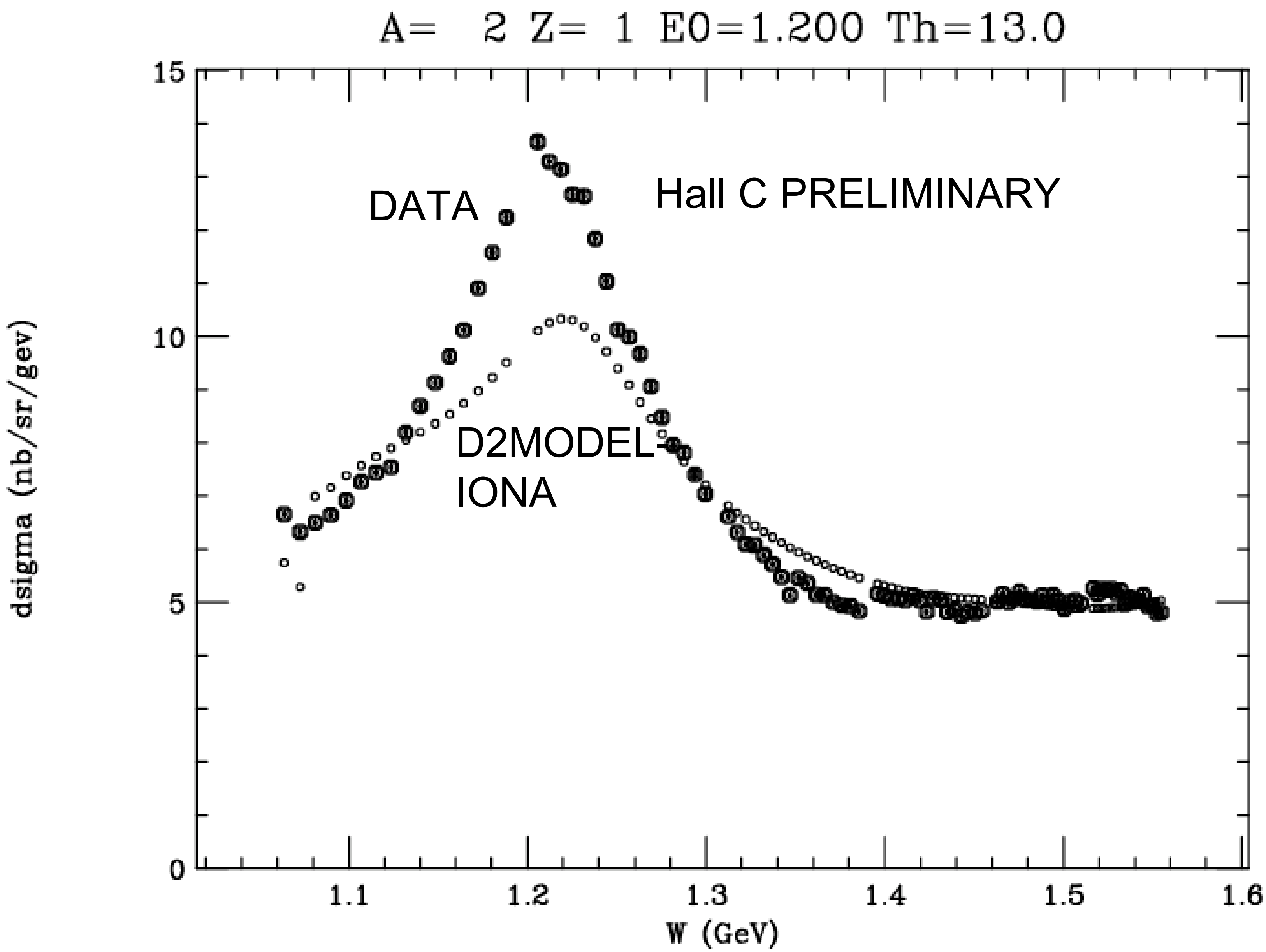


Different epsilon values for same Q^2 and W^2, allows LT separation.

New data compliments data from experiment E02-109 (lower Q^2, similar W and ε range).

Inelastic scattering on deuteron

- ➢Previous fits did not specifically include Fermi smearing effect : fixed widths of resonances were used rather than ones that increase with momentum transfer q.
- ➢This can be seen on next slide for the lowest Q^2 (about 0.1 GeV^2) preliminary radiated cross section data from the Jlab E03-109 experiment.



A= 2 Z= 1 E0=1.200 Th=13.0
DATA
Hall C PRELIMINARY
D2MODEL-
IONA
dsigma (nb/sr/gev)
W (GeV)
0
5
10
15
1.1
1.2
1.3
1.4
1.5
1.6

Fermi-smearing

- Nucleons move in nuclei with characteristic "Fermi" momentum controlled by nuclear density.

- Typical values are 0.05 GeV for D, 0.15 GeV for ^3He, 0.2 GeV for 3<A<12, and 0.24 GeV for A>12.

- In inclusive electron scattering, "smears" out invariant mass W by:

$$W^2 = M^2 + 2M\nu - Q^2 + 2\vec{q} \cdot \vec{p_f}$$

More on Fermi smearing

- So true W and measured W differ by magnitude of momentum transfer vector times struck nucleon momentum

- For deuteron, used Paris wave function to estimate probability of finding a nucleon with a given value of p*cos(θ), where θ is angle between q and p.

- For A>2, used Gaussian distribution truncated at 3 sigma for inelastic (plus small binding energy correction).

Basic structure of deuteron fit

P.E. Bosted and M.E. Christy, arXiv: 0711.0159 (2007)

- Subtract off quasi-elastic contributions using PWIA calculation, Paris wave function.

- Next, extract F_1 from cross section using proton fit for $R=\sigma_L/\sigma_T$, assuming R is same for proton and neutron.

- Underlying fit parameters describe F_1 from the sum of a free proton plus a free neutron. Functional from similar to proton fit (relativistic Breit-Wigner resonances plus polynomial non-resonant background).

Basic structure of the fit

- Also speeded up code by pre-calculating resonance parameters (for example branching ratios for single pion, double pion, or eta) as a function of W and storing for later use.

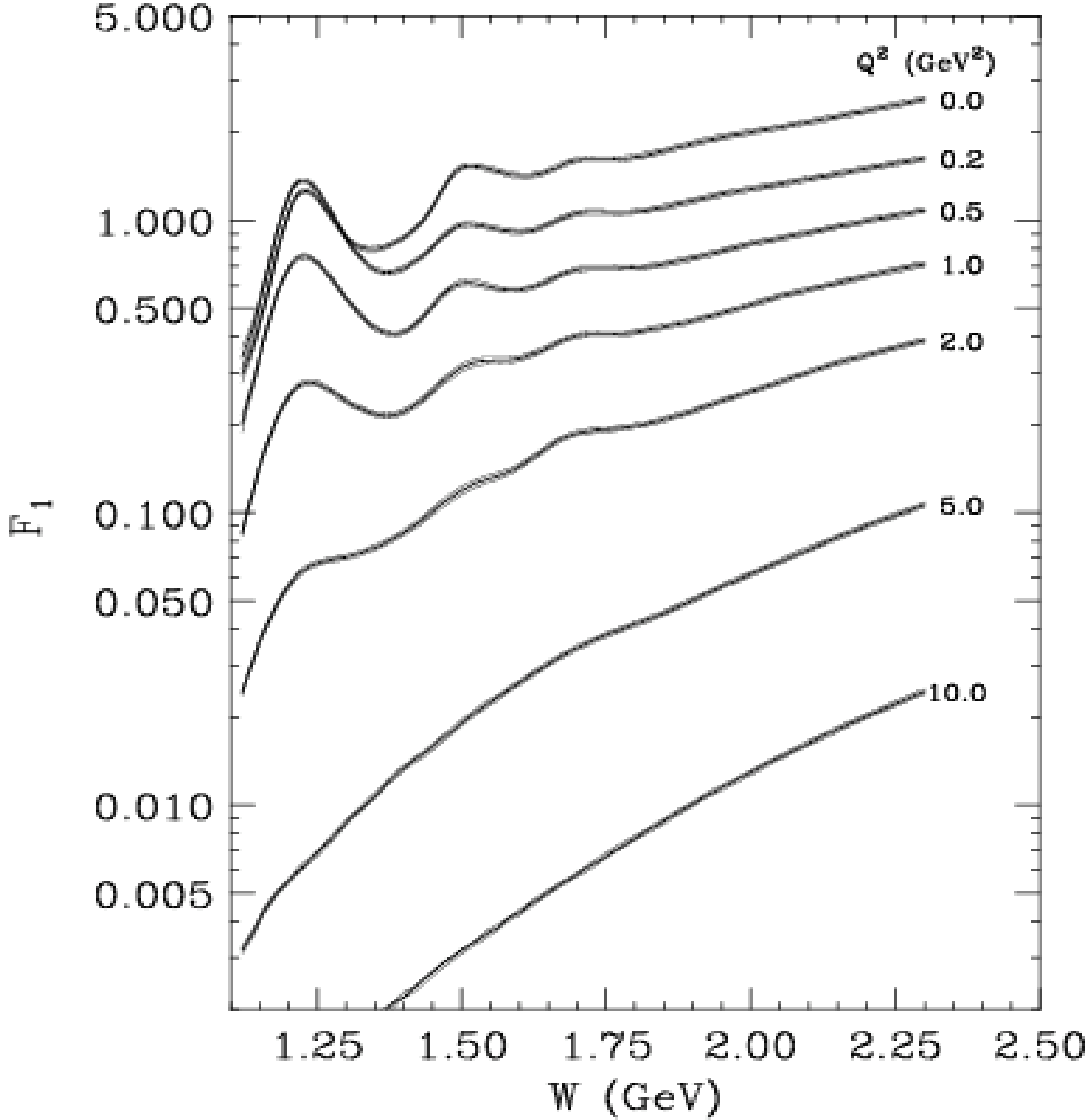
- Used 1/10 of data to get starting parameters, then full data to refine results.

Basic structure of the fit

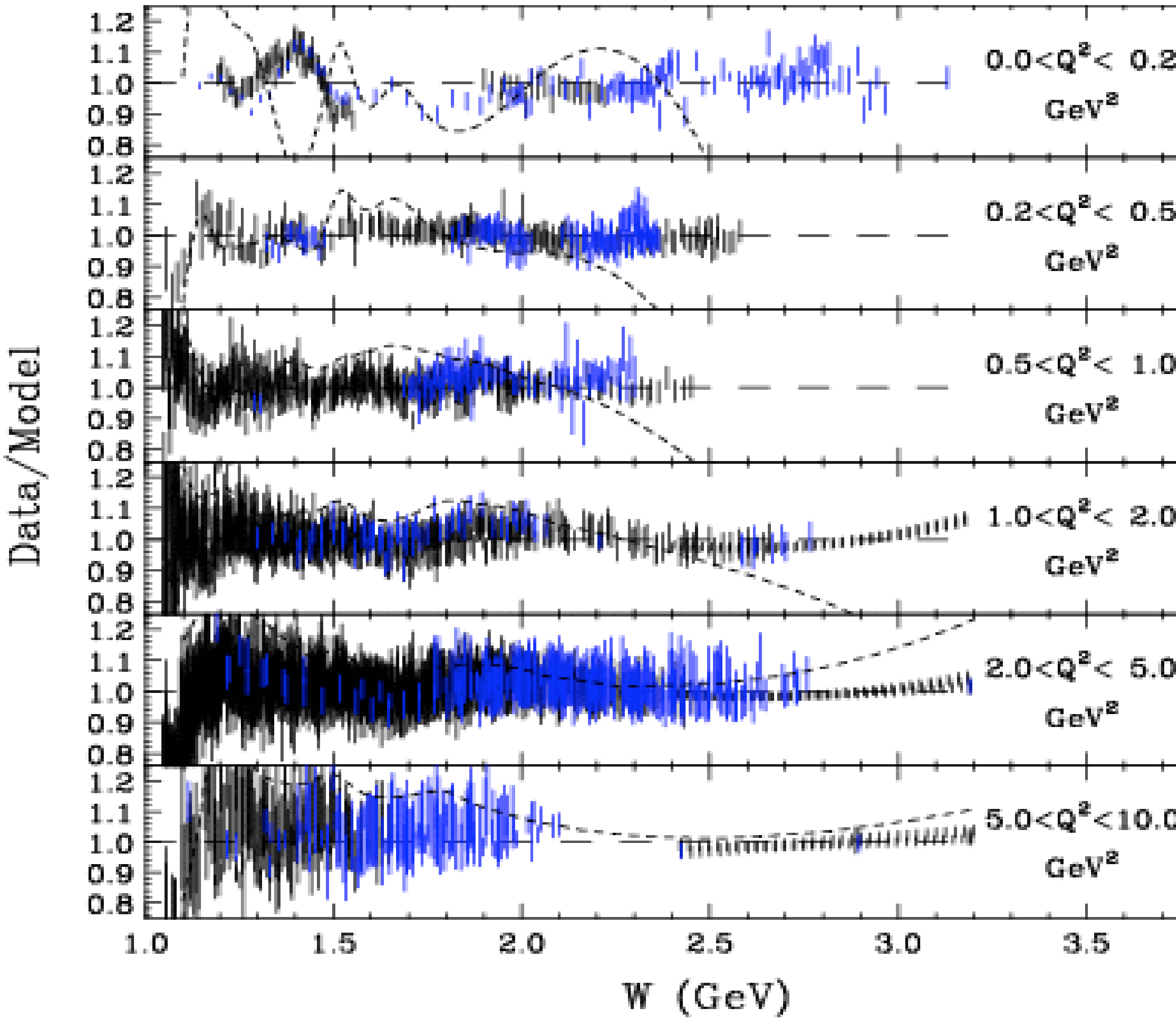
- **Also speeded up code by pre-calculating resonance parameters (for example branching ratios for single pion, double pion, or eta) as a function of W and storing for later use.**

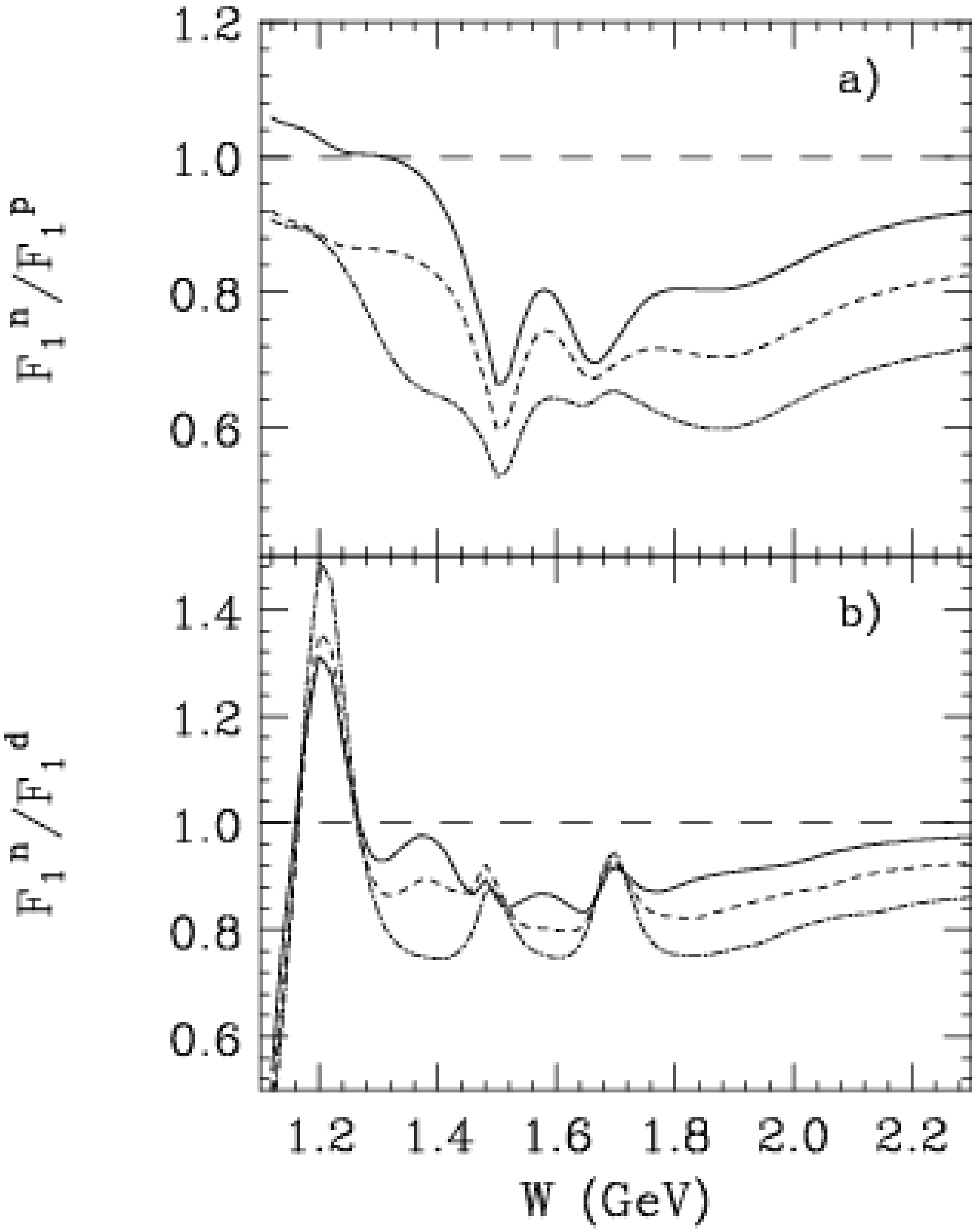
- **Used 1/10 of data to get starting parameters, then full data to refine results.**

Deuteron F_1 fit results



Ratios deuteron data to fit (blue is low ε)

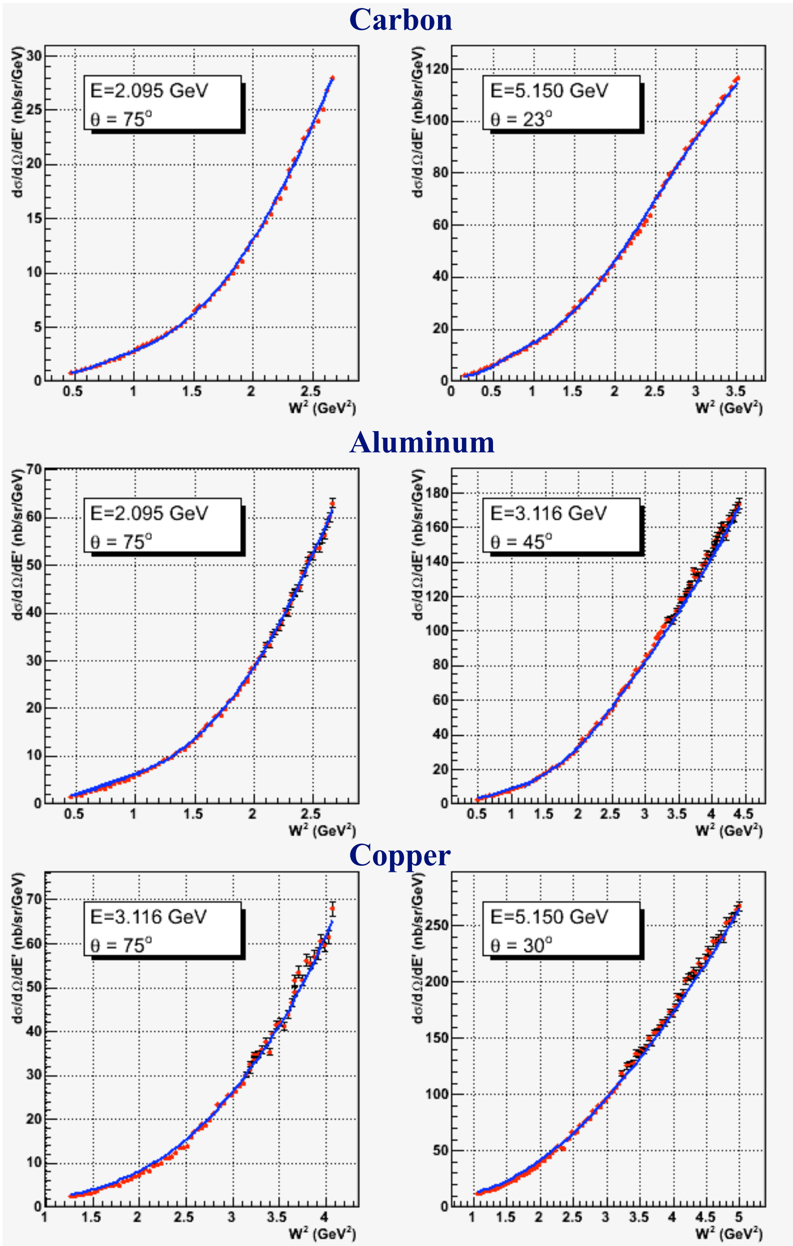


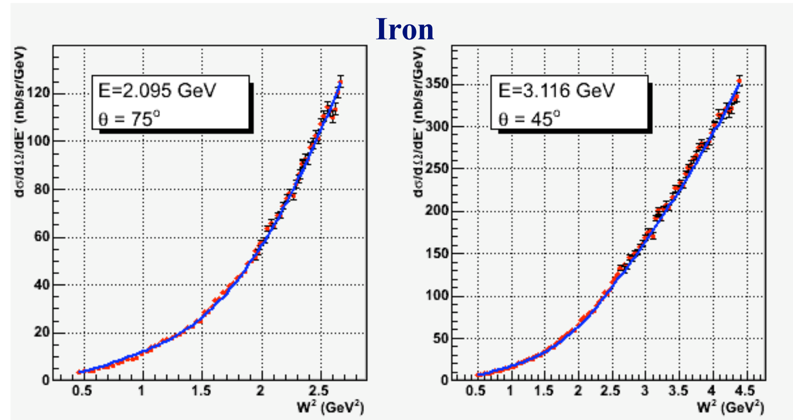


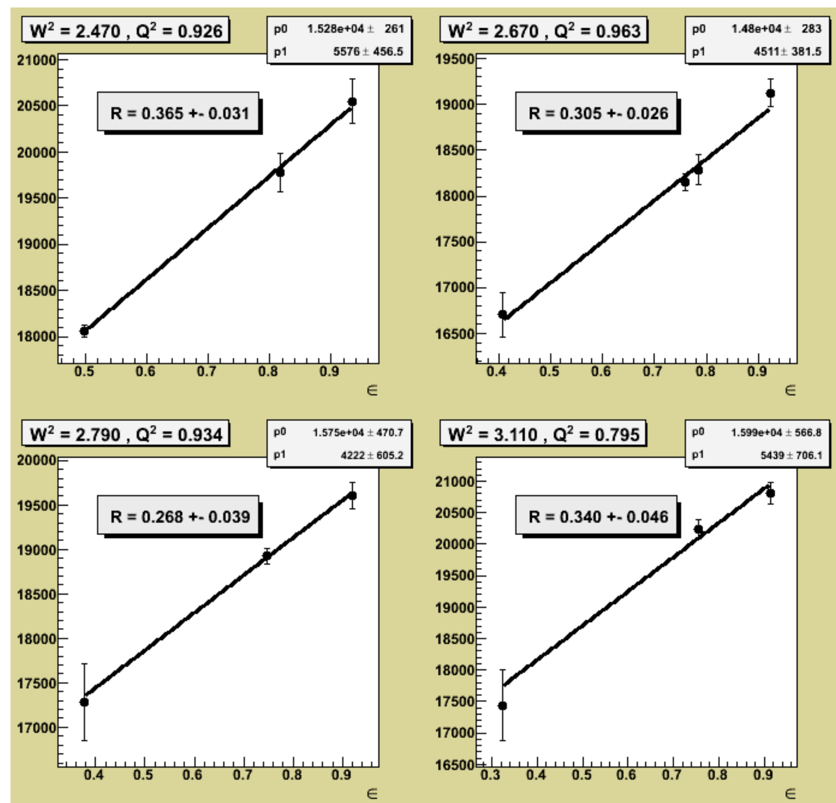
n/p ratios at Q^2=0.5, 1, 2 GeV^2

n/d ratios at Q^2=0.5, 1, 2 GeV^2 (will be tested in Jlab “Bonus” Experiment).

Preliminary Results for Jlab E06-109

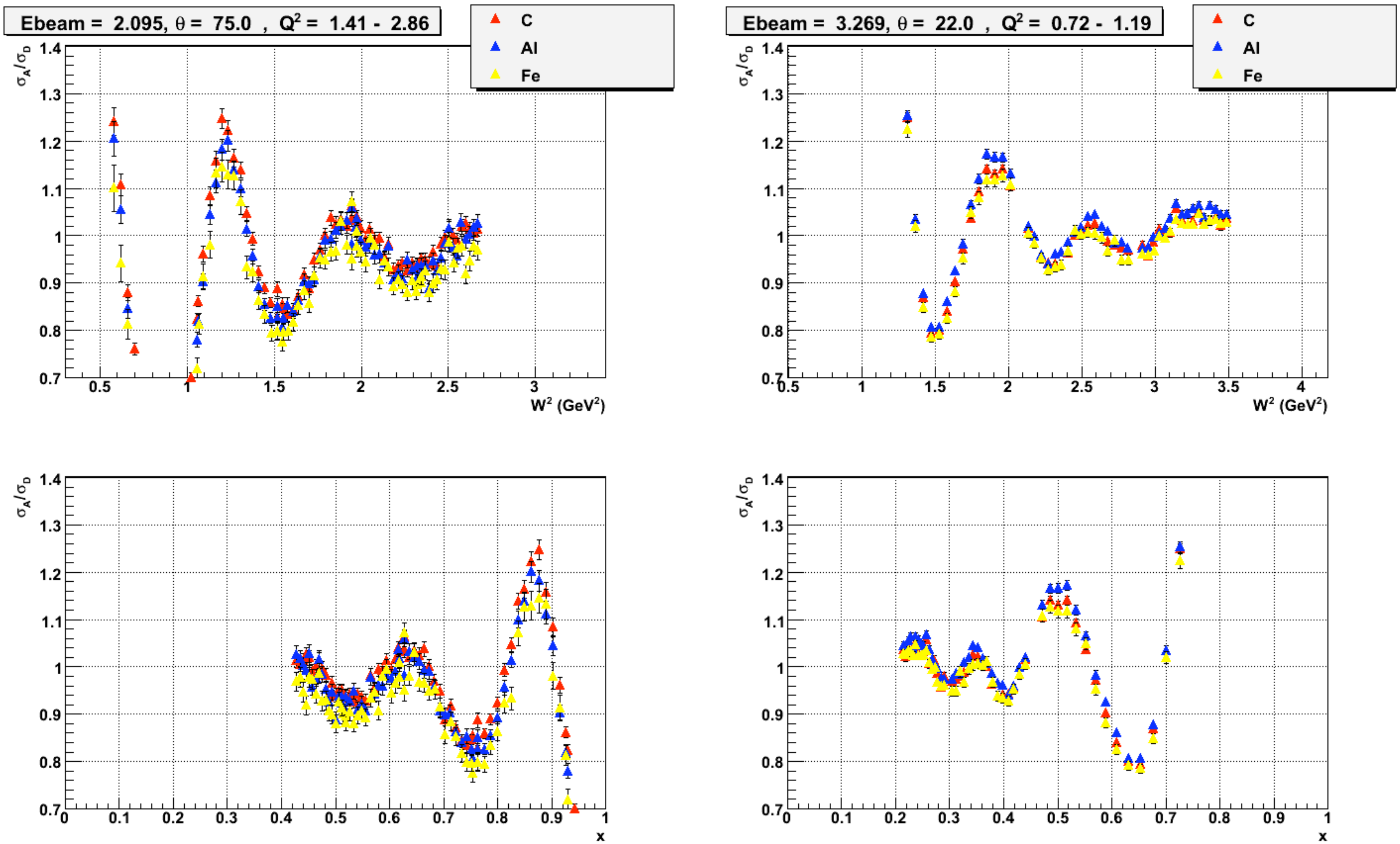






Jlab E06-109 Cross Section Ratios A/D

Similar for C/D, Al/D, and Fe/D



Fit for A>2: "Basic fit"

- Start off with Fermi-smearing of free neutron and proton. Quasi-elastic done using Donnelly, Day, and Sick treatment of off-mass shell form factors. Inelastic done using simple Gaussian smearing

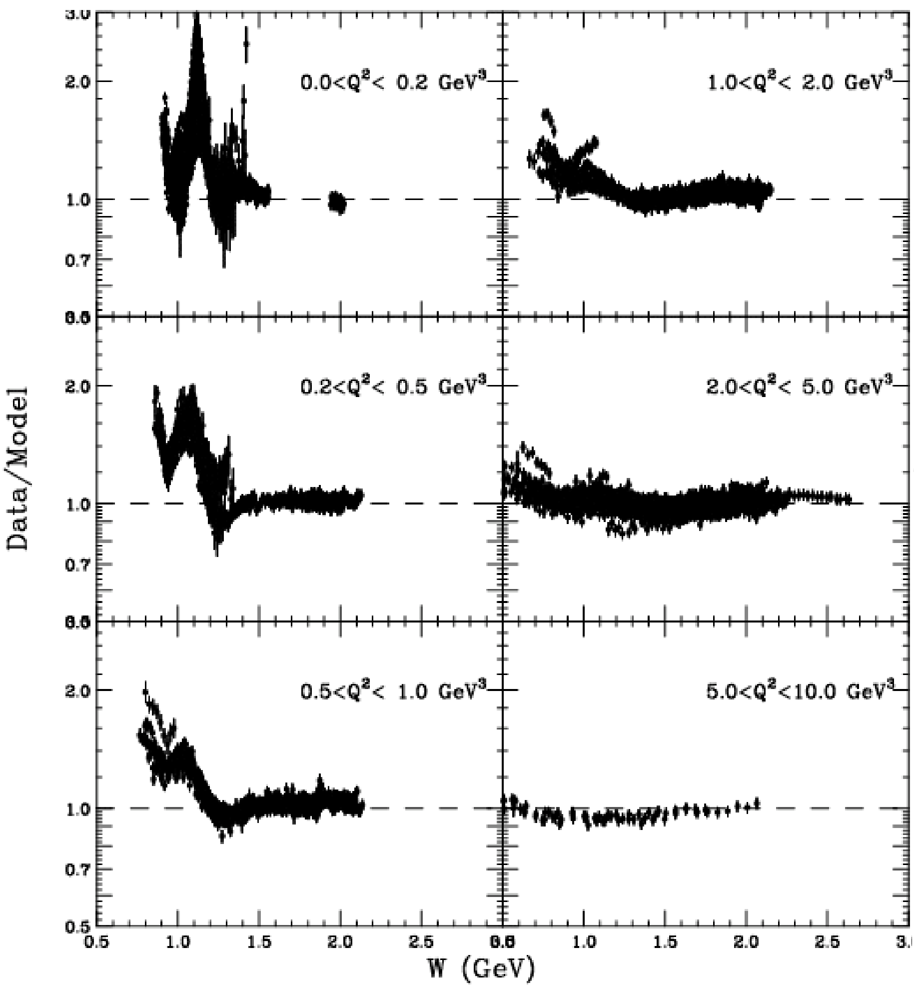
- For each nucleus, fit "average Fermi momentum" parameter k_F and "average binding energy" E_s (typically 0.2 GeV and 0.02 GeV respectively).

- Apply fit to "EMC effect" using SLAC/NMC fits and assuming only depends on x.

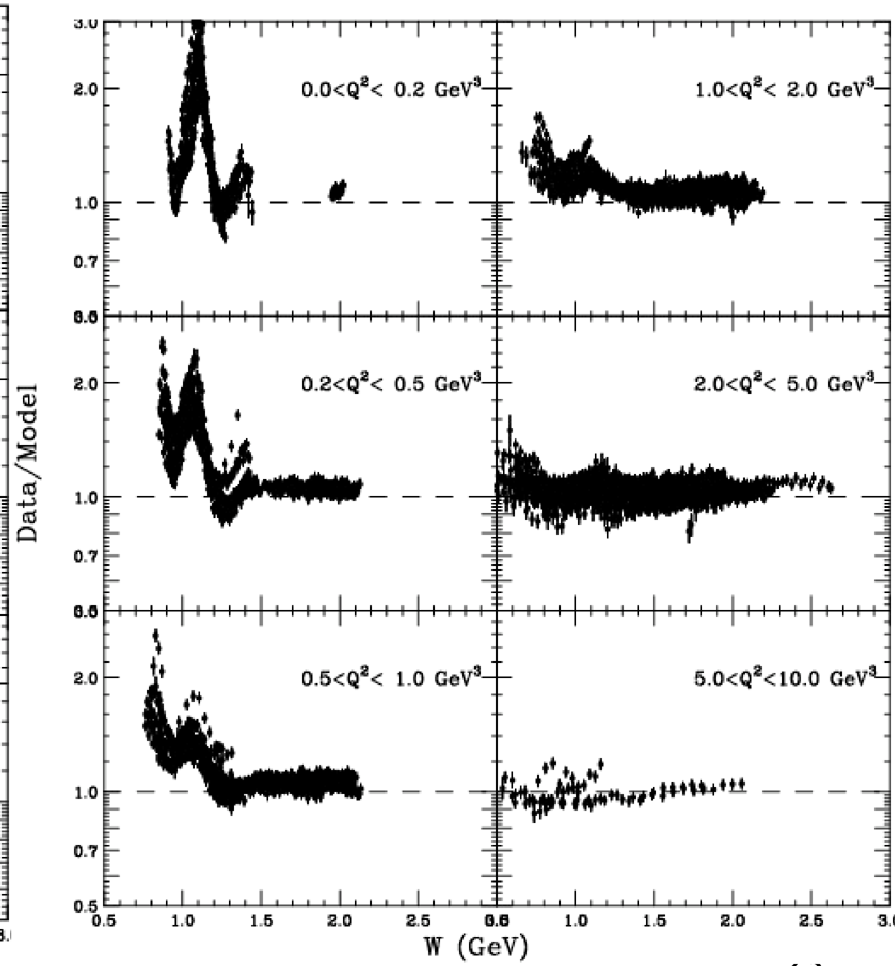
Ratios of Data to Basic Fit

Note problems low Q^2 and low W (q.e., Delta regions)

CARBON



IRON



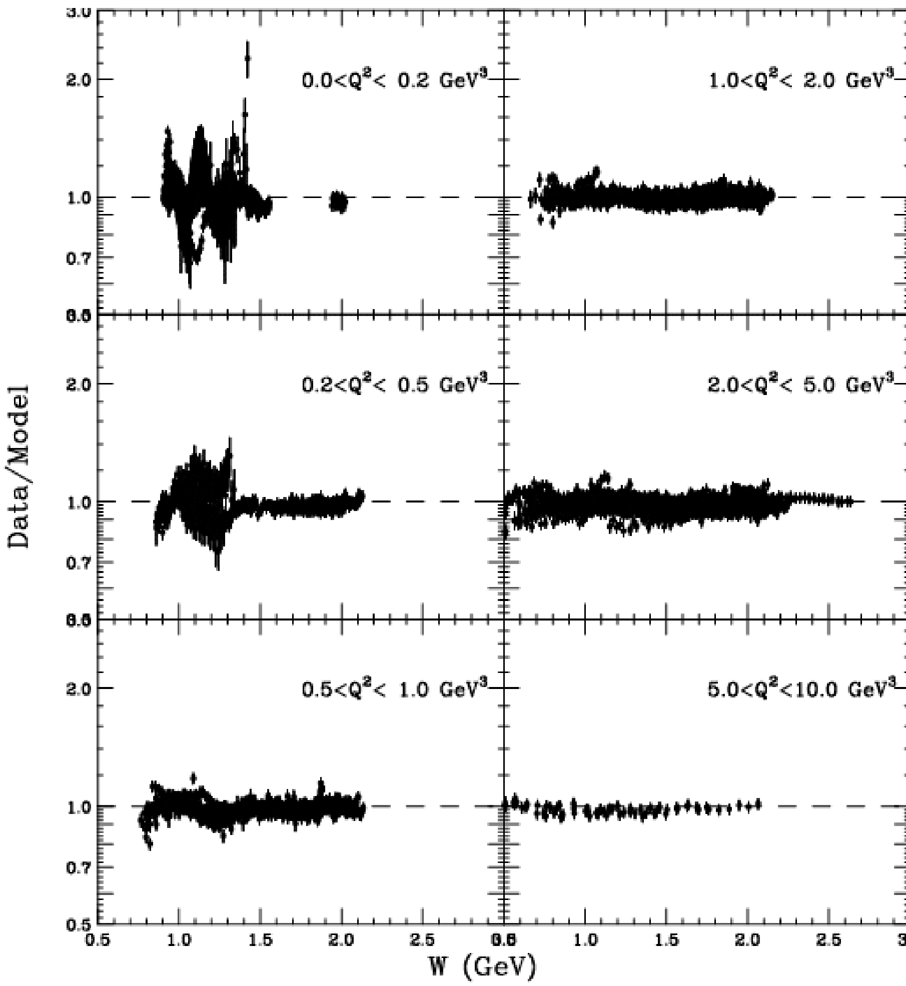
Improvements to Basic Fit

- 7-parameter modified Gaussian added to F_1 to fill in dip between q.e. and Delta(1232).
- 5-parameter polynomial in scaling variable “y” modifies the q.e. response
- 5-th order polynomial in x and 2nd order in W polynomial (suppresed by dipole-like form) modify F_1 inelastic.
- R for q.e. shifted down by -0.37.
- R for inelastic shifted by 0.014 A.

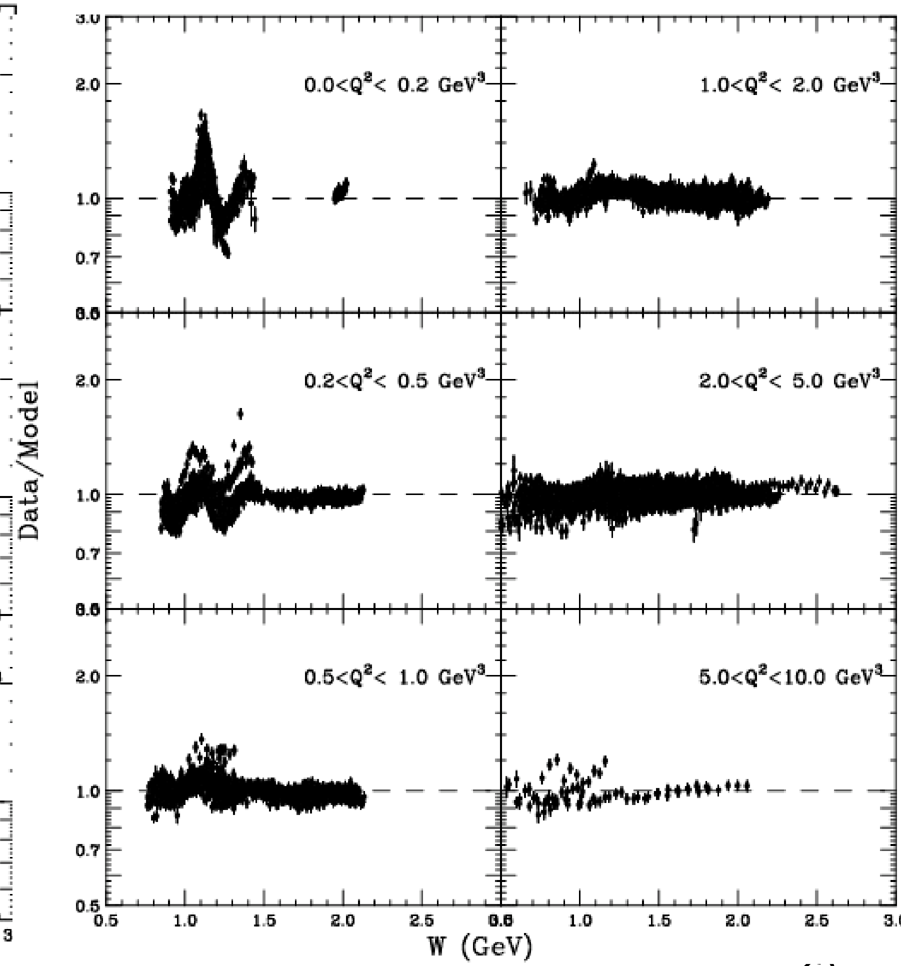
Ratios of Data to Improved Fit

Still few small wiggles at low Q^2. Waiting final E02-109.

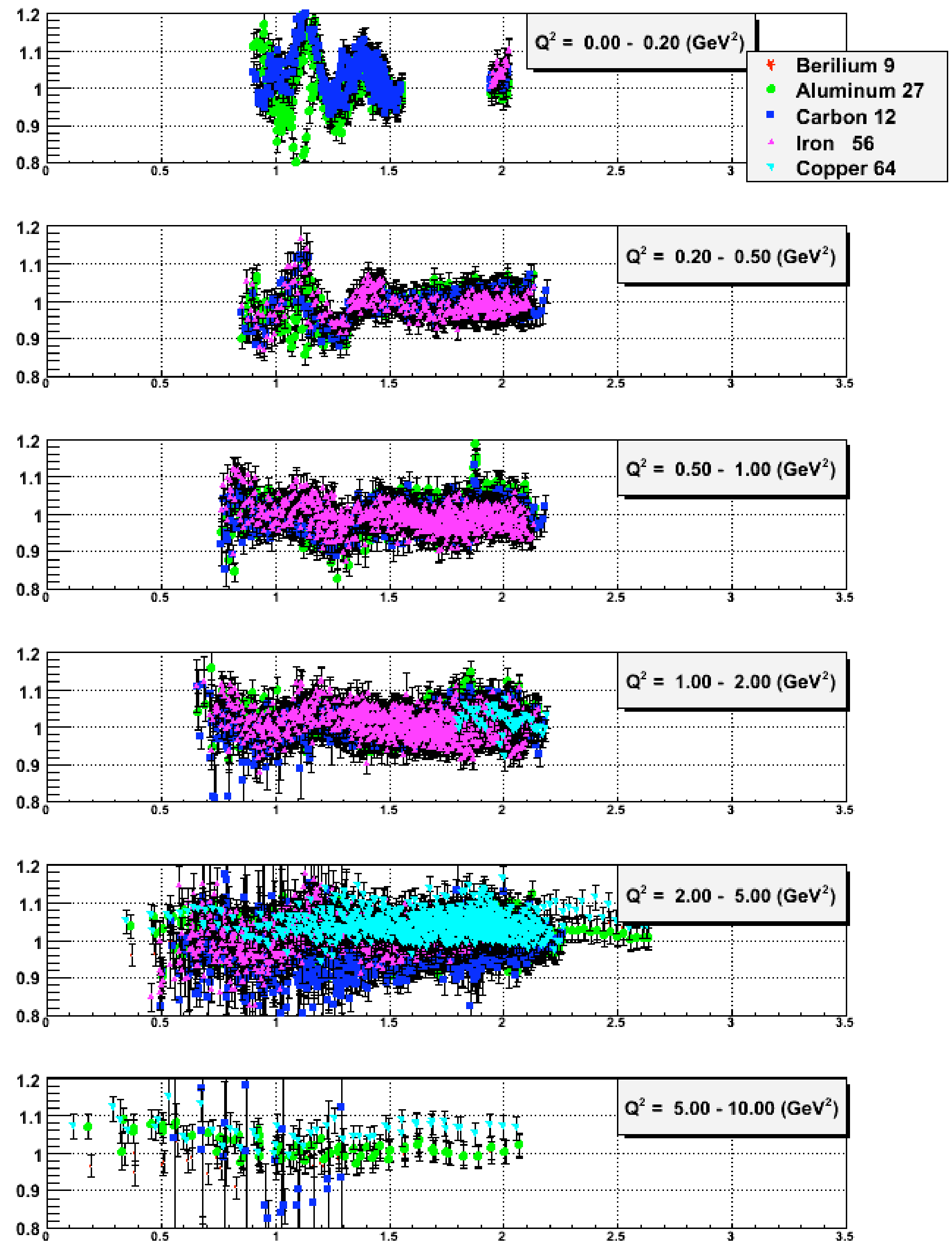
CARBON



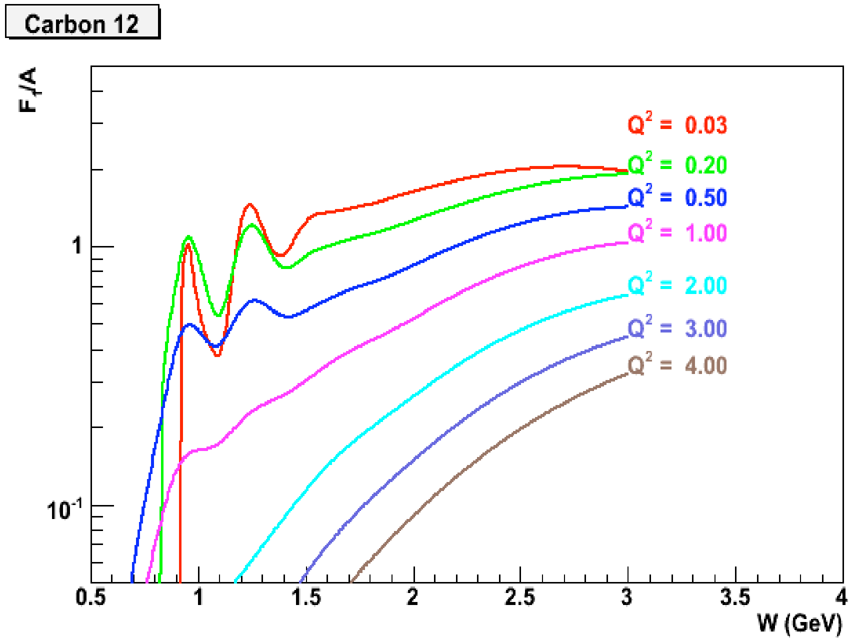
IRON

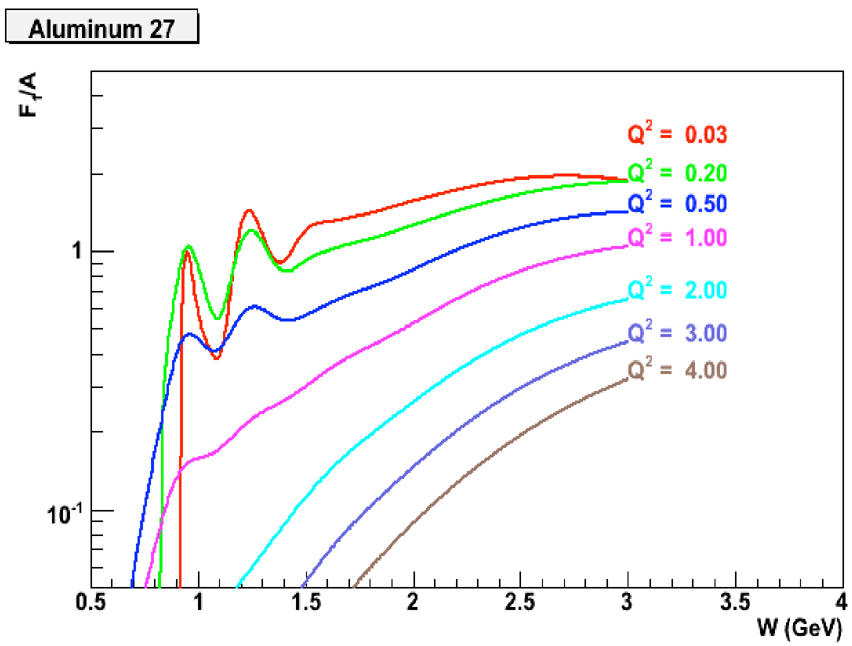


Ratio of data/fit by nucleus, from Be to Cu. Some A-dependent parameters allow for good agreement (strength of dip correction and shift in R,…)



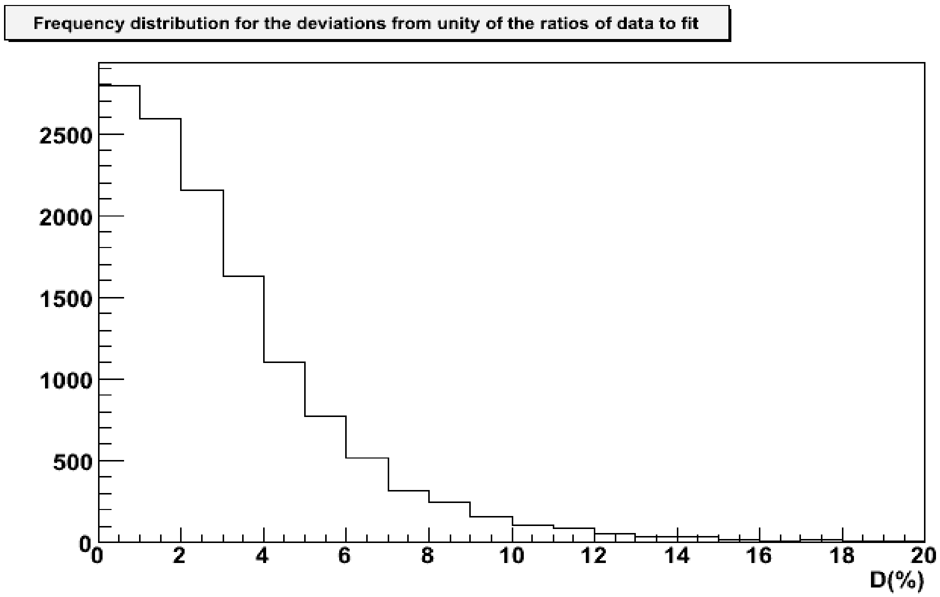
W and Q^2 dependence of F_1 for two nuclei. Note that compared to the deuteron, there is almost no structure above the Delta(1232) resonance.



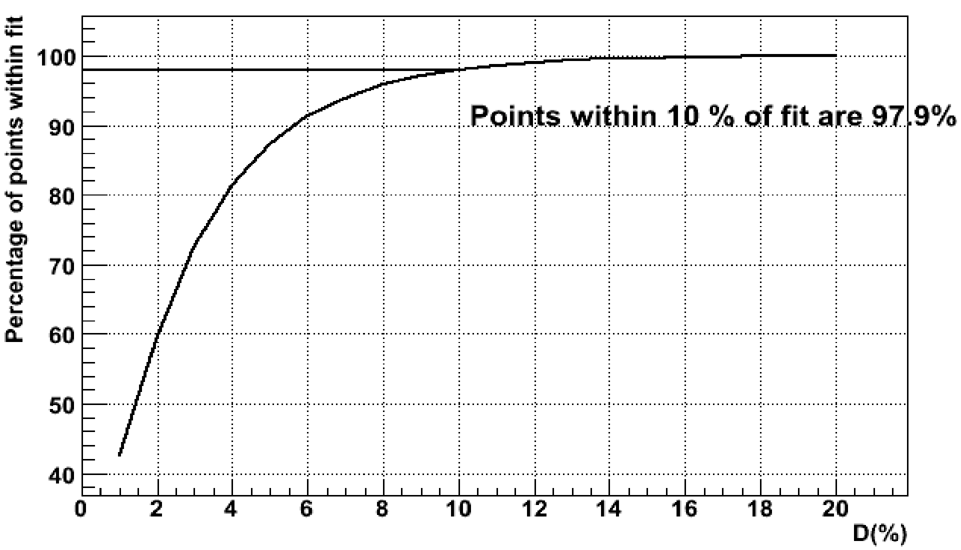


Improved Fit - Results (for A>4)

Frequency distribution for the deviations from unity of the ratios of data to fit.



Percentage of points which agree with the fit within given deviation from unity.



Summary

- Empirical fit for A>1 and free neutron in agreement with most data points within 10% for $0<Q^2<10$ GeV^2 and $0<W<3$ GeV.
- New Jefferson Lab data over wide kinematic range and for many nuclei very useful in constraining the fits.
- Still working on ^{3}He and ^{4}He to correct some problems at low W.
- FORTRAN code available www.jlab.org/~bosted/

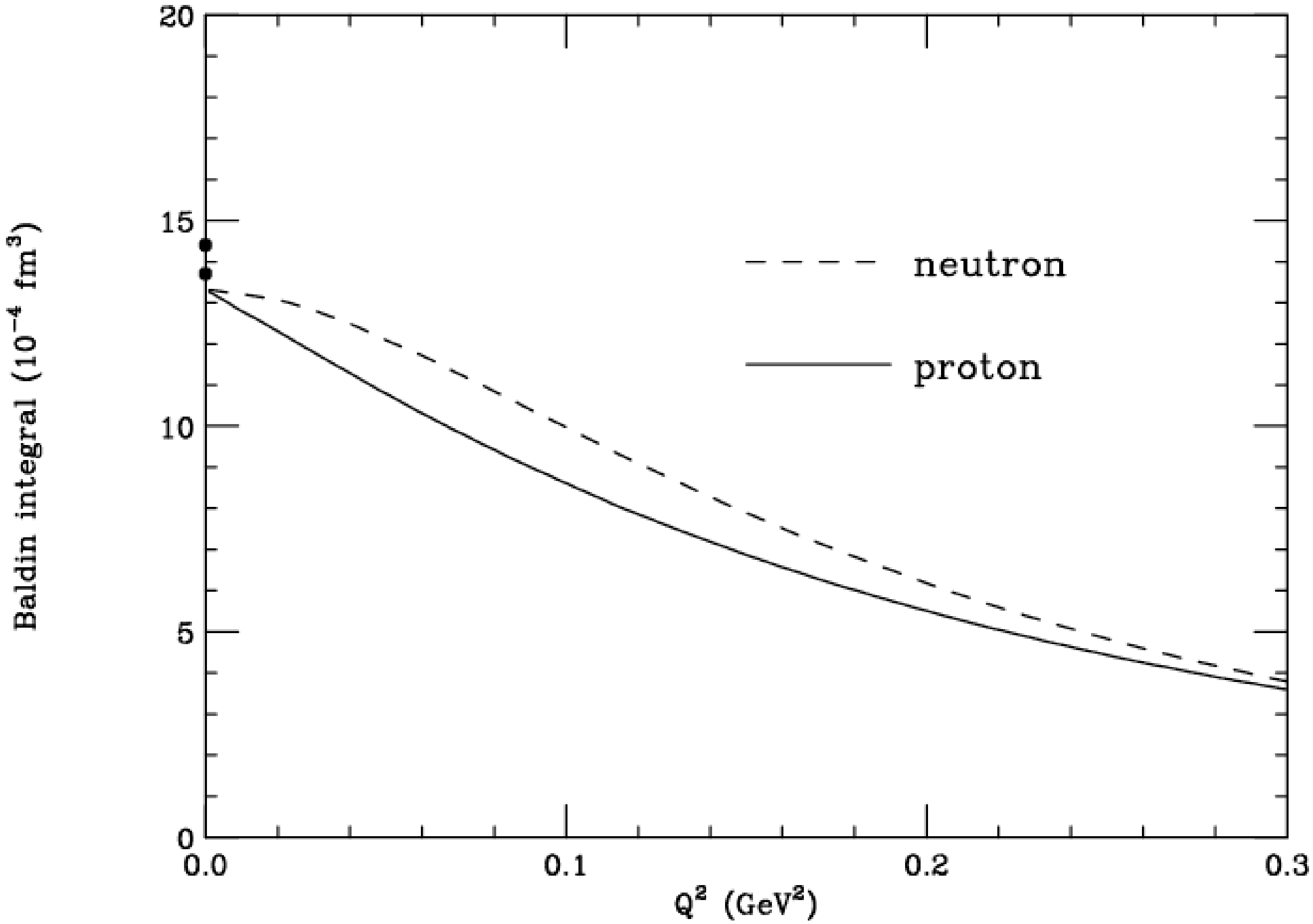
Outlook

- Allow R for proton and deuteron to be different as function of W and Q^2.
- Allow for R for A>2 to differ from deuteron in W and Q^2-dependent way (presently only putting fixed shifts).
- Include “free neutron” data from BONUS experiment in fitting.
- Use final cross sections from Jlab Hall C and A experiments once all corrections complete and iteration on radiative corrections done.
- Extend to higher Q^2 with 12 GeV JLab.

Backup Slides

Application to extended Baldin sum rule

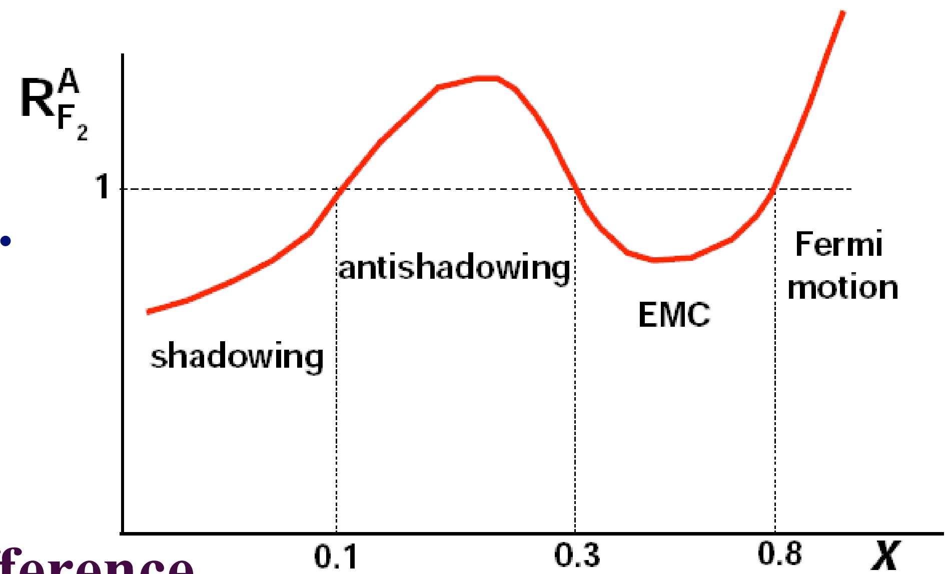
Relates nucleon polarizability to integral of $2xF_1$ from zero to pion threshold. Q^2 dependence should be calculable (analog of GDH sum rule).



Global Fit – EMC effect

Nuclear structure functions are different from structure functions of nuclei.

$$R^A_{F_2}(x,Q^2) = \frac{F_2^A(x,Q^2)}{AF_2^{nucleon}(x,Q^2)}$$



In our fit we correct that difference.

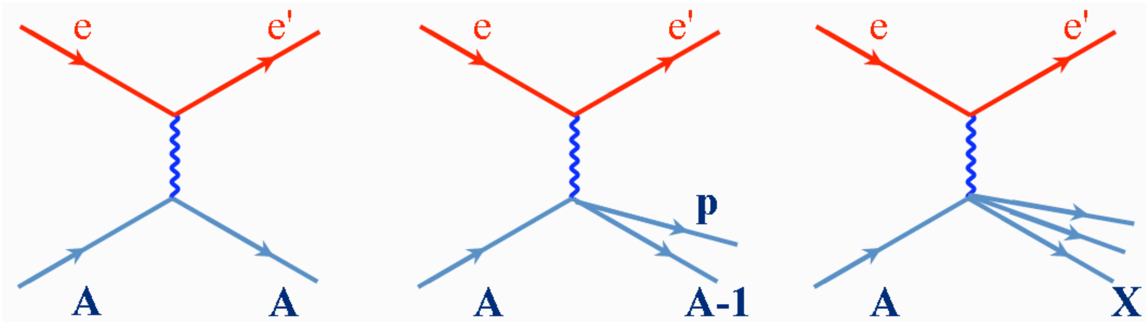
For x>0.125 SLAC-E139 fit was used.

For x<0.0085-0.09 Amaudruz et al Z. Phys C. 51,387(91).

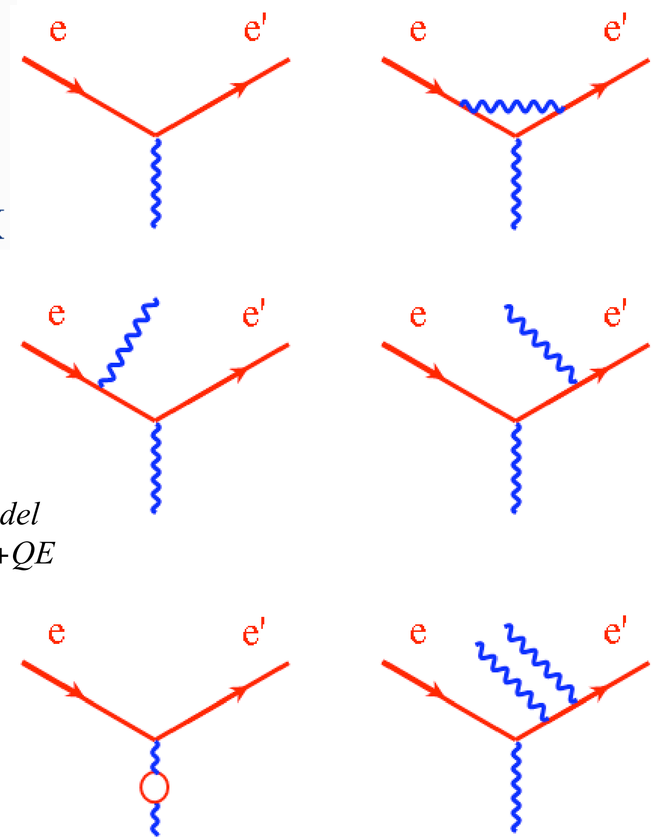
$$F_1^A = M_P W_1^A EMCCORR(x,A) \qquad F_2^A = M_P W_2^A EMCCORR(x,A)$$

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 (E')^2}{Q^4}\left[\cos^2\frac{\theta}{2} W_2(\nu,q^2) + 2W_1(\nu,Q^2)\sin^2\frac{\theta}{2}\right]$$

Radiative and Bin Centering Corrections



Elastic **Quasi-elastic** **Inelastic**



IE+ QE rad. corrected CS

$$\sigma^{exp}_{IE+QE}(\theta, E') = (\sigma^{exp}_{All} - \sigma^{model}_{RadEl}) \Big/ (\sigma^{model}_{RadAll} - \sigma^{model}_{RadEl}) \times \sigma^{model}_{IE+QE}$$

Bin correction factor

$$BC(\theta_i, E'_j) = \sigma^{model}_{RadAll}(\theta_o, E'_j) \Big/ \sigma^{model}_{RadAll}(\theta_i, E'_j)$$

Bin centered CS, FINAL !

$$\frac{d^2\sigma(\theta_o, E_j')}{d\Omega dE'} = \frac{\sum_i^n \sigma(\theta_i, E'_j) BC(\theta_i, E'_j) \Big/ (\Delta\sigma(\theta_i, E'_j) BC(\theta_i, E'_j))^2}{\sum_i^n 1 \Big/ \Delta\sigma(\theta_i, E'_j) BC(\theta_i, E'_j)}$$

Fit A>2 includes new data from CLAS for $^{15}N/C$ and $^{4}He/C$

(CLAS Collaboration, P.E. Bosted, R, Fersch et al., arXiv: 0712.2438 (2007)