# How simple can a nuclear model be, and still be right ?

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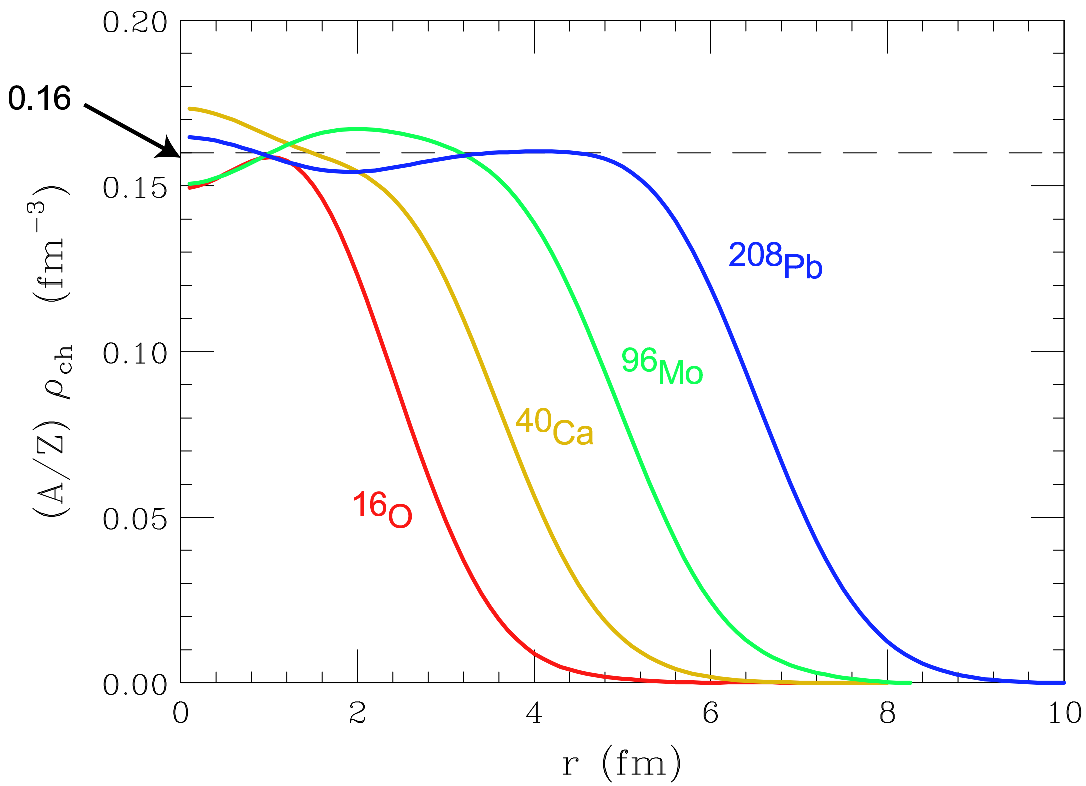
- Summary & prospects

# Basic facts on atomic nuclei

⋆ saturation of nuclear densities indicates that nucleons (just like molecules of a *van der Waals liquid*) cannot be packed too tightly



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- ⋆ probability of finding two *non interacting* nucleons separated by a distance $r$, in isospin symmetric nuclear matter at average density $\bar{\rho}$

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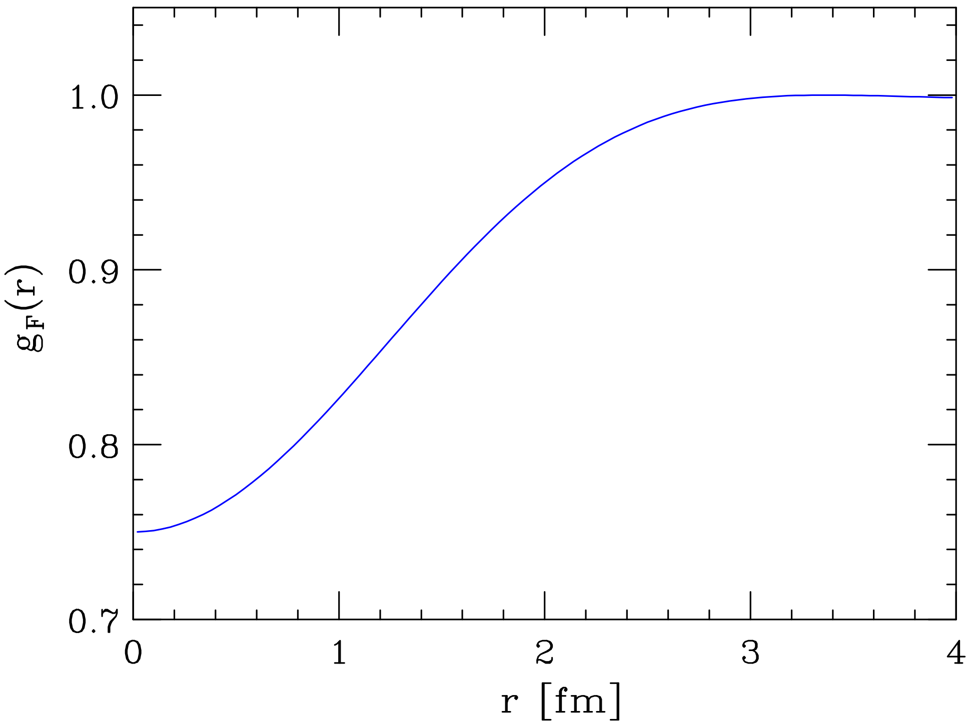
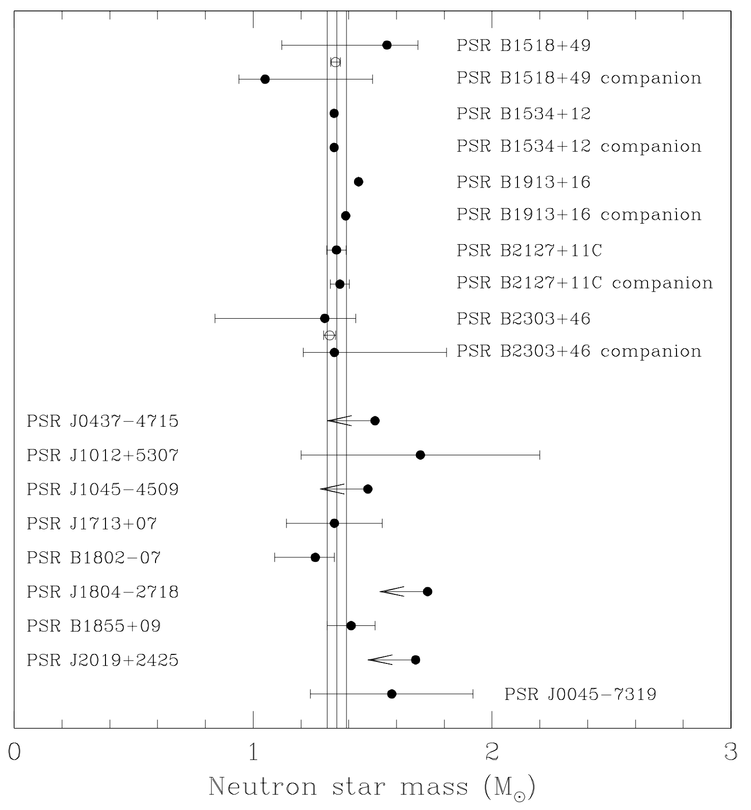


⋆ modeling neutron matter as a non interacting Fermi gas (FG) leads to predict a maximum neutron star mass $\sim 0.8\ M_\odot$ (Oppenheimer & Volkoff, 1939).




⋆ most observed masses are close to $1.4\ M_\odot$. In this instance, the FG model is certainly too simple to be right

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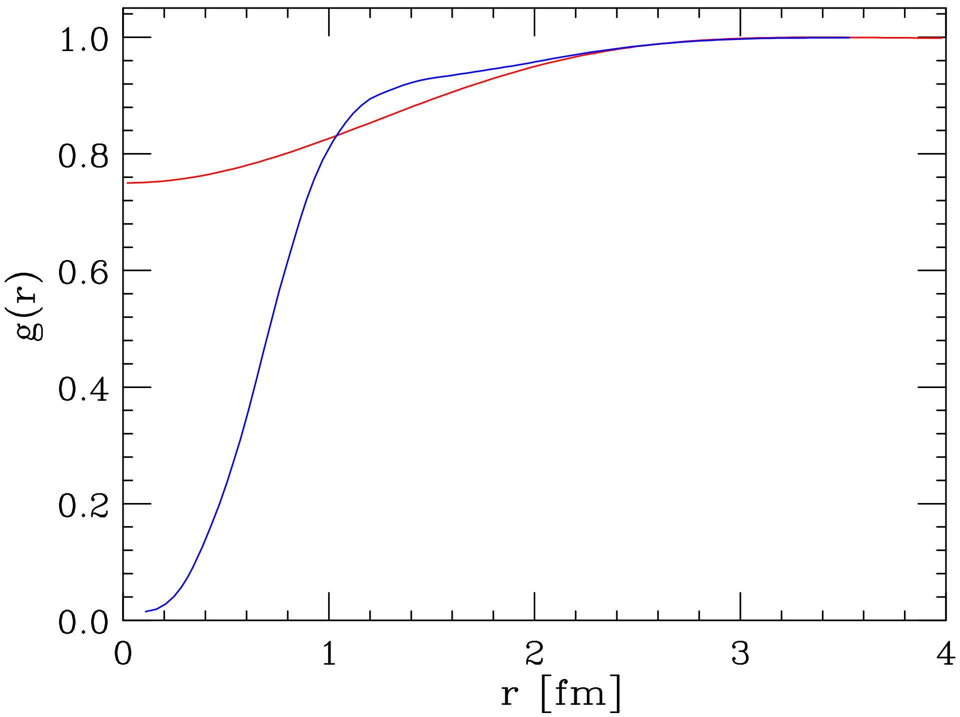

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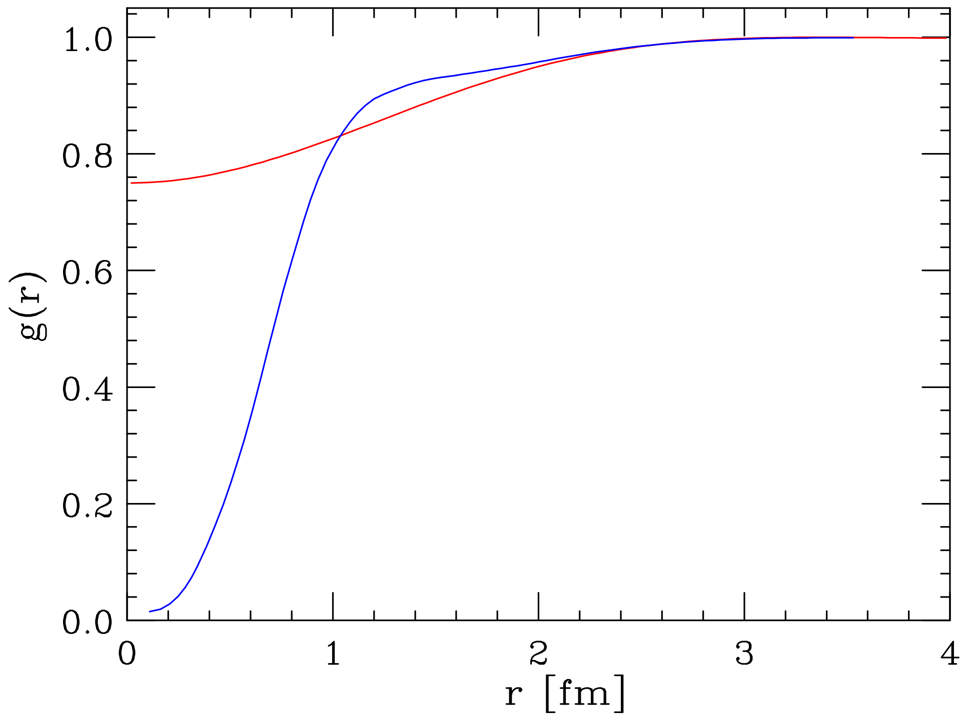


⋆ NN forces induce *strong correlations* in coordinate space, which in turn give rise to *high momentum and high removal energy* components in the nuclear wave function

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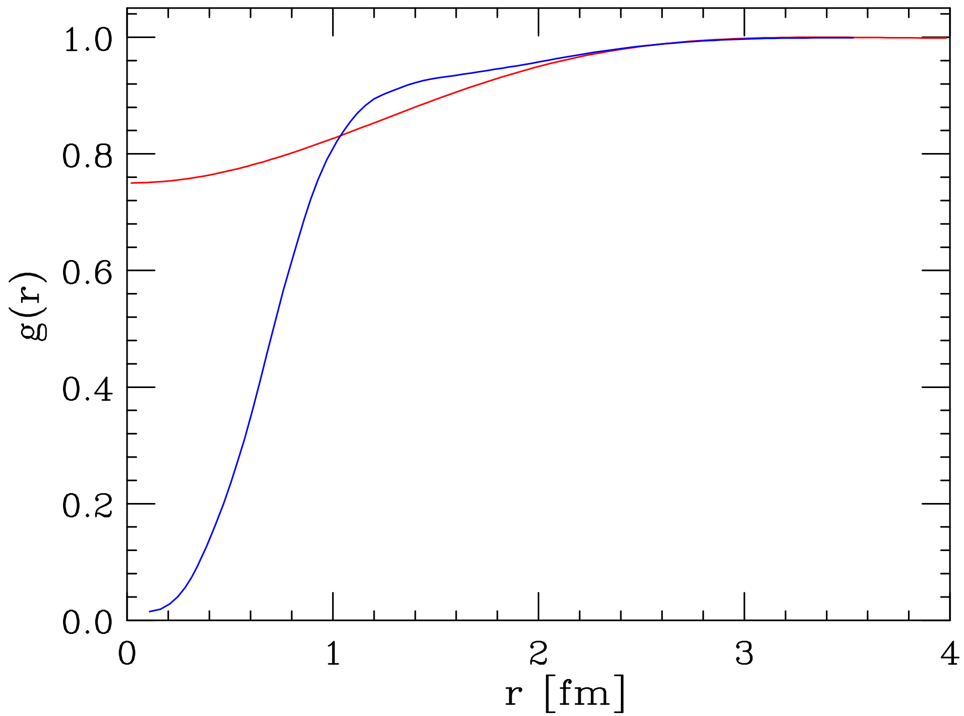


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⋆ is nuclear dynamics needed to explain the electron scattering cross sections ?

# Early applications of the FG model to $e - A$ scattering (A.D. 1971)

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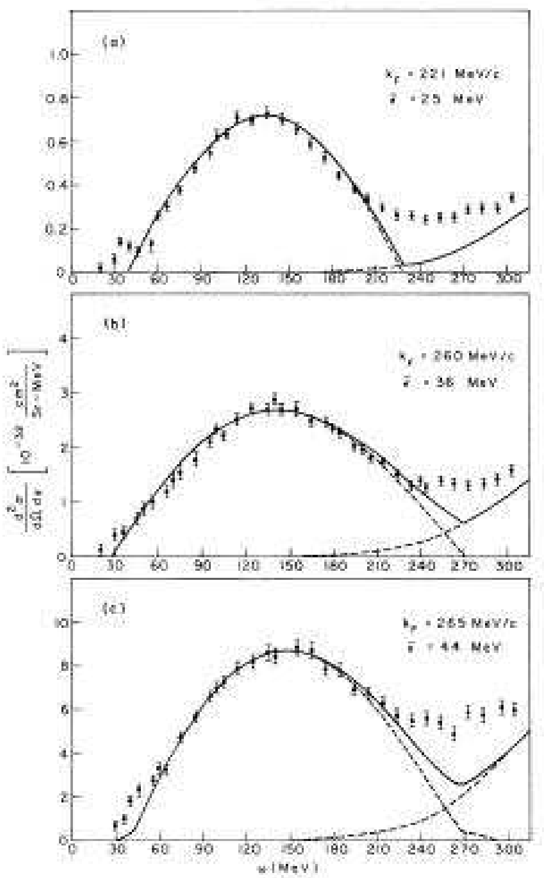

## The crisis (A.D. 1980)

* People lived happily for about 10 years, until experimentalists managed to measure the longitudinal and transverse nuclear responses

$$\frac{d^2\sigma}{d\Omega_{e'} dE_{e'}} = \left(\frac{d\sigma}{d\Omega_{e'}}\right)_M \left[\frac{Q^4}{|\mathbf{q}|^4} R_L(|\mathbf{q}|,\omega) + \left(\frac{1}{2}\frac{Q^2}{|\mathbf{q}|^2} + \tan^2\frac{\theta}{2}\right) R_T(|\mathbf{q}|,\omega)\right]$$

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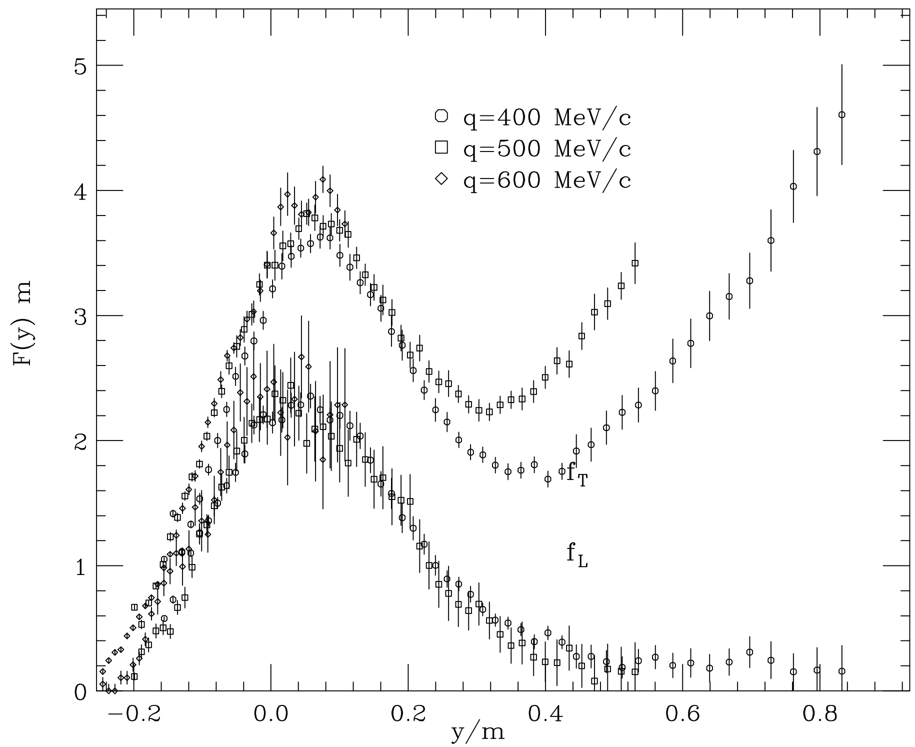

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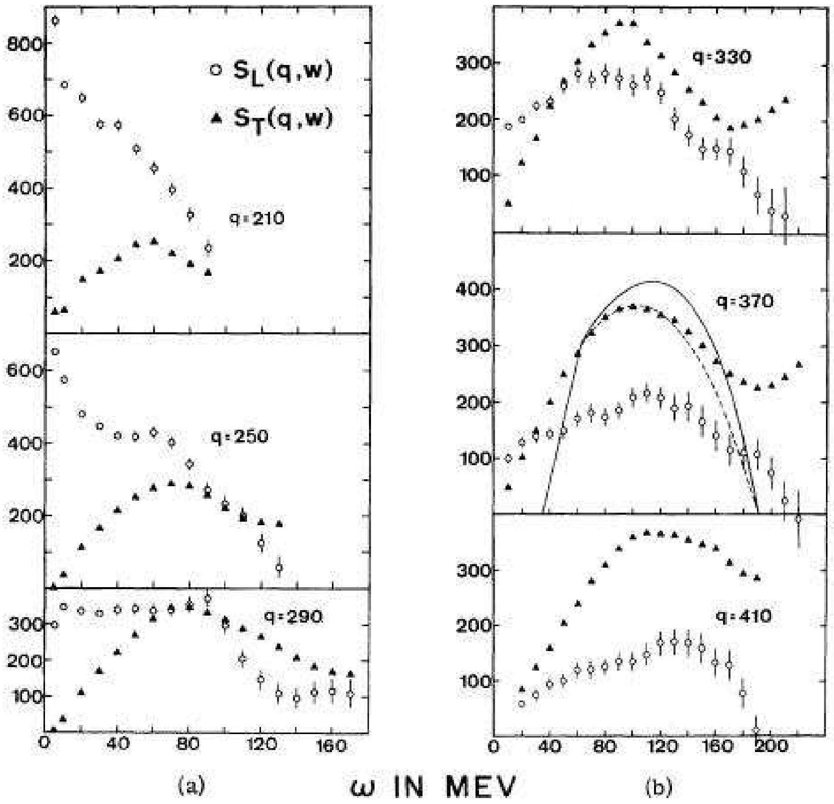


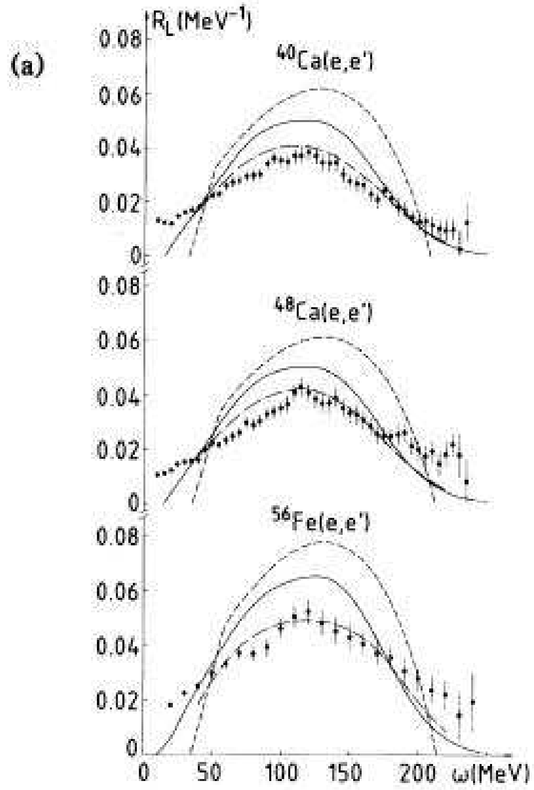

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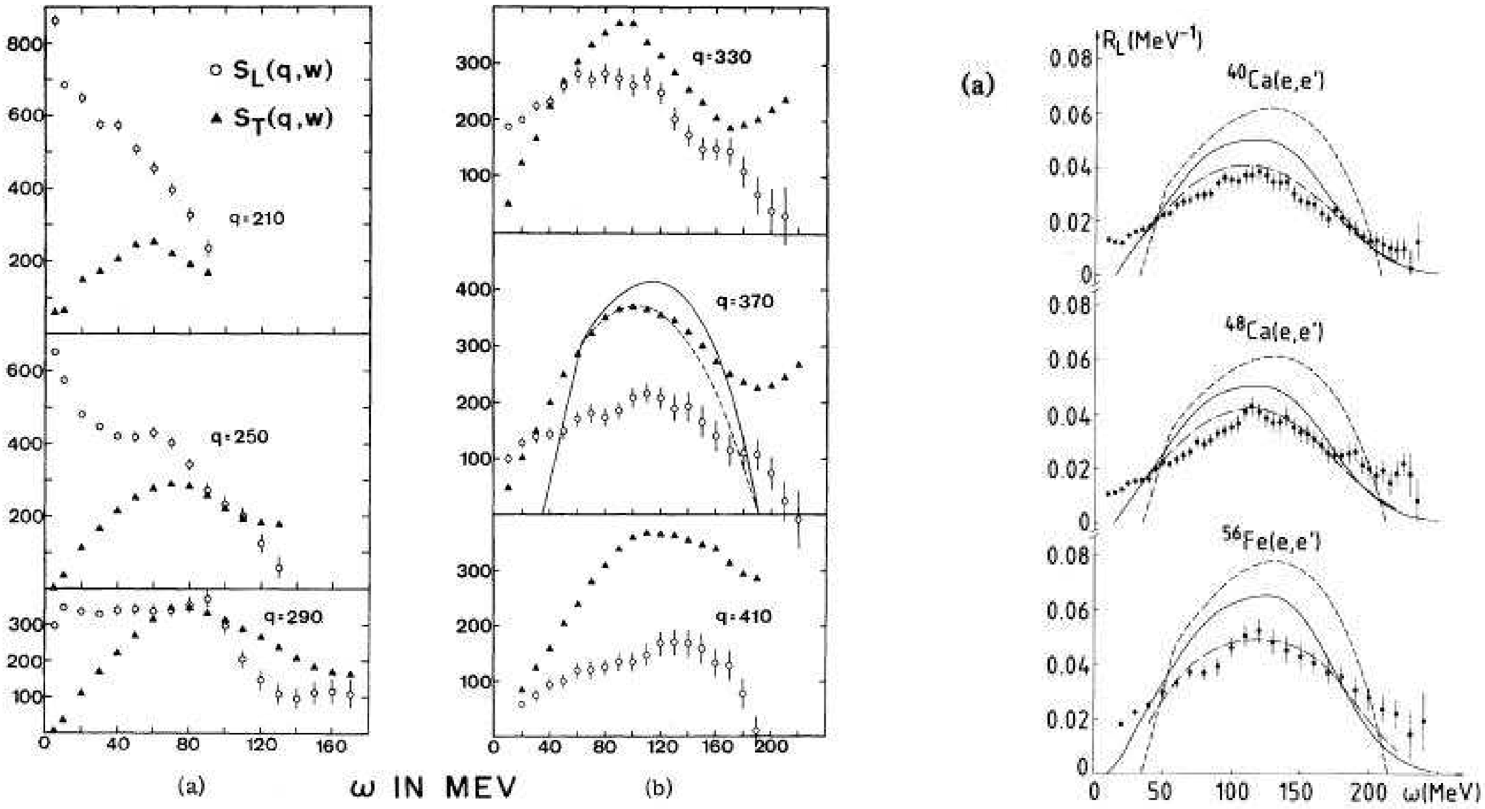


- ⋆ Nuclear dynamics must be included in the picture !

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- ⋆ Dynamics determined from the properties of two- and three-nucleon systems (exactly solvable)

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{j>i} v_{ij} + \sum_{k>j>i} V_{ijk}$$

where $v_{ij}$ is a realistic NN potential (e.g. the ANL $v_{18}$) and $V_{ijk}$ is needed to reproduce the energies of the the three-nucleon systems

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- ⋆ Calculations of nuclear observables *do not* involve any adjustable parameters

## Nucleon-nucleon scattering in free space

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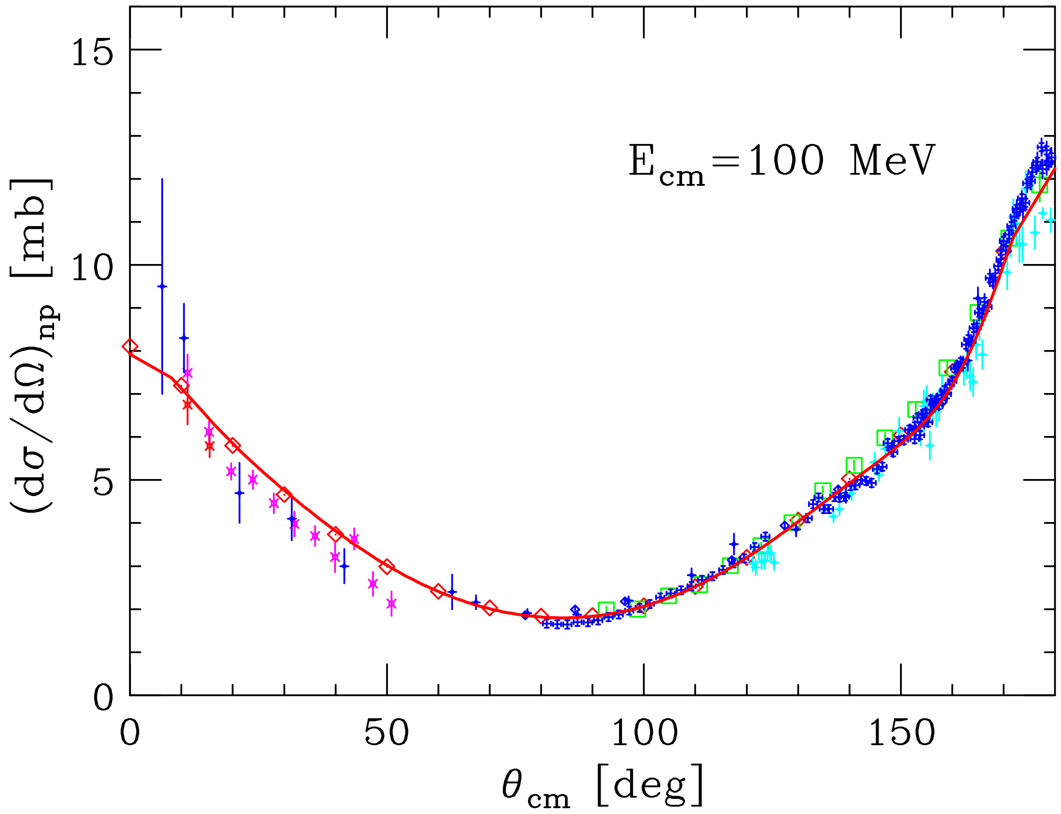
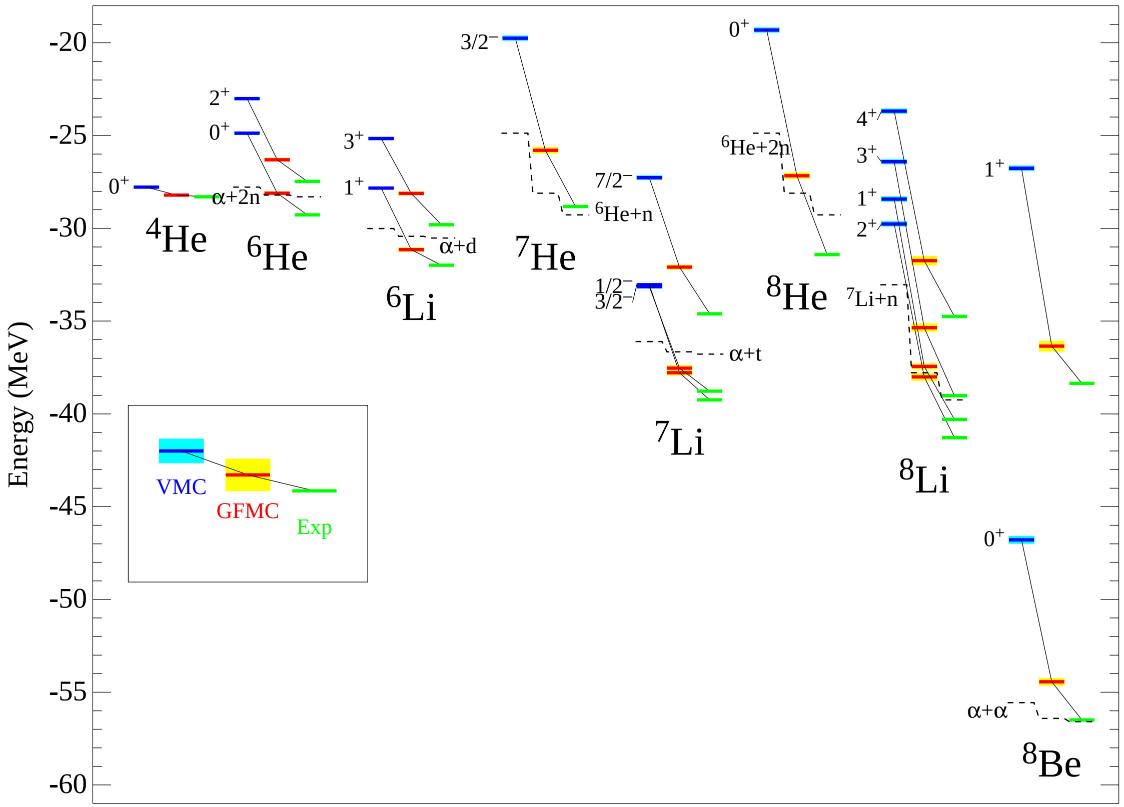
- ⋆ $np$ differential x-section



⋆ Ground and low-lying excited states of nuclei with $A \leq 8$. No approximations involved in the solution of the Schrödinger equation

$$H|n\rangle = E_n|n\rangle$$




$$\Delta = \frac{|E_n^{GFMC} - E_n^{exp}|}{E_n^{exp}} \lesssim 5\%$$

⋆ Note: these calculations are now doable for nuclei with $A \leq 12$.

# Dynamical effects on the nuclear cross section

⋆ Impulse approximation (reasonble at $|\mathbf{q}|^{-1} \ll r_0 = (3\bar{\rho}/4\pi)^{1/2}$ )

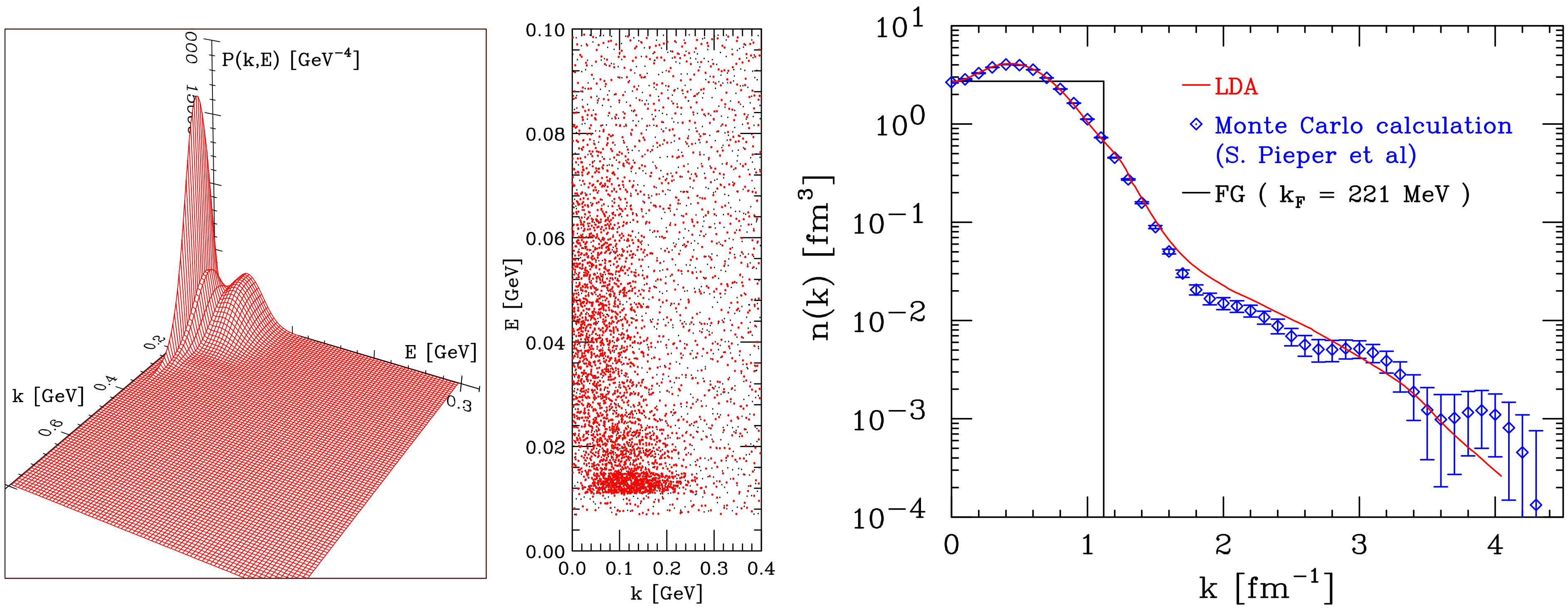
$$J_A^\mu = \sum_{i=1}^{A} j_i^\mu$$

$$d\sigma_A(\mathbf{q},\omega) = \int d^3k \, dE \, d\sigma_N(\mathbf{q},\omega,\mathbf{k},E) P_h(\mathbf{k},E) \, P_p(\mathbf{k}+\mathbf{q},\omega - E)$$

▷ $P_h$: initial state dynamics. Energy-momentum distribution of the struck particle

▷ $P_p$: final state dynamics. Energy-momentum distribution of the outgoing particle

# Hole spectral function and momentum distribution of Oxygen

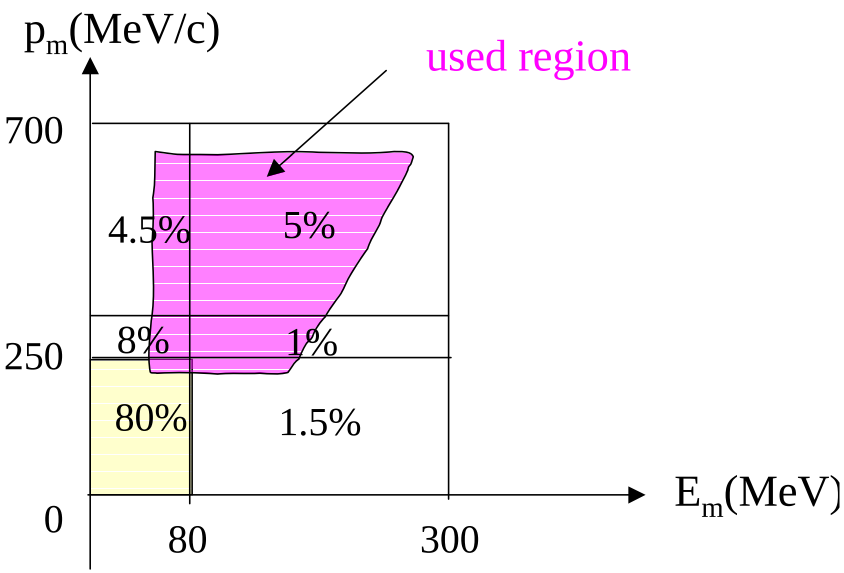


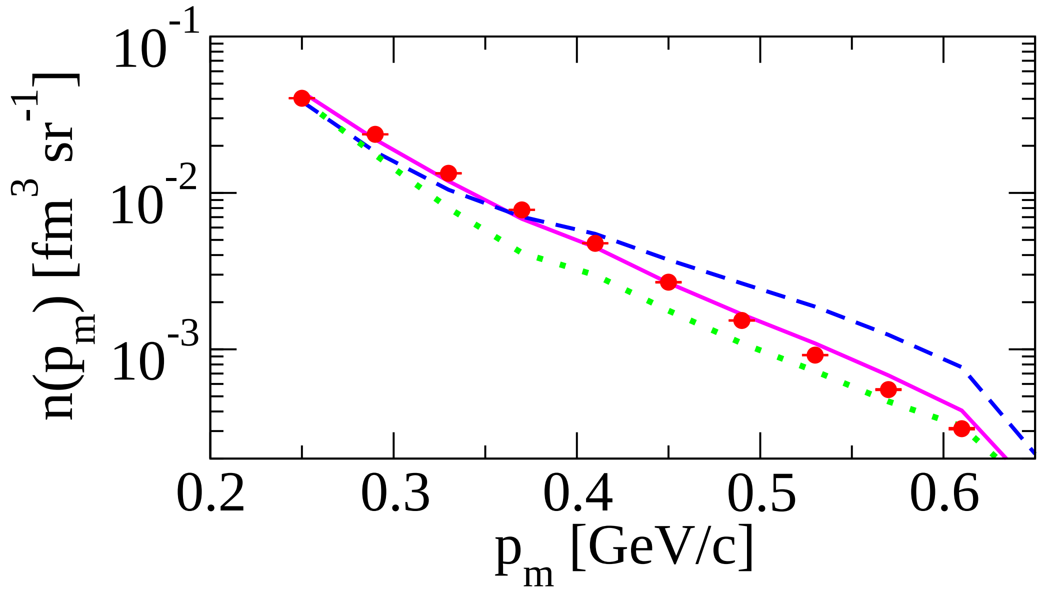

- FG model: $P_h(\mathbf{k}, E) \propto \theta(k_F - |\mathbf{k}|)\,\delta(E - \sqrt{|\mathbf{k}|^2 + m^2} + \epsilon)$
- shell model states account for $\sim$ 80% of the strenght
- the remaining $\sim$ 20%, arising from NN correlations, is located at high momentum *and* large removal energy ($\mathbf{k} \gg k_F, E \gg \epsilon$)

## JLab E97-006 data. Carbon target

- due to the strong correlation between high momentum and high removal energy the spectral function exhibits a pronounced ridge located at

$$E \sim E_{thr} + \frac{A-2}{A-1} \frac{\mathbf{k}^2}{2m}$$



# Particle spectral function

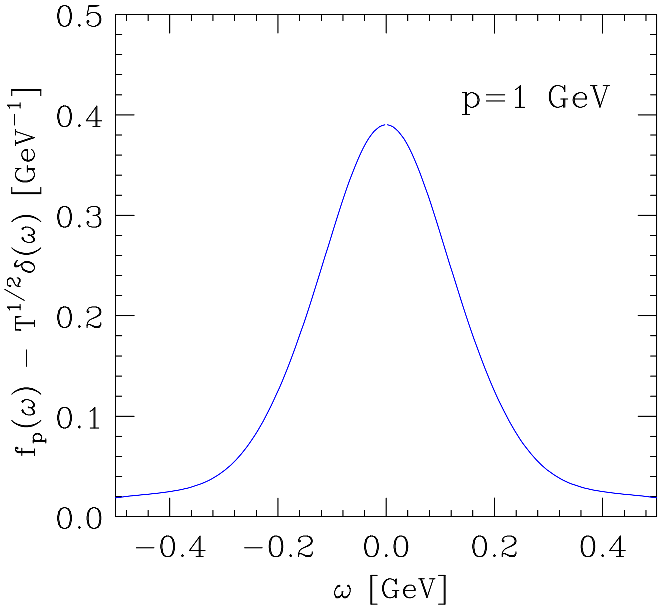
- FG model:

  $$P_p(\mathbf{k}+\mathbf{q},\omega-E) \propto \theta(|\mathbf{k}+\mathbf{q}|-k_F)\,\delta(\omega-E-\sqrt{|\mathbf{k}+\mathbf{q}|^2+m^2})$$

  - ▷ accounts for Pauli blocking
  - ▷ dynamical correlations, not included, must be consistently taken into account (gauge invariance)

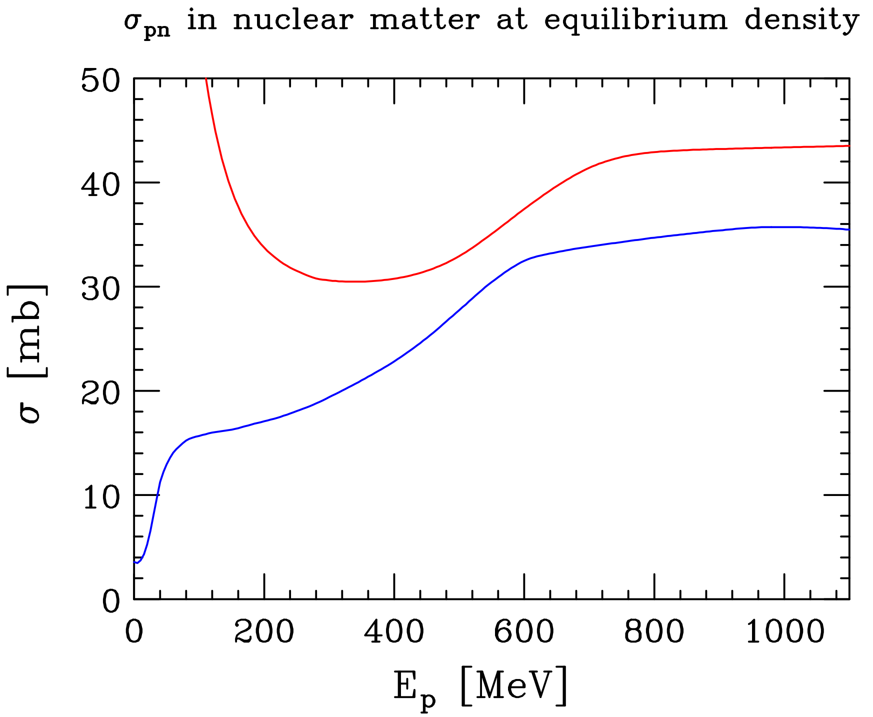
- nuclear many-body theory + eikonal approximation

$$\delta(\omega-E-\sqrt{|\mathbf{k}+\mathbf{q}|^2+m^2})$$
$$\to f(\omega-E-\sqrt{|\mathbf{k}+\mathbf{q}|^2+m^2}))$$



# What do we need ?

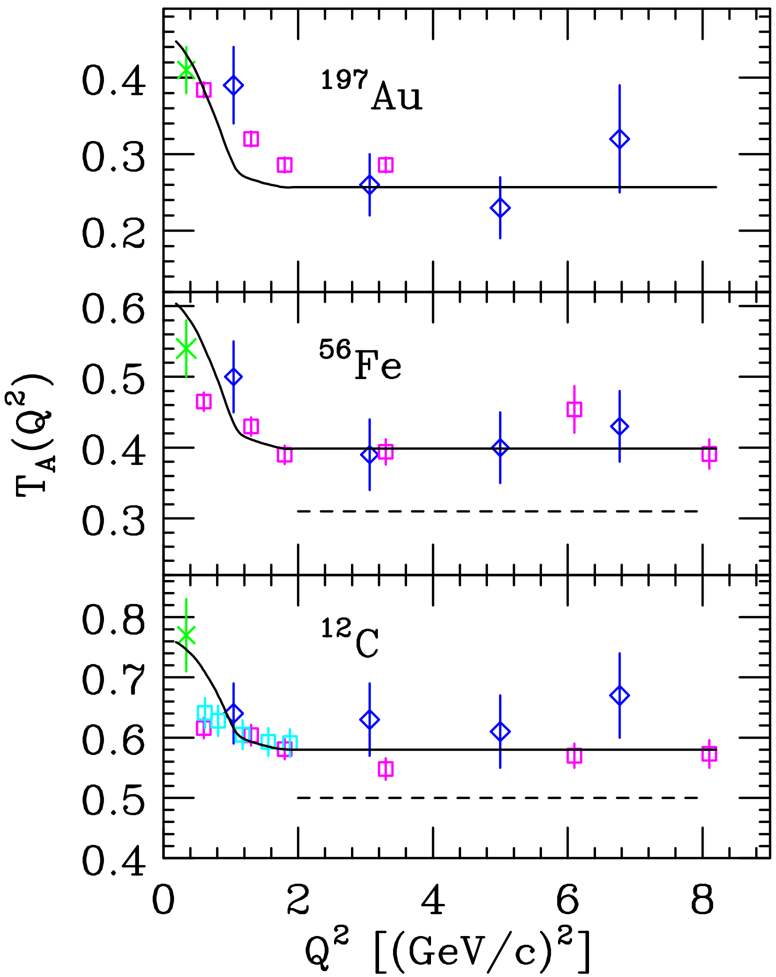
- NN scattering cross section *in the nuclear medium*




- Distribution of the spectator particles in coordinate space. Strongly affected by NN correlations.

# Nuclear transparency measured in $(e, e'p)$

- $\star$ recall: no FSI $\rightarrow T_A \equiv 1$

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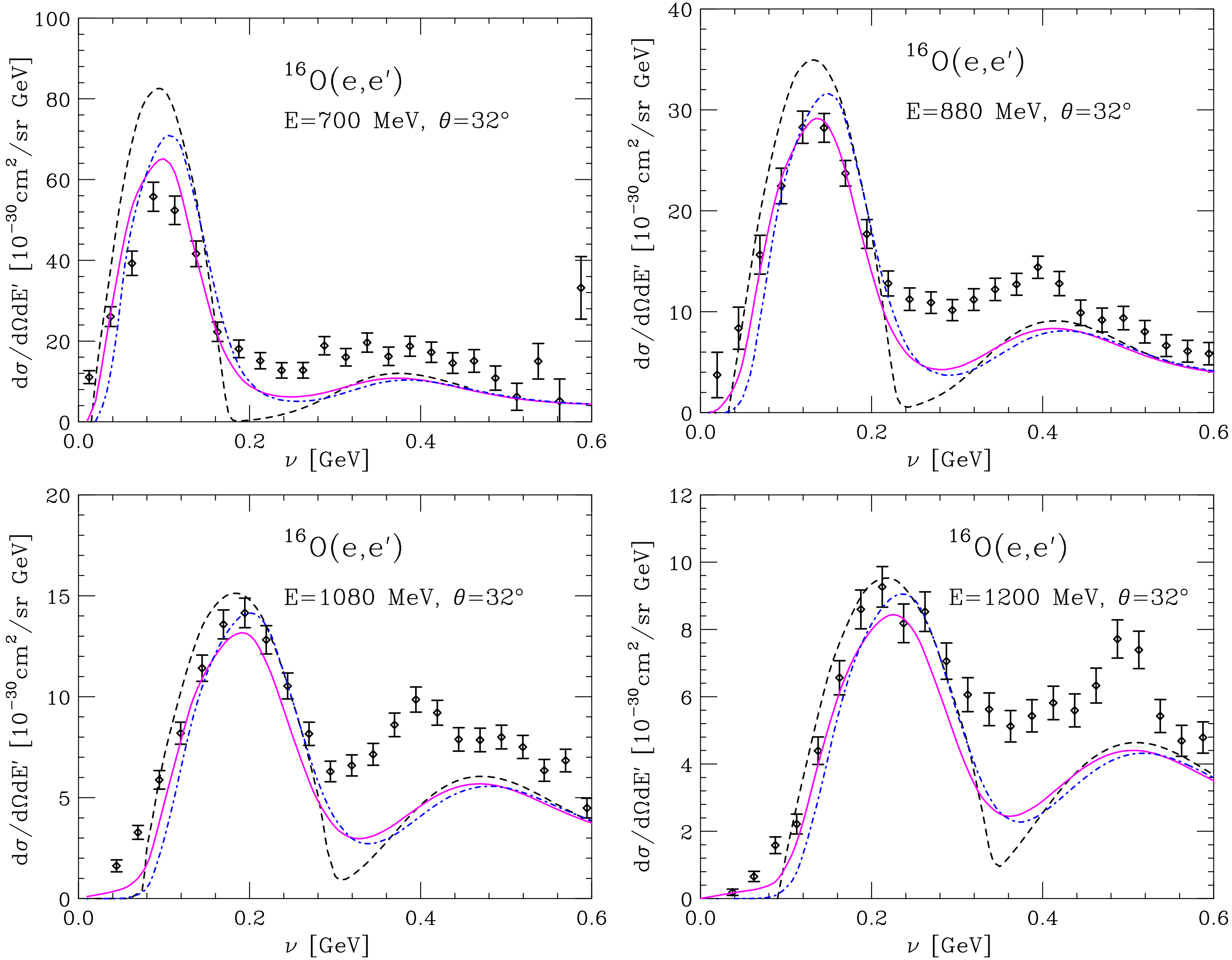
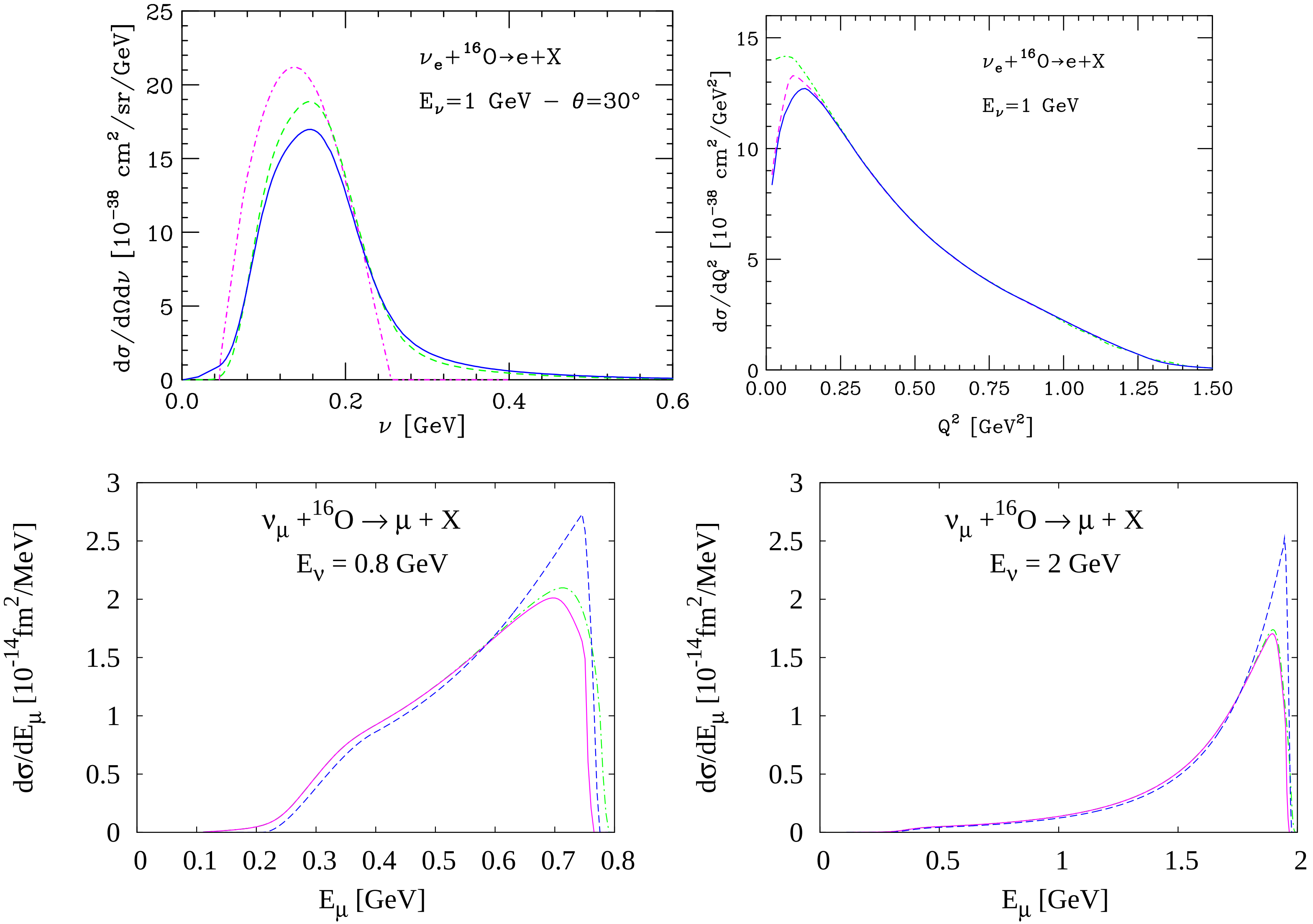
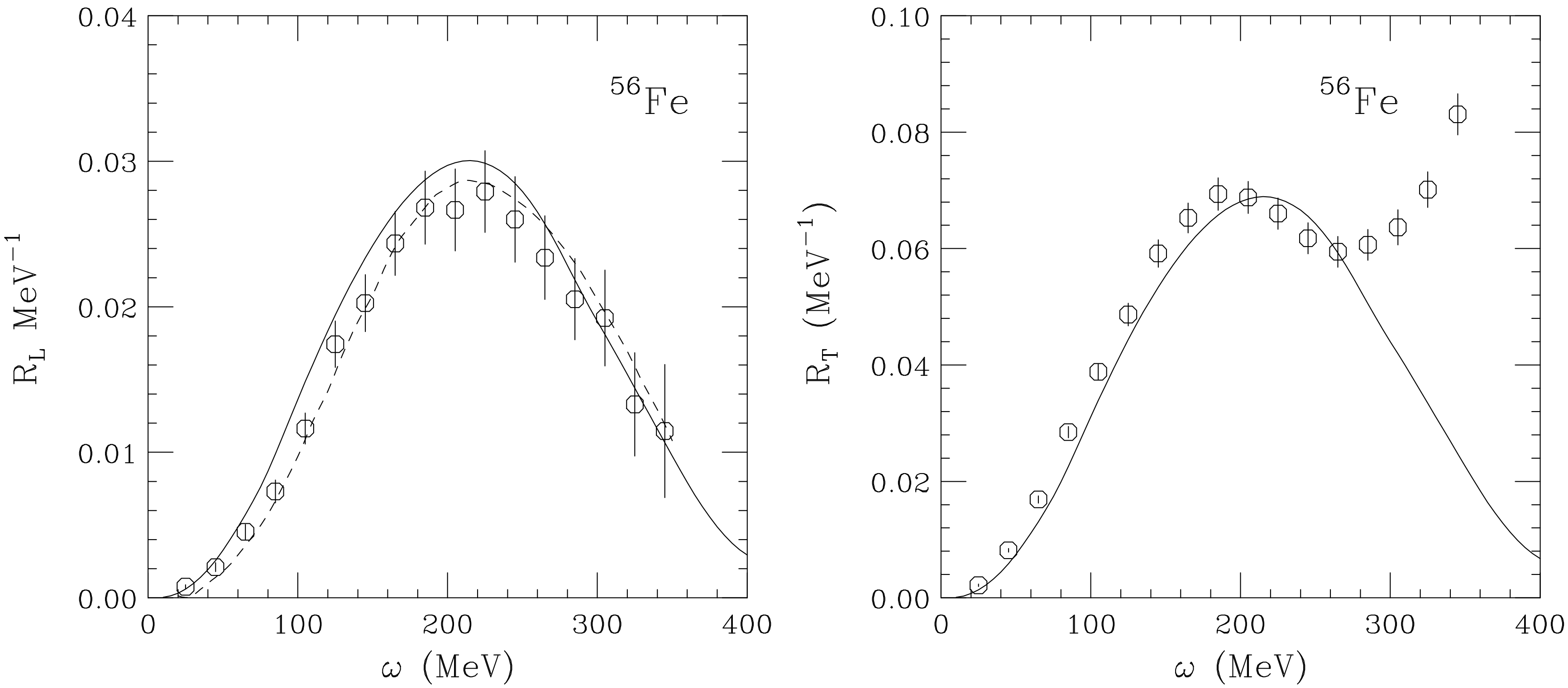
# Comparison to Oxygen data @ $0.2 \lesssim Q^2 \lesssim 0.6$ GeV$^2$



# Results for $^{16}O\ (\nu_e, e)$ scattering



# Crisis ? What crisis ?




- longitudinal and transvere response of Iron at $|\mathbf{q}| = 570$ MeV
- calculations by A. Fabrocini and S. Fantoni, *involving no adjustable parameters*

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- ⋆ Breakdown of the impulse approximation.
  - ▷ Antisymmetrization of the final state
  - ▷ Appearance of long range correlations leading to excitation of collective modes

## Summary & prospects: *vox clamantis in deserto*

- ⋆ Realistic nuclear models should incorporate all the available dynamical information
- ⋆ The emerging picture, confirmed by electron scattering data, suggests that atomic nuclei are strongly correlated systems
- ⋆ Oversimplified models may lead to totally wrong predictions