
Calibration of the Pythia 8.2 Top Quark Mass Using 2-Jettiness in e^+e^-

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Outline

- Introduction
- Monte Carlo generators and the top quark mass
- Calibration of the Monte Carlo top mass parameter
- Preliminary detailed results of first serious systematic analysis
- Summary, future plans

In collaboration with:

M. Butenschön

B. Dehnadi,

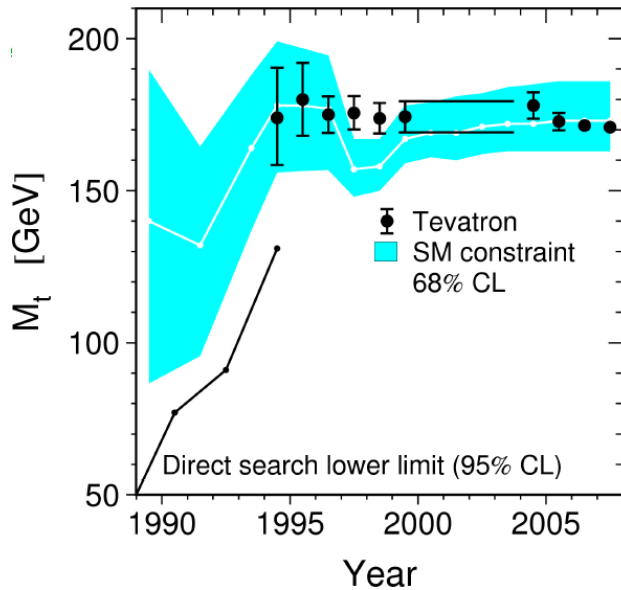
V. Mateu,

M. Preisser

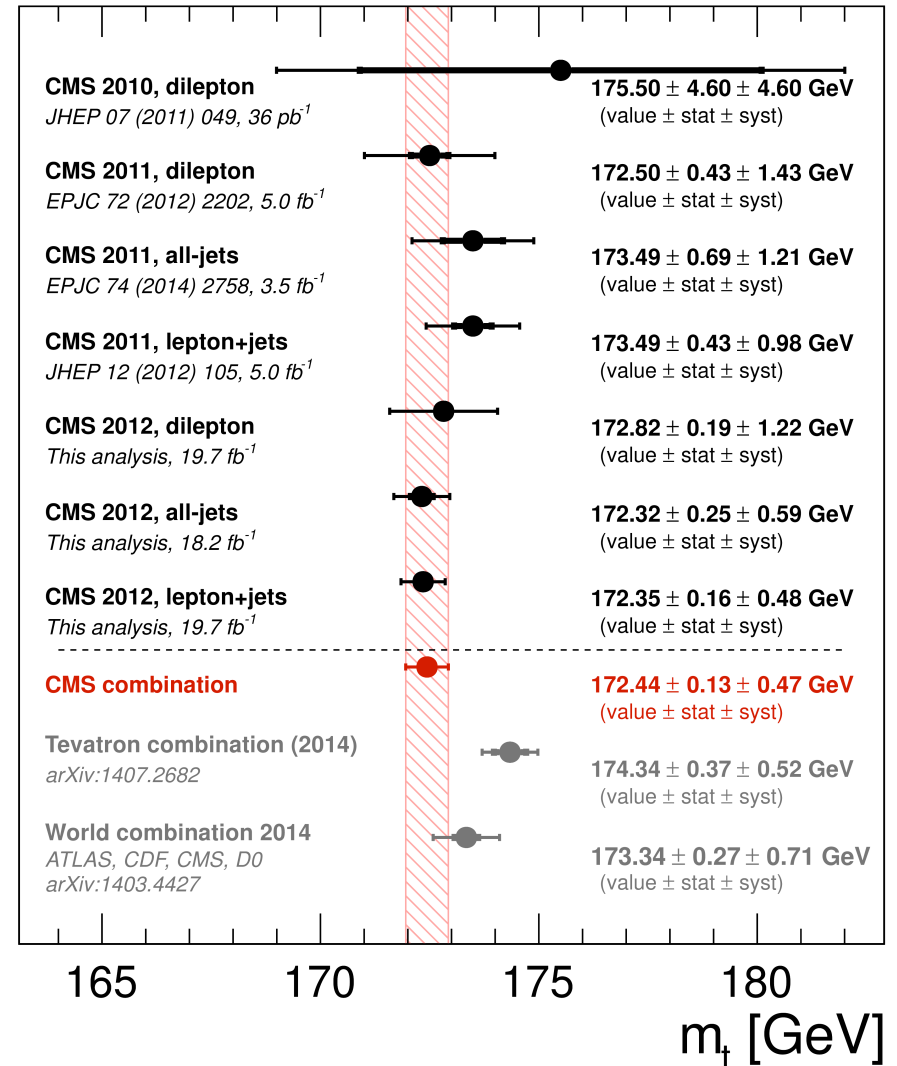
I. Stewart



A small history on top mass reconstruction



- Many individual measurements with uncertainty below 1 GeV.
- Some discrepancies between LHC and Tevatron
- Reached <500MeV range.



Main Top Mass Measurements Methods

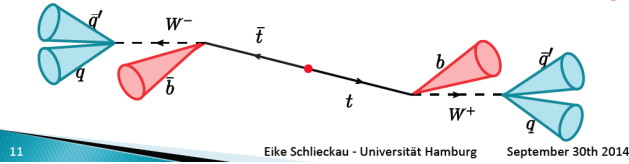
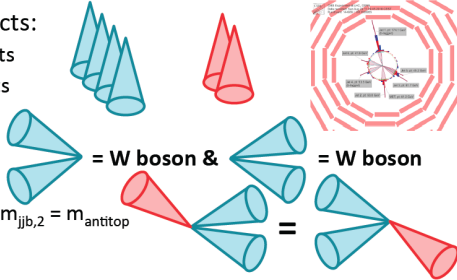
LHC+Tevatron

Direct Reconstruction:

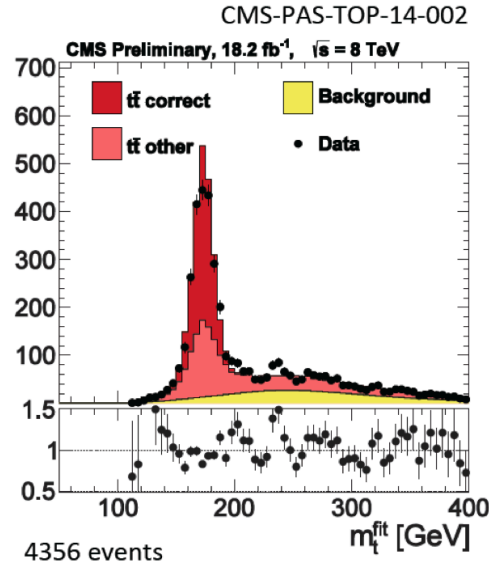
Kinematic Fit

Selected objects:

- 4 untagged jets
- 2 b-tagged jets

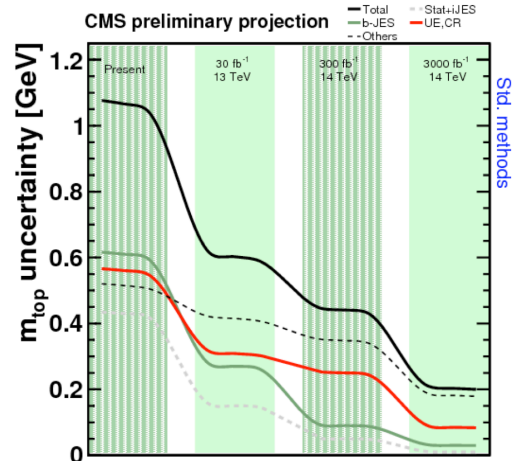


11 Eike Schlieckau - Universität Hamburg September 30th 2014



Determination of the best-fit value of the Monte-Carlo top quark mass parameter

kinematic mass determination

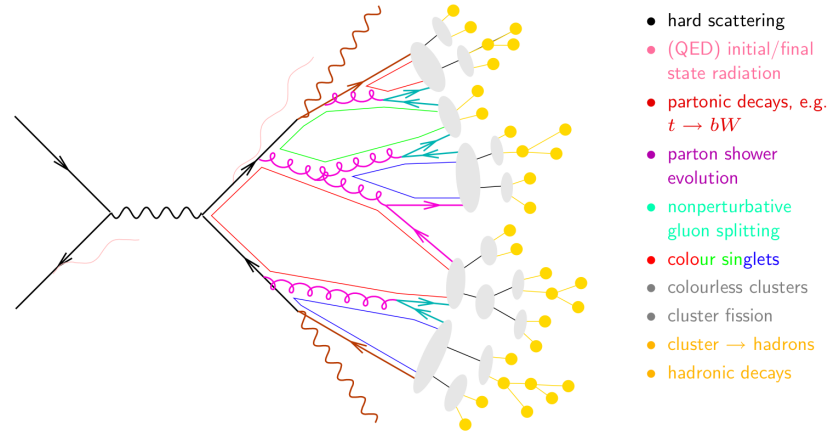


← $\Delta m_t \sim 200 \text{ MeV}$ (projection)

- ⊕ High top mass sensitivity
- ⊖ Precision of MC ?
- ⊖ Meaning of m_t^{MC} ?

$\Delta m_t \sim 0.5 \text{ GeV}$

Monte-Carlo Event Generators



- Full simulation of all processes (all experimental aspects accessible)
- QCD-inspired: partly first principles QCD \Leftrightarrow partly model (observable-dependent)
- Description power of data better than intrinsic theory accuracy.
- Top quark: treated like a real particle ($m_t^{\text{MC}} \approx m_t^{\text{pole}} + ?$).

But pole mass ambiguous by $O(1 \text{ GeV})$ due to confinement.

Better mass definition needed.

Uncertainty (a): But how precise is modelling? \rightarrow Part of exp. Analyses

Uncertainty (b): What is the meaning of MC QCD parameters? \rightarrow

Depends strictly speaking on the observable, because of model character of MCs !

Must be addressed for each type of observable (until we have better MCs).

MC Top Quark Mass

AHH, Stewart 2008
AHH, 2014

$$m_t^{\text{MC}} = m_t^{\text{MSR}}(R = 1 \text{ GeV}) + \Delta_{t,\text{MC}}(R = 1 \text{ GeV})$$

$$\Delta_{t,\text{MC}}(1 \text{ GeV}) \sim \mathcal{O}(1 \text{ GeV})$$

- small size of $\Delta_{t,\text{MC}}$
- Renormalon-free
- little parametric dependence on other parameters

MSR Mass Definition

MS Scheme: $(\mu > \bar{m}(\bar{m}))$

$$\bar{m}(\bar{m}) - m^{\text{pole}} = -\bar{m}(\bar{m}) [0.42441 \alpha_s(\bar{m}) + 0.8345 \alpha_s^2(\bar{m}) + 2.368 \alpha_s^3(\bar{m}) + \dots]$$

MSR Scheme: $(R < \bar{m}(\bar{m}))$



$$m_{\text{MSR}}(R) - m^{\text{pole}} = -R [0.42441 \alpha_s(R) + 0.8345 \alpha_s^2(R) + 2.368 \alpha_s^3(R) + \dots]$$

$$m_{\text{MSR}}(m_{\text{MSR}}) = \bar{m}(\bar{m})$$

$\Rightarrow m_{\text{MSR}}(R)$ Short-distance mass that smoothly interpolates all R scales

Calibration of the MC Top Mass

Method:

- ✓ 1) Strongly mass-sensitive hadron level observable (as closely as possible related to reconstructed invariant mass distribution !)
- ✓ 2) Accurate analytic hadron level QCD predictions at \geq NLL/NLO with full control over the quark mass scheme dependence.
- ✓ 3) QCD masses as function of m_t^{MC} from fits of observable.
- 4) Cross check observable independence

$$m_t^{\text{MC}} = m_t^{\text{MSR}}(R = 1 \text{ GeV}) + \Delta_{t,\text{MC}}(R = 1 \text{ GeV})$$

$$\Delta_{t,\text{MC}}(1 \text{ GeV}) = \bar{\Delta} + \delta\Delta_{\text{MC}} + \delta\Delta_{\text{pQCD}} + \delta\Delta_{\text{param}}$$

Monte Carlo errors:

- different tunings
- parton showers
- color reconnection
- Intrinsic error, ...

QCD errors:

- perturbative error
- scale uncertainties
- electroweak effects

Parametric errors:

- strong coupling α_s
- Non-perturbative parameters

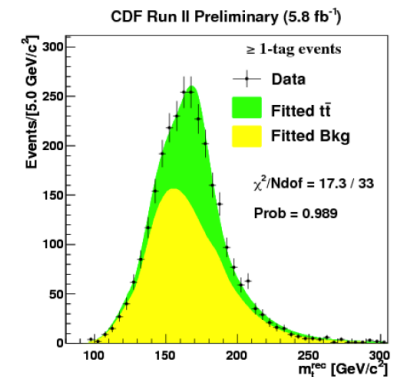
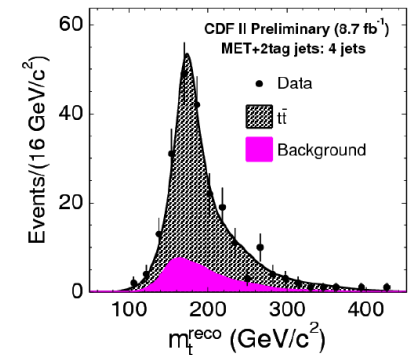
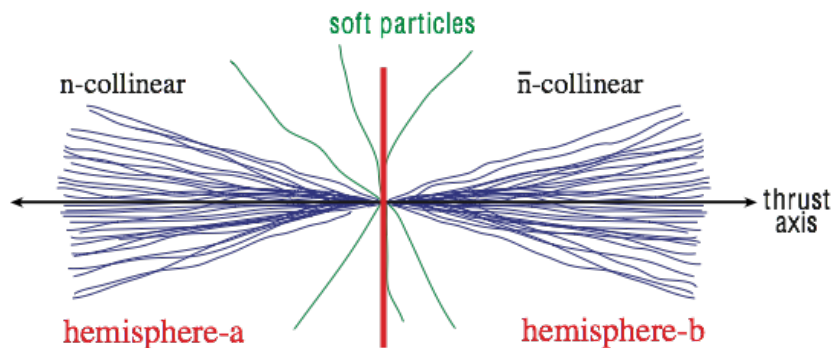
Thrust Distribution

Observable: 2-jettiness in e^+e^- for $Q \sim p_T \gg m_t$ (boosted tops)

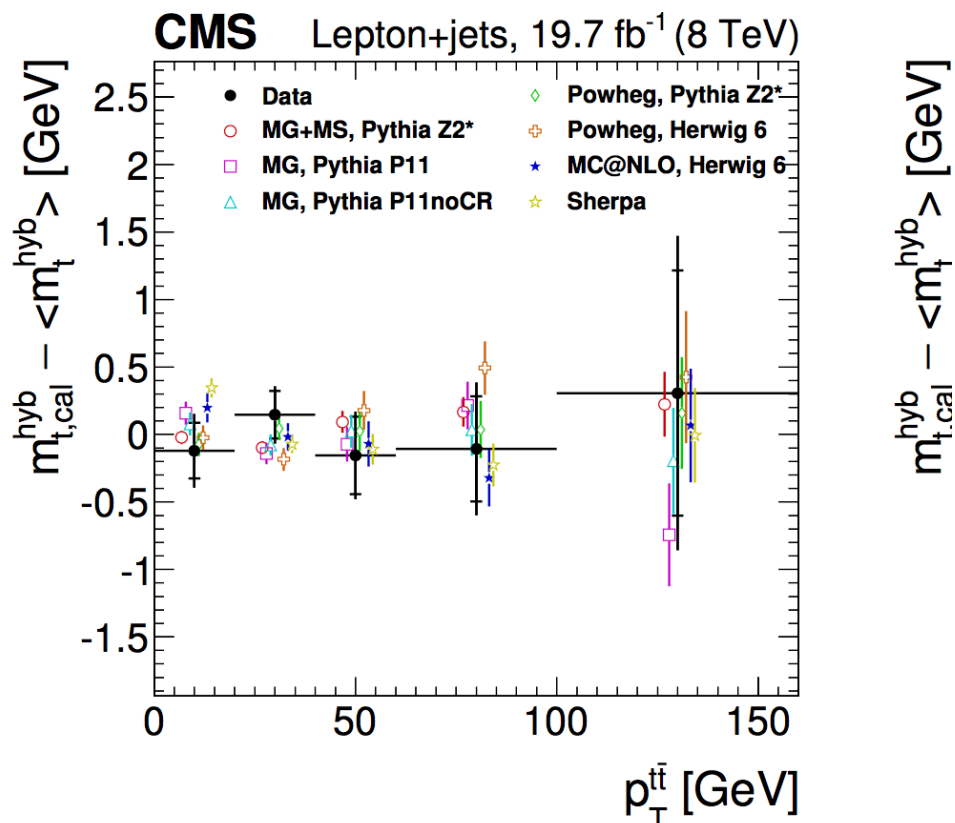
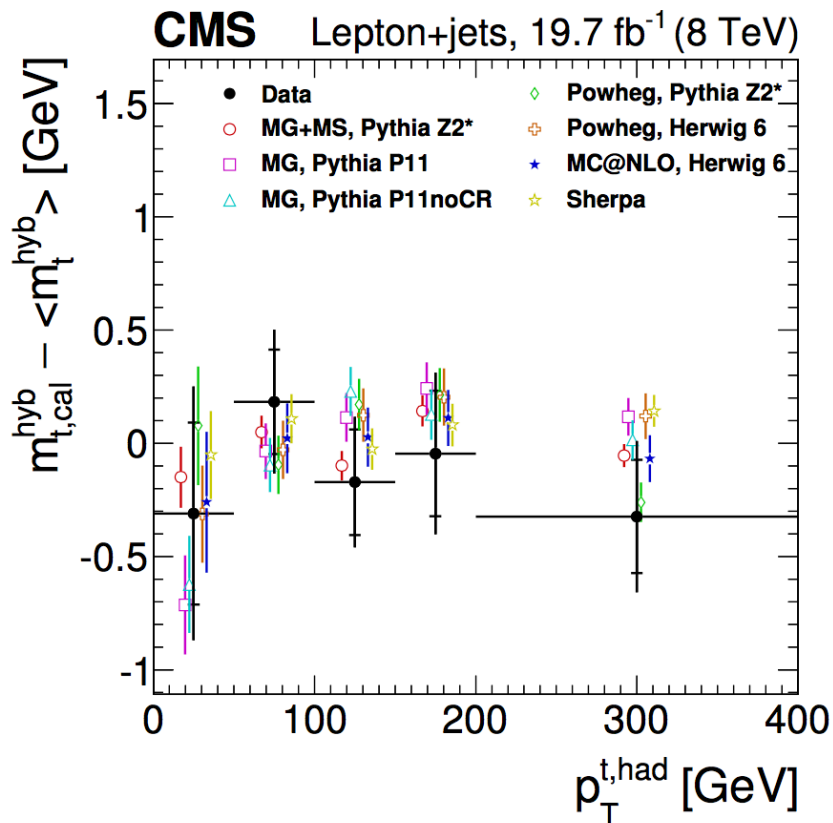
$$\tau = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{n} \cdot \vec{p}_i|}{Q}$$

$$\xrightarrow{\tau \rightarrow 0} \frac{M_1^2 + M_2^2}{Q^2}$$

Invariant mass distribution in the resonance region of wide hemisphere jets !

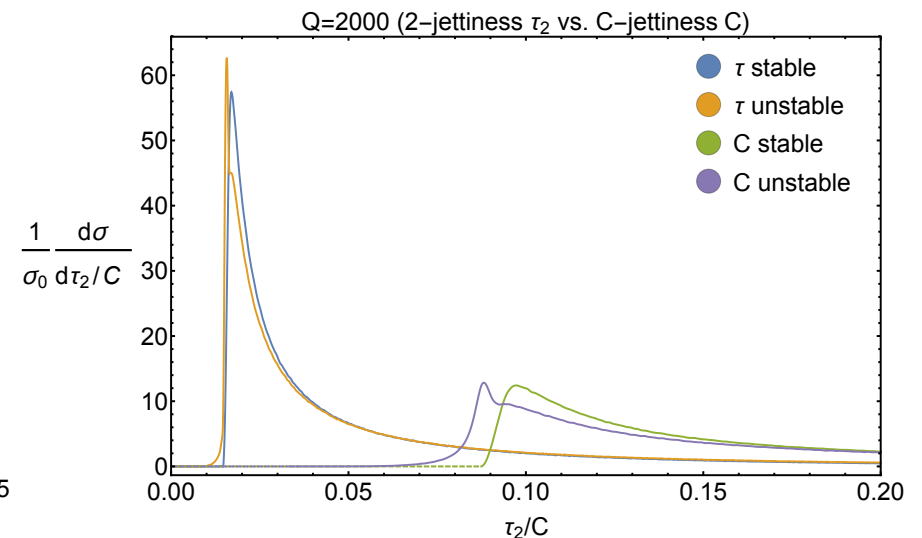
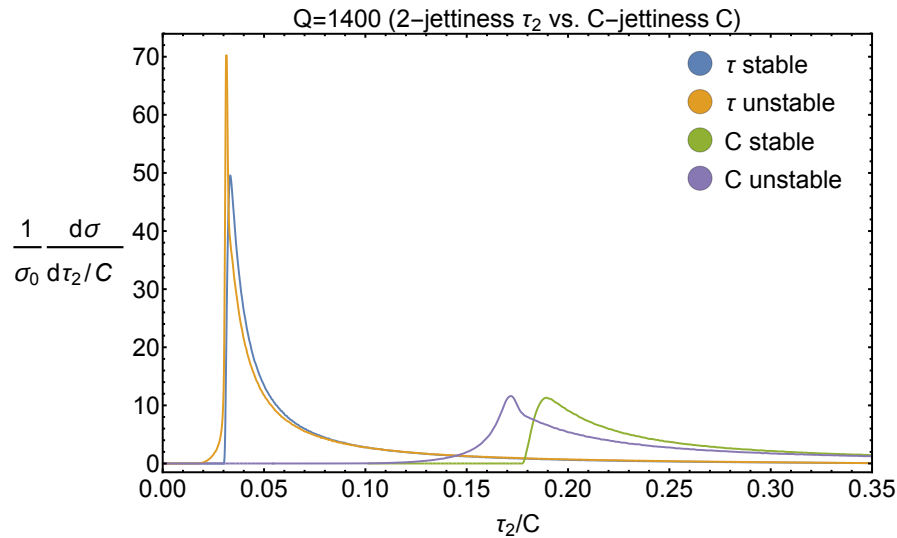
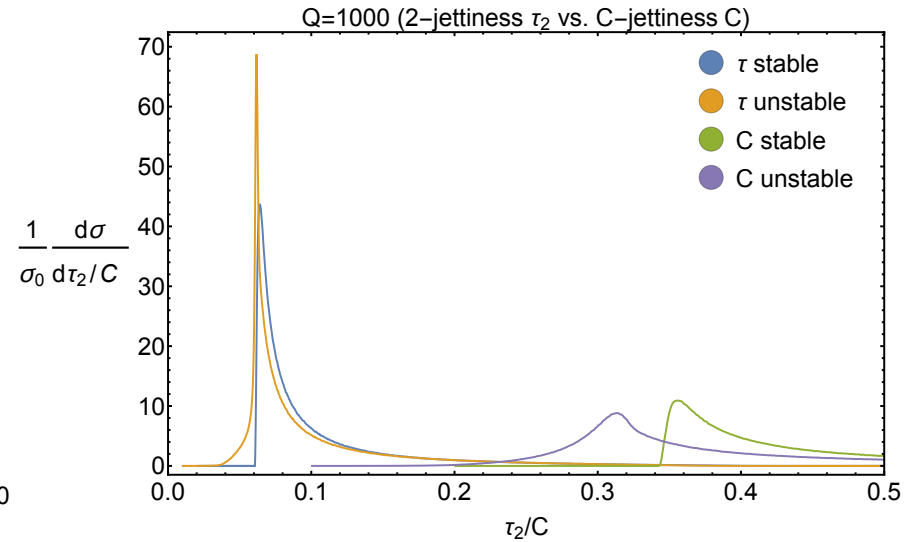
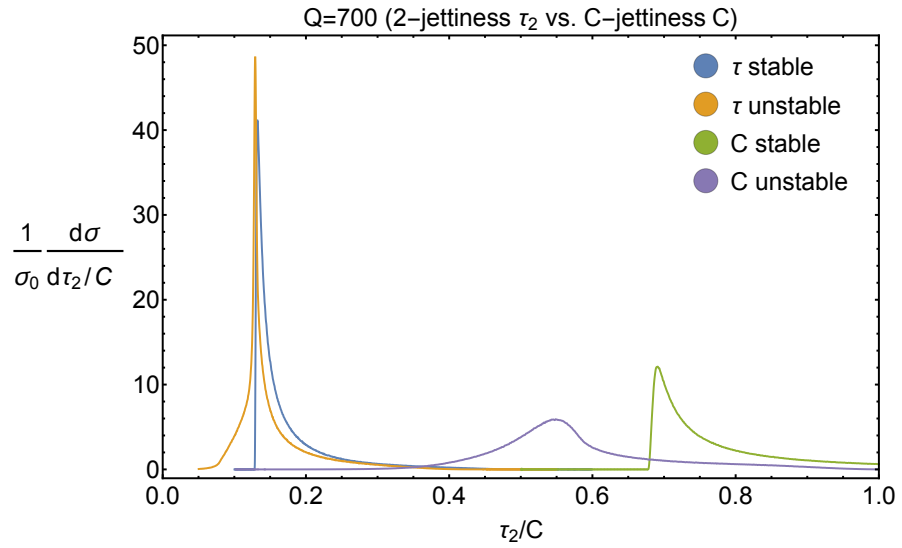


Boosted Top Mass Measurements at CMS



- Top mass from reconstruction of boosted tops consistent with low p_T results.
- More precise studies possible with more statistics from Run2.

Event Shape Distributions (Pythia 8.2)



Factorization for Event Shapes

$$\frac{d\sigma}{d\tau} = Q^2 \sigma_0 H_0(Q, \mu) \int dl J_0(Ql, \mu) S_0(Q\tau - l, \mu)$$

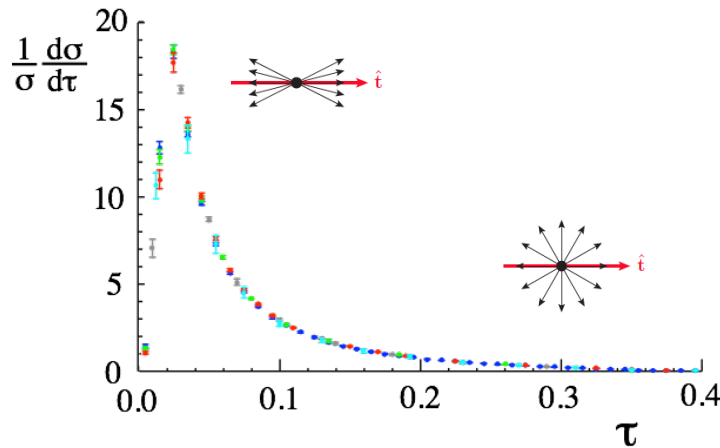
Massless quarks:

Korshemski, Sterman 1995-2000

Bauer, Fleming, Lee, Sterman (2008)

Becher, Schwartz (2008)

Abbate, AHH, Fickinger, Mateu, Stewart 2010



Extension to massive quarks:

- VFNS for final state jets (with massive quarks): log summation incl. mass
- Boosted fat top jets

Fleming, AHH, Mantry, Stewart 2007

Gritschacher, AHH, Jemos, Mateu Pietrulewicz 2013-2014

Butenschön, Dehnadi, AHH, Mateu 2016 (to appear soon)

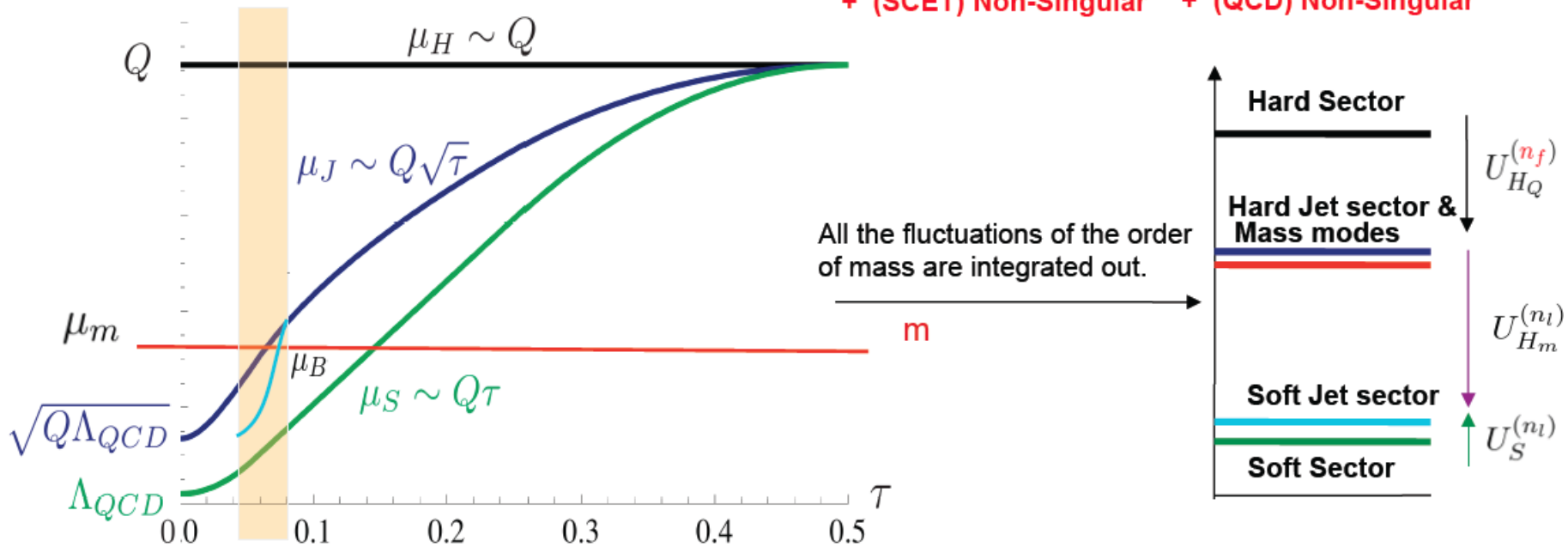
➔ NNLL + NLO + non-singular + hadronization + renormalon-subtraction

b(oosted)HQET Factorization

$$\left| \frac{1}{\sigma_0} \frac{d\hat{\sigma}(\tau)}{d\tau} \right|^{\text{bHQET}} = Q H_Q^{(n_f)}(Q, \mu_Q) U_{H_Q}^{(n_f)}(Q, \mu_Q, \mu_m) H_m^{(n_f)}(\bar{m}^{(n_f)}, \mu_m) U_{H_m}^{(n_f)}\left(\frac{Q}{\bar{m}^{(n_f)}}, \mu_m, \mu_B\right) \quad n_f = n_l + 1$$

$$\int ds \int dk B^{(n_l)}\left(\frac{s}{m_J^{(n_l)}}, \mu_B, m_J^{(n_l)}\right) U_S^{(n_l)}(k, \mu_B, \mu_S) S_{\text{part}}^{(n_l)}\left(Q\tau - Q\tau_{\text{MIN}} - \frac{s}{Q} - k, \mu_S\right)$$

+ (SCET) Non-Singular + (QCD) Non-Singular



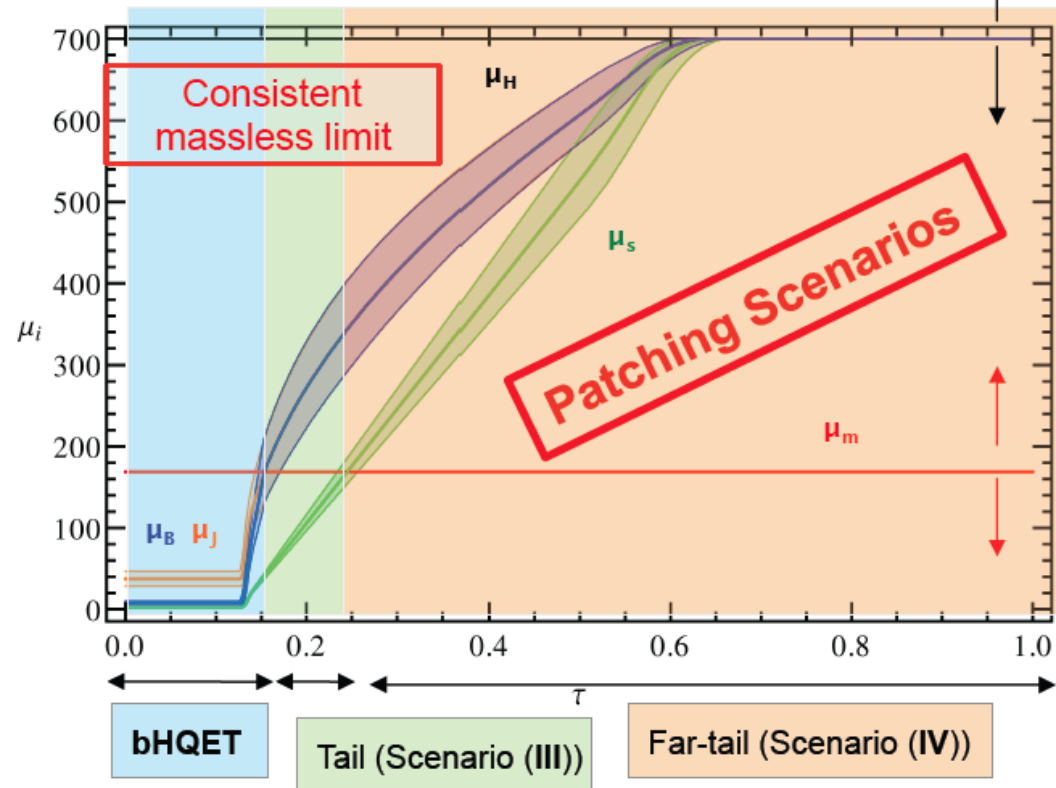
> Matching coefficient of SCET and bHQET have a large log from secondary corrections.

Profile Functions

Profile functions should sum up large logarithms and achieve smooth transition between the peak, tail and far-tail.

$$\log\left(\frac{Q}{\mu_H}\right) \quad \log\left(\frac{m_J}{\mu_m}\right) \quad \log\left(\frac{\mu_J^2}{Q\mu_s}\right) \quad \log\left(\frac{m_J\mu_B}{Q\mu_s}\right) \quad \log\left(\frac{Q(\tau - \tau_{\min}) + 2\Lambda_{\text{QCD}}}{\mu_s}\right)$$

$Q = 700 \text{ GeV}$

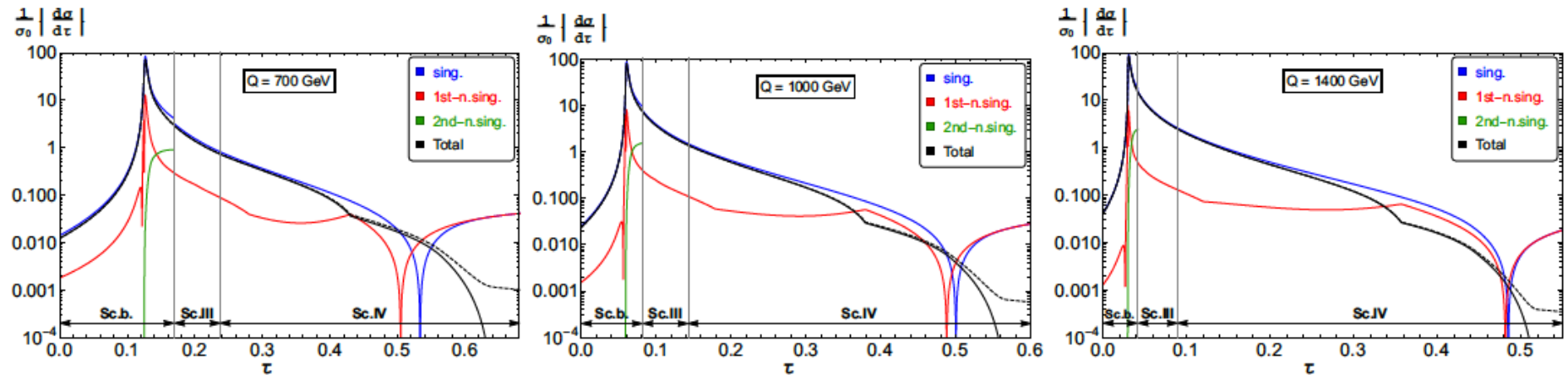


Scales Variation

- ✓ Generalized to arbitrary mass values
- ✓ Compatible with massless profiles

Proper scale variations are essential in reliable estimation of missing higher order terms.

Large Log Resummation



(a) Unstable Top Production

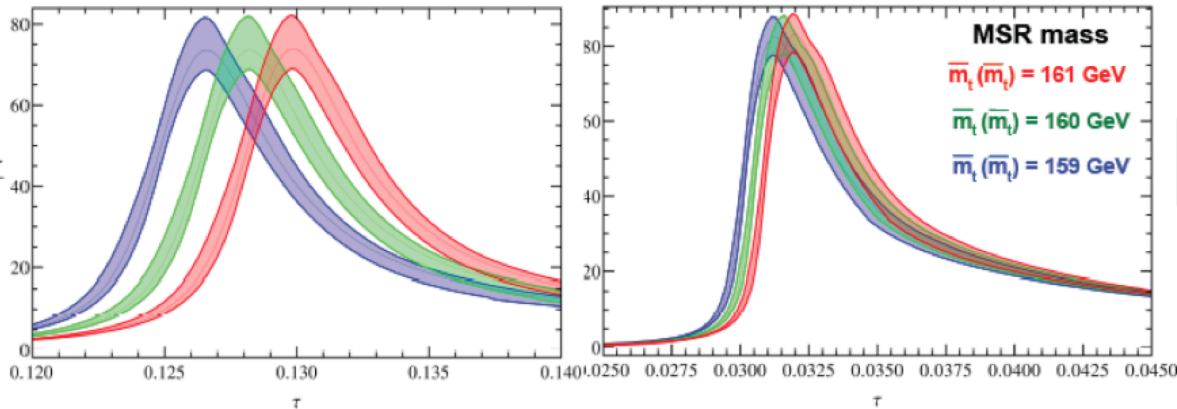
- Contributions contained in singular factorized cross section more than 1 order of magnitude larger than the non-singular contributions
- Confirmation that for $Q=700$ GeV the top quark are already boosted that that the correct treatment of large collinear logs is important.

2-Jettiness for Top Production (QCD)

$$\frac{d\sigma}{d\tau_2} = f(\underbrace{m_t^{\text{MSR}}(R)}_{\text{any scheme possible}}, \underbrace{\alpha_s(M_Z), \Omega_1, \Omega_2, \dots}_{\text{Non-perturbative}}, \underbrace{\mu_h, \mu_j, \mu_s, \mu_m, R, \Gamma_t}_{\text{renorm. scales finite lifetime}})$$

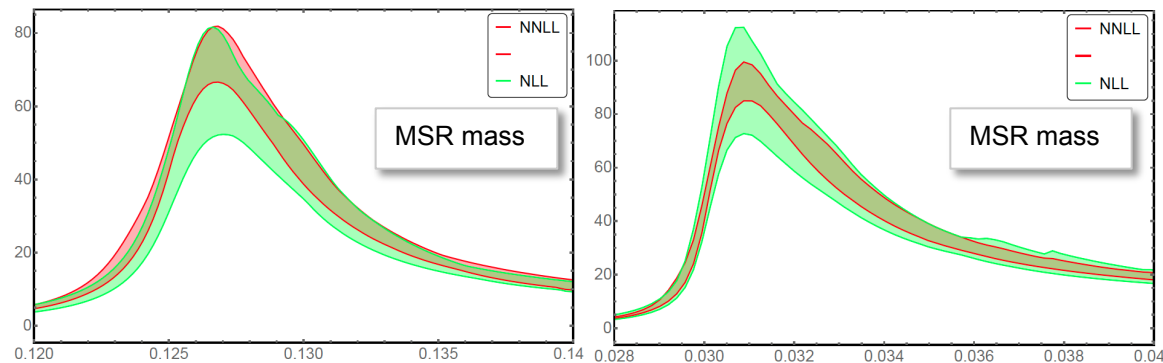
Q=700 GeV

Q=1400 GeV



Q=700 GeV

Q=1400 GeV



- Higher mass sensitivity for lower Q (p_T)
- Finite lifetime effects included
- Dependence on non-perturbative parameters
- Convergence: $\Omega_{1,2,\dots}$
- Good convergence
- Reduction of scale uncertainty (NLL to NNLL)
- Control over whole distribution

Fit Procedure Details

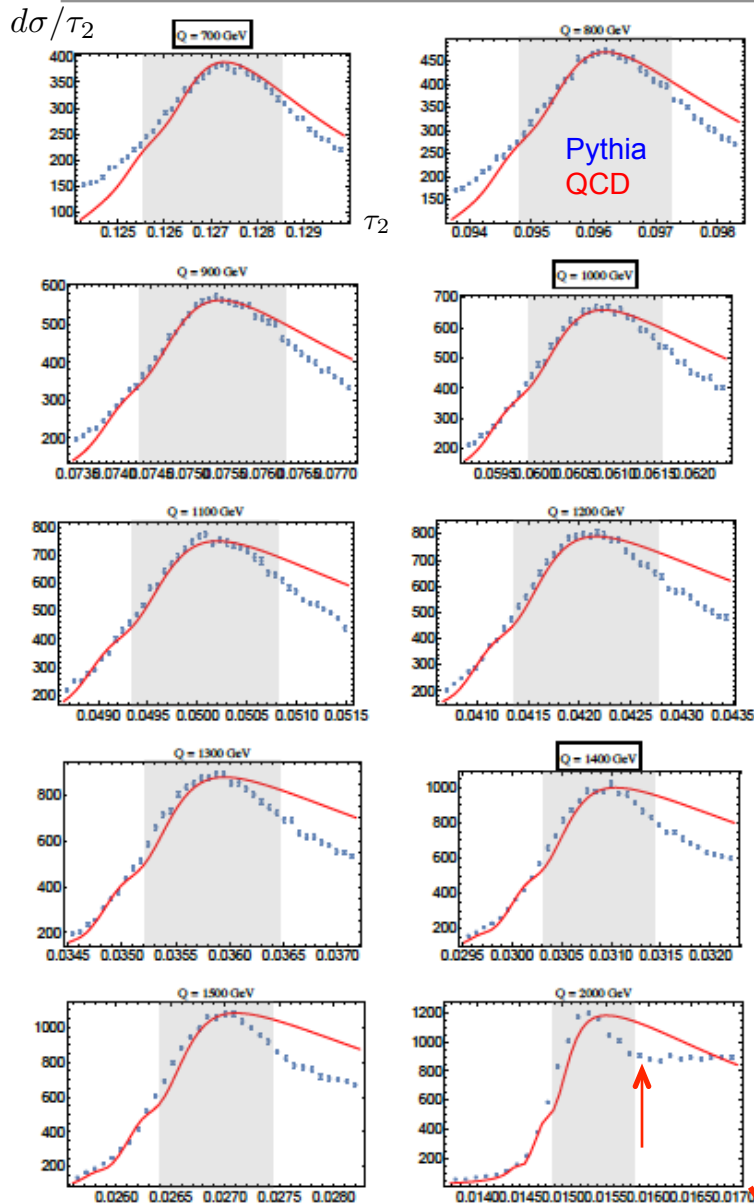
$$\frac{d\sigma}{d\tau_2} = f(\underbrace{m_t^{\text{MSR}}(R)}_{\text{any scheme possible}}, \underbrace{\alpha_s(M_Z), \Omega_1, \Omega_2, \dots}_{\text{Non-perturbative}}, \underbrace{\mu_h, \mu_j, \mu_s, \mu_m}_{\text{renorm. scales}}, \underbrace{R, \Gamma_t}_{\text{finite lifetime}})$$

QCD parameters measured from Pythia

- Fit parameters: $m_t^{\text{MSR}}(R), \alpha_s(M_Z), \Omega_1, \Omega_2, \dots,$
- Perturbative error: fits for 500 randomly picked sets of renor. scales
- Tunings: 1 (“very old”), 3 (“LEP”), 7 (“Monash”)
- Top quark width: $\Gamma_t =$ dynamical (default), 0.7, 1.4, 2.0 GeV
- External smearing (Detector effects): $\Omega_{1,\text{smear}} = 0, 0.5, \dots, 3.0, 3.5, \text{ GeV}$ (just for cross checks)
- Pythia masses: $m_t^{\text{Pythia}} = 170, \dots, 175 \text{ GeV}$
- Strong coupling: $\alpha_s(M_Z) = 0.114, 0.116, 0.118, 0.120, 0.122$
- Fit possible for any order / mass scheme (so far NLL+NNLL / MSR)

Number of fits entering the first analysis: $2.8 \cdot 10^6$

Peak Fits



Default renormalization scales; $\Gamma_t=1.4$ GeV, tune 3, $\Omega_{1,smear}=0$ GeV, $m_t^{Pythia}=170$ GeV, $Q=\{700, 1000, 1400\}$ GeV, peak fit (60/80)%, normalized to fit range

- Good agreement of Pythia 8.2 with NNLL+NLO QCD description
- Pythia statistics: 10^6 events
- Discrepancies in distribution tail and for higher energies (Pythia is less reliable where fixed-order results valid, well reliable in soft-collinear limit)
- **Pythia kink issue ?**
- Excellent sensitivity to the top quark mass.
- Tree-Level:

$$\tau_2^{\text{peak}} = 1 - \sqrt{1 - \frac{4m_t^2}{Q^2}}$$

tune = 3

$m_{MC} = 170$.

$\Gamma_t = -1$. GeV

$\alpha = 0.118$

$m^{SR}(5 \text{ GeV}) = 169.138 \pm 0.099$

$\frac{\chi^2}{\text{dof}} = 35.36$

$\Omega_1 = 0.434 \pm 0.060$ GeV

$\Omega_2 = 0.473 \pm 0.060$ GeV

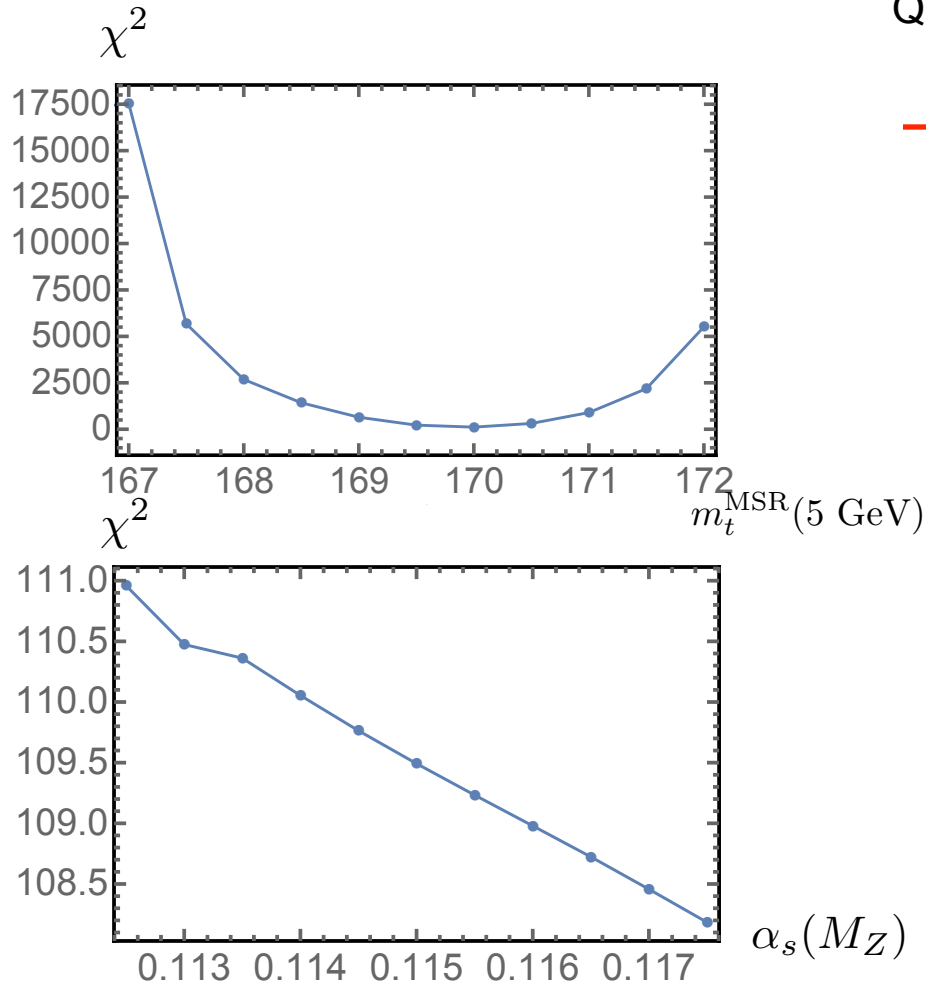
$\Omega_3 = -0.158 \pm 0.300$ GeV

$\Omega_4 = -2.226 \pm 1.000$ GeV

Preliminary

Peak Fits

Default renormalization scales; $\Gamma_t=1.4$ GeV, tune 7, $\Omega_{1,\text{smear}}=2.5$ GeV, $m_t^{\text{Pythia}}=171$ GeV, $Q=\{700, 1000, 1400\}$ GeV, peak fit (60/80)%

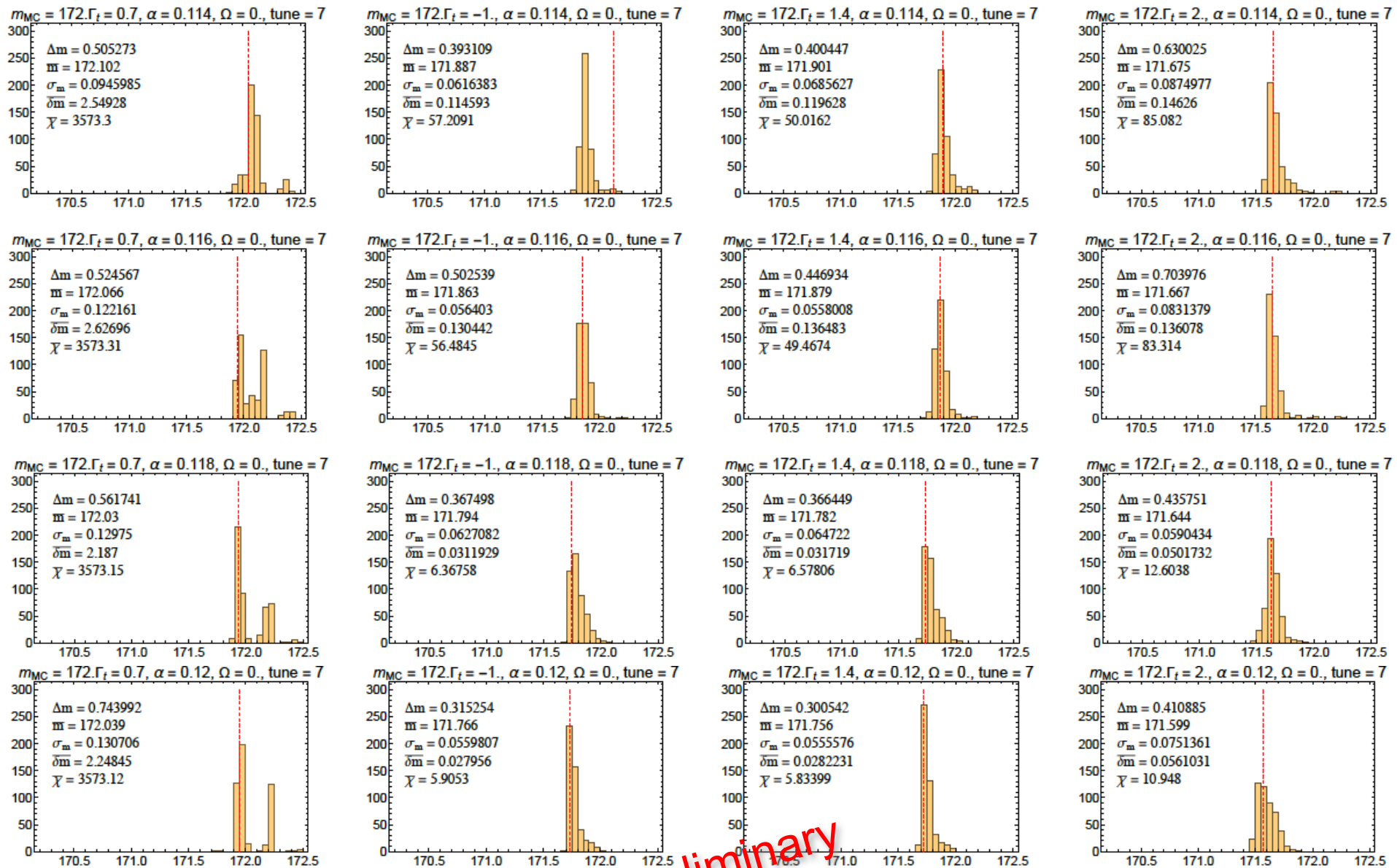


→ $\chi^2_{\text{min}} \sim O(100)$

- Very strong sensitivity to m_t
- Low sensitivity to strong coupling
- Take strong coupling as input
- χ^2_{min} and δm_t^{stat} do not have any physical meaning
- We use rescaled χ^2/dof (PDG prescription) to define “intrinsic MC compatibility uncertainty”

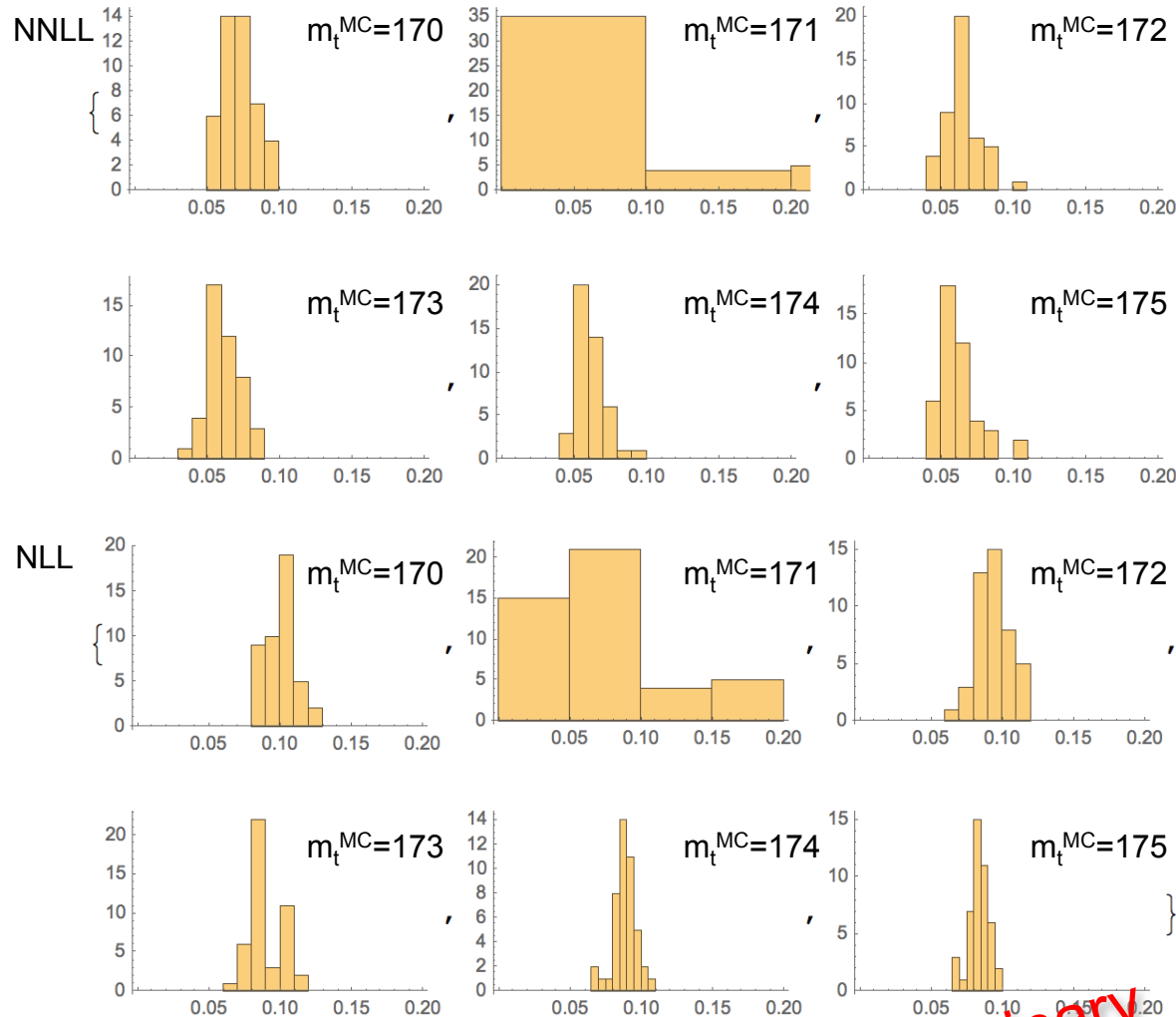
Preliminary

Peak Fits: $m_t^{\text{MSR}}(1 \text{ GeV})$



Peak Fits: $m_t^{\text{MSR}}(1 \text{ GeV})$

Distribution of standard deviations: each from scan over 500 profile functions



- Measure for scale error
- NNLL: 60-75 MeV
- NLL: 90-110 MeV
- Probably to be multiplied by factor 2 for scale uncertainty

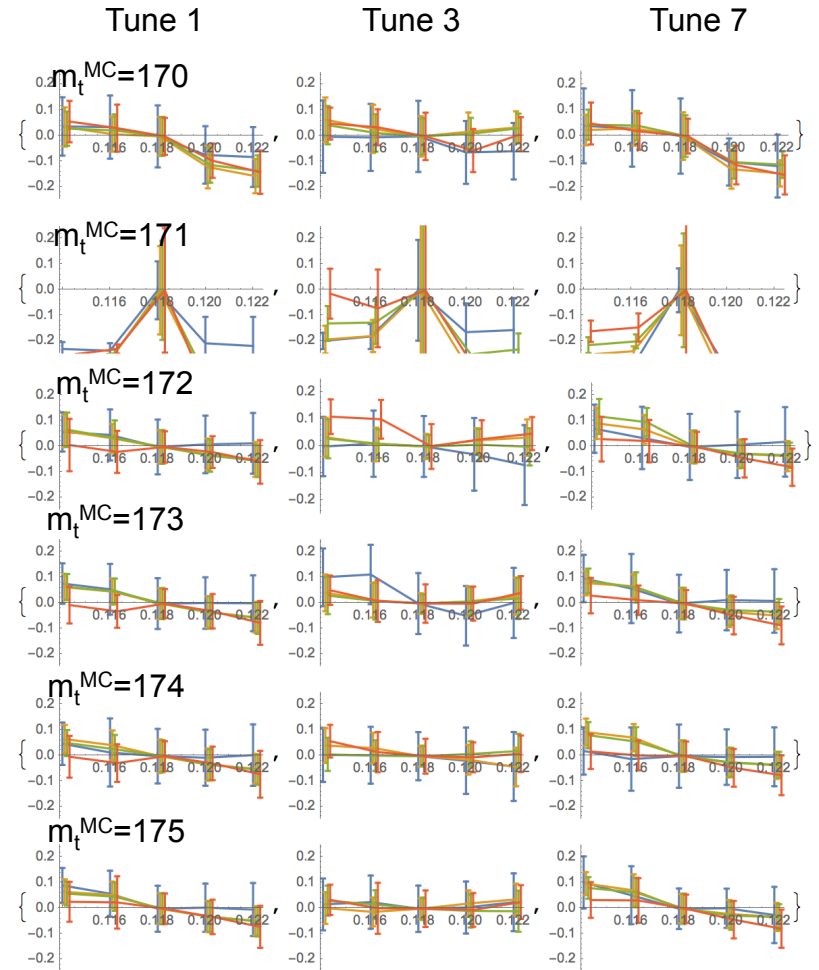
Preliminary

Peak Fits: $m_t^{\text{MSR}}(1 \text{ GeV})$

Parametric dependence on strong coupling

$$m_t^{\text{MSR}}[\alpha_s(M_Z)] - m_t^{\text{MSR}}[0.118]$$

- Small sensitivity of $m_t^{\text{MSR}}(1\text{GeV})$ on $\alpha_s(M_Z)$. [$\sim 50 \text{ MeV}$ error] ✓



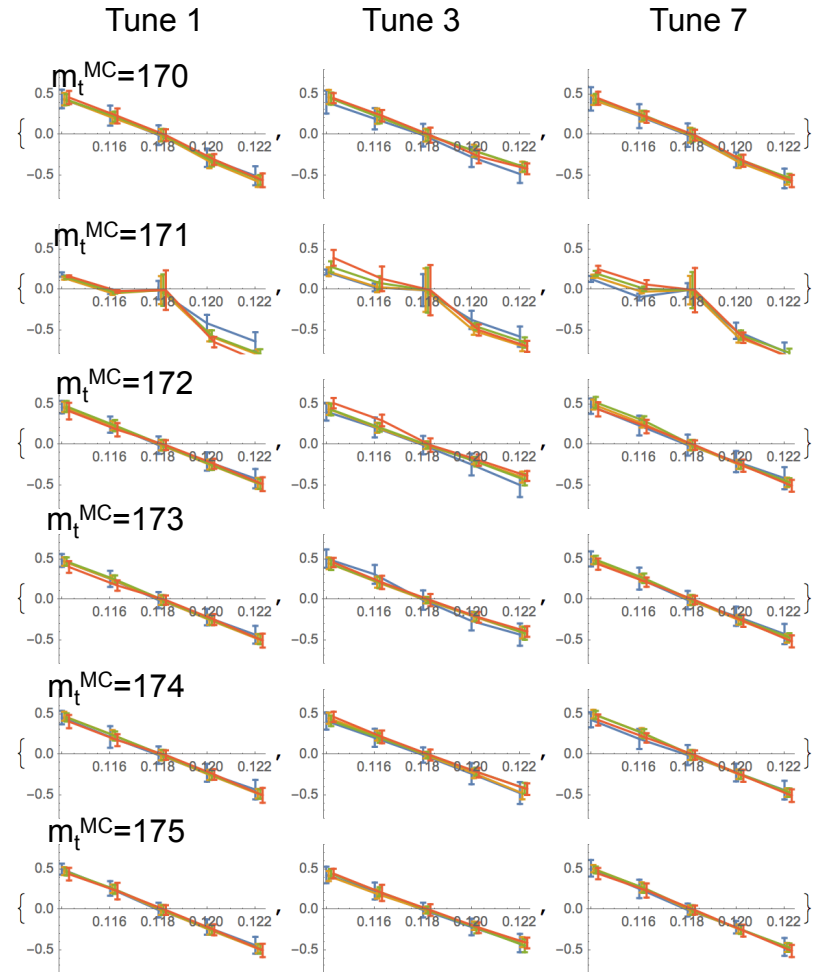
Preliminary

Peak Fits: $m_t^{\text{MSR}}(1 \text{ GeV})$

Parametric dependence on strong coupling

$$m_t(m_t)[\alpha_s(M_Z)] - m_t(m_t)[0.118]$$

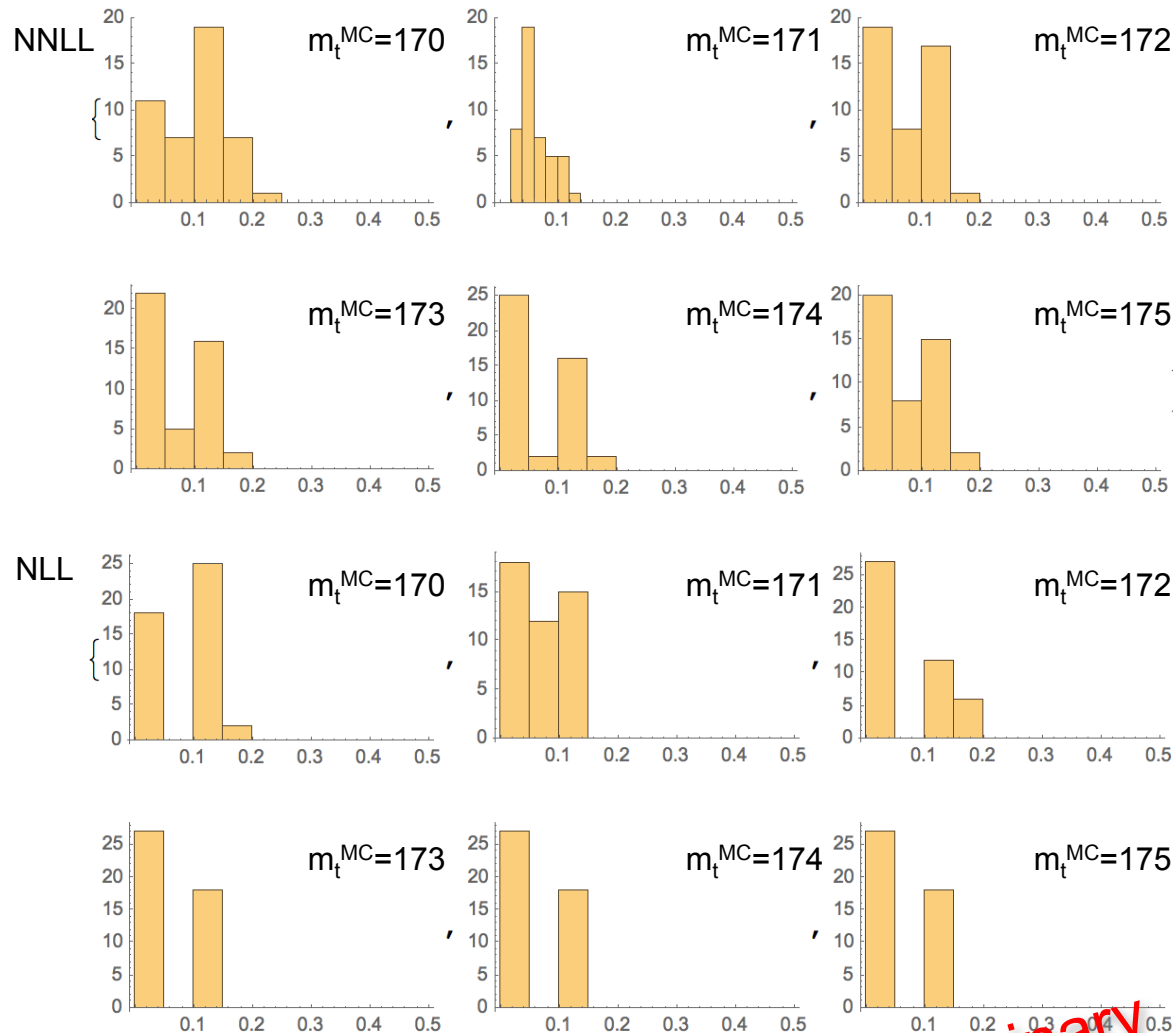
- Large sensitivity of MSbar mass on $\alpha_s(M_Z)$. [not an error, but calculated from MSR mass] ✓
- The MC top mass IS FAR AWAY from the MSbar mass.



Preliminary

Peak Fits: $m_t^{\text{MSR}}(1 \text{ GeV})$

Intrinsic MC Compatibility Error (distribution of mean values)



- Measure for intrinsic MC uncertainty
- NNLL: $\sim 150 \text{ MeV}$
- NLL: $\sim 150 \text{ MeV}$
- Probably never before accounted in reconstruction analyses
- Measure for ultimate precision (MC dependent !)

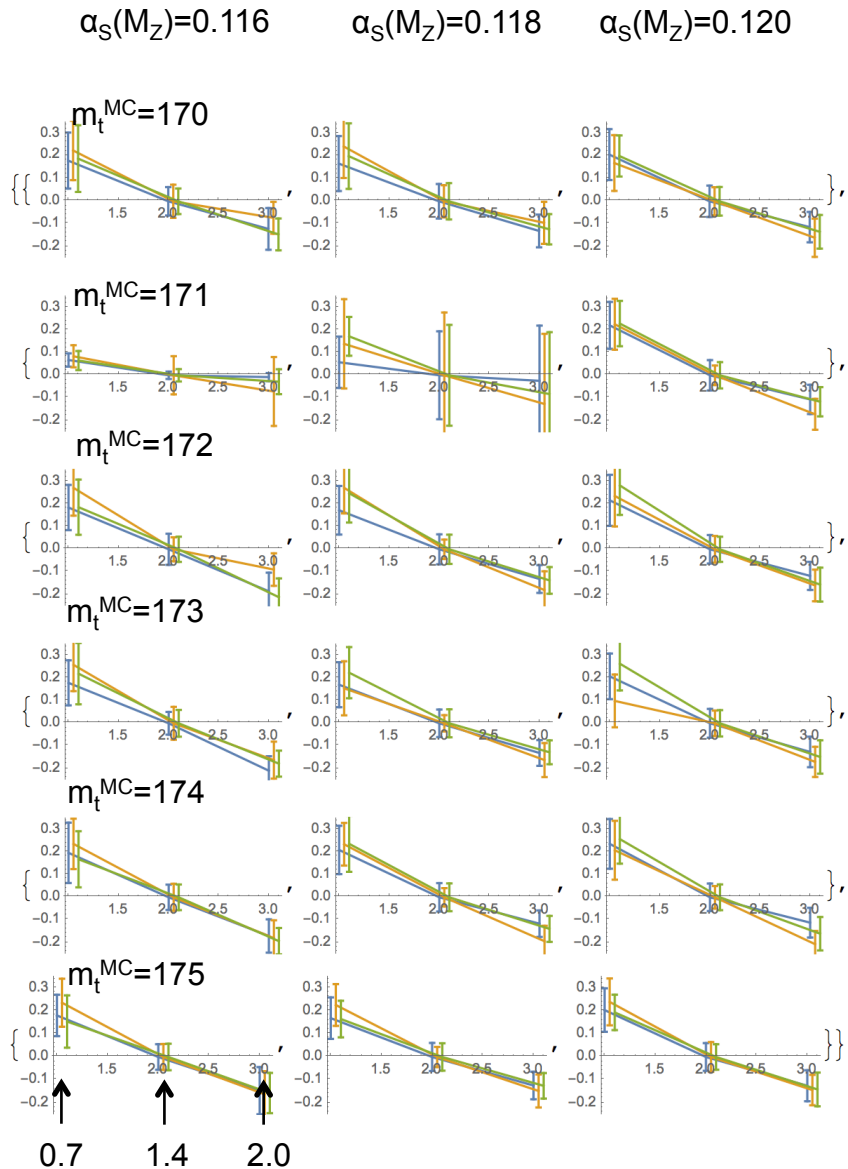
Peak Fits: $m_t^{\text{MSR}}(1 \text{ GeV})$

Top width dependence

$$m_t^{\text{MSR}}[\Gamma_t] - m_t^{\text{MSR}}[\Gamma_t=1.4]$$

- Clear sensitivity to top width value.
- Can be interpreted as observable dependence or MC modelling dependence.
- Should be accounted for as an additional uncertainty [$\sim 150 \text{ MeV}$].

Preliminary



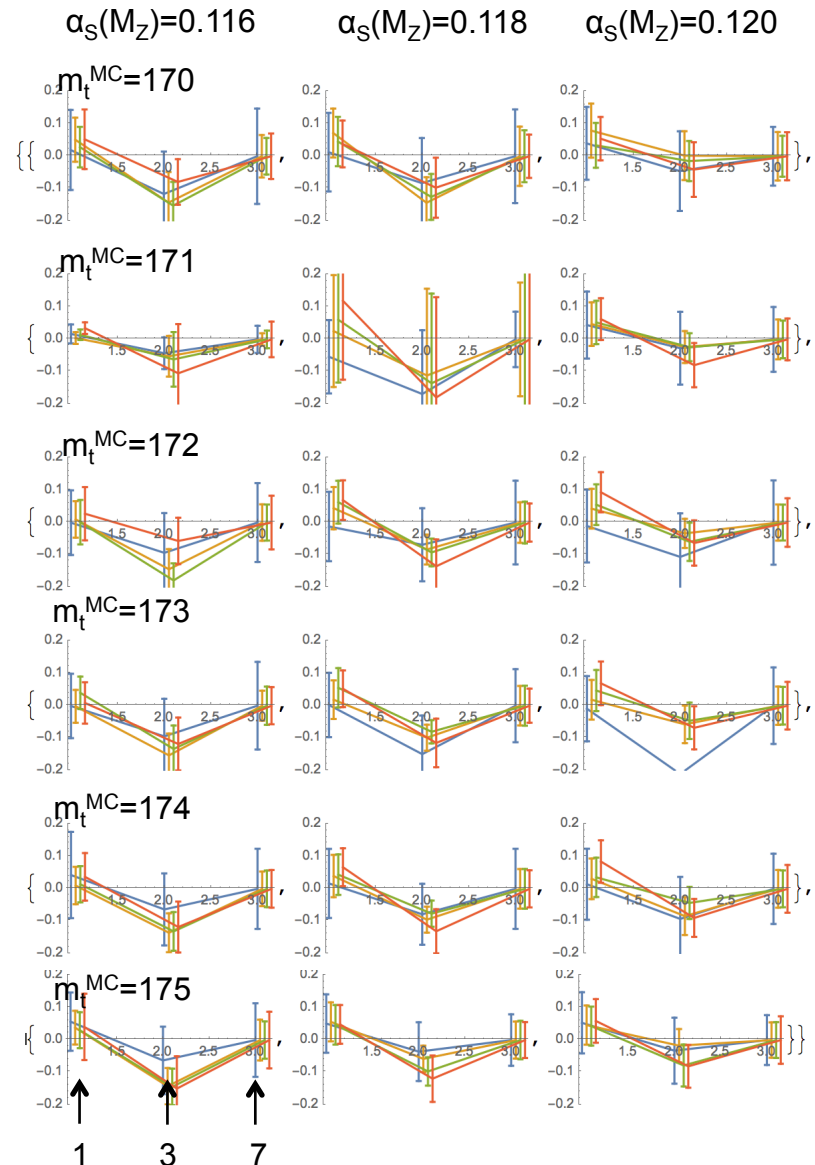
Peak Fits: $m_t^{\text{MSR}}(1 \text{ GeV})$

Tune dependence

$$m_t^{\text{MSR}}[\text{tune}] - m_t^{\text{MSR}}[\text{tune 7}]$$

- Clear sensitivity to tune.
- MC top mass is tune-dependent !
- Tune-dependence is not an error !
- Opposite dependence should be visible in MC top mass determinations from experimental data.
(highly nontrivial validation)

Preliminary



Summary

- First serious precise MC top quark mass calibration based on e^+e^- 2-jettiness (large p_T): preliminary results.
- NNLL+NLO QCD calculations based on an extension of the SCET approach concerning massive quark effects (all large logs incl. $\ln(m)$'s summed systematically).
- The Monte Carlo top mass calibration in terms of $m_t^{\text{MSR}}(1\text{GeV})$:
 - Scale dependence (NNLL): ~ 150 MeV
 - α_s dependence ($\delta\alpha_s=0.002$): ~ 50 MeV
 - Intrinsic MC error: ~ 150 MeV
 - Observable dependence: ~ 150 MeV

Preliminary !!!

- MC top mass is tune-dependent and MC dependent !

Using MC top mass calibration might eliminate these error sources from the experimental analyses.

Confirmation of the dependence predicted by calibration provides highly non-trivial cross check concerning the universality of the calibration.

Outlook & Plans

- Full verified error analysis @ NNLL+NLO on the way
 - Different sets of Q (p_T) values
 - Different fit ranges
 - Bug fixes
- Calibration Package for public use
 - Calibration $m_t^{\text{MC}} \rightarrow m_t^{\text{MSR}}(1\text{GeV})$
 - Code $m_t^{\text{MSR}}(1\text{GeV}) \rightarrow$ any other scheme
- Heavy jet mass, C-parameter (NNLL), pp-2-jettiness analysis (NLL) w.i.p.
- NNNLL+NNLO (2-jettiness for e^+e^-) w.i.p
- Mass (+ Yukawa coupling) conversions w. QCD + electroweak

Backup Slides

Pole Mass from MSR Mass

$$\alpha_s(M_Z) = 0.118$$
$$n_f = 5$$

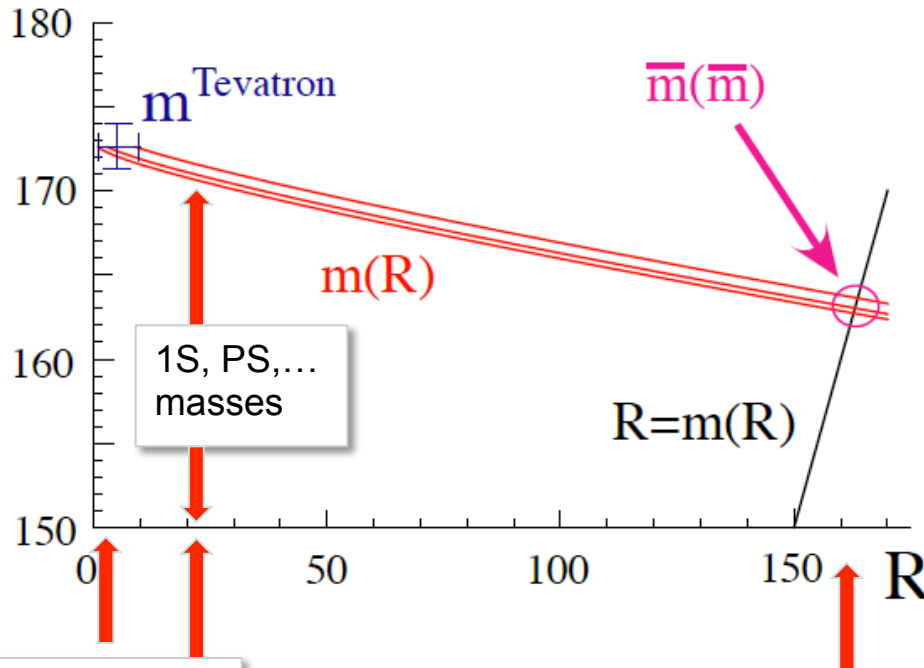
$$m_t^{\text{pole}} - m_t^{\text{MSR}}(1 \text{ GeV}) = \begin{array}{cccc} \mathcal{O}(\alpha_s) & \mathcal{O}(\alpha_s^2) & \mathcal{O}(\alpha_s^3) & \mathcal{O}(\alpha_s^4) \\ 0.173 & + 0.138 & + 0.159 & + 0.23 \text{ GeV} \leftarrow \text{calculated} \\ + 0.53 & + 1.43 & + 4.54 & + 16.6 \text{ GeV} \leftarrow \text{extrapolated} \\ + 68.6 & + 317.7 & + 1629 & + 9158 \text{ GeV} \end{array}$$

- Size of terms consistent with scale error estimate of calibration.
- No stable determination of pole mass.

MSR Mass Definition

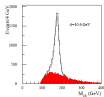
AH, Stewart: arXiv:0808.0222

$$m_t^{\text{MC}} = m_t^{\text{MSR}}(3_{-2}^{+6} \text{ GeV}) = m_t^{\text{MSR}}(3 \text{ GeV})_{-0.3}^{+0.6}$$



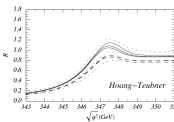
Good choice for R:

Of order of the typical scale of the observable used to measure the top mass.



Peak of invariant mass distribution, endpoints

Top-antitop threshold at the ILC



Total cross section, e.w. precision obs., Unification, MSbar mass

Masses Loop-Theorists Like to use

Total cross section (LHC/Tev):

$$m_t^{\text{MSR}}(R = m_t) = \bar{m}_t(\bar{m}_t)$$

$$M_t = M_t^{(O)} + M_t(0)\alpha_s + \dots$$

- more inclusive
- sensitive to top production mechanism (pdf, hard scale)
- indirect top mass sensitivity
- large scale radiative corrections

Threshold cross section (ILC):

$$m_t^{\text{MSR}}(R \sim 20 \text{ GeV}), m_t^{1S}, m_t^{\text{PS}}(R)$$

$$M_t = M_t^{(O)} + \langle p_{\text{Bohr}} \rangle \alpha_s + \dots$$

$$\langle p_{\text{Bohr}} \rangle = 20 \text{ GeV}$$

Inv. mass reconstruction (ILC/LHC):

$$m_t^{\text{MSR}}(R \sim \Gamma_t), m_t^{\text{jet}}(R)$$

$$M_t = M_t^{(O)} + \Gamma_t \alpha_s + \dots$$

$$\Gamma_t = 1.3 \text{ GeV}$$

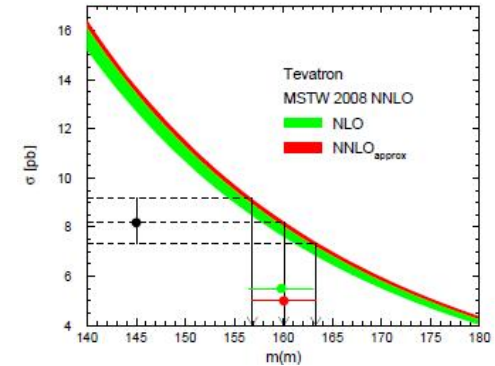
- more exclusive
- sensitive to top final state interactions (low scale)
- direct top mass sensitivity
- small scale radiative corrections



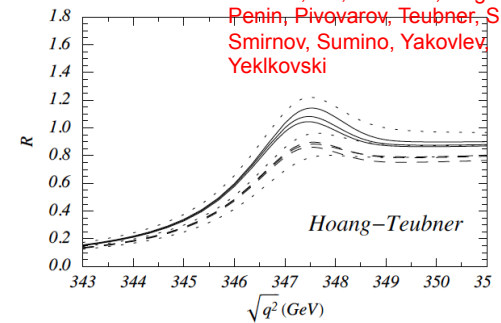
Mass schemes related to different computational methods

Relations computable in perturbation theory

Langenfeld, Moch, Uwer



Beneke, AH, Melnikov, Nagano, Penin, Pivovarov, Teubner, Signer, Smirnov, Sumino, Yakovlev, Yeklkovski



Fleming, AH, Mantry, Stewart

