Physical constants

neutron mass	$mc^2 = 939.565379(21)$ MeV
Planck conversion constant	$\hbar c = 197.3269718(44)$ MeV fm
Avogadro constant	$N_A = 6.02214129(27) \times 10^{23}$ mol^{-1}
Boltzmann constant	$k_B = 1.3806488(13) \times 10^{-23}$ J/K
Atomic mass unit	$u = 931.494028(23)$ MeV/c^2

Nuclear properties

Nucleus	nat. ab.	b [fm]	$\sigma_a^{\rm th}$ [barn]	atomic mass [u]
^{1}H	99.99%	-3.74	0.333	1.0078250322
^{2}H	0.015%	6.67	0.00052	2.0141017781
^{3}H		4.792	0	3.0160492779
^{3}He	10^{-4}%	5.74	5333	3.0160293201
^{4}He	100%	3.26	0	4.0026032541
^{6}Li	7.5%	2.00	940	6.0151228874
^{7}Li	92.5%	-2.22	0.0454	7.0160034366
^{10}B	20%	-0.1	3835	10.012936949
^{11}B	80%	6.65	0.0055	11.009305355
^{14}N	99.6%	9.37	1.91	14.003074004
^{19}F	100%	5.654	0.0096	18.998403163
^{27}Al	100%	3.449	0.231	26.981538531
^{56}Fe	91.7%	9.94	2.59	55.934936326

Physical properties of materials

material	ρ [g/cm^3]	M [g/mol]
aluminum	2.70	27.0
boron	2.34	10.8

1 Kinetics of neutron induced reaction

Calculate the kinetic energy of the products for the following reactions induced by slow neutrons:

$$n + {}^3\text{He} \rightarrow t + p \tag{1}$$
$$n + {}^6\text{Li} \rightarrow \alpha + t \tag{2}$$
$$n + {}^{10}\text{B} \rightarrow {}^7\text{Li} + \alpha. \tag{3}$$

2 Transmission of Aluminum windows

Aluminium foils are used as vacuum separation windows in UCN experiments (for gazeous detectors for exmple). Calculate the Fermi potential of aluminium, and the corresponding critical velocity. Calculate the transmission coefficient

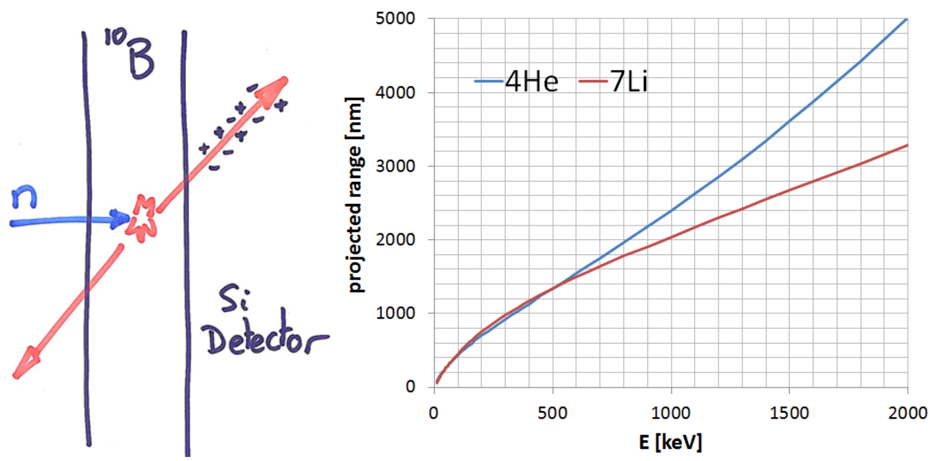

Figure 1: Principle of a boron conversion layer to detect neutrons (left) and Range of ^{4}He and ^{7}Li ions in Boron (right).

of a 30 μm thick aluminum foil, for UCNs at normal incidence at energies 100 neV and 200 neV. We recall that the neutron capture cross section σ_a follows the $1/v$ rule: $\sigma_a = \sigma_a^{\rm th}\, v_{\rm th}/v$, where $v_{\rm th} = 2200$ m/s, and v is the velocity *inside* aluminum.

3 UCN detectors: boron conversion layer

The principle of a neutron detector using a Boron conversion layer is depicted in fig. 1. An incoming neutron is converted into charged particles according to the following nuclear reactions:

$$
\begin{aligned}
{}^{10}\mathrm{B} + n &\rightarrow \alpha(1.47 \text{ MeV}) + {}^7\,\mathrm{Li}(0.84 \text{ MeV}) + \gamma(0.48 \text{ MeV}) \quad (94\%) \\
{}^{10}\mathrm{B} + n &\rightarrow \alpha(1.78 \text{ MeV}) + {}^7\,\mathrm{Li}(1.01 \text{ MeV}) \qquad (6\%)
\end{aligned}
$$

- Calculate the Fermi potential of (i) natural boron (ii) isotopically pure ^{10}B. Is it necessary to use isotopically pure boron to detect UCNs?
- We consider a conversion layer of thickness 200 nm. We assume that the reaction products are produced uniformly in the depth of the conversion layer. Calculate the mean distance travelled by a reaction product inside the conversion layer.
- The range of the reaction products is shown in fig. 1. Calculate the mean energy of the reaction products at the exit of the conversion layer.
- Calculate the efficiency of the detector to UCNs (take $v = 3$ m/s), and to thermal neutrons ($v = 2200$ m/s).

4 Lifetime of UCNs in a gas

Most of UCN experiments are performed in vacuum. To understand why this is necessary, let us discuss the behaviour of UCNs in a gas. Notice that the molecules in a gas at room temperature are much faster than UCNs. We recall the Maxwell-Boltzmann distribution of molecular velocities V:

$$dP(V) = \frac{4}{\sqrt{\pi}} \frac{V^2}{V_{\rm th}^3} e^{-(V/V_{\rm th})^2} dV \tag{4}$$

where $V_{\rm th} = \sqrt{2k_BT/M}$ is the most probable velocity and M is the mass of a molecule.

- Calculate the most probable velocity $V_{\rm th}$ of helium at room temperature.

We also recall the flux of molecules with velocity V:

$$\frac{d\Phi}{dV} = nVP(V) \tag{5}$$

where n is the number density of molecules.

We have to consider two different processes leading to UCN losses: absorption and up-scattering.

Absorption: the case of nitrogen We want to calculate the lifetime τ of a neutron at rest in a nitrogen athmosphere (1 bar). We will consider only the neutron capture (n,γ). The cross section for this process, at a N_2 molecule, is given by $\sigma = 2\sigma_a^{\rm th} v_{\rm th}/V$, where V is the relative velocity between the molecule and the neutron.

- Calculate the number density n of N_2 molecules.
- Establish the relation $\tau^{-1} = 2n\sigma_a^{\rm th} v_{\rm th}$, calculate τ.
- Estimate the mean free path of UCNs in a nitrogen atmosphere (take $v = 3$ m/s).
- Calculate the mean free path of UCNs (take $v = 3$ m/s) before capture in a nitrogen atmosphere with the assumption that all nitrogen nuclei are at rest. Compare with the previous estimate.

Upscattering: the case of helium In the case of ^{4}He, the neutron absorption cross section is strictly zero. Still UCNs can be lost in helium at room temperature because of scattering.

- Calculate the typical de Broglie wavelenght of UCNs (take $v = 3$ m/s).
- Calculate the typical distance between molecules in a gas at room temperature as a function of the pressure P.

- In what pressure range can the gas be considered as an optical medium for UCNs?

Now we consider a UCN bottle filled with 1 mbar of pure ^{4}He. We assume that a UCN is lost when colliding with a helium nucleus.

- Calculate the helium number density n. Calculate the scattering cross section σ.
- Establish the relation $\tau^{-1} = \frac{2}{\sqrt{\pi}} n\sigma \, V_{\mathrm{th}}$, calculate τ.
- Compare with the most recent experimental result $P\tau = 418 \pm 2$ mbar s.

Systematic effect in neutron lifetime measurements In air, the UCN losses are quantified by $P\tau \approx 100$ mbar s. In UCN experiments measuring the neutron lifetime, the presence of air due to non-perfect vacuum is a source of systematic error. What residual pressure correspond to a systematic effect of $\Delta\tau_n = 1$ s?