Low x, 2016

Gyöngyös, 6-11 June

Relating low-x DIS structure functions and hadronic total cross sections

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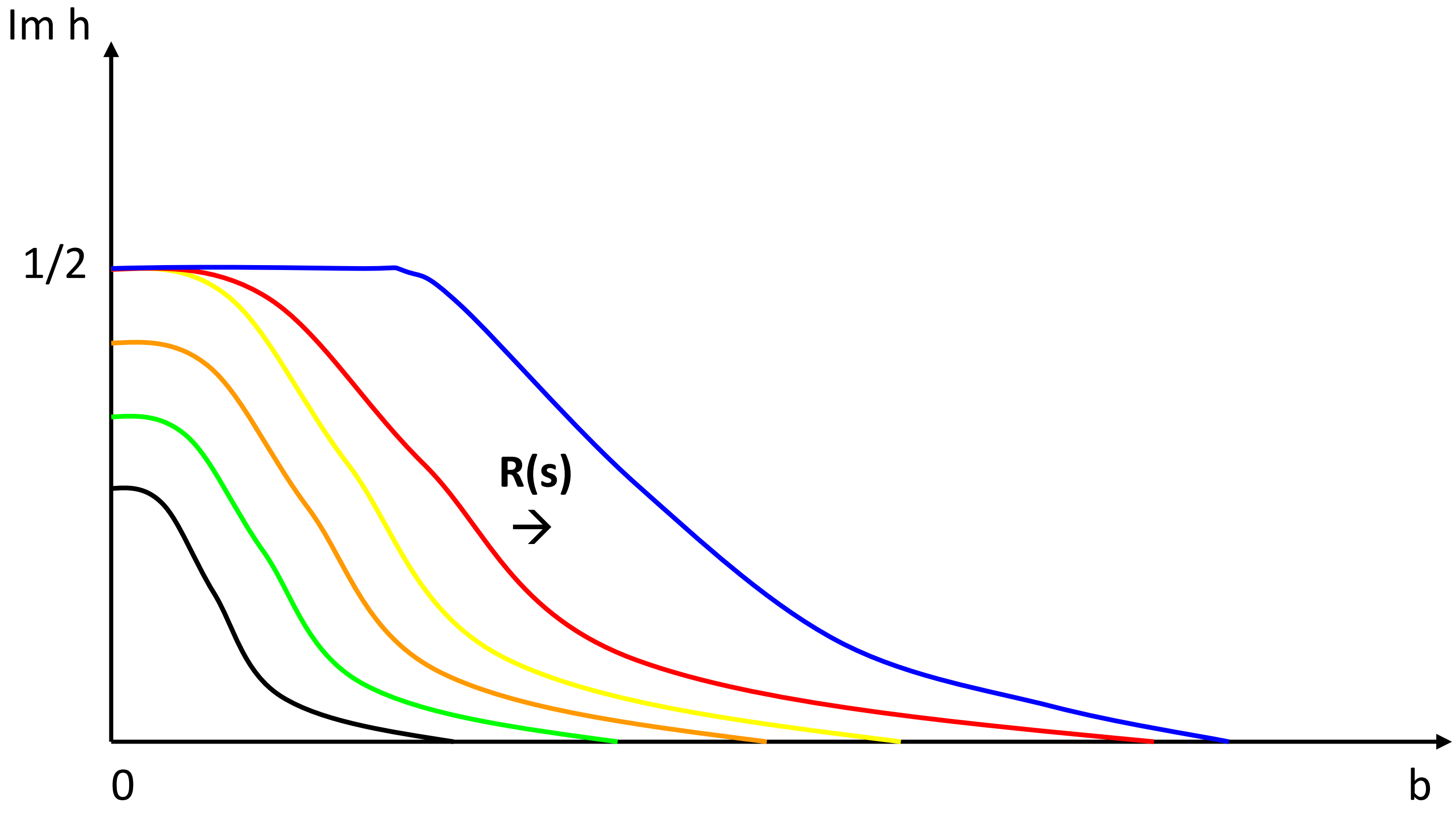
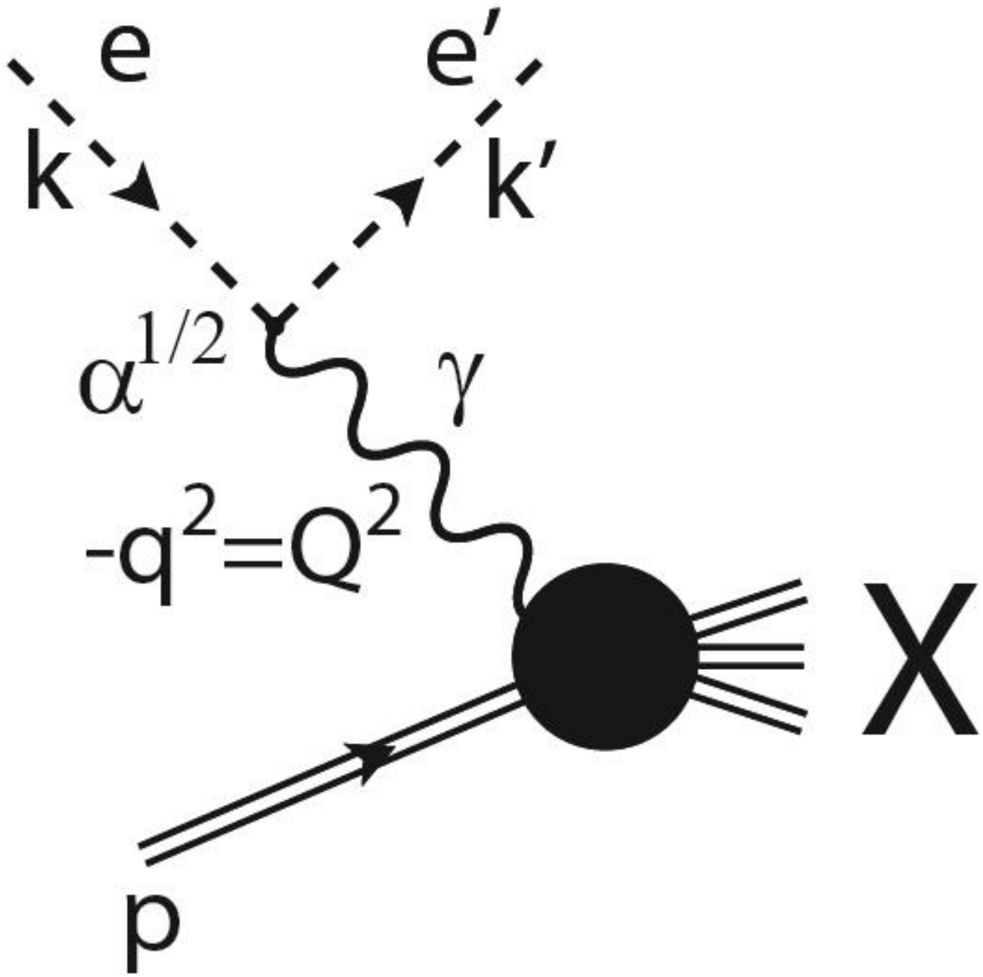

Im h
1/2
R(s)
→
0
b



e
e′
k
k′
α^1/2
γ
-q^2=Q^2
X
p

The basic object of the theory

$$A(s,t,Q^2)$$

$$A(s,t,Q^2=m^2)$$ (on mass shell)

$$\Im m A(s,t=0,Q^2) \sim F_2$$ DIS

Reconstruction of the DVCS amplitude from DIS

$$F_2 \sim \Im m A(\gamma^* p \to \gamma^* p)\big|_{t=0} \to \Im m A(\gamma^* p \to \gamma p)\big|_{t=0}$$

$$\to A(\gamma^* p \to \gamma p)\big|_{t=0} \to A(\gamma^* p \to \gamma p)$$

or

$$\Im m A(\gamma^* p \to \gamma^* p)\big|_{t=0} \sim F_2(x_B,Q^2) = x_B q(x_B,Q^2)$$

$$q(x_B,Q^2) \to q(\xi,\eta,t,x_B,Q^2) \rightarrow$$

?

$$\rightarrow \xi q(\xi,\eta,t,x_B,Q^2) = GPD(\xi,\eta,t,x_B,Q^2)$$

Kinematics:

$$W^2 = s = (p+q)^2 = Q^2(1-x)/x + m^2 \approx Q^2/s.$$

$$\sigma_t(s) = \frac{4\pi}{s} Im A(s, t=0); \quad \frac{d\sigma}{dt} = \frac{\pi}{s^2}|A(s,t)|^2; \quad \boldsymbol{n(s)};$$

$$\sigma_{el} = \int_{t_{min\approx -s/2 \approx \infty}}^{t_{thr.\approx 0}} \frac{d\sigma}{dt}; \quad \sigma_{in} = \sigma_t - \sigma_{el}; \quad B(s,t) = \frac{d}{dt} \ln\left(\frac{d\sigma}{dt}\right);$$

$$A_{pp}^{p\bar{p}}(s,t) = P(s,t) \pm O(s,t) + f(s,t) \pm \omega(s,t) \rightarrow_{LHC} \approx P(s,t) \pm O(s,t),$$

where P, O, f. ω are the Pomeron, odderon and non-leading Reggeon contributions.

α(0)\C	+	-
1	P	O
1/2	f	ω

$$\int_0^1 dx[F_2^V(x,Q^2) + F_2^S(x,Q^2)],$$

with typically logarithmic scaling violation parametrizations know at those times, e.g.

$$F_2^S(x) = F_2(x)\Big[1 - \epsilon \ln\Big(\frac{Q^2}{Q_0^2}\Big) \ln\Big(\frac{x}{x_0}\Big)\Big],$$

or

$$F_2^S(x) = F_2(x)\Big[1 + \epsilon\Big(\frac{Q^2}{Q_0^2}\Big)^{f(x)}\Big], \quad f(x) = x_0 - x,$$

with the following values of the parameters: $a = 0.25,\ b = 1.35,\ c = 0.2,\ \epsilon = 0.05$ and $q_0^2 = 3\ \mathrm{GeV}^2$,

Sum rules in Q^2: Jan Kwieciński, Phys. Letters, **120B** (1983) 418;
L.L. Jenkovszky and B.V. Struminsky, Yad. Fizika, 38 (1983) 1568.

VMD:

$$\int_{Q_1^2}^{Q_2^2} F_2(x,Q^2)dQ^2 = \frac{1}{4\pi}\sum_V \frac{\sigma_{VM}}{\gamma_V^2}(m_V^2)^2,$$

$$\Gamma(V \to e^+e^-) = \frac{\alpha m_V}{2\gamma_V^2}.$$

Sum rule:

$$\int_{Q_1^2}^{Q_2^2} F_2(x,Q^2)dQ^2 = \int_{Q_2^2}^{Q_{as}^2} F_2^{as}(x,Q^2)dQ^2.$$

By setting $\sigma_{\rho N} \approx \sigma_{\omega N} \approx \sigma_{\pi N}$, one gets (optionally)

$$\sigma_{\pi N}^t = 24[1 + 0.1\ln\left(\frac{s}{Q_0^2}\right)]mbn$$

.

Direct, s-channel point of view: additive quark model, single and multiple scattering of partons (Glauber).

Additive quark model relations (Levin-Frankfurt, 70-ies):

$$\sigma_{pp}^t = \sigma_0 n_A n_B. \tag{1}$$

While this simple rule is confirmed *e.g.* by the ratio 2/3 of meson-baryon to baryon-baryon scattering in a fairly wide range of intermediate energies, *e.g.* $\sigma_{\pi p}^t/\sigma_{pp}^t \approx 0.67$ at $\sqrt{s} \approx 10$ GeV, it is progressively violated as the energy increases. It was suggested in Ref. [2, 3] that while the constant components of the cross sections, obeying the above quark rule are determined by constituents quarks their rise comes from the increasing number of see quarks.

In Refs. [2, 3] the rise of hadronic total cross sections was related the proliferation of see quarks and gluons in colliding hadron, hence Eq. (1) modifies as

$$\sigma_{pp}^t = (n_v + n_s)^2, \tag{2}$$

where n_v and n_s is the number of valence and see quarks and gluons in the proton. The number of see quarks and gluons was related to the logarithmic scaling violation, resulting in

The fraction of momenta carried by quarks can be calculated from the integrals

$$\int_0^1 dx F_2^v(s, Q^2) = 0.423,$$

$$\int_0^1 dx F_2^s(s, Q^2) = 0.01 + 0.001 \ln(s/Q_0^2).$$

Consequently,

$$\sigma_{pp}^{tot} \approx \sigma_0 n_{v_1} n_{v_2} (1 + 0.016 \ln(s/Q_0^2).$$

We remind that $x \sim Q^2/s$.

Unitarity bounds in DIS SF and in hadronic total cross sections.

An advanced model for DIS SF: P. Desgrolard, L. Jenkovszky and F. Paccanoni: EPJ C **7** (1999) 263; hep-ph/9803286

Interpolating between "soft" (VMD, Pomeron, Δ~0.1) and hard (DGLAP, Δ~0.4) regimes:
1) DGLAP evolution, x fixed, Q^2→infty; 2) Gauge inv.: x fixed, Q^2→0; 3) x→0, Q fixed (Regge)

$$F_2^{(S,0)}(x,Q^2) = A\left(\frac{Q^2}{Q^2+a}\right)^{1+\widetilde{\Delta}(Q^2)} e^{\Delta(x,Q^2)},$$

with "effective power"

$$\widetilde{\Delta}(Q^2) = \epsilon + \gamma_1 \ell n\left(1+\gamma_2 \ell n\left[1+\frac{Q^2}{Q_0^2}\right]\right),$$

$$\Delta(x,Q^2) = \left(\widetilde{\Delta}(Q^2)\ell n \frac{x_0}{x}\right)^{f(Q^2)},$$

$$f(Q^2) = \frac{1}{2}\left(1 + e^{-Q^2/Q_1^2}\right).$$

At small and moderate values of Q^2 (to be specified from the fits, see below), the exponent $\widetilde{\Delta}(Q^2)$ (3.2) may be interpreted as a Q^2-dependent "effective Pomeron intercept".

The function $f(Q^2)$ has been introduced in order to provide for the transition from the Regge behavior, where $f(Q^2) = 1$, to the asymptotic solution of the GLAP evolution equation, where $f(Q^2) = 1/2$.

Large Q^2, fixed x:

$$F_2^{(S,0)}(x, Q^2 \to \infty) \to A \ \exp\sqrt{\gamma_1 \ell n \ell n \frac{Q^2}{Q_0^2} \ \ell n \frac{x_0}{x}} \ ,$$

h is the asymptotic solution of the GLAP evolution equation (see Sec. 1).

Low Q^2, fixed x:

$$F_2^{(S,0)}(x, Q^2 \to 0) \to A \ e^{\Delta(x, Q^2 \to 0)} \ \left(\frac{Q^2}{a}\right)^{1+\widetilde{\Delta}(Q^2 \to 0)}$$

$$\widetilde{\Delta}(Q^2 \to 0) \to \epsilon + \gamma_1 \gamma_2 \left(\frac{Q^2}{Q_0^2}\right) \ \to \ \epsilon,$$

$$f(Q^2 \to 0) \to 1,$$

ıce

$$F_2^{(S,0)}(x, Q^2 \to 0) \to A \ \left(\frac{x_0}{x}\right)^{\epsilon} \ \left(\frac{Q^2}{a}\right)^{1+\epsilon} \ \propto (Q^2)^{1+\epsilon} \ \to 0 \ ,$$

:quired by gauge invariance.

ired by gauge invariance.

w x, fixed Q^2:

$$F_2^{(S,0)}(x \to 0, Q^2) \; = \; A \left(\frac{Q^2}{Q^2+a} \right)^{1+\widetilde{\Delta}(Q^2)} e^{\Delta(x\to 0,Q^2)}.$$

$$f(Q^2) \sim 1 \; ,$$

en $Q^2 \ll Q_1^2$, we get the standard (Pomeron-dominated) Regge behavior (with a Q^2 dependence in t
n intercept)

$$F_2^{(S,0)}(x \to 0, Q^2) \to A \; \left(\frac{Q^2}{Q^2+a} \right)^{1+\widetilde{\Delta}(Q^2)} \; \left(\frac{x_0}{x} \right)^{\widetilde{\Delta}(Q^2)} \; \propto x^{-\widetilde{\Delta}(Q^2)}.$$

in this approximation, the total cross-section for (γ, p) scattering as a function of the center of r

$$\sigma_{\gamma,p}^{tot,(0)}(W) = 4\pi^2\alpha \; \left[\frac{F_2^{(S,0)}(x,Q^2)}{Q^2} \right]_{Q^2\to 0} \; = \; 4\pi^2\alpha \; A \; a^{-1-\epsilon} \; x_0^{\epsilon} \; W^{2\epsilon}.$$

[T, we multiply the singlet part of the above structure function $F_2^{(S,0)}$
actor to get

$$F_2^{(S)}(x,Q^2) = F_2^{(S,0)}(x,Q^2)\ (1-x)^{n(Q^2)},$$

$$n(Q^2) = \frac{3}{2}\left(1+\frac{Q^2}{Q^2+c}\right),$$

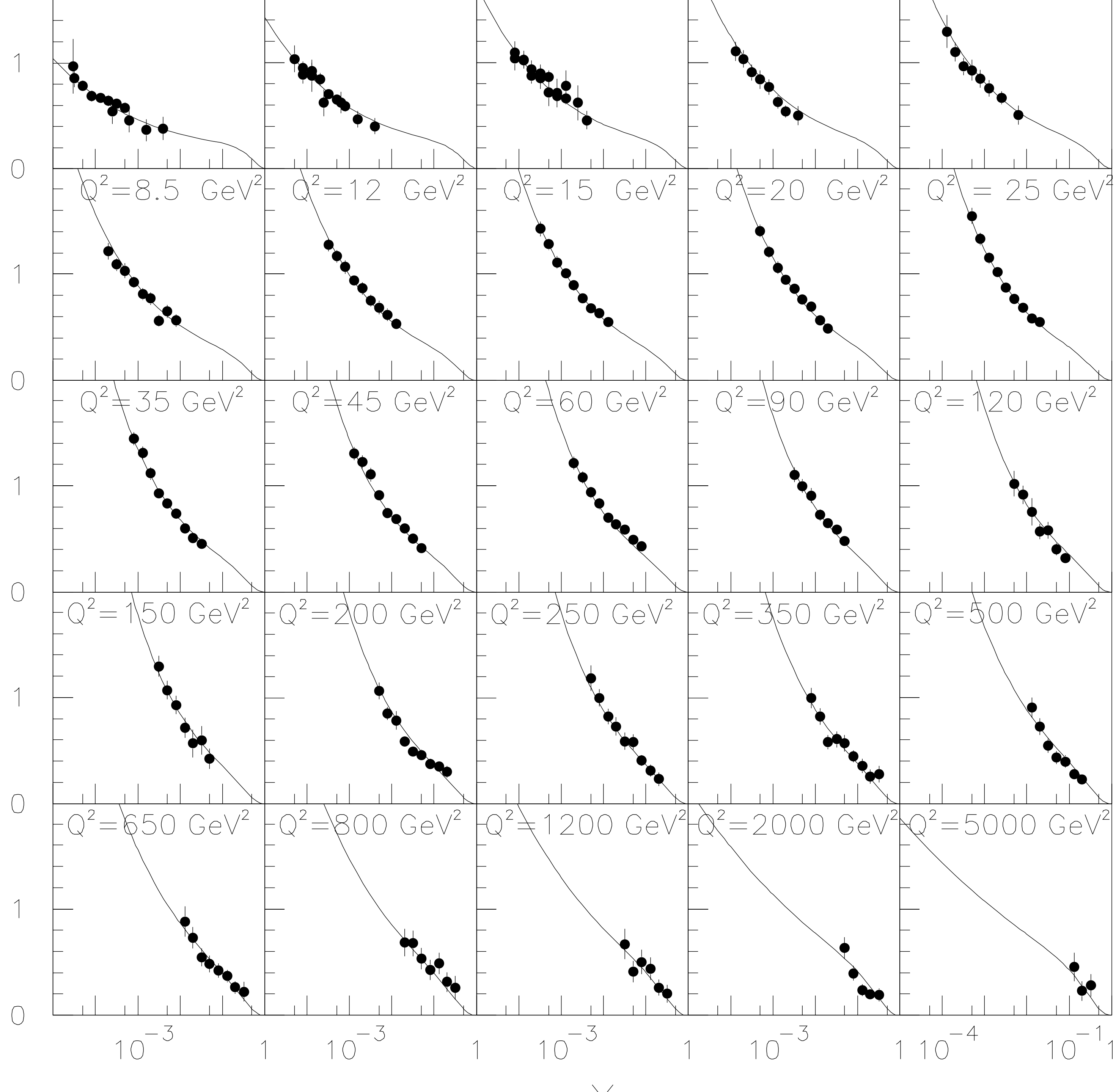
GeV^2 [4a].

e nonsinglet (NS) part of the structure function, also borrowed from CK

$$F_2^{(NS)}(x,Q^2) = B\ (1-x)^{n(Q^2)}\ x^{1-\alpha_r}\ \left(\frac{Q^2}{Q^2+b}\right)^{\alpha_r}.$$

ers that appear with this addendum are c, B, b and α_r. The final and c
unction thus becomes

$$F_2(x,Q^2) = F_2^{(S)}(x,Q^2) + F_2^{(NS)}(x,Q^2).$$



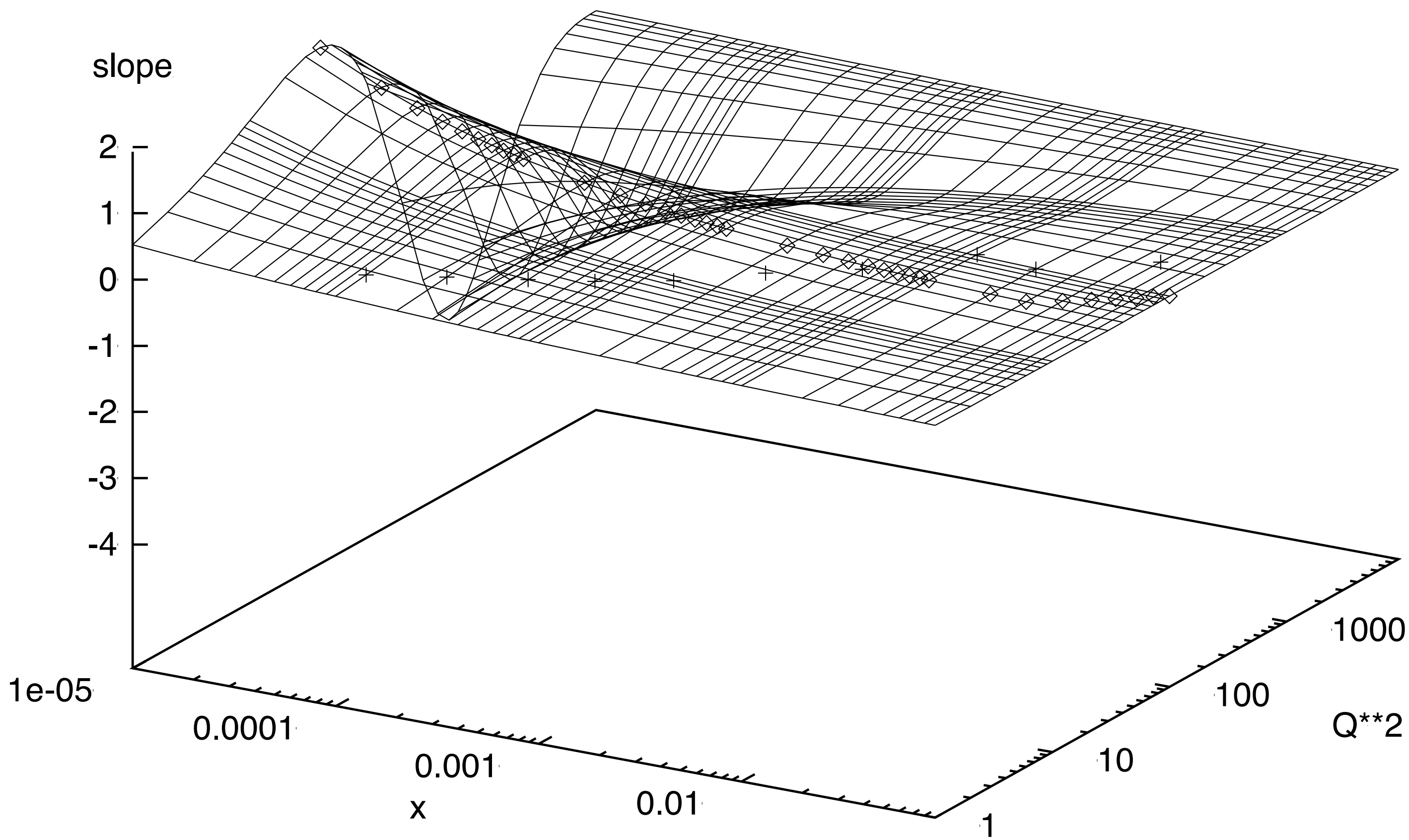
Q²=8.5 GeV²
Q²=12 GeV²
Q²=15 GeV²
Q²=20 GeV²
Q² = 25 GeV²
Q²=35 GeV²
Q²=45 GeV²
Q²=60 GeV²
Q²=90 GeV²
Q²=120 GeV²
Q²=150 GeV²
Q²=200 GeV²
Q²=250 GeV²
Q²=350 GeV²
Q²=500 GeV²
Q²=650 GeV²
Q²=800 GeV²
Q²=1200 GeV²
Q²=2000 GeV²
Q²=5000 GeV²
0
1
10⁻³
10⁻⁴
10⁻¹
x

Saturation and slopes

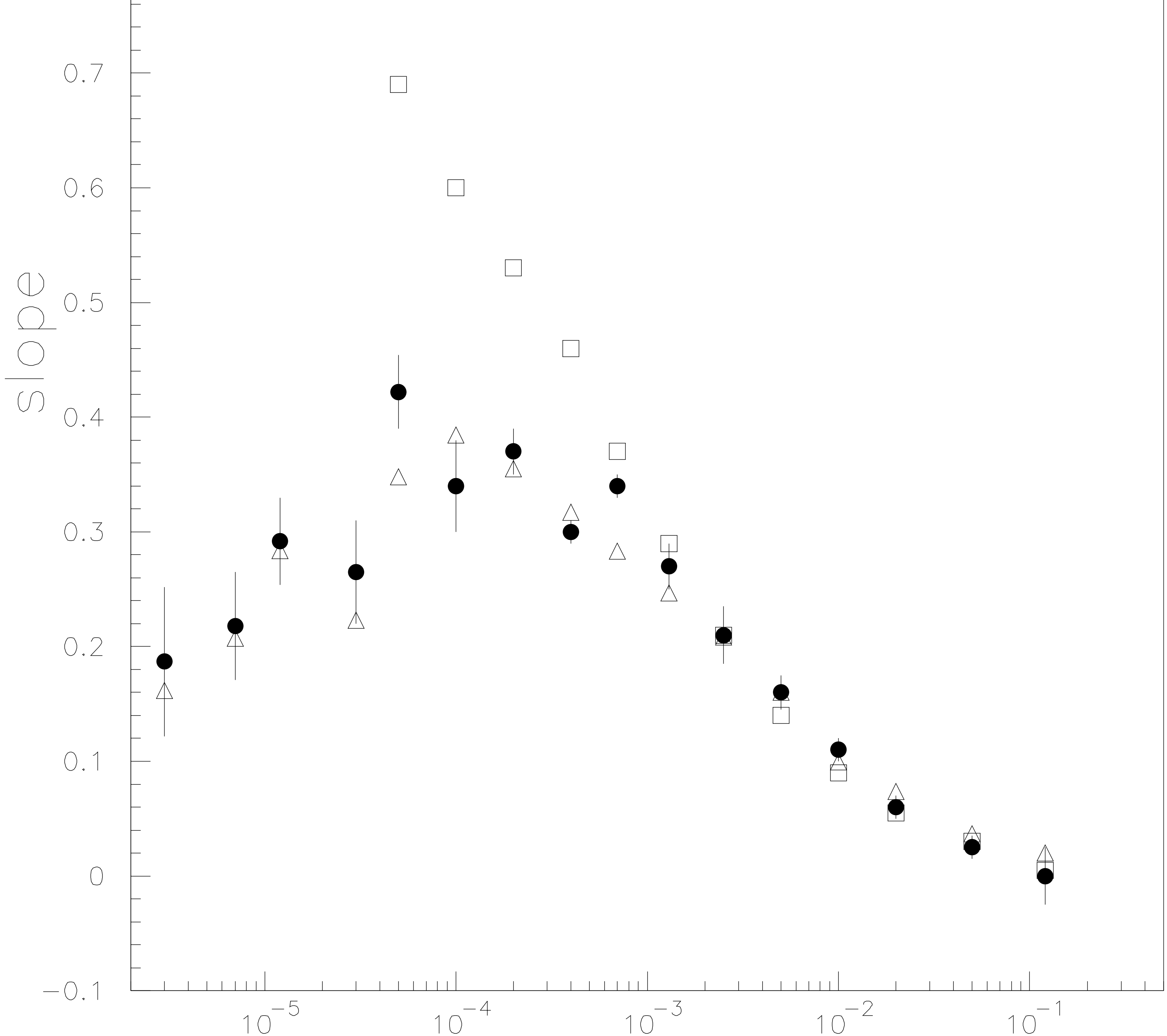
Logarithmic derivatives (slopes) B are sensitive measures of the changing trends/regimes (*unitarity and saturation*):

$$B_Q(x,Q^2)=\frac{\partial F_2(x,Q^2)}{\partial(\ln Q^2)},$$

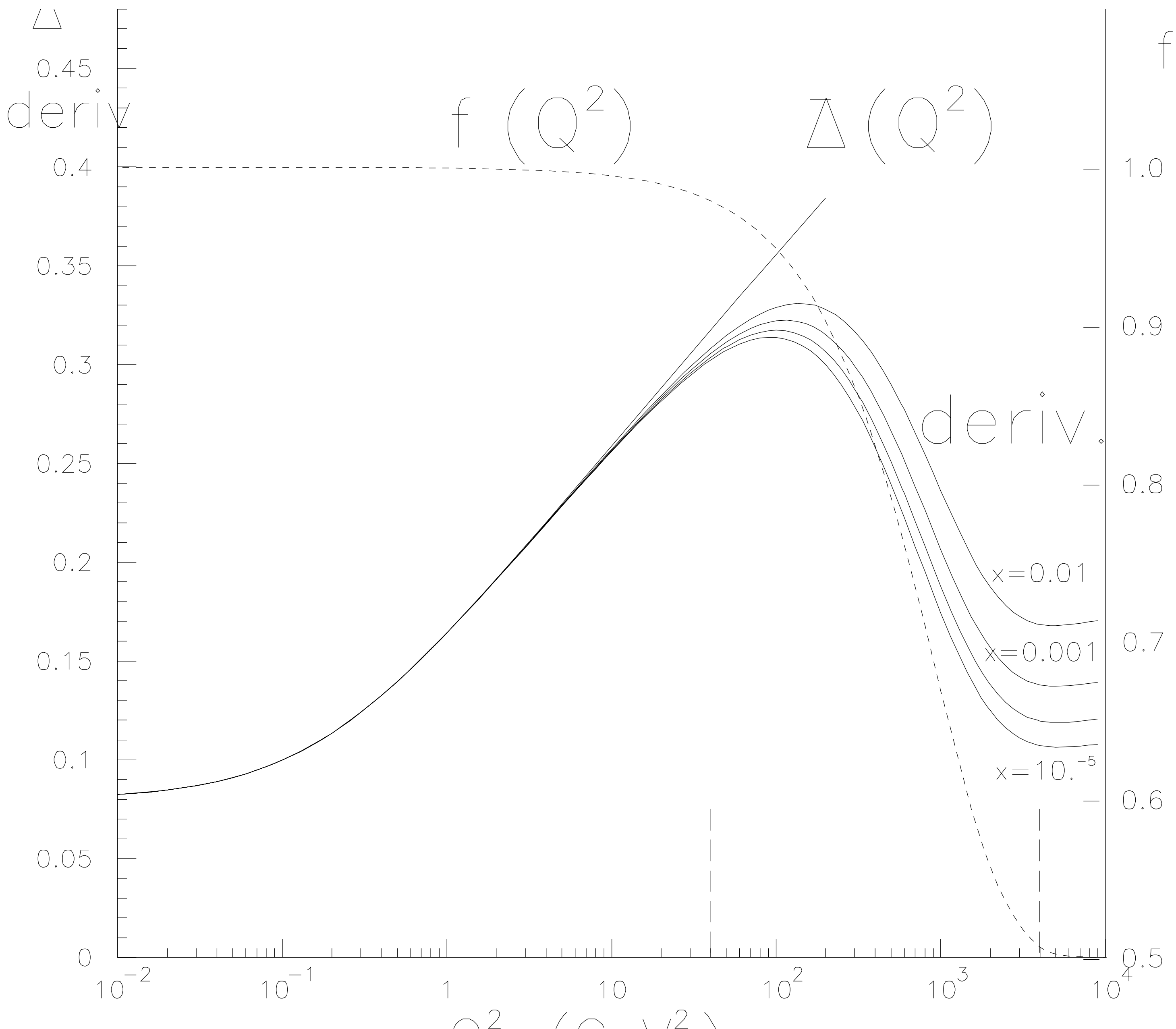
$$B_x(x,Q^2)=\frac{\partial F_2(x,Q^2)}{\partial(\ln 1/x)},$$



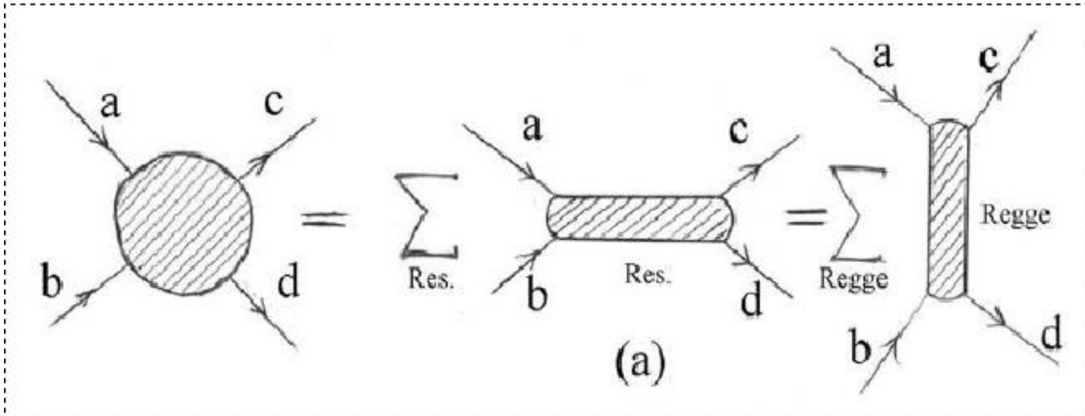
slope
2
1
0
-1
-2
-3
-4
1e-05
0.0001
0.001
0.01
x
1
10
100
1000
Q**2

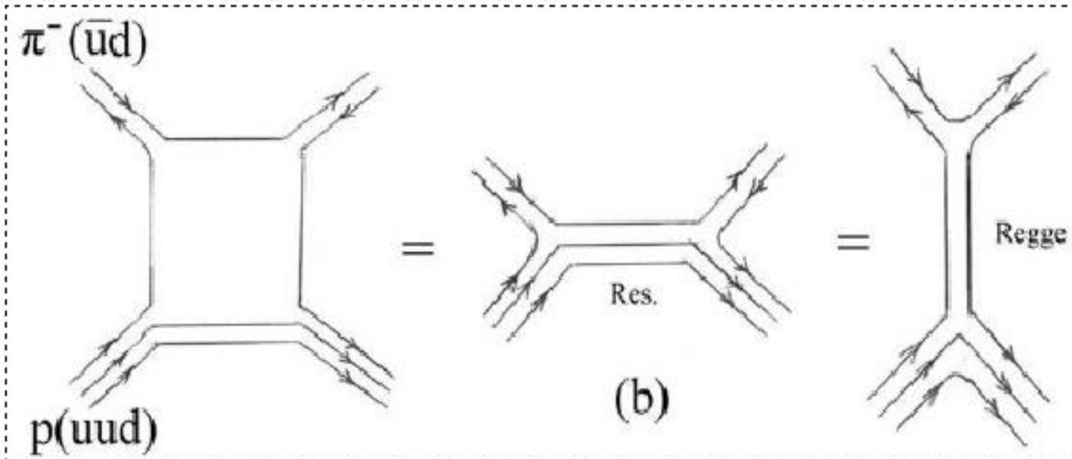


slope
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
−0.1
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}



Δ
deriv
f
f (Q²)
Δ(Q²)
deriv.
x=0.01
x=0.001
x=10.⁻⁵
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
1.0
0.9
0.8
0.7
0.6
0.5
10⁻²
10⁻¹
1
10
10²
10³
10⁴


a
c
b
d
a
c
Res.
b
Res.
d
a
c
Regge
Regge
b
d
(a)


π⁻ (ūd)
p(uud)
Res.
Regge
(b)

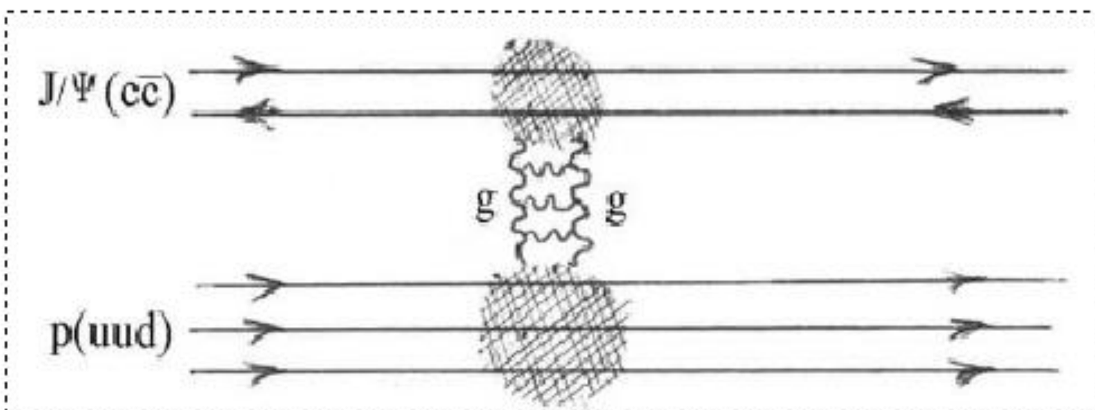
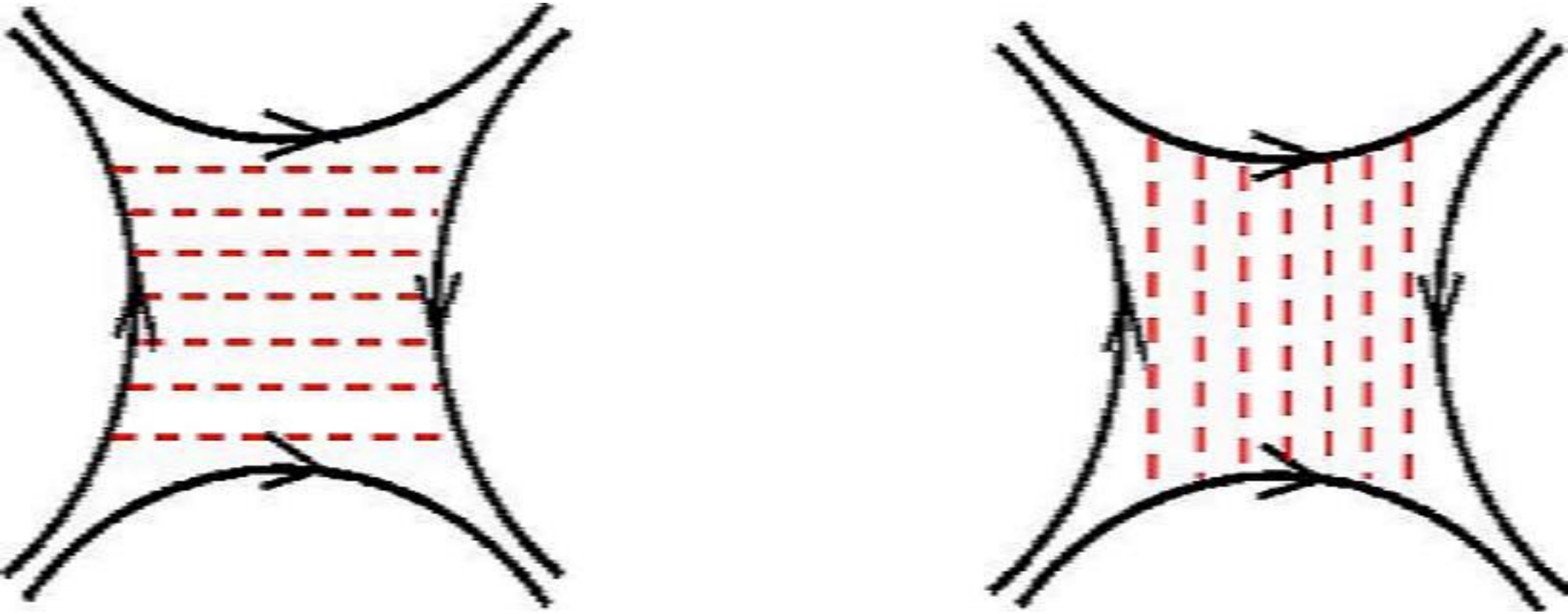
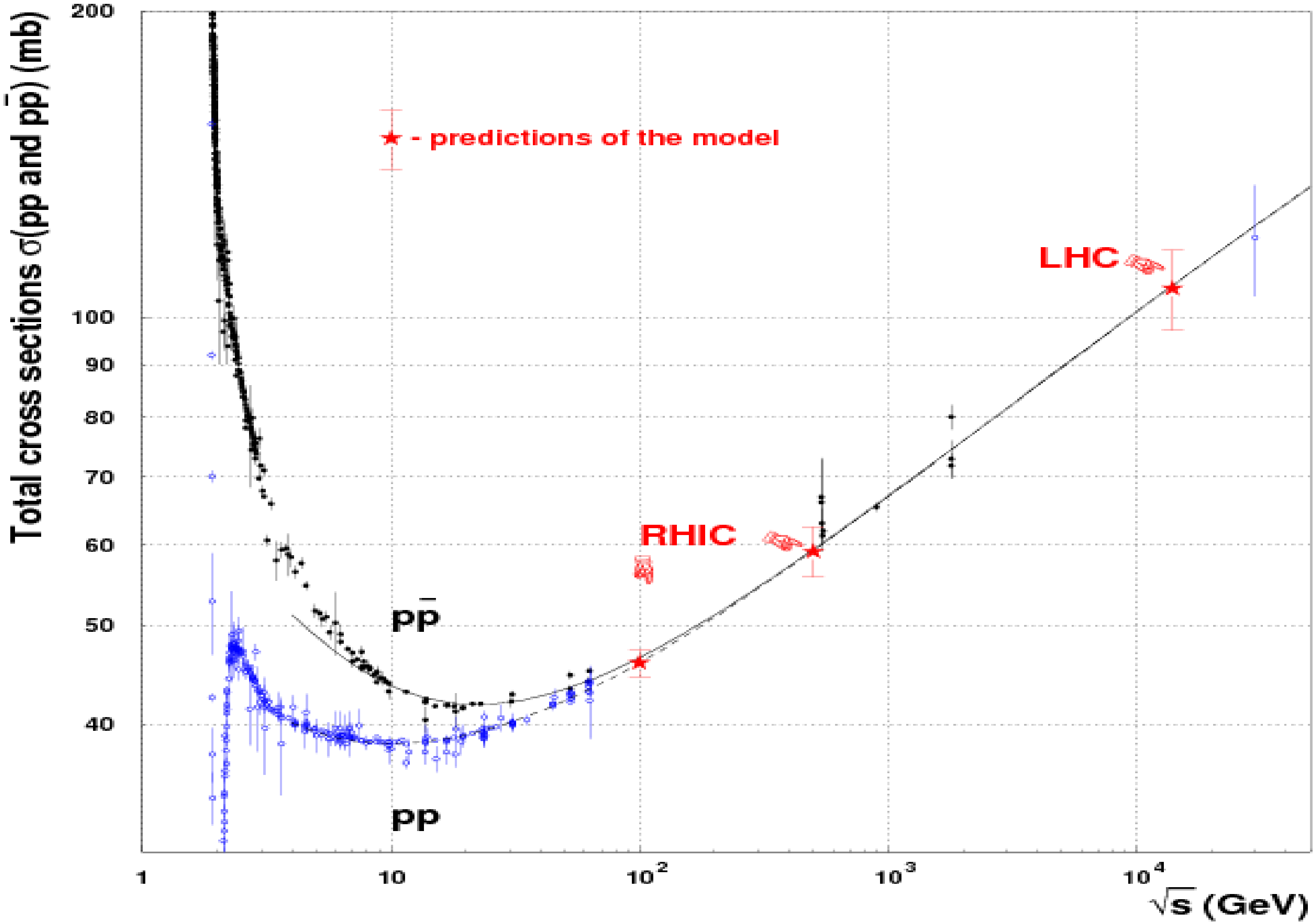

J/Ψ (cc̄)
g
g
p(uud)

TABLE I: Two-component duality

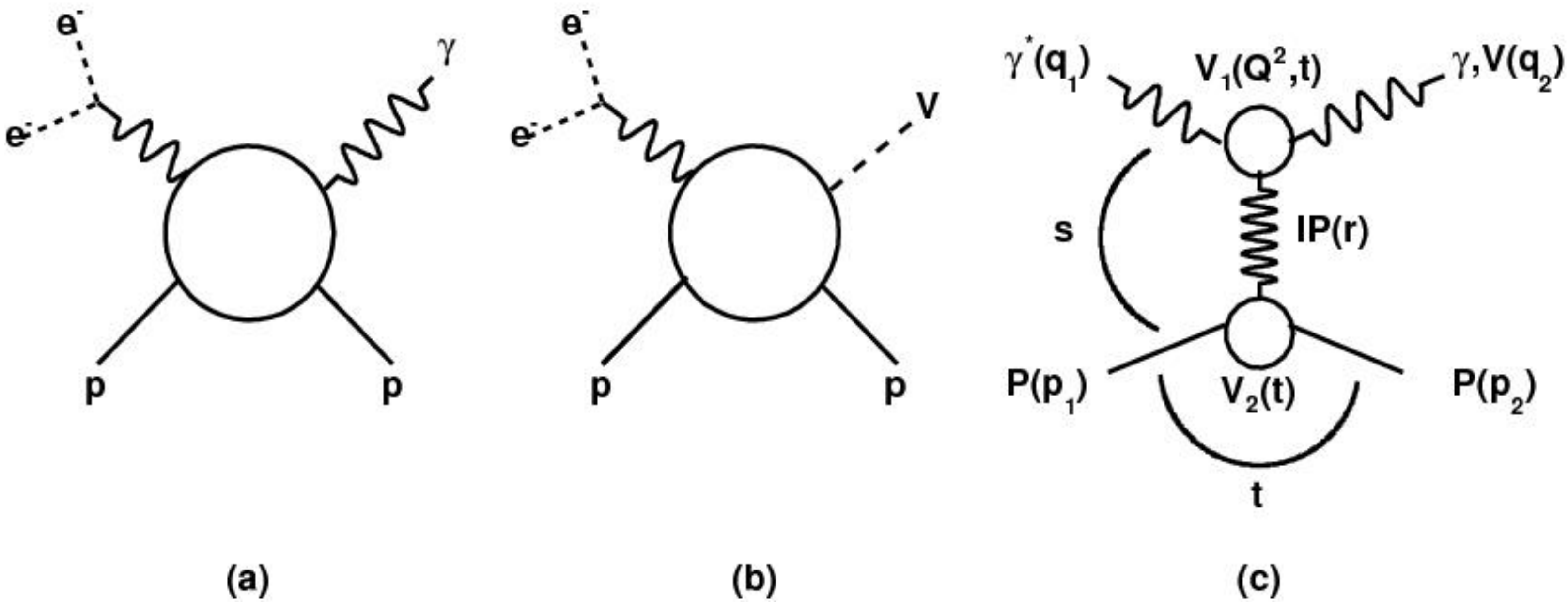
$\mathcal{I}mA(a+b\to c+d) =$	R	Pomeron
$s-$channel	$\sum A_{Res}$	Non-resonant background
$t-$channel	$\sum A_{Regge}$	Pomeron $(I=S=B=0;\ C=+1)$
Duality quark diagram	Fig. 1b	Fig. 2
High energy dependence	$s^{\alpha-1},\ \alpha<1$	$s^{\alpha-1},\ \alpha\geq 1$





Total cross sections σ(pp and p̄p) (mb)
200
100
90
80
70
60
50
40
★ - predictions of the model
LHC
RHIC
pp̄
pp
1
10
10²
10³
10⁴
√s (GeV)

LHC and HERA: parton distribution functions (PDF) and diffraction (the Pomeron) at HERA (“hard”?) and at the LHC (“soft”?);



Diagrams of DVCS (a) and VMP (b) amplitudes and their Regge-factorized form (c)

Thank you!