



Alexander von Humboldt  
Stiftung / Foundation

# An investigation of the HERA combined data at low $Q^2$

Submitted to PRD [[arXiv:1604.02299](https://arxiv.org/abs/1604.02299)]



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Low  $x$  meeting  
Károly Róbert College,  
Gyöngyös, Hungary 2016

# Low $Q^2$ data in HERAPDF2.0

Eur.Phys.J.C75 (2015) 12, 580 [[arxiv:1506.06042](https://arxiv.org/abs/1506.06042)]

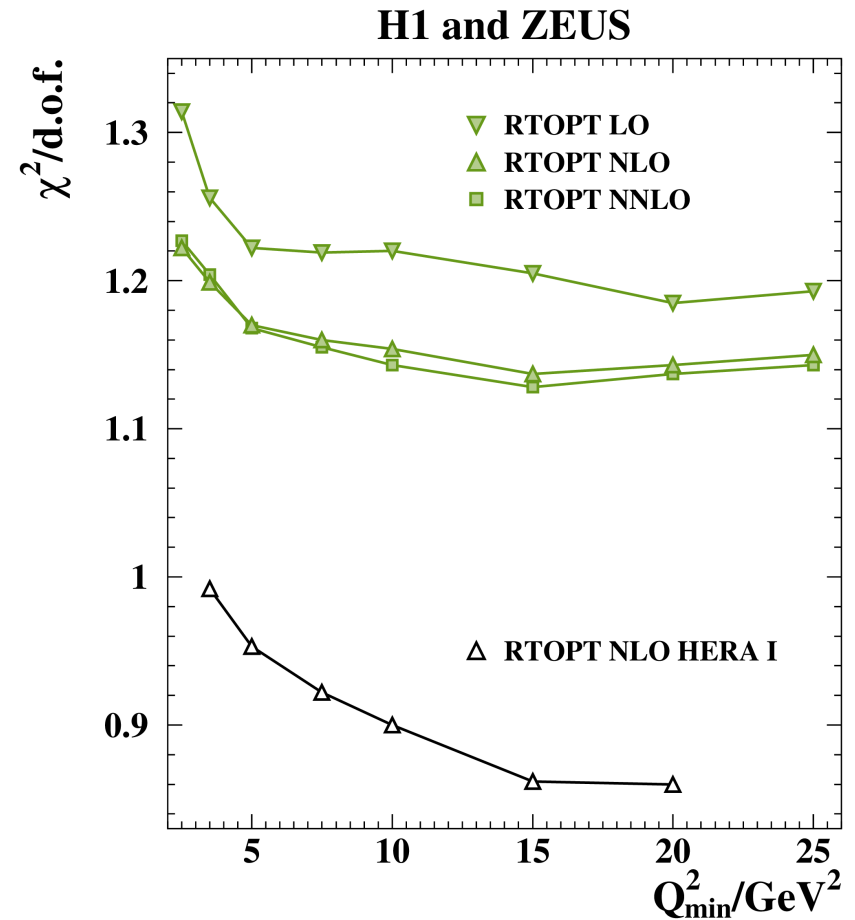
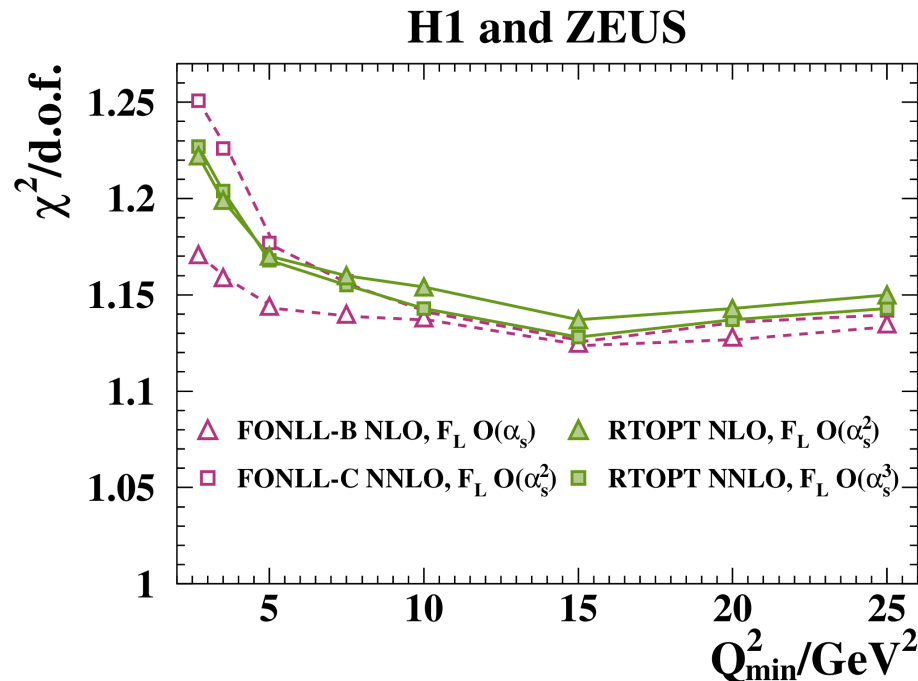
➡ DGLAP data description gets worse at low scales.

➡ The case is observed for various orders of calculation & HF schemes.

➡  $Q^2_{\min} = 3.5 \text{ GeV}^2$   
HERAPDF2.0

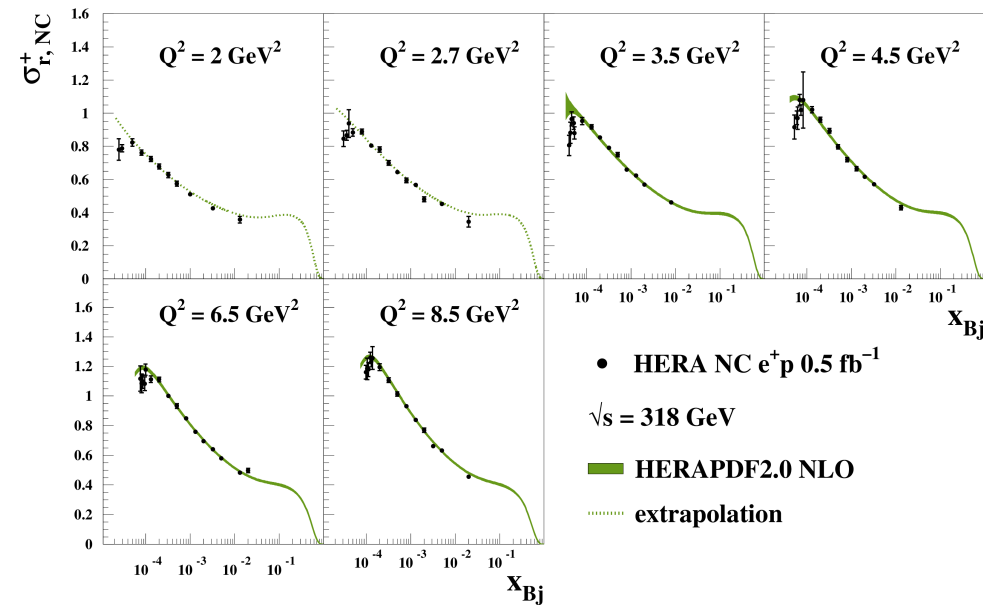
NLO  $\frac{\chi^2}{ndf} = \frac{1356}{1131} \approx 1.20$

NNLO  $\frac{\chi^2}{ndf} = \frac{1363}{1131} \approx 1.21$



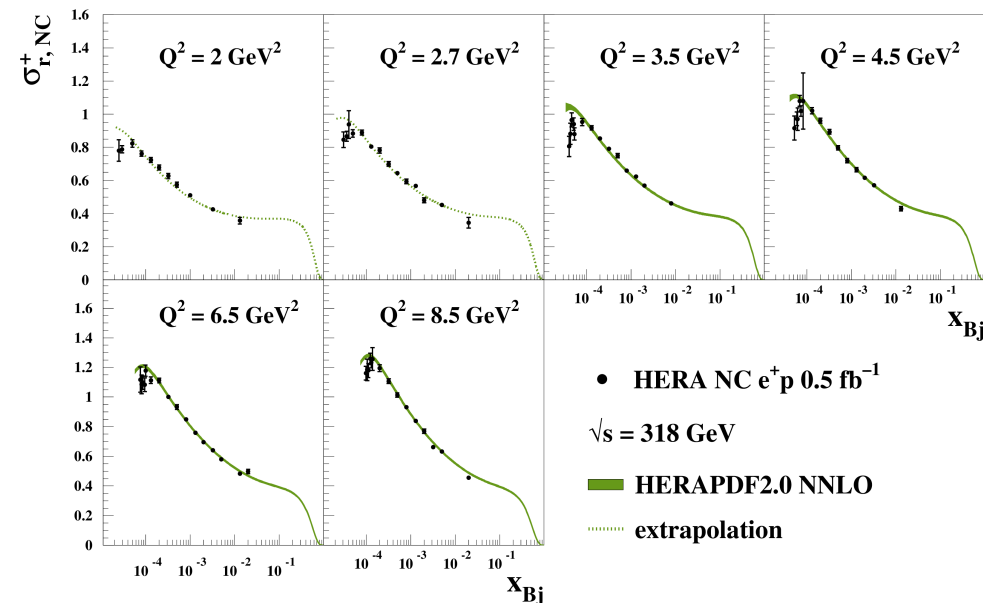
# Low $Q^2$ data in HERAPDF2.0

Eur.Phys.J.C75 (2015) 12, 580 [[arxiv:1506.06042](https://arxiv.org/abs/1506.06042)]



◆ Data between  $Q^2 = 3.5 \text{ GeV}^2$  and  $Q^2 = 15 \text{ GeV}^2$  create one third of the excess  $\chi^2/\text{d.o.f.}$

◆ Two thirds originate from the data with  $Q^2 > 150 \text{ GeV}^2$  (fluctuations)



Is cutting harder an option?!

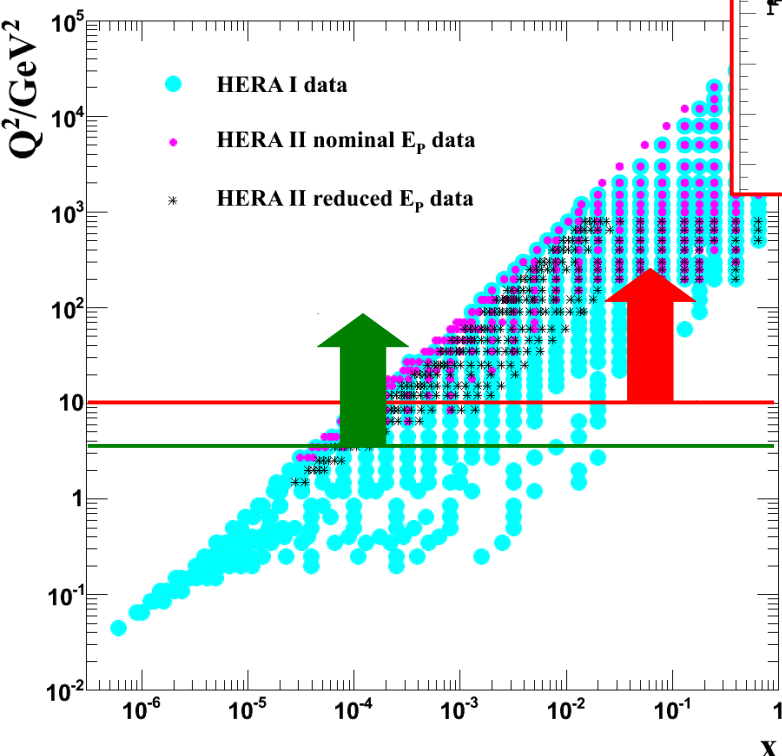
# Low $Q^2$ data in HERAPDF2.0

◆ Cut on low  $Q^2$  implies cut on low  $x$

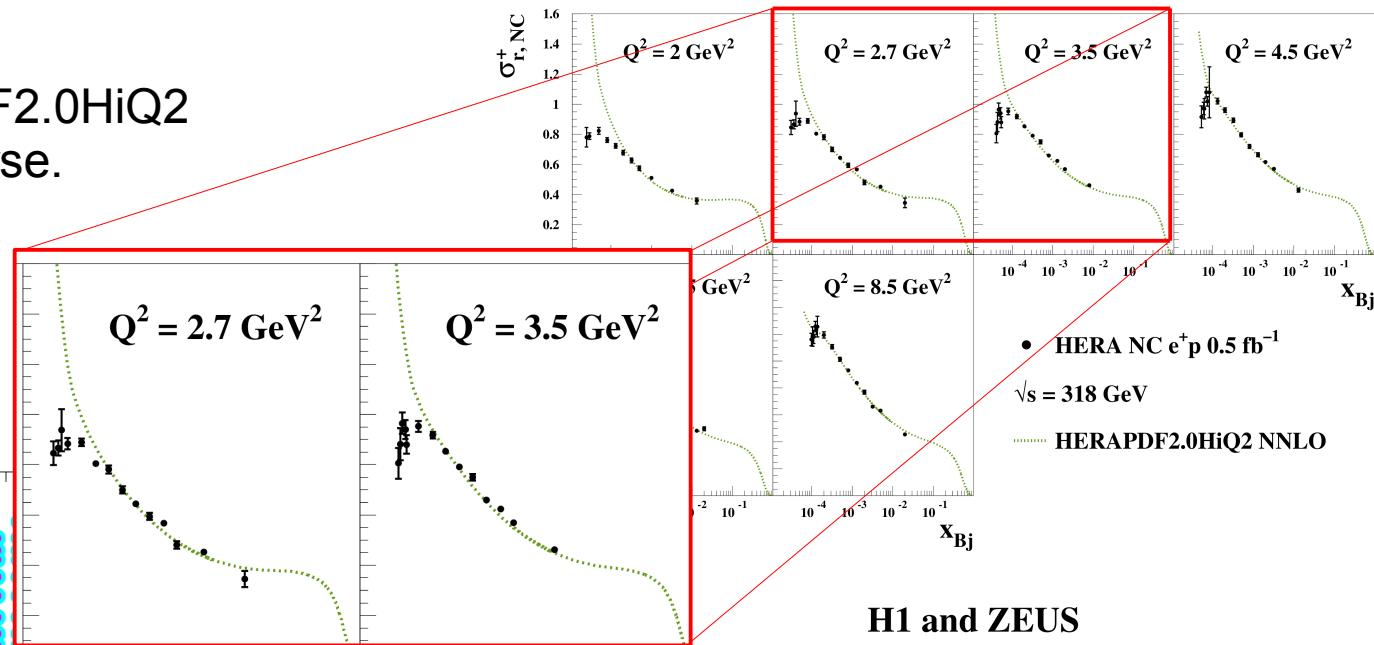
Eur.Phys.J.C75 (2015) 12, 580 [[arxiv:1506.06042](https://arxiv.org/abs/1506.06042)]

◆ Predictions from HERAPDF2.0HiQ2 describe low  $Q^2$  data even worse.

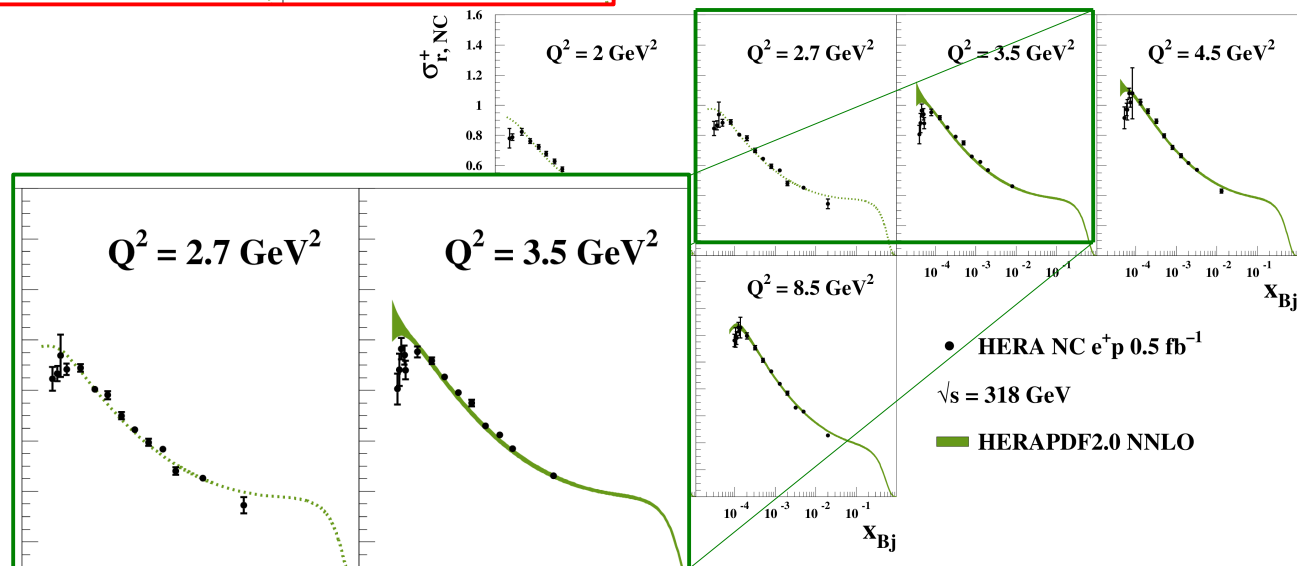
◆ Although high scale predictions **do not change!**



H1 and ZEUS



H1 and ZEUS



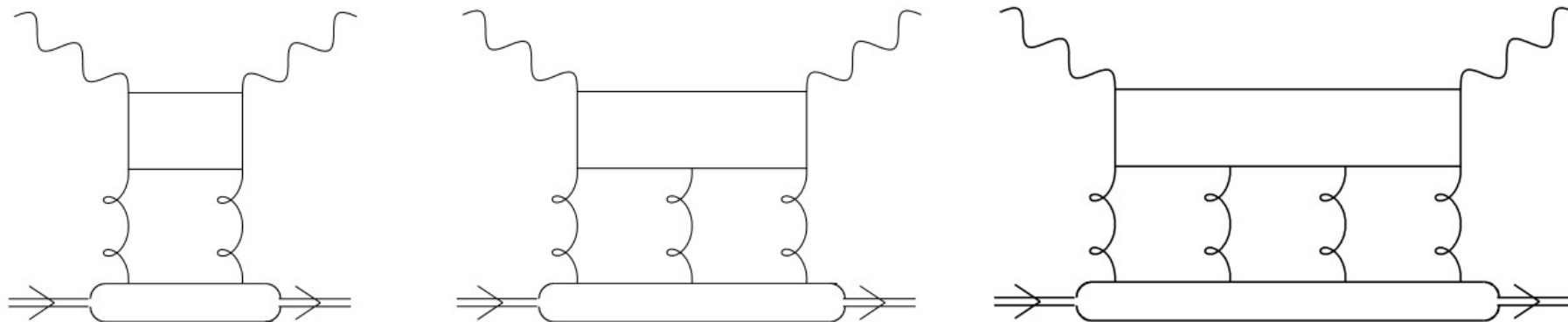


# Higher-twist correction

- ◆ The problem might be in absence of **higher twist consideration** in evolution equations

May be visualized as gluon leaders with recombining gluons

Eur. Phys. J. C 71, 121 (2000), [\[hep-ph/0003042v4\]](#)



$$\sigma_{r,NC}^{\pm} = F_2 - \frac{y^2}{Y_+} F_L$$

Cross section — a linear combination of structure functions

- ◆ Introduce simple correction factors to each of structure functions

- ◆ Higher twist terms expected to contribute to  $F_L$

$$F_L \frac{4\pi^2\alpha}{Q^2(1-x)} = \sigma_L$$

- ◆ ...and cancel in  $F_2$

$$F_2 \frac{4\pi^2\alpha}{Q^2(1-x)} = \sigma_T + \sigma_L$$

$$F_2^{HT} = F_2^{DGLAP} \left( 1 + \frac{A_2^{HT}}{Q^2} \right)$$

$$F_L^{HT} = F_L^{DGLAP} \left( 1 + \frac{A_L^{HT}}{Q^2} \right)$$

# Higher-twist correction effect

HERAPDF2.0

**NLO**  $\frac{\chi^2}{ndf} = \frac{1356}{1131} \approx 1.20$

**NNLO**  $\frac{\chi^2}{ndf} = \frac{1363}{1131} \approx 1.21$

➡ Introducing  $F_2^{HT} = F_2^{DGLAP} \left(1 + \frac{A_2^{HT}}{Q^2}\right)$  gives almost no effect:

HHT@F<sub>2</sub>

**NLO**  $\frac{\chi^2}{ndf} = \frac{1354}{1130} \approx 1.20$

**NNLO**  $\frac{\chi^2}{ndf} = \frac{1357}{1130} \approx 1.20$

$A_2^{HT} = 0.14 \pm 0.10 \text{ GeV}^2$   
 $A_2^{HT} = 0.12 \pm 0.07 \text{ GeV}^2$

corr. factors consistent with 0



➡ Introducing  $F_L^{HT} = F_L^{DGLAP} \left(1 + \frac{A_L^{HT}}{Q^2}\right)$  helps a lot more:

HHT@F<sub>L</sub>

**NLO**  $\frac{\chi^2}{ndf} = \frac{1329}{1130} \approx 1.18$

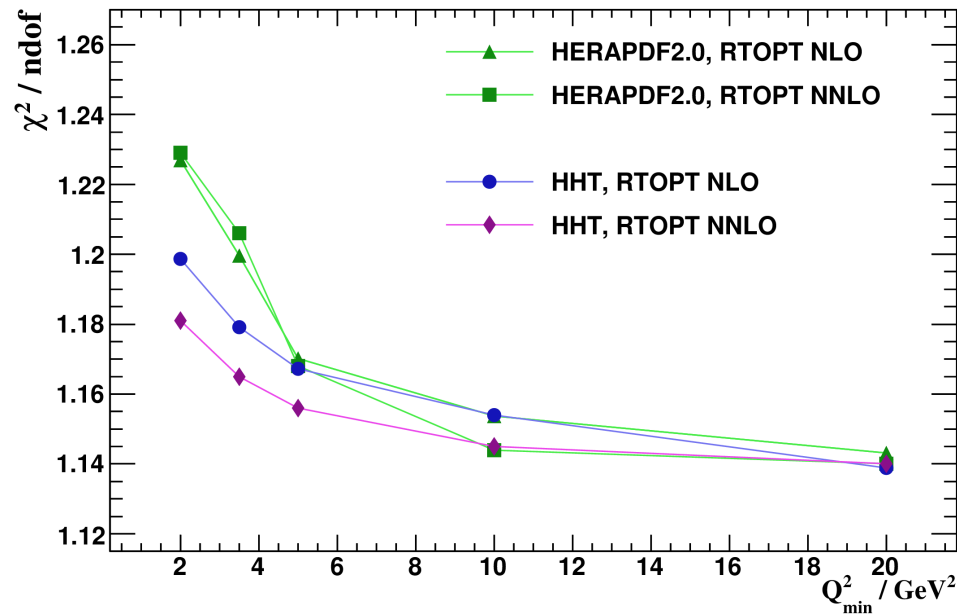
**NNLO**  $\frac{\chi^2}{ndf} = \frac{1316}{1130} \approx 1.16$

$\Delta \chi^2 = 27$   
 $\Delta \chi^2 = 47$

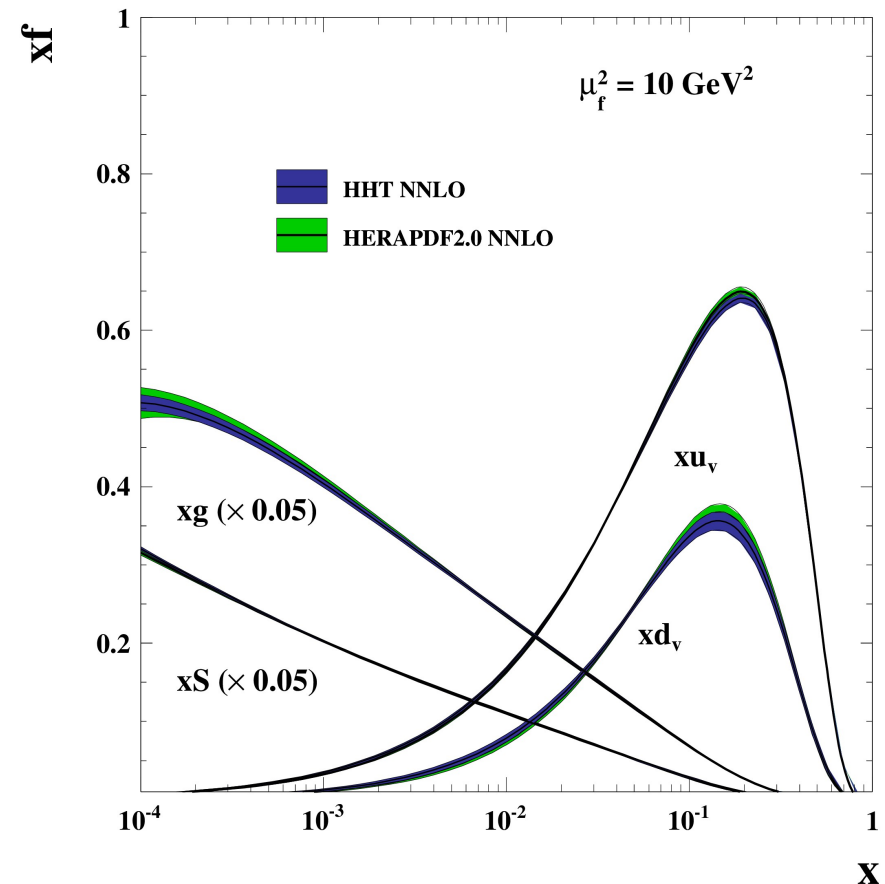
$A_L^{HT} = 4.2 \pm 0.7 \text{ GeV}^2$   
 $A_L^{HT} = 5.5 \pm 0.6 \text{ GeV}^2$

# Higher-twist correction effect

◆  $Q_{\min}^2$  dependence flattens significantly



◆ and PDFs almost do not change!



HHT@F<sub>L</sub>

NLO  $\frac{\chi^2}{\text{ndf}} = \frac{1329}{1130} \approx 1.18$

NNLO  $\frac{\chi^2}{\text{ndf}} = \frac{1316}{1130} \approx 1.16$

$\Delta \chi^2 = 27$

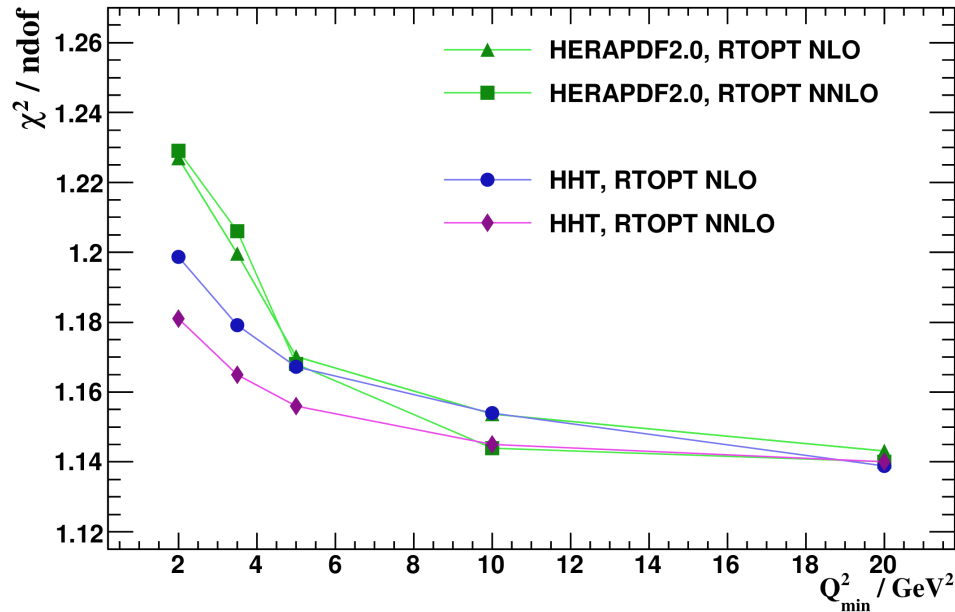
$\Delta \chi^2 = 47$

$A_L^{\text{HT}} = 4.2 \pm 0.7 \text{ GeV}^2$

$A_L^{\text{HT}} = 5.5 \pm 0.6 \text{ GeV}^2$

# Higher-twist correction effect

◆  $Q_{\min}^2$  dependence flattens significantly

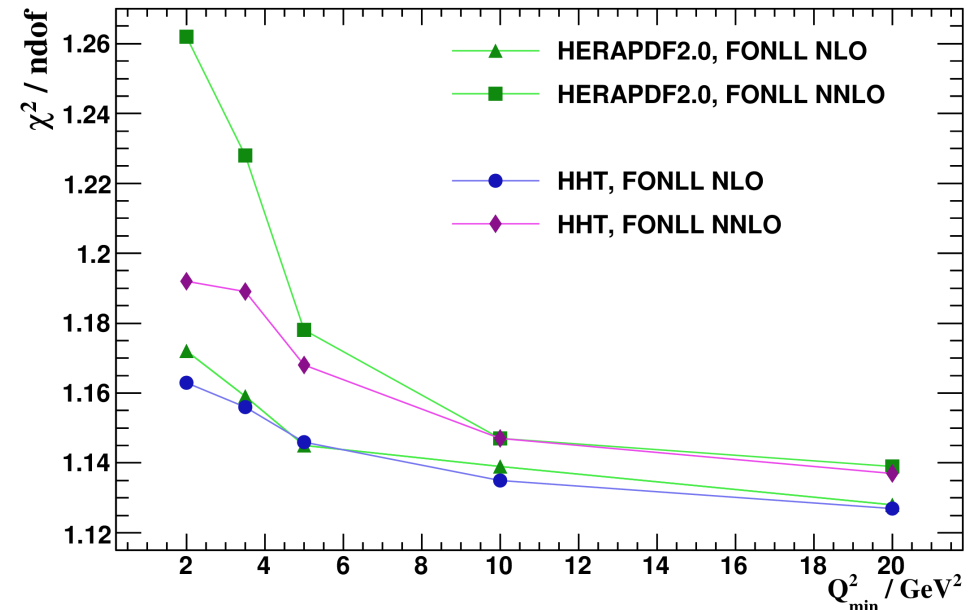


◆ and it also does within various HF schemes!

FONLL scheme:

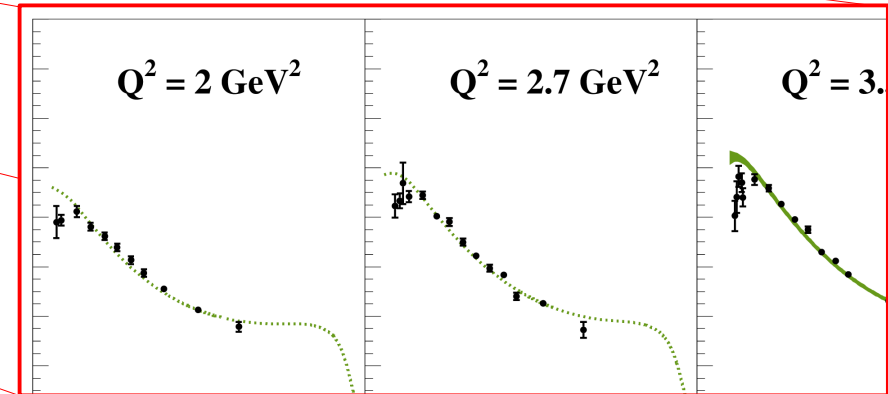
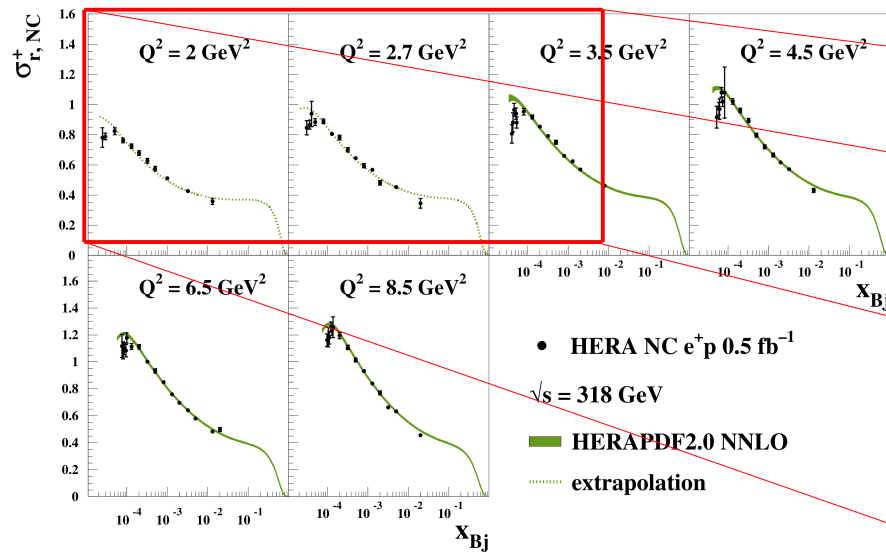
◆ Not much of a gain @NLO ( $F_L^{\text{FONLL}} \propto \alpha_s$ )

◆ Substantial improvement @NNLO ( $F_L^{\text{FONLL}} \propto \alpha_s^2$ )



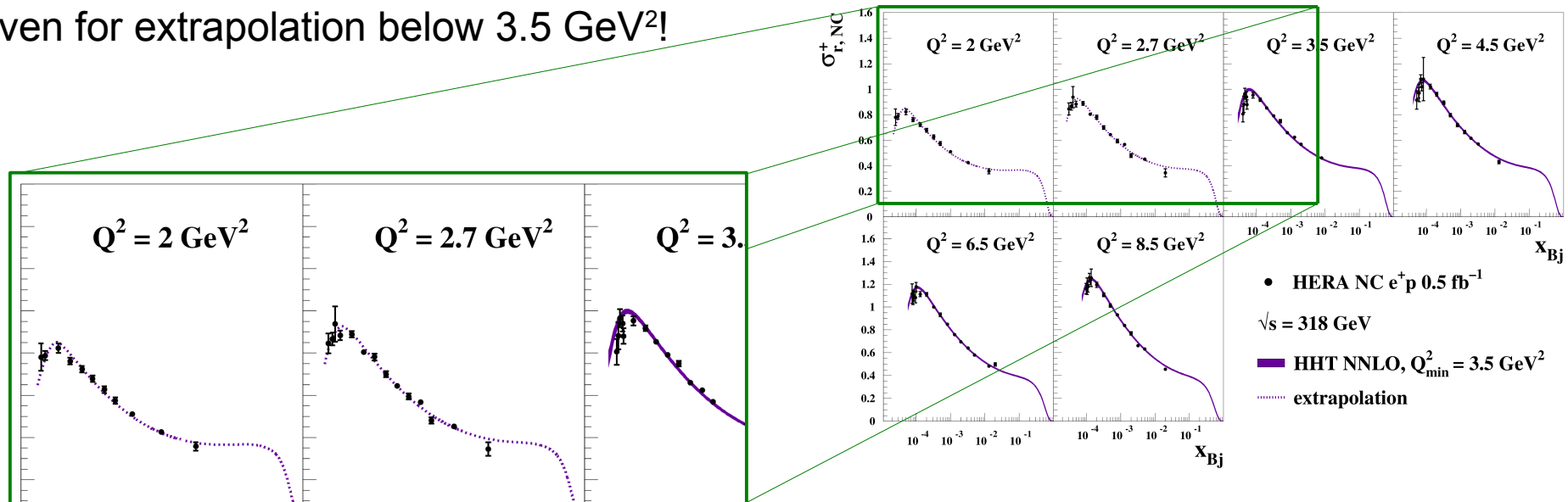


# HHT: data description



Low  $Q^2$  description improves remarkably

...even for extrapolation below  $3.5 \text{ GeV}^2$ !



# HHT: data description

What about the fitting data over  $Q^2_{\min} = 2 \text{ GeV}^2$  then?

$$Q^2_{\min} = 3.5 \text{ GeV}^2 \quad \begin{array}{l} \text{NLO} \quad \frac{\chi^2}{ndf} = \frac{1329}{1130} \approx 1.18 \\ \text{NNLO} \quad \frac{\chi^2}{ndf} = \frac{1316}{1130} \approx 1.16 \end{array}$$

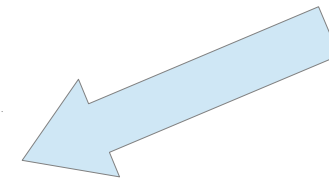
$$A_L^{\text{HT}} = 4.2 \pm 0.7 \text{ GeV}^2$$

$$A_L^{\text{HT}} = 5.5 \pm 0.6 \text{ GeV}^2$$

$$Q^2_{\min} = 2 \text{ GeV}^2 \quad \begin{array}{l} \text{NLO} \quad \frac{\chi^2}{ndf} = \frac{1398}{1170} \approx 1.19 \\ \text{NNLO} \quad \frac{\chi^2}{ndf} = \frac{1381}{1170} \approx 1.18 \end{array}$$

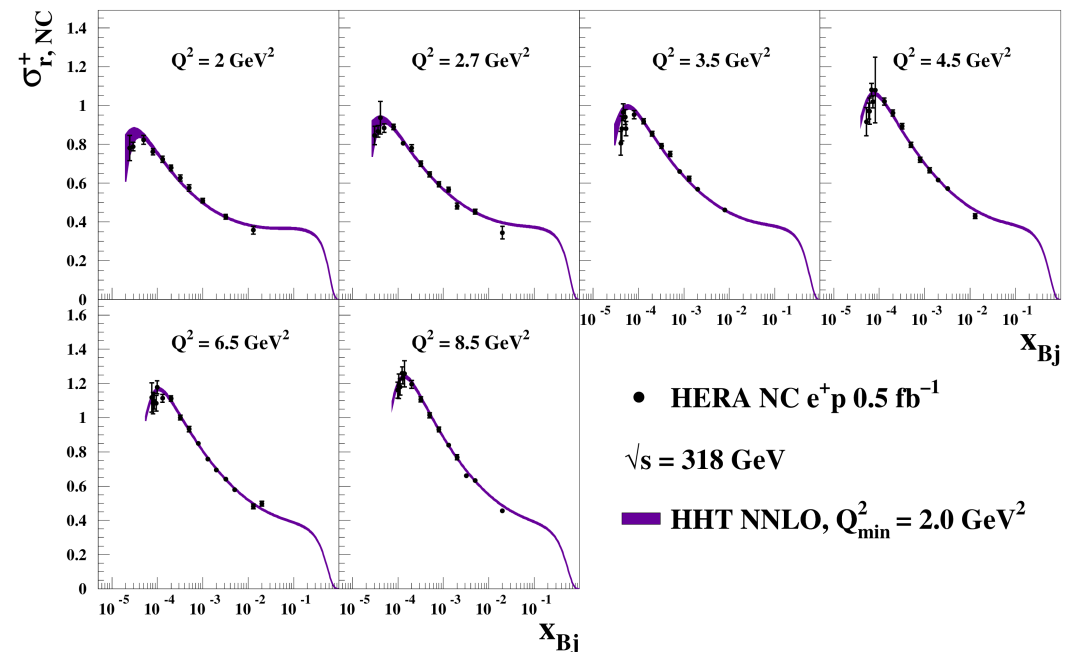
$$A_L^{\text{HT}} = 4.0 \pm 0.6 \text{ GeV}^2$$

$$A_L^{\text{HT}} = 5.2 \pm 0.7 \text{ GeV}^2$$



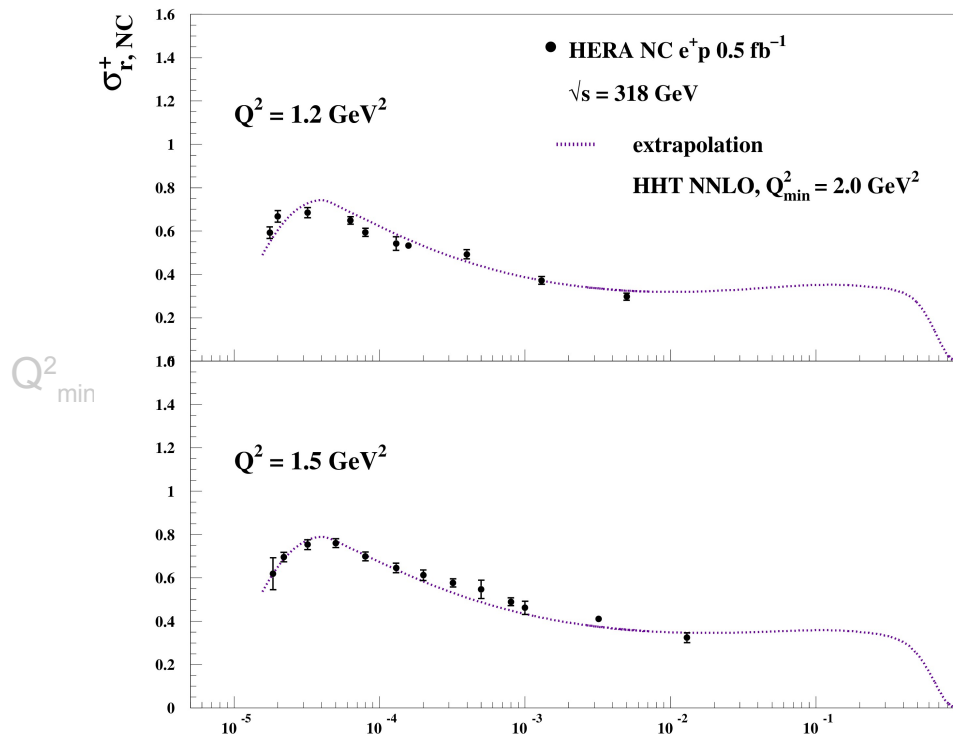
Excellent data description!

Although  $\chi^2/\text{dof}$  is somewhat higher



# HHT: data description

What about the fitting data over  $Q^2_{\min} = 2 \text{ GeV}^2$  than?



Excellent data description!

Although  $\chi^2/\text{dof}$  is somewhat higher

$$\frac{\chi^2}{ndf} = \frac{1329}{1130} \approx 1.18$$

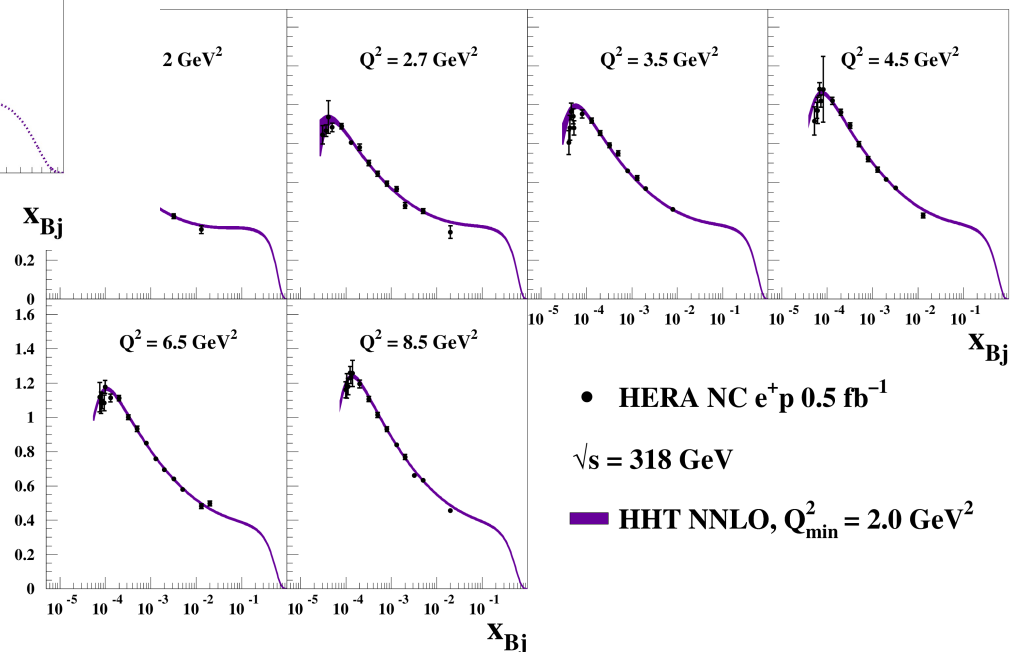
$$\frac{\chi^2}{ndf} = \frac{1316}{1130} \approx 1.16$$

$$A_L^{\text{HT}} = 4.2 \pm 0.7 \text{ GeV}^2$$

$$A_L^{\text{HT}} = 5.5 \pm 0.6 \text{ GeV}^2$$

$$3 \pm 0.6 \text{ GeV}^2$$

$$2 \pm 0.7 \text{ GeV}^2$$



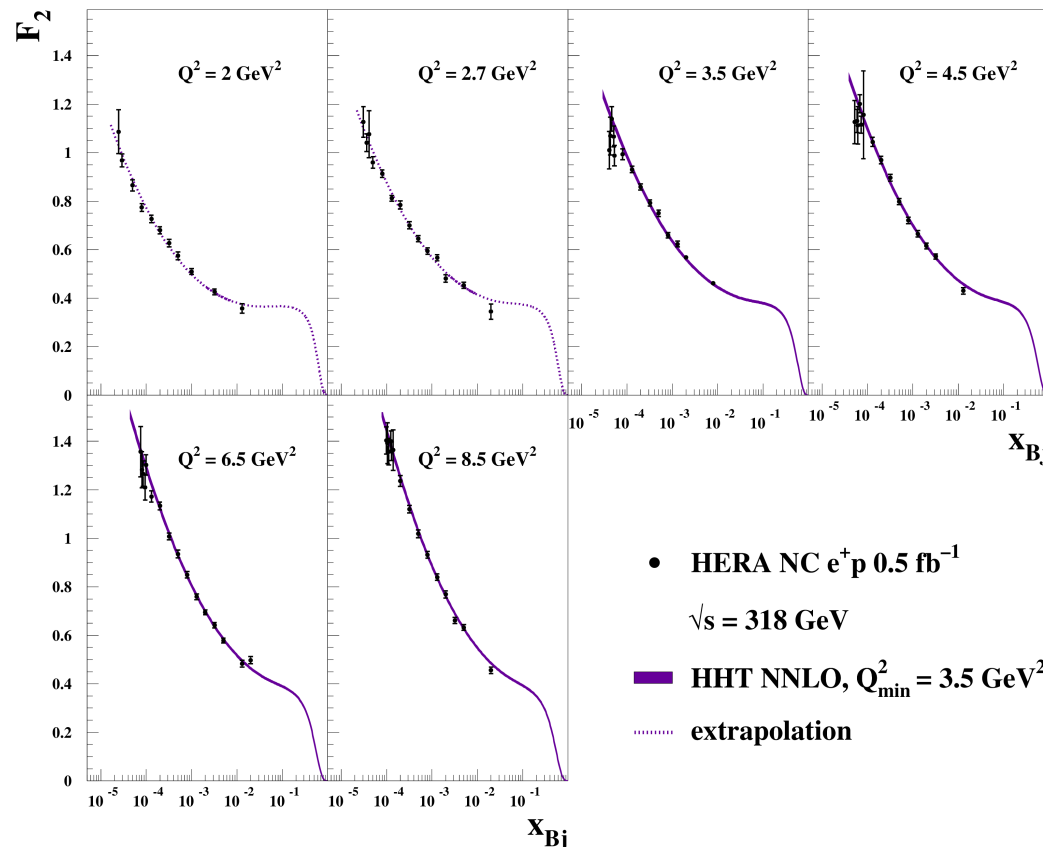
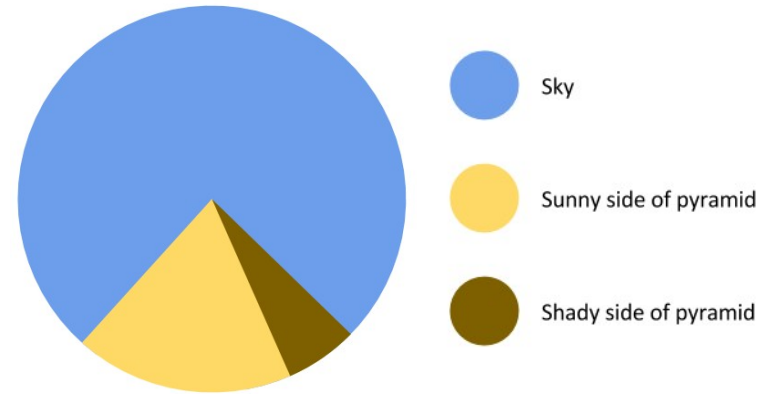
# HHT: prediction components

◆  $\sigma$  predictions do great job! What are the constituents of this?

$$\sigma_{r,NC}^{\pm} = F_2 - \frac{y^2}{Y_+} F_L$$

+HHT

$$\sigma_{r,NC}^{\pm} = F_2 - \frac{y^2}{Y_+} F_L \left( 1 + \frac{A_L^{HT}}{Q^2} \right)$$



◆  $F_2$  looks reasonable as well

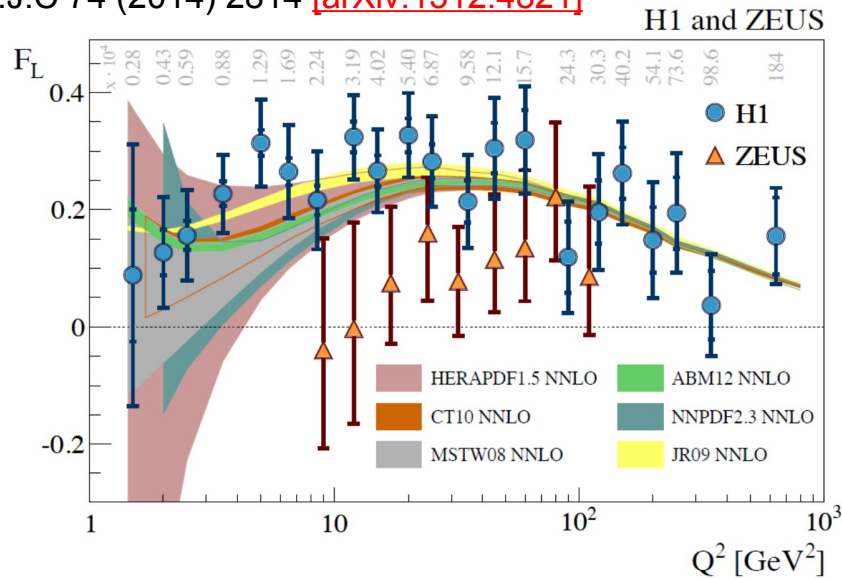
◆ Prediction describes extracted  $F_2$  well.

$$F_2^{extr} = F_2^{pred} \frac{\sigma_r^{meas}}{\sigma_r^{pred}}$$

How about  $F_L$ ?

# HHT: $F_L$ structure function

Eur.Phys.J.C 74 (2014) 2814 [[arXiv:1312.4821](https://arxiv.org/abs/1312.4821)]

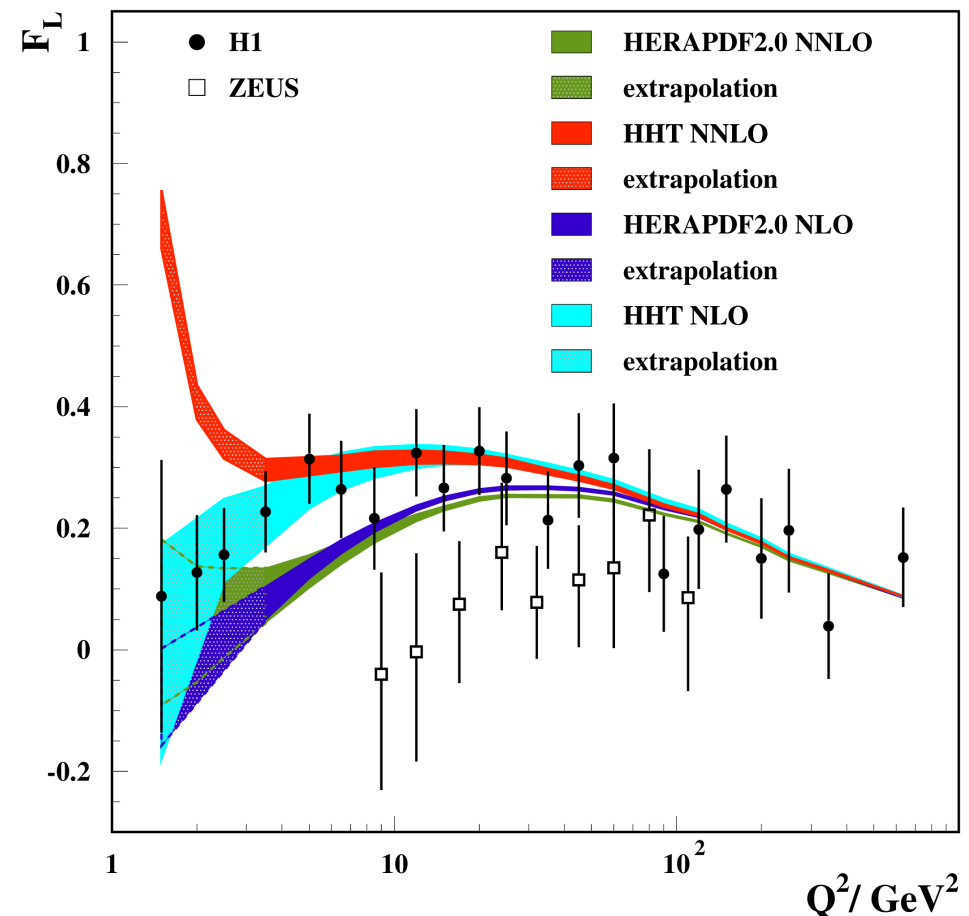


Current study:

- Way smaller uncertainties
- $F_L$  from HHT is larger both @ NLO and NNLO
- $F_L$ @NNLO shows a **dramatic upturn at low  $Q^2$**

Previous studies:

- Large uncertainties
- Predictions indicate very similar behavior





# HHT: $F_L$ structure function

$$xg(x, Q^2) \approx 1.77 \frac{3\pi}{2\alpha_s(Q^2)} F_L(x, Q^2)$$

◆  $F_L$  wants to be larger

◆  $F_L$  is directly related to gluon PDF

...can try to drop negative gluon term in  $xg$  parametrization

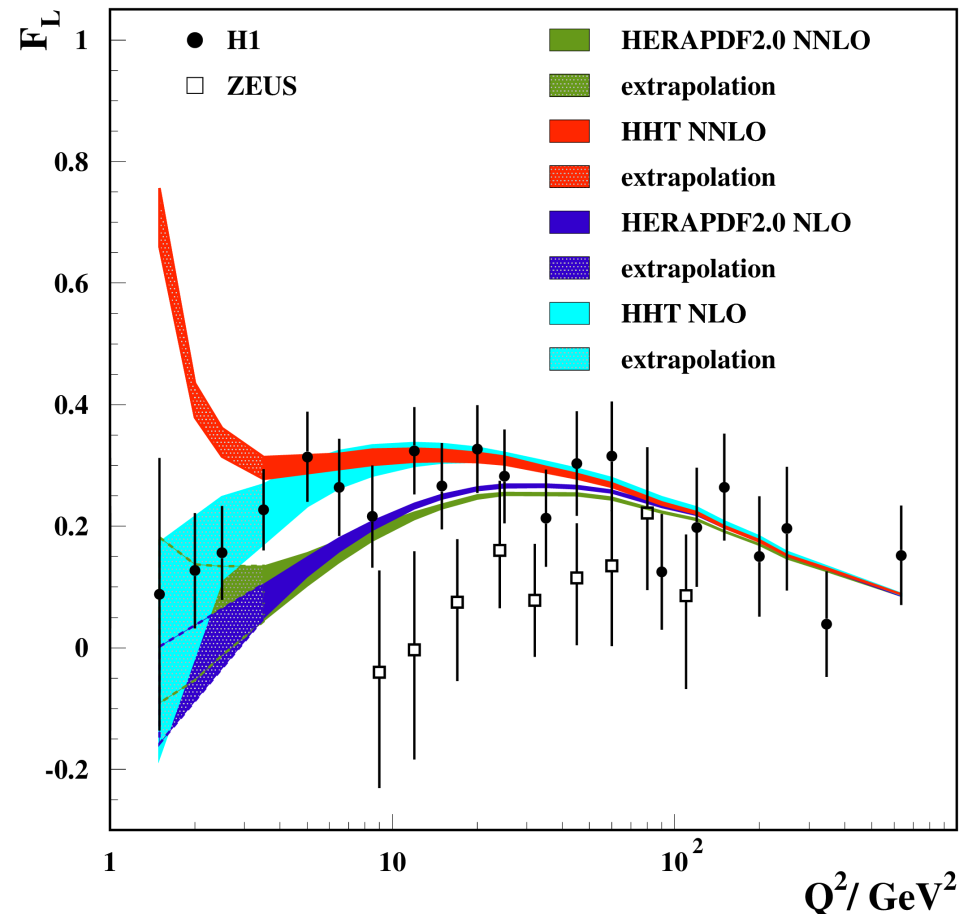
$$xg(x) = A_g x_g^B (1-x)_g^C - \cancel{A_g' x_g^{B'} (1-x)_g^{C'}}$$

HHT@NNLO  $\frac{\chi^2}{ndf} = \frac{1316}{1130} \approx 1.16$

HHT<sub>AG</sub>@NNLO  $\frac{\chi^2}{ndf} = \frac{1350}{1132} \approx 1.20$

HERAPDF2.0<sub>AG</sub>@NNLO  $\frac{\chi^2}{ndf} = \frac{1385}{1132} \approx 1.22$

◆ Negative gluon term is definitely needed!



# HHT: prediction components

Another perspective: data at constant W and various Q<sup>2</sup>:

◆ Discrepancies observed at low x => should appear at high W

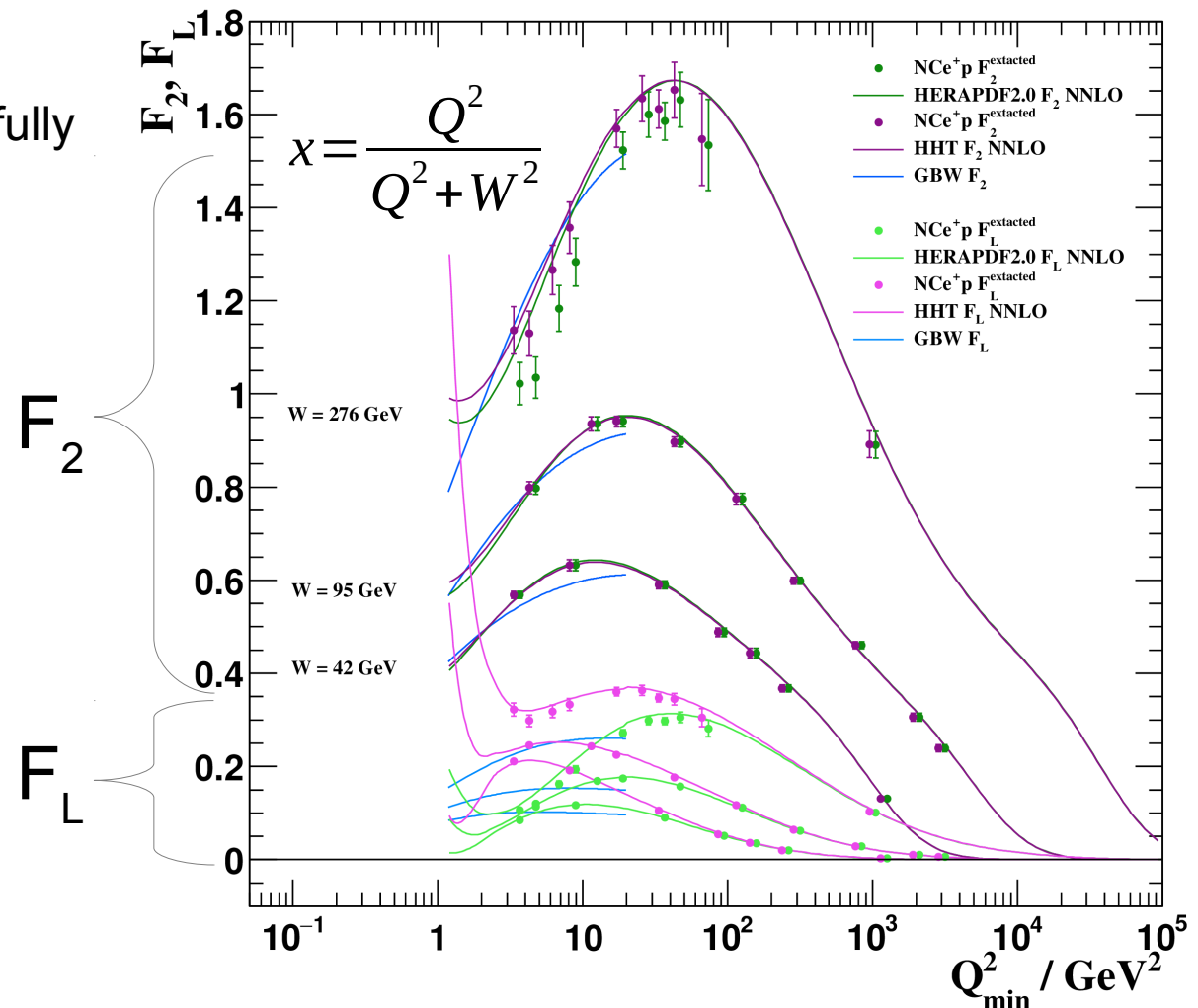
◆ HHT describes F<sub>2</sub><sup>extr</sup> more successfully

$$F_{2/L}^{extr} = F_{2/L}^{pred} \frac{\sigma_r^{meas}}{\sigma_r^{pred}}$$

◆ Also Golec-Biernat, Wusthoff dipole model is shown:

• GBW agrees well with HHT at region of applicability.

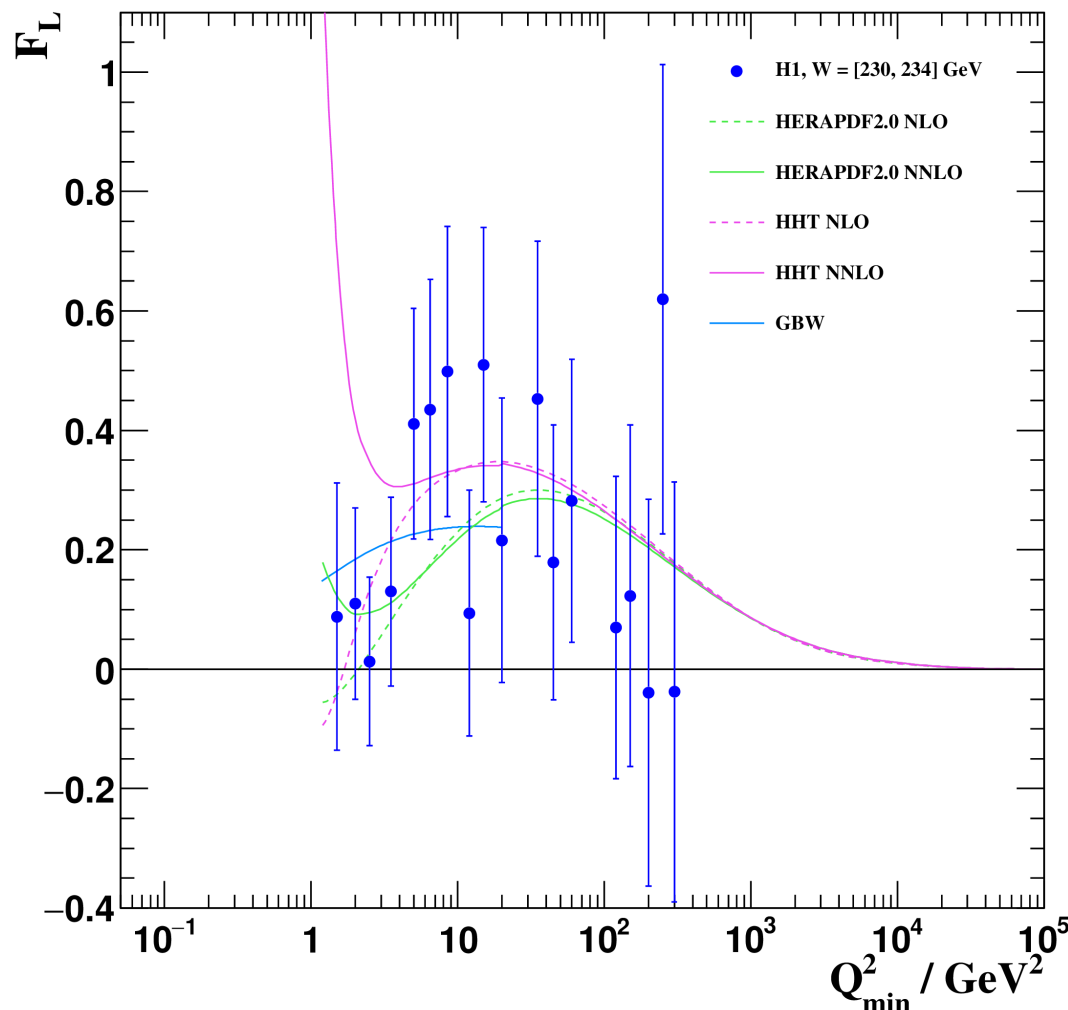
• GBW and HHT start to disagree when either exceeds its relevant region



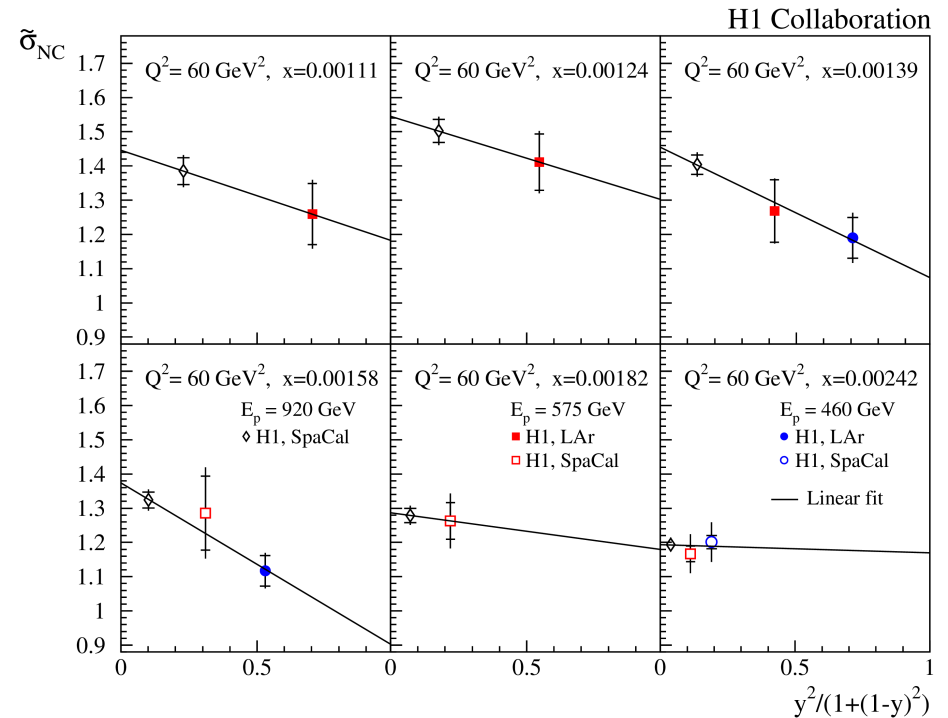
# HHT: $F_L$ structure function

◆  $F_L^{extr} = F_L^{pred} \frac{\sigma_r^{meas}}{\sigma_r^{pred}}$  is highly model dependent

◆ Direct measurements of  $F_L$  exist:



Eur.Phys.J.C 74 (2014) 2814 [[arXiv:1312.4821](https://arxiv.org/abs/1312.4821)]

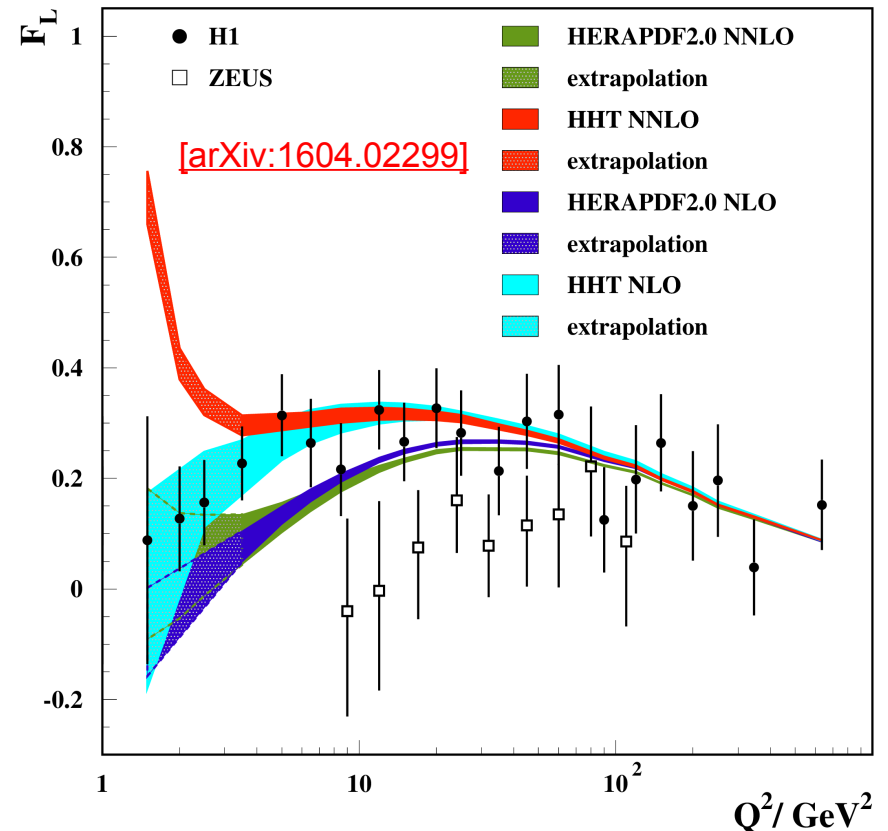


◆ Model independent data does not show any upturn at low  $Q^2$

◆ More investigations required for proper  $F_L$  predictions

# Summary

- ◆ HHT successfully describe inclusive ep cross sections data down to  $Q^2 \sim 2 \text{ GeV}^2$
- ◆ Addition of HT corrections flattens  $\chi^2(Q^2)$  dependence substantially
- ◆ HT correction does not change PDFs much
- ◆  $F_L$  from HHT demonstrates unphysical upturn at low scales
- ◆ The HHT approach might be too simplistic therefore requires more studies.

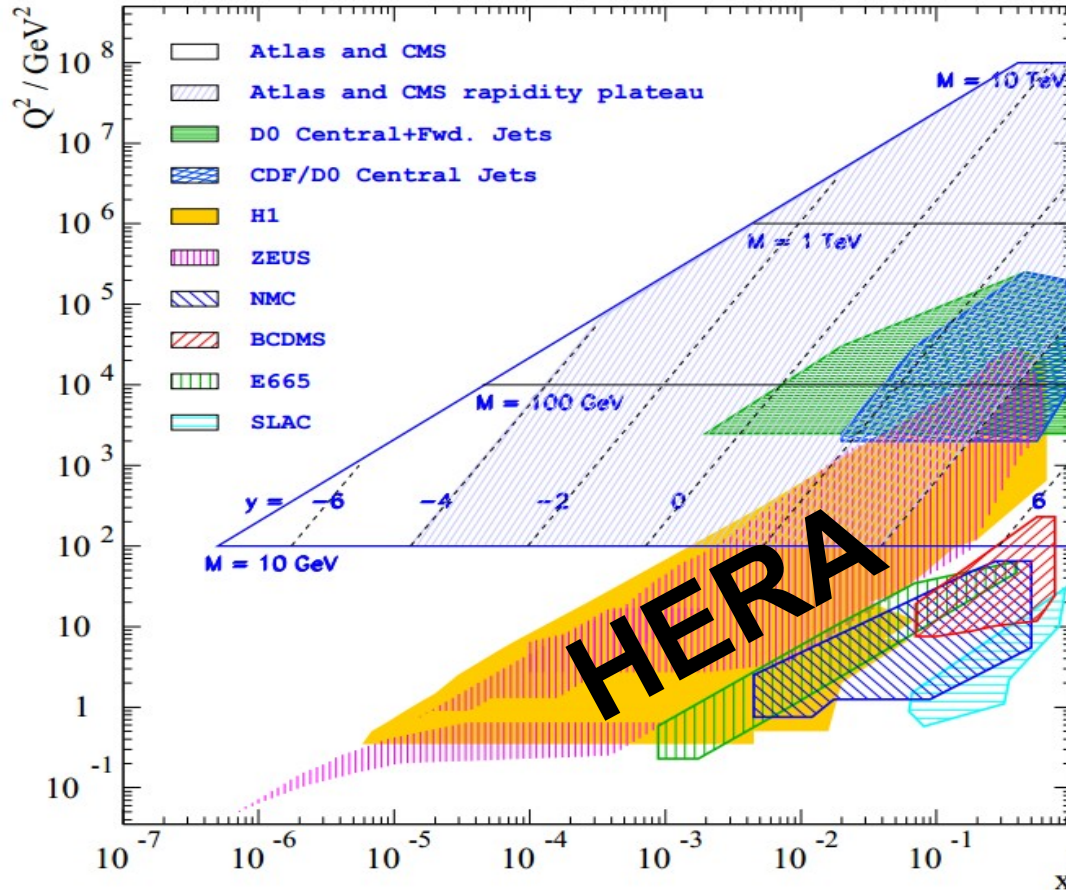


# Backup

not necessarily useful...



# HERA collider



$$Q^2 = -q^2 = -(k - k')^2$$

$$x_{Bj} = \frac{Q^2}{2pq} \quad y = \frac{pq}{pk}$$

$$s = (p + k)^2 \quad Q^2 = xys$$

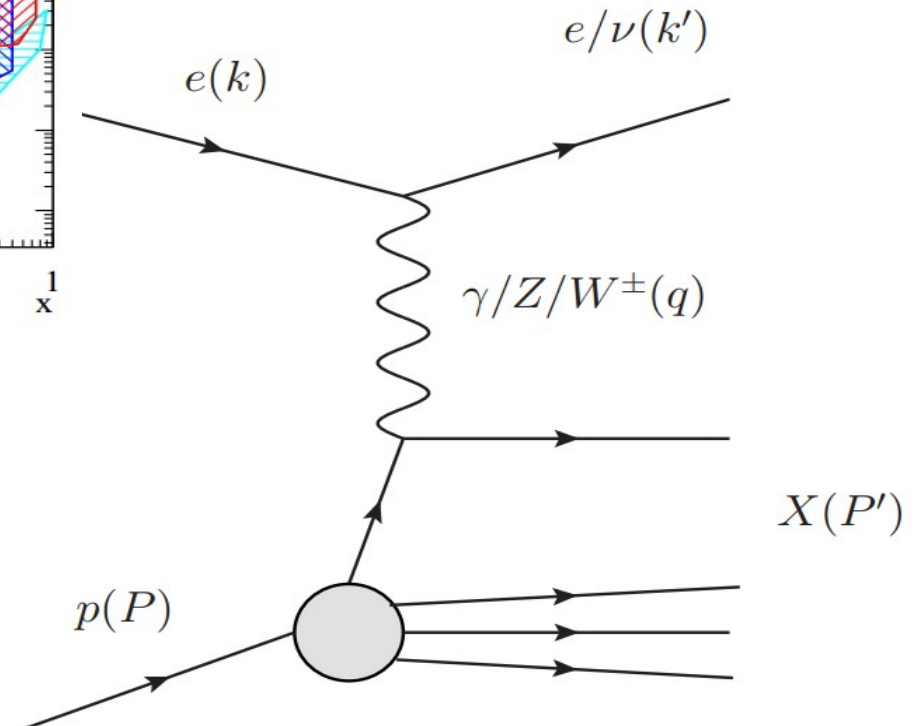
$$E_p = 920 (460, 575) \text{ GeV}$$

$$E_e = 27.5 \text{ GeV}$$

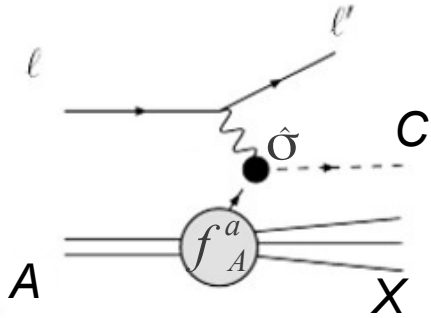
$$\sqrt{s} = 318 (225, 252) \text{ GeV}$$

Experimental achievements:

$\sim 0.5 \text{ fb}^{-1}$  DIS data from each experiment

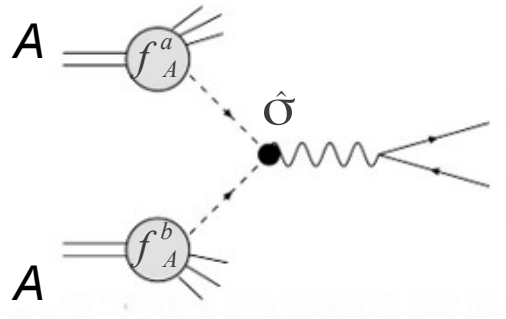


# PDFs for the precision measurements



 **Factorisation theorem: PDFs + hard-scattering cross section**

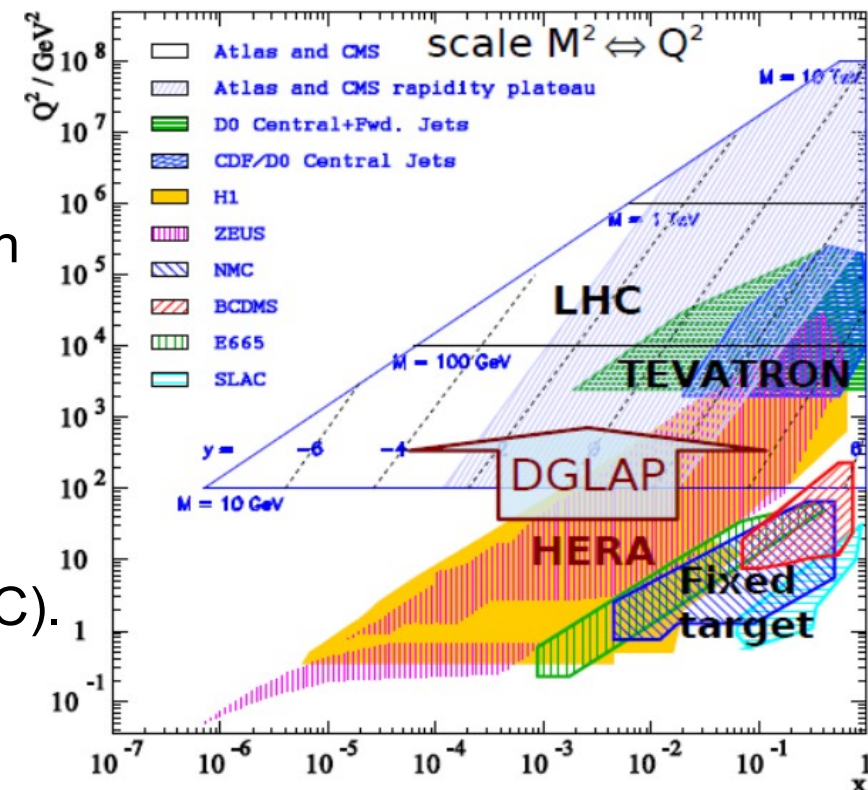
$$\sigma_{A \rightarrow C}^i(q, p) = \sum_a \int_x^1 d\xi f_A^a(\xi, \mu) \hat{\sigma}_{a \rightarrow C}^i(q, \xi p, \mu, \alpha_s)$$



➡ PDFs are **universal** => essential for precision measurements.

➡ HERA data is a core of every PDF determination

- + Covers wide kinematic range
- + Probes linear combination of quarks.
- + Sensitive to the quark flavor decomposition (CC).
- + Information on the gluon content of proton

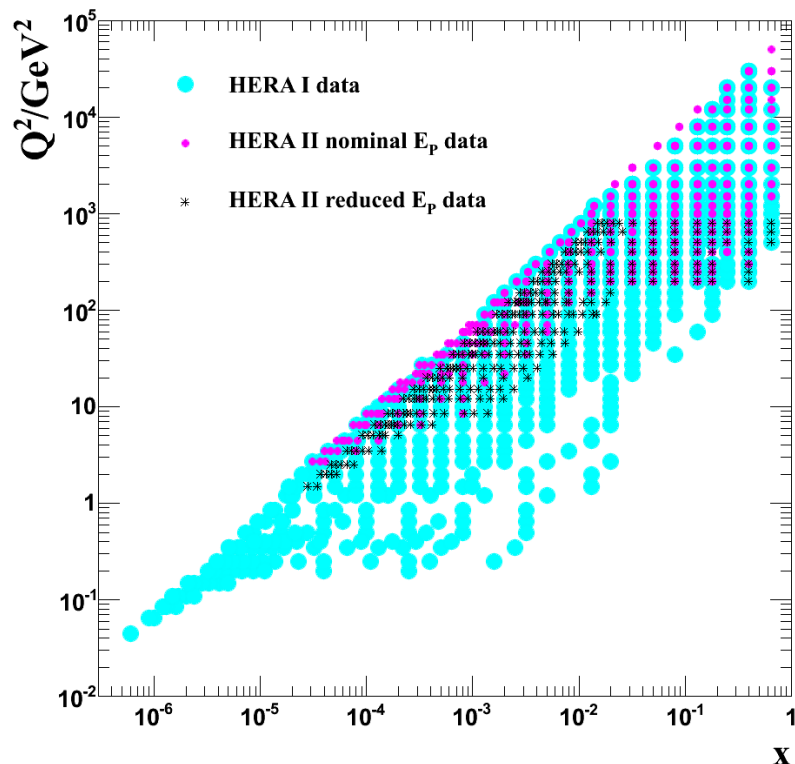


# Full HERA data combination

HERAPDF1.0

HERAPDF1.5

HERAPDF2.0



Data Set		$x_{Bj}$ Grid	$Q^2$ [GeV <sup>2</sup> ] Grid	$\mathcal{L}$	$e^+/e^-$	$\sqrt{s}$
		from	to	pb <sup>-1</sup>		GeV
HERA I $E_p = 820$ GeV and $E_p = 920$ GeV data sets						
H1 svx-mb	95-00	0.000005	0.02	0.2 12	2.1	$e^+p$ 301, 319
H1 low $Q^2$	96-00	0.0002	0.1	12 150	22	$e^+p$ 301, 319
H1 NC	94-97	0.0032	0.65	150 30000	35.6	$e^+p$ 301
H1 CC	94-97	0.013	0.40	300 15000	35.6	$e^+p$ 301
H1 NC	98-99	0.0032	0.65	150 30000	16.4	$e^-p$ 319
H1 CC	98-99	0.013	0.40	300 15000	16.4	$e^-p$ 319
H1 NC HY	98-99	0.0013	0.01	100 800	16.4	$e^-p$ 319
H1 NC	99-00	0.0013	0.65	100 30000	65.2	$e^+p$ 319
H1 CC	99-00	0.013	0.40	300 15000	65.2	$e^+p$ 319
ZEUS BPC	95	0.000002	0.00006	0.11 0.65	1.65	$e^+p$ 300
ZEUS BPT	97	0.0000006	0.001	0.045 0.65	3.9	$e^+p$ 300
ZEUS SVX	95	0.000012	0.0019	0.6 17	0.2	$e^+p$ 300
ZEUS NC	96-97	0.00006	0.65	2.7 30000	30.0	$e^+p$ 300
ZEUS CC	94-97	0.015	0.42	280 17000	47.7	$e^+p$ 300
ZEUS NC	98-99	0.005	0.65	200 30000	15.9	$e^-p$ 318
ZEUS CC	98-99	0.015	0.42	280 30000	16.4	$e^-p$ 318
ZEUS NC	99-00	0.005	0.65	200 30000	63.2	$e^+p$ 318
ZEUS CC	99-00	0.008	0.42	280 17000	60.9	$e^+p$ 318
HERA II $E_p = 920$ GeV data sets						
H1 NC $^{1.5p}$	03-07	0.0008	0.65	60 30000	182	$e^+p$ 319
H1 CC $^{1.5p}$	03-07	0.008	0.40	300 15000	182	$e^+p$ 319
H1 NC $^{1.5p}$	03-07	0.0008	0.65	60 50000	151.7	$e^-p$ 319
H1 CC $^{1.5p}$	03-07	0.008	0.40	300 30000	151.7	$e^-p$ 319
H1 NC med $Q^2$ $^{*y.5}$	03-07	0.0000986	0.005	8.5 90	97.6	$e^+p$ 319
H1 NC low $Q^2$ $^{*y.5}$	03-07	0.000029	0.00032	2.5 12	5.9	$e^+p$ 319
ZEUS NC	06-07	0.005	0.65	200 30000	135.5	$e^+p$ 318
ZEUS CC $^{1.5p}$	06-07	0.0078	0.42	280 30000	132	$e^+p$ 318
ZEUS NC $^{1.5}$	05-06	0.005	0.65	200 30000	169.9	$e^-p$ 318
ZEUS CC $^{1.5}$	04-06	0.015	0.65	280 30000	175	$e^-p$ 318
ZEUS NC nominal $^{*y}$	06-07	0.000092	0.008343	7 110	44.5	$e^+p$ 318
ZEUS NC satellite $^{*y}$	06-07	0.000071	0.008343	5 110	44.5	$e^+p$ 318
HERA II $E_p = 575$ GeV data sets						
H1 NC high $Q^2$	07	0.00065	0.65	35 800	5.4	$e^+p$ 252
H1 NC low $Q^2$	07	0.0000279	0.0148	1.5 90	5.9	$e^+p$ 252
ZEUS NC nominal	07	0.000147	0.013349	7 110	7.1	$e^+p$ 251
ZEUS NC satellite	07	0.000125	0.013349	5 110	7.1	$e^+p$ 251
HERA II $E_p = 460$ GeV data sets						
H1 NC high $Q^2$	07	0.00081	0.65	35 800	11.8	$e^+p$ 225
H1 NC low $Q^2$	07	0.0000348	0.0148	1.5 90	12.2	$e^+p$ 225
ZEUS NC nominal	07	0.000184	0.016686	7 110	13.9	$e^+p$ 225
ZEUS NC satellite	07	0.000143	0.016686	5 110	13.9	$e^+p$ 225

◆ All inclusive DIS results are final and published!

# HERAPDF2.0: settings for QCD fit

◆ QCD fits are performed using **HERAFitter** package

◆ PDFs (**14p**) are parametrised at  $Q_0^2 = 1.9 \text{ GeV}^2$

$$xg(x) = A_g x^{B_g} (1-x)^{C_g} - A'_g x^{B'_g} (1-x)^{C'_g},$$

$$xu_v(x) = A_{u_v} x^{B_{u_v}} (1-x)^{C_{u_v}} (1 + E_{u_v} x^2),$$

$$xd_v(x) = A_{d_v} x^{B_{d_v}} (1-x)^{C_{d_v}},$$

$$x\bar{U}(x) = A_{\bar{U}} x^{B_{\bar{U}}} (1-x)^{C_{\bar{U}}} (1 + D_{\bar{U}} x),$$

$$x\bar{D}(x) = A_{\bar{D}} x^{B_{\bar{D}}} (1-x)^{C_{\bar{D}}}.$$

◆  $A_{u_v}, A_{d_v}, A_g$  are constrained by **QCD sum rules**

◆  $x\bar{u} \xrightarrow{x \rightarrow 0} x\bar{d}$  ◆  $A_{\bar{U}}, A_{\bar{D}}$  are constrained via  $x\bar{s} = f_s x\bar{D}$

◆ PDF evolution is performed using **DGLAP** equations

◆ Heavy flavour coefficients are obtained within **GM VFNS (RT OPT)**

$$\chi^2 = \sum_i \frac{[\mu_i - m_i (1 - \sum_j \gamma_j^i b_j)]^2}{\delta_{i,uncor}^2 m_i^2 + \delta_{i,stat}^2 \mu_i m_i (1 - \sum_j \gamma_j^i b_j)} + \sum_j b_j^2 + \sum_i \ln \frac{\delta_{i,uncor}^2 m_i^2 + \delta_{i,stat}^2 \mu_i m_i}{\delta_{i,uncor}^2 \mu_i^2 + \delta_{i,stat}^2 \mu_i^2}$$



