On Lam-Tung relation breaking at the LHC

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Plan: to make full use of forward Drell-Yan process at the LHC to measure higher twist contributions

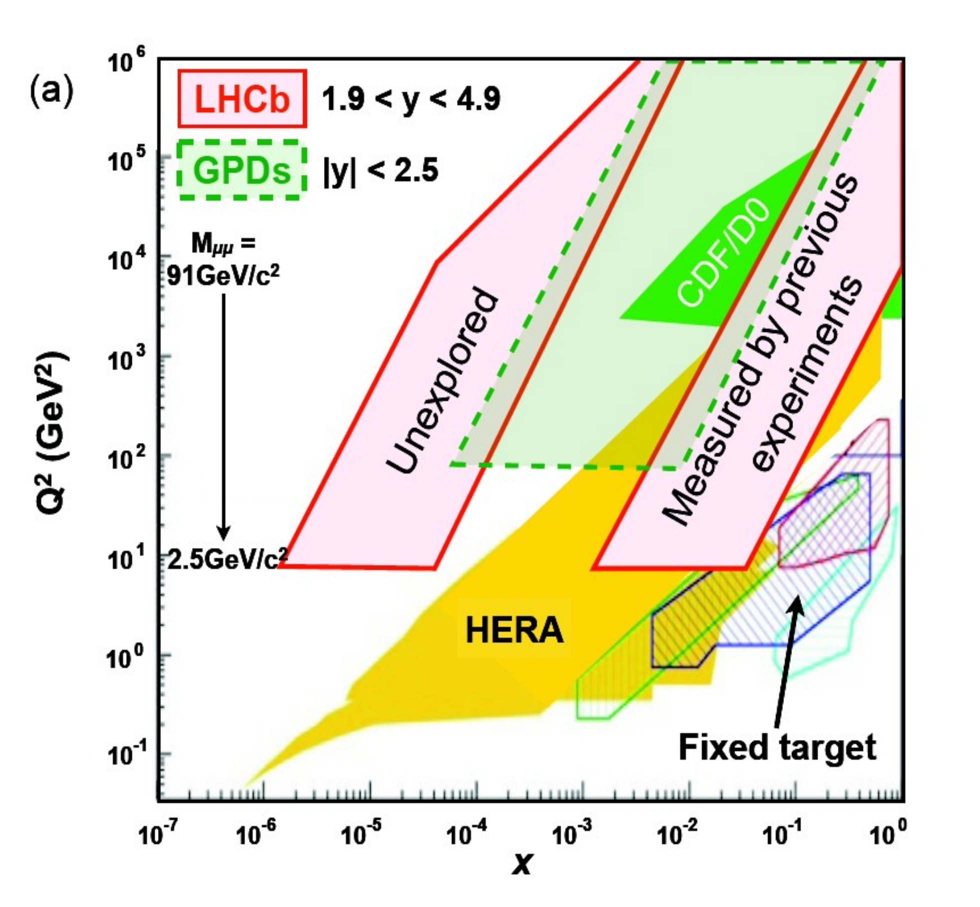
- Forward Drell-Yan: kinematics, observables
- Drell-Yan structure functions
- Lam-Tung relation in QCD
- Dipole picture of forward Drell-Yan
- Twist decomposition
- Results
- Conclusions

Work done with

D. Brzemiński, Mariusz Sadzikowski and Tomasz Stebel

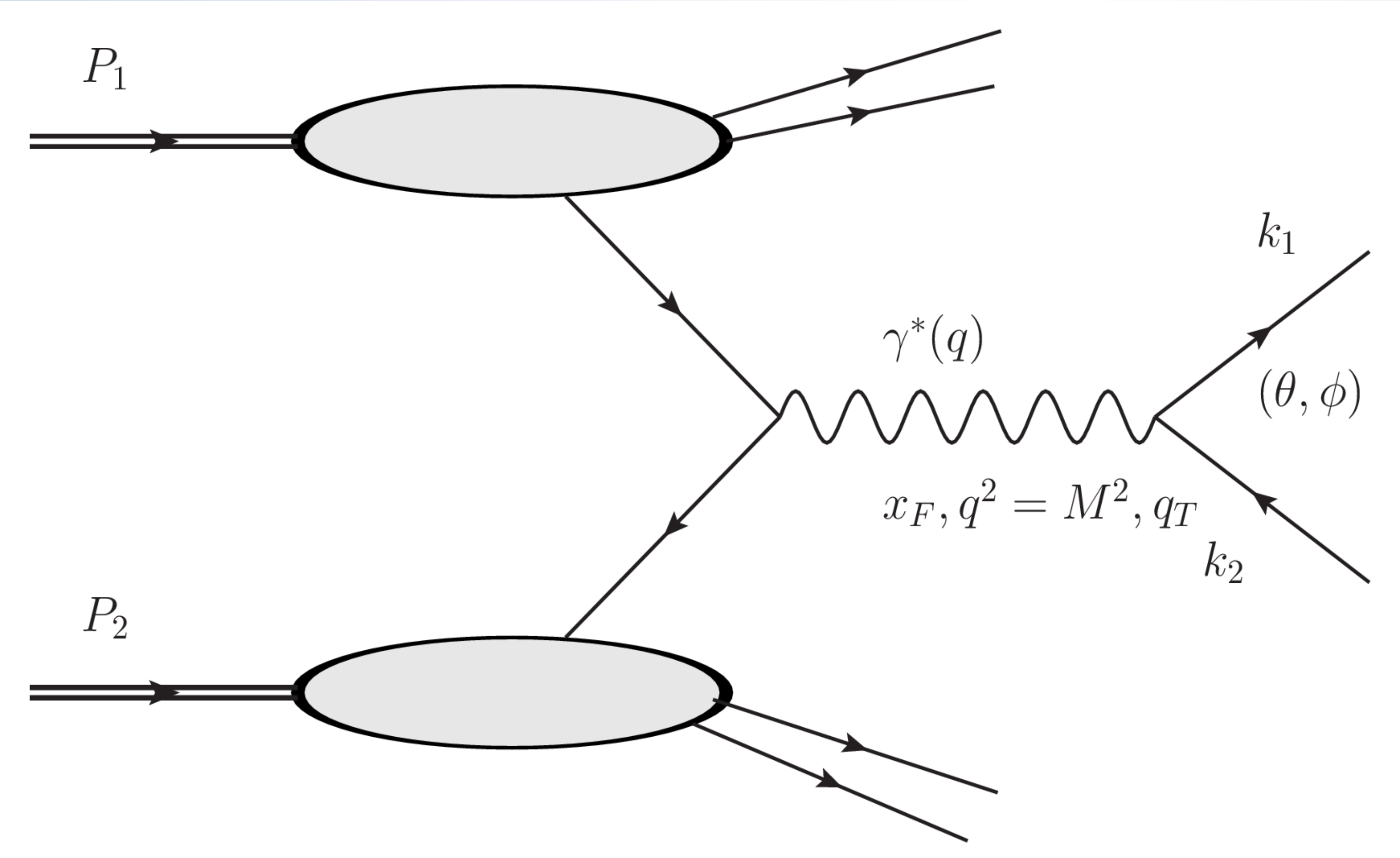
Forward Drell-Yan at LHC: kinematical reach and use

- Forward Drell-Yan may be used to measure parton densties down to $x < 10^{-6}$ at $M^2 \sim 10\ \mathrm{GeV}^2$
- Possible effects of multiple scattering and higher twists (small x enhancement of multiple gluon exchange): competition of $1/M^2$ and $x^{-\lambda}$ terms
- Needed to be controlled theoretically to avoid systematic errors of parton determination
- Potentially → measurement of higher twists.



Advantage: 4 independent structure functions

Drell-Yan kinematics



Drell-Yan structure functions:

- Lepton angular distributions: 4 Drell-Yan structure functions (W_a – frame dependent)

$$\frac{d\sigma}{dx_F dM^2 d\Omega d^2 q_T} = \frac{\alpha_{\rm em}^2}{2(2\pi)^4 M^4}\left[(1-\cos^2\theta)W_L + (1+\cos^2\theta)W_T + (\sin^2\theta\cos 2\phi)W_{TT} + (\sin 2\theta\cos\phi)W_{LT}\right]$$

- Invariant structure functions:

$$W^{\mu\nu} = -T_1\,\tilde{g}^{\mu\nu} + T_2\,\tilde{P}^\mu\tilde{P}^\nu - T_3\,\frac{1}{2}\left(\tilde{P}^\mu\tilde{p}^\nu + \tilde{p}^\mu\tilde{P}^\nu\right) + T_4\,\tilde{p}^\mu\tilde{p}^\nu$$

$$\tilde{g}^{\mu\nu} = g^{\mu\nu} - q^\mu q^\nu/q^2,\ \tilde{P}^\mu = \tilde{g}^{\mu\nu}P_\nu/\sqrt{S},\ \tilde{p}^\mu = \tilde{g}^{\mu\nu}p_\nu/\sqrt{S}$$

$$P = P_1 + P_2,\ p = P_1 - P_2$$

Lam-Tung relation

- DY helicity structure functions: projections of DY amplitudes on virtual photon polarization states

- Lam-Tung relation (1980, 1982): vanishing combination of DY structure functions at leading twist up to NNLO: $W_L - 2W_{TT} = 0$

- At twist 4 – non-zero contribution → enhanced higher twist contributions

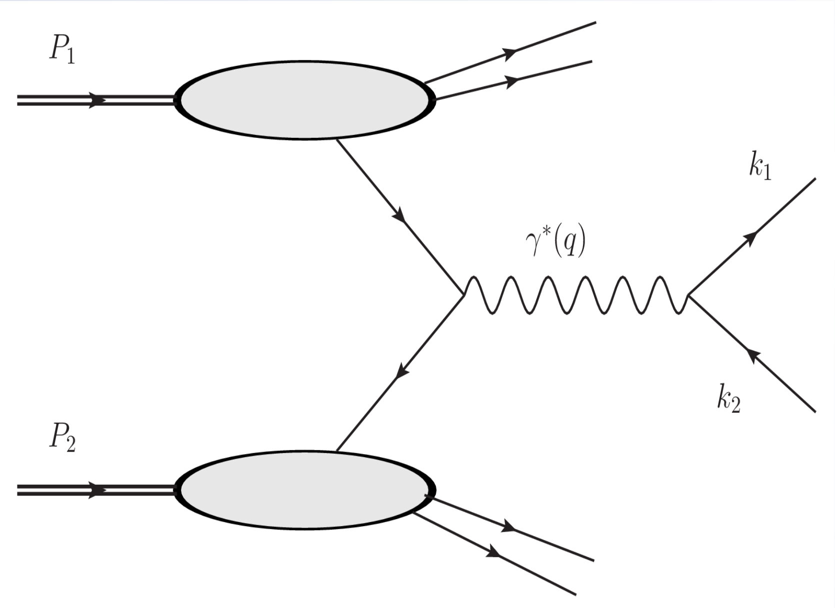
- Also interesting: Lam-Tung relation breaking by higher order QCD effects (not covered in this talk)

Motivation to study higher twist effects:

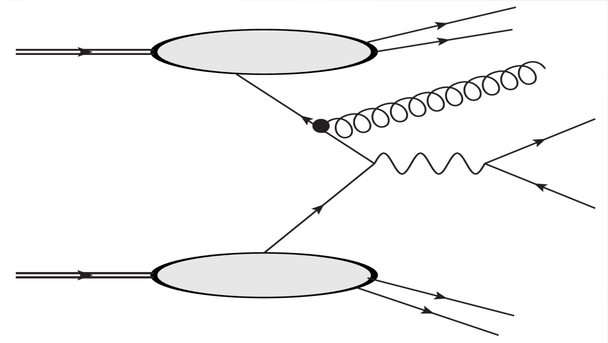
- Fundamental: access new information about the proton structure
- Theoretical: understanding QCD at small x: multiple scattering, higher twist evolution, higher orders in QCD
- Pragmatic: possible significant corrections to precise parton determination, dependent on x and Q^2
- Phenomenological: understanding the data (see ATLAS talk on Thursday)
- In general: opportunity for small x physics: at the LHC region of very small $x \sim 10^{-6}$ may be probed for perturbative scales $\sim 10\ GeV^2$

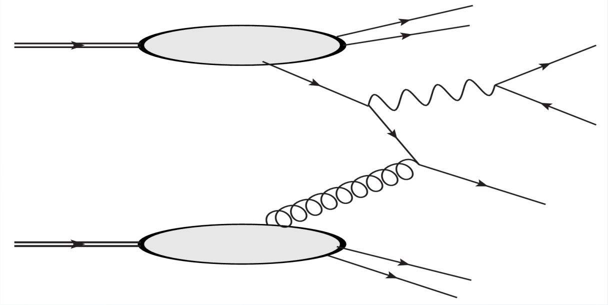
Leading diagrams of Drell-Yan

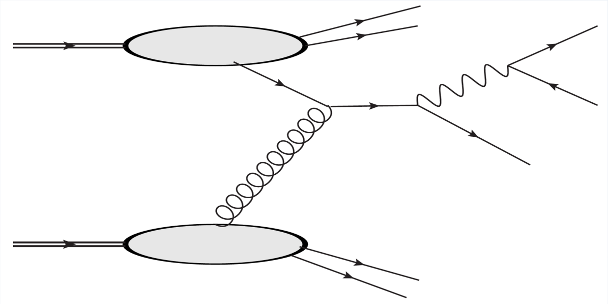
- Leading Order



- NLO

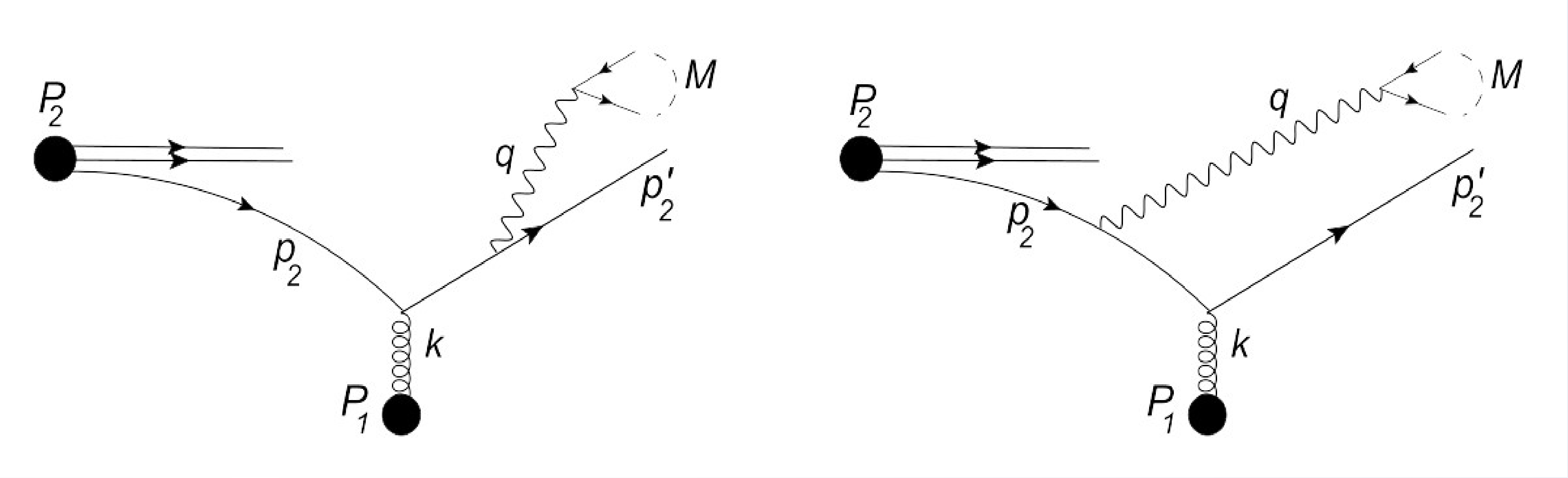






Leading diagrams of forward Drell-Yan

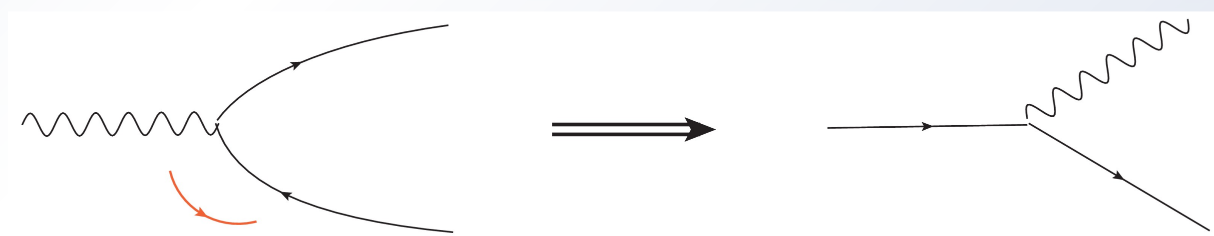
- Asymmetric kinematics: x_2 >> x1
- Dominance of the quark see → driven by gluon evolution
- Good approximation: gluon evolution followed by splitting to quark (anti-quark) in the last step



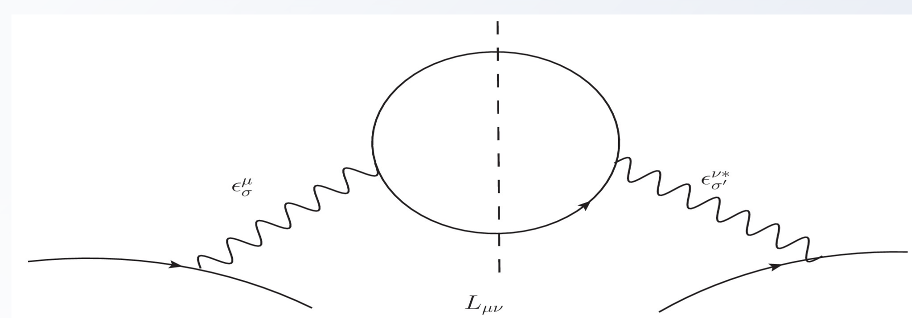
Forward Drell-Yan in dipole formulation

- Large energy limit: conservation of transverse positions in scattering
- "Effective color dipole" emerges from interference of photon emission before and after scattering, γ^* carries fraction z of p^+ of incident quark

- "Crossed" photon wave function:



- Interference of photon helicity states through leptonic tensor



Forward Drell-Yan in dipole formulation

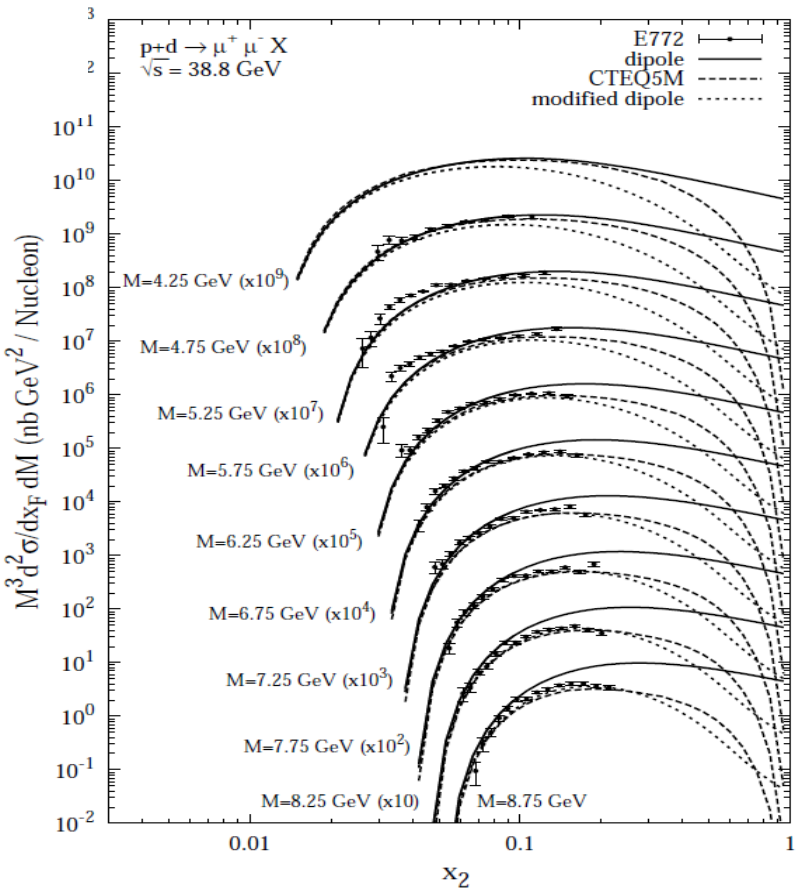
$$\sigma^f_{T,L}(qp \to \gamma^* X) = \int d^2r \, W^f_{T,L}(z, r, M^2, m_f) \, \sigma_{q\bar{q}}(x_2, zr)$$

$$W_T^f = \frac{\alpha_{em}}{\pi^2} \left\{ [1 + (1-z)^2] \, \eta^2 K_1^2(\eta r) + m_f^2 \, z^4 K_0^2(\eta r) \right\}$$

$$W_L^f = \frac{2\alpha_{em}}{\pi^2} M^2 (1-z)^2 K_0^2(\eta r) \, ,$$

Formalism proposed and developed by:

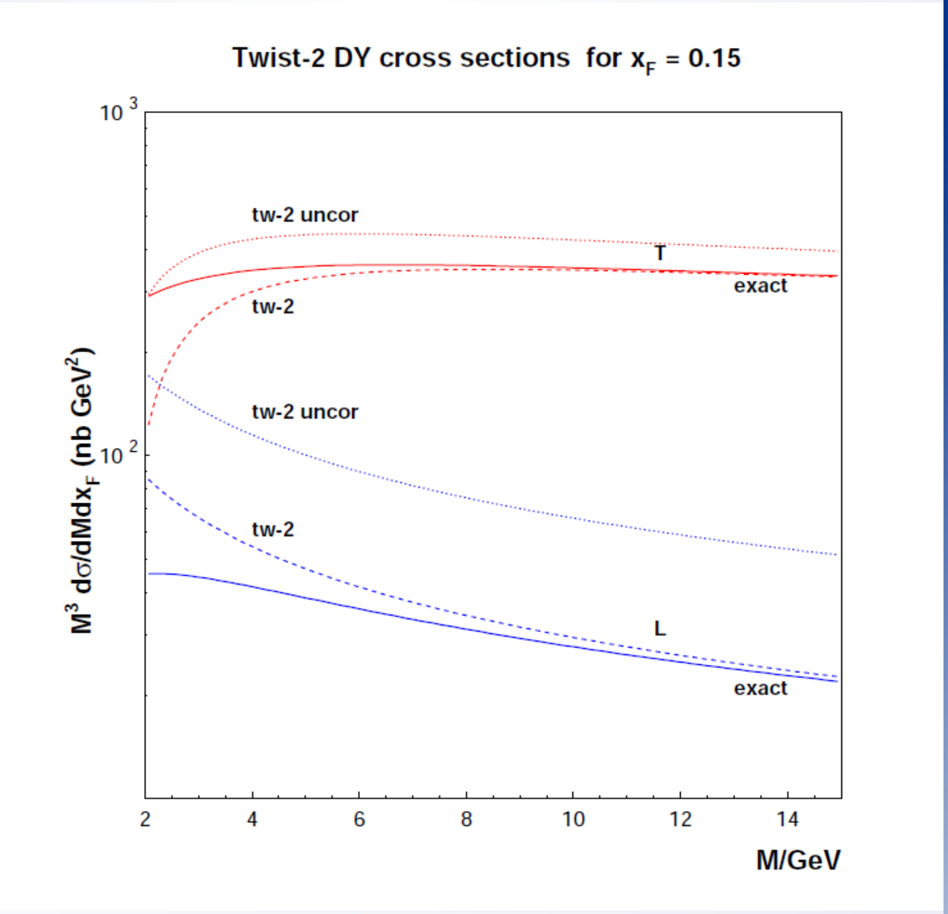
- Brodsky, Hebecker, Quack (1997)
- B. Z. Kopeliovich, J. Raufeisen, A. V. Tarasov (2001)
- Gelis, Jalilian-Marian (2002)
- Raufeisen, Peng, Nayak (2002): plot →
- V. Goncalves et al, A. Szczurek et al.



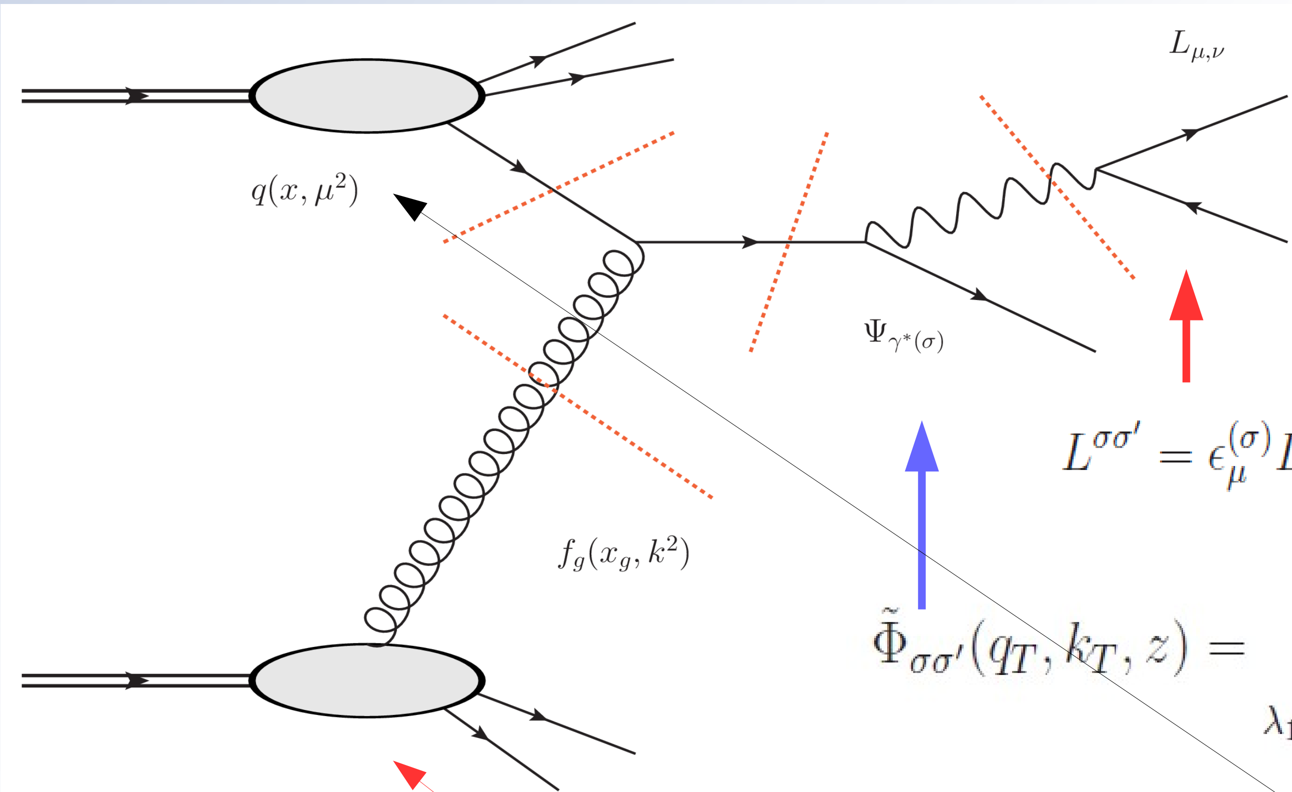
Inclusive forward Drell-Yan at LHC: higher twist corrections

- Golec-Biernat, Lewandowska, Staśto, 2010 (plot): first analysis of twist content of forward Drell-Yan within the GBW saturation model for dipole cross-section, using the technique of Bartels, Golec-Biernat and Peters done for the inclusive cross-section (in qT and the lepton azimuthal angle)

- Predictions for the LHC (plot) large higher twist corrections within kinematical range of LHC (LHCb)



Forward Drell-Yan cross-section in kT factorisation



$$L^{\sigma\sigma'} = \epsilon_\mu^{(\sigma)} L^{\mu\nu}\, \epsilon_\nu^{(\sigma')\dagger}, \quad L^{\mu\nu} = -g^{\mu\nu} + \frac{\kappa^\mu \kappa^\nu}{\kappa^2}$$

$$\tilde{\Phi}_{\sigma\sigma'}(q_T, k_T, z) = \sum_{\lambda_1,\lambda_2=+,-} A^{(\sigma)}_{\lambda_1,\lambda_2}(\vec{q}_T)^\dagger A^{(\sigma')}_{\lambda_1,\lambda_2}(\vec{q}_T)$$

$$\frac{d\sigma}{dx_F dM^2 d\Omega d^2 q_T} = \frac{\alpha_{\rm em}}{(2\pi)^2 (P_1\cdot P_2)^2\, M^2\, x_F^2 (1-z)} L^{\sigma\sigma'}(\Omega) \int_{x_F}^1 dz\, \wp(x_F/z)$$
$$\times \int d^2 k_T \frac{2\pi\alpha_s}{3} \frac{f(x_g, k_T^2)}{k_T^4} \tilde{\Phi}_{\sigma\sigma'}(q_T, k_T, z)$$

Mellin representation of forward Drell-Yan structure functions:

- Standard procedure: r-space → Mellin moments space

$$W_i = \int_{x_F}^{1} dz\, \wp(x_F/z) \int_{\mathcal{C}} \frac{ds}{2\pi i}\, \tilde{\sigma}(-s) \left(\frac{z^2 Q_0^2}{\eta_z^2} \right)^s \hat{\Phi}_i(q_T, s, z)$$

$$\eta_z^2 = M^2(1-z)$$

$$\hat{\Phi}_i(q_T, s, z) = \frac{2(2\pi)^4 M^4}{\alpha_{\rm em}^2} \int d^2r \left(\frac{\eta_z^2}{4z^2}\, r \right)^s \Phi_i(q_T, r, z)$$

$$\tilde{\sigma}(-s) = \int_0^\infty \frac{d\rho^2}{\rho^2} (\rho^2)^{-s}\ \hat{\sigma}(\vec{\rho})$$

Two descriptions of dipole cross-section:

- Phenomenological: eikonal GBW model: twist 2n contribution enhanced by $x^{-n\lambda}$ at small x

- BFKL description (new): based on twist decomposition of LL BFKL cross-section:
- Higher twist effects extracted from singularities of the BFKL kernel in Mellin space, $\chi(\gamma)$, at integer values of `anomalous dimensions' γ
- $\sigma(\gamma) \sim \exp(c \log(1/x)\, \alpha_s\, \chi(\gamma))$ $\rightarrow$ essential singularities of Mellin cross-section
- Saddle point treatment of BFKL amplitudes $\rightarrow$ power suppression x^{δ} of higher twist terms in LL BFKL amplitudes

Previous findins: Mellin representation of DY impact factors

- Mellin transforms of impact factors for all DY structure functions found, e.g.:

$$\hat{\Phi}_L(q_T, s, z) = \frac{2}{z^2}\left\{\frac{2\Gamma^2(s+1)}{1+q_T^2/\eta_z^2}\ {}_2F_1\left(s+1, s+1, 1, -\frac{q_T^2}{\eta_z^2}\right) - \Gamma(s+1)\Gamma(s+2)\ {}_2F_1\left(s+1, s+2, 1, -\frac{q_T^2}{\eta_z^2}\right)\right\}$$

$$\hat{\Phi}_{TT}(q_T, s, z) = \frac{1}{2z^2}\left\{\frac{2\pi}{\Gamma(1-s)\sin\pi s\ q_T^2/\eta_z^2}\left(1+\frac{q_T^2}{\eta_z^2}\right)^{-s-3}\Gamma(s+2)\right.$$
$$\left[\left(1+\frac{q_T^2}{\eta_z^2}\right)\left(1+\frac{q_T^2}{\eta_z^2}(s+2)\right)\ {}_2F_1\left(-s+1, s+1, 1, \frac{q_T^2}{q_T^2+\eta_z^2}\right)\right.$$
$$\left. - \left(1+2\frac{q_T^2}{\eta_z^2}(s+1)\right)\ {}_2F_1\left(-s+1, s+2, 1, \frac{q_T^2}{q_T^2+\eta_z^2}\right)\right]$$
$$\left. - \frac{4q_T^2/\eta_z^2}{1+q_T^2/\eta_z^2}\Gamma(s+1)\Gamma(s+2)\ {}_2F_1\left(s+1, s+2, 2, -\frac{q_T^2}{\eta_z^2}\right)\right\}$$

- Necessary for twist analysis, but useful also in BFKL approach

Previous findings: twist decomposition of helicity structure functions in GBW

- Twist 2

$$W_L^{(2)} = \sigma_0 \frac{Q_0^2}{M^2} \int_{x_F}^1 dz\ \wp(x_F/z) \frac{4M^6\ q_T^2(1-z)^2}{\left[q_T^2 + M^2(1-z)\right]^4}$$

$$W_T^{(2)} = \sigma_0 \frac{Q_0^2}{M^2} \int_{x_F}^1 dz\ \wp(x_F/z)\left[1+(1-z)^2\right] \frac{M^4\left[q_T^4 + M^4(1-z)^2\right]}{2\left[q_T^2 + M^2(1-z)\right]^4}$$

$$W_{TT}^{(2)} = \sigma_0 \frac{Q_0^2}{M^2} \int_{x_F}^1 dz\ \wp(x_F/z) \frac{2M^6\ q_T^2(1-z)^2}{\left[q_T^2 + M^2(1-z)\right]^4}$$

$$W_{LT}^{(2)} = \sigma_0 \frac{Q_0^2}{M^2} \int_{x_F}^1 dz\ \wp(x_F/z)(2-z)\ \frac{M^5\ q_T\left[-q_T^2 + M^2(1-z)\right](1-z)}{\left[q_T^2 + M^2(1-z)\right]^4}$$

Twist decomposition of helicity structure functions I GBW (2)

- Twist 4:

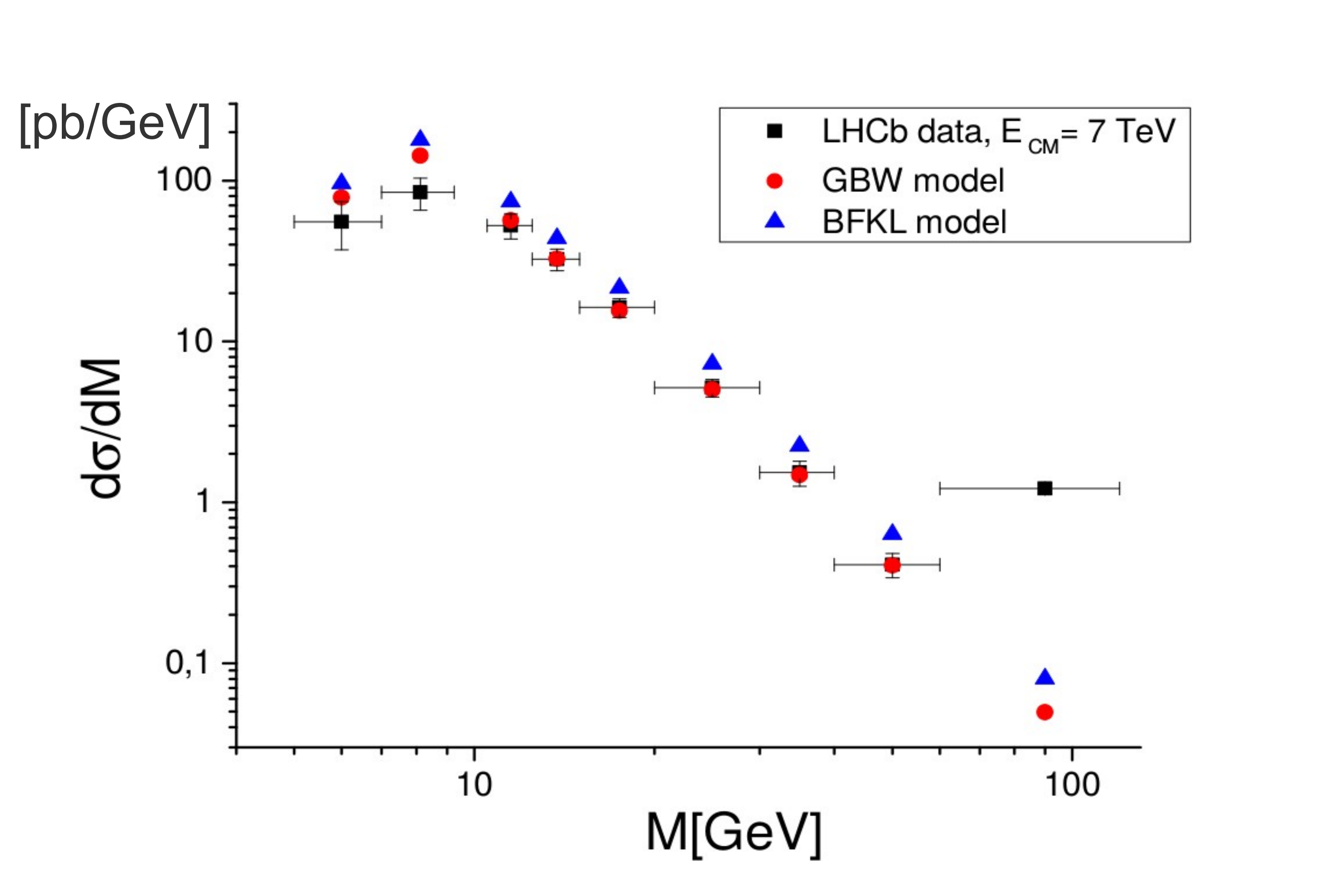
$$W_L^{(4)} = \sigma_0 \frac{Q_0^4}{M^4} \int_{x_F}^1 dz\ \wp(x_F/z) z^2 \times \times \frac{4M^8 \left[7q_T^2 - 10M^2 q_T^2 (1-z) + M^4(1-z)^2\right](1-z)^2}{\left[q_T^2 + M^2(1-z)\right]^6}$$

$$W_T^{(4)} = \sigma_0 \frac{Q_0^4}{M^4} \int_{x_F}^1 dz\ \wp(x_F/z) \left[1 + (1-z)^2\right] z^2 \times \times \frac{M^6 \left[q_T^2 - 2M^2(1-z)\right]\left[q_T^4 - 4M^2 q_T^2(1-z) + M^4(1-z)^2\right]}{\left[q_T^2 + M^2(1-z)\right]^6}$$

$$W_{TT}^{(4)} = \sigma_0 \frac{Q_0^4}{M^4} \int_{x_F}^1 dz\ \wp(x_F/z) z^2 \frac{12M^8 q_T^2 \left[q_T^2 - 2M^2(1-z)\right](1-z)^2}{\left[q_T^2 + M^2(1-z)\right]^6}$$

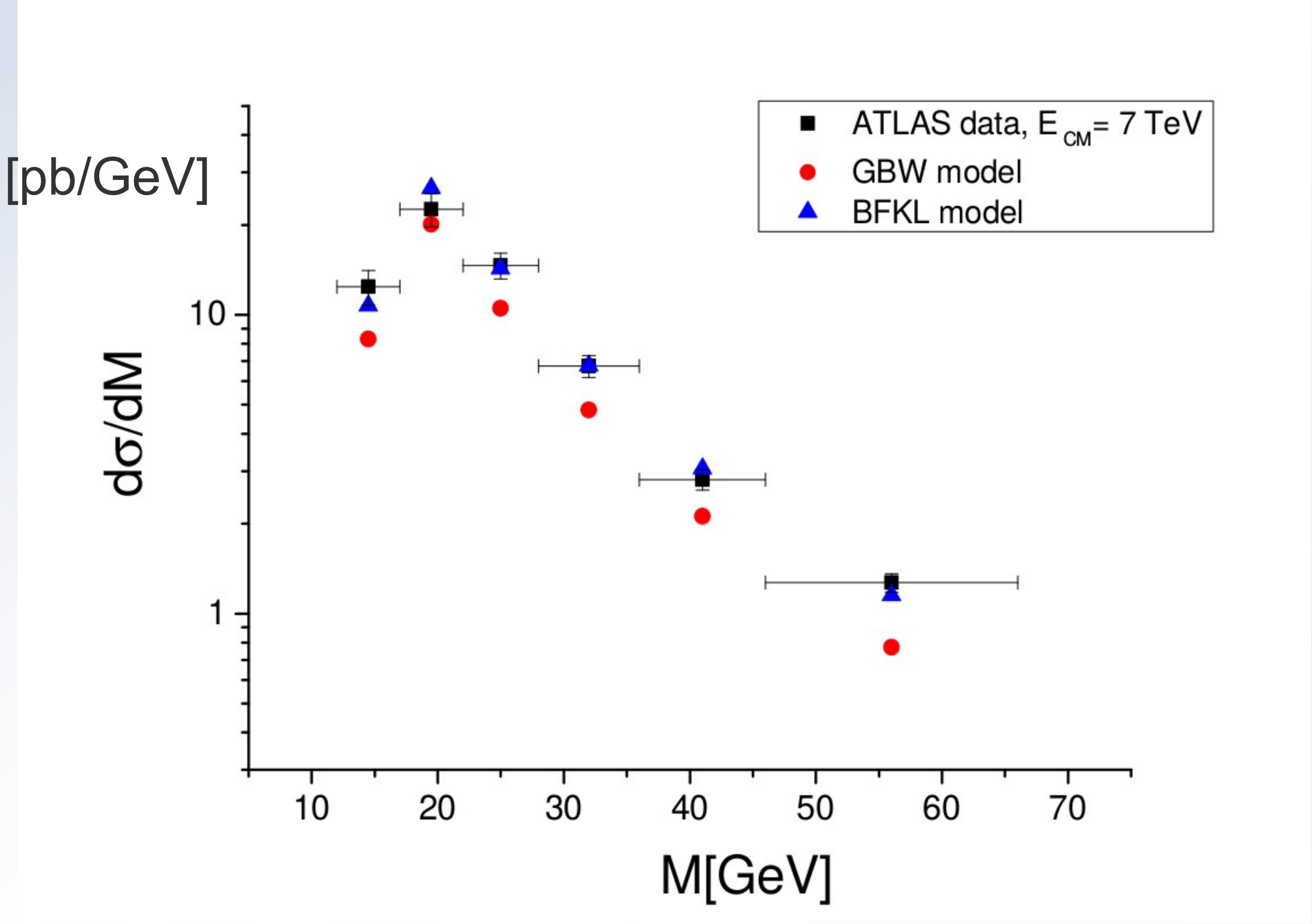
$$W_{LT}^{(4)} = \sigma_0 \frac{Q_0^4}{M^4} \int_{x_F}^1 dz\ \wp(x_F/z)(2-z)\ z^2 \times \times \frac{2M^7\ q_T \left[-2q_T^2 + M^2(1-z)\right]\left[q_T^2 - 5M^2(1-z)\right](1-z)}{\left[q_T^2 + M^2(1-z)\right]^6}$$

Results: inclusive Drell-Yan at the LHC: LHCb data



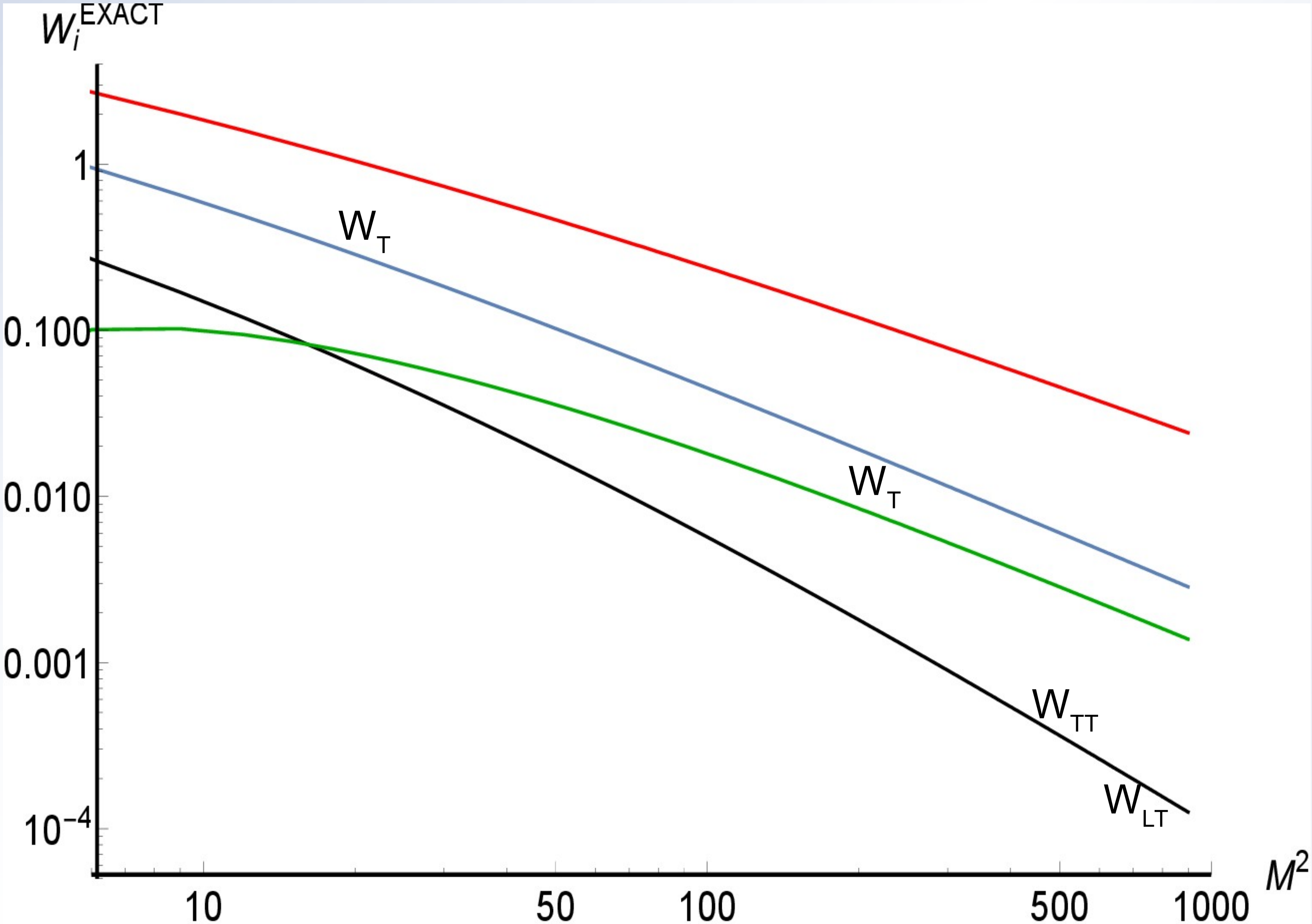
- Good description in terms of GBW, BFKL somewhat above data

Results: inclusive Drell-Yan at the LHC: ATLAS data



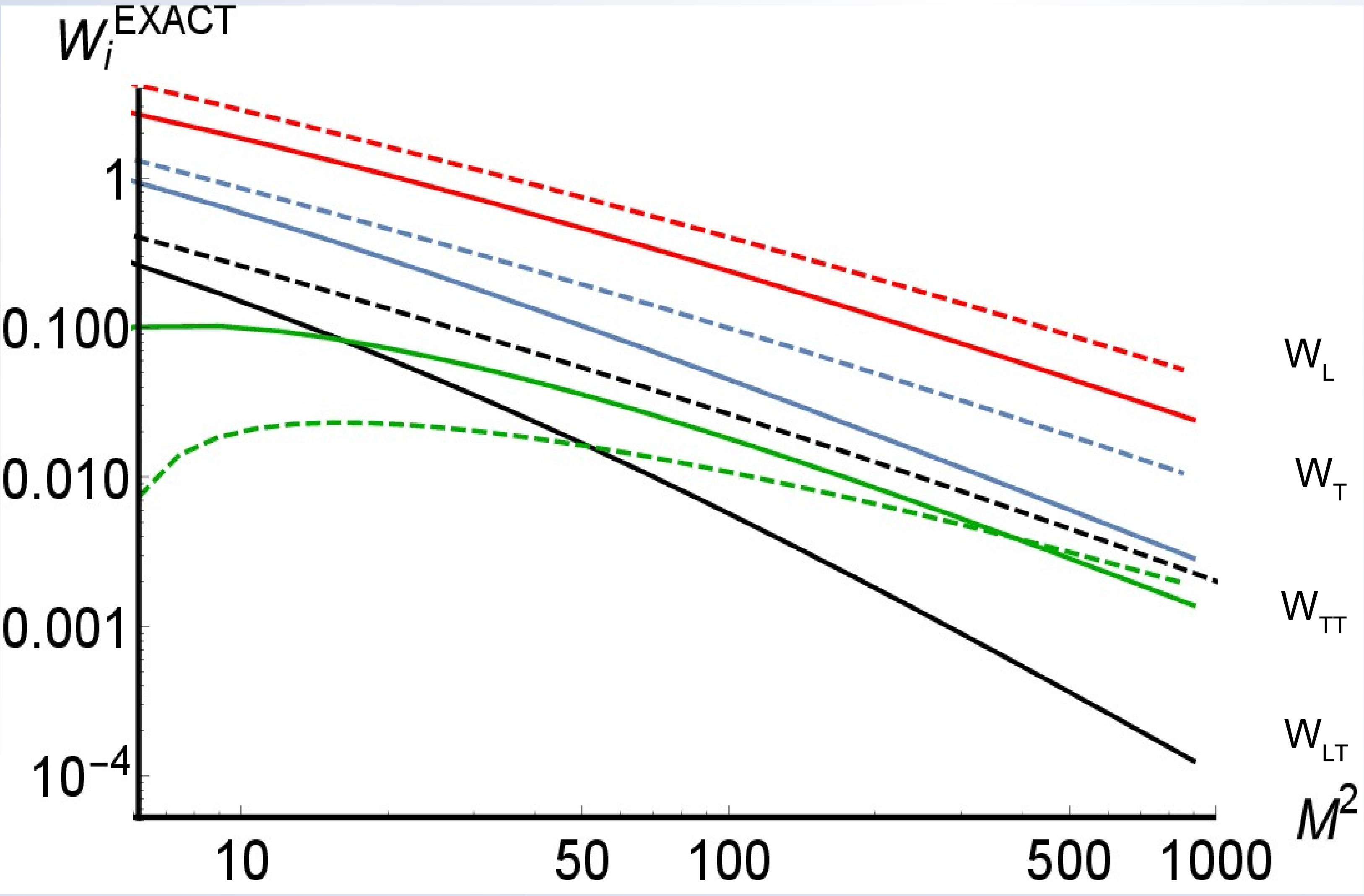
- Good description in terms of BFKL, GBW somewhat below data
- However: central kinematics → expected terms beyond forward DY approximation

DY at LHC: structure functions from GBW



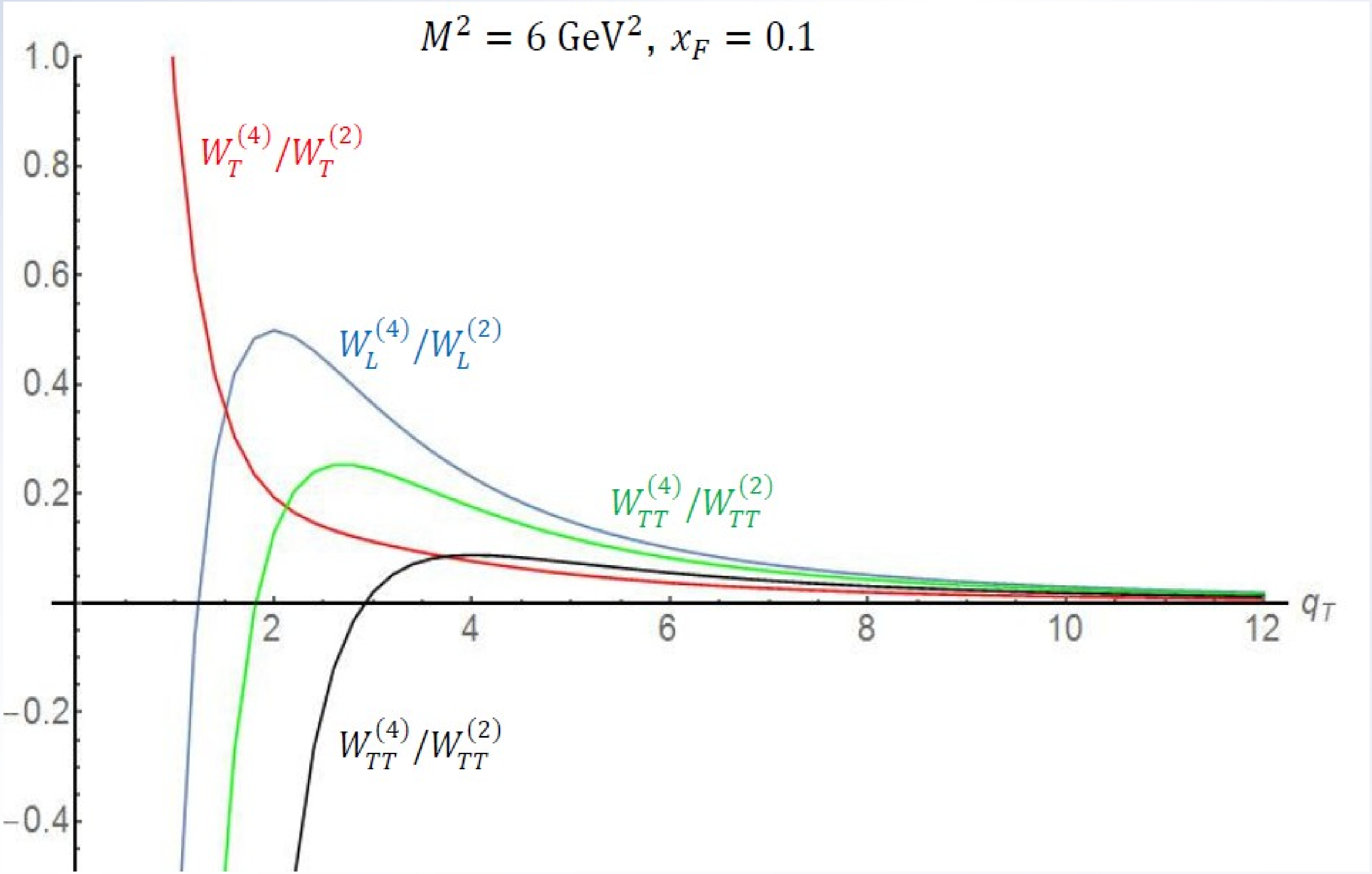
DY at LHC: structure functions from BFKL vs GBW

- Similar behavior of W_T and W_L ,significant differences in “inteference” structure functions



Twist-4 correction to DY structure functions from GBW

- Steep decrease of twist-4 correction with q_T



Lam-Tung relation in dipole model

- Lam-Tung relation: at leading twist up to NNLO:

$$W_L - 2W_{TT} = 0$$

- Holds in the dipole model at twist 2
- At twist 4 – non-zero contribution → enhanced higher twist contributions

$$W_L^{(4)} - 2W_{TT}^{(4)} = \sigma_0 \frac{Q_0^4}{M^4} \int_{x_F}^1 dz\ \wp(x_F/z) z^2 \frac{4M^8(1-z)^2}{[q_T^2 + M^2(1-z)]^4}$$

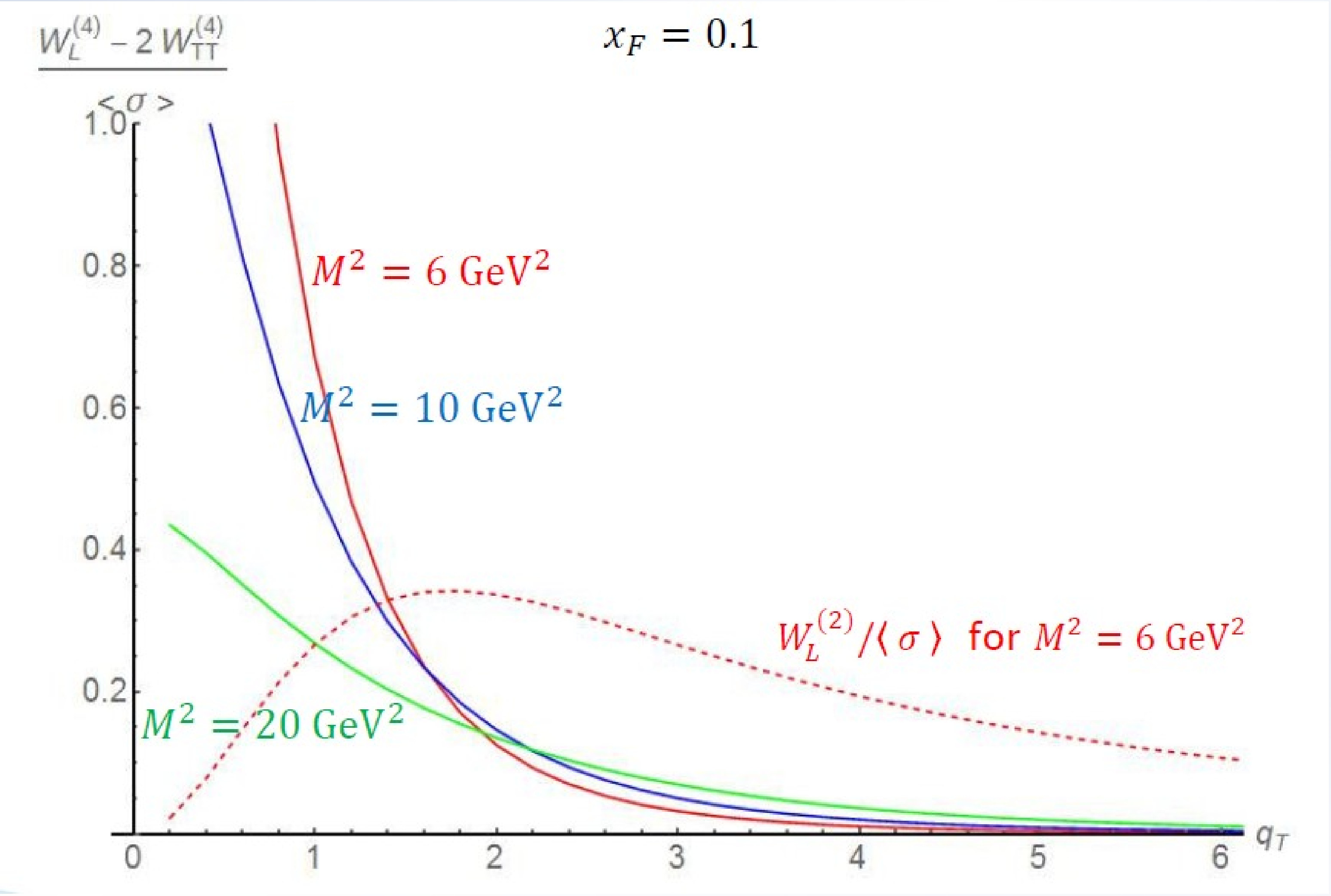
- qT-integrated cross-section also shows breaking of Lam-Tung relation

$$\int \left(W_L^{(4)} - 2W_{TT}^{(4)}\right) d^2q_T = 2\pi\sigma_0 M^2 \left(\tilde{W}_L^{(4)} - 2\tilde{W}_{TT}^{(4)}\right)$$
$$= 2\pi\sigma_0 \frac{Q_0^4}{M^2} \left\{ \frac{1}{18}\wp(x_F)\left[-19 + 12\gamma_E + 12\ln\left(\frac{M^2(1-x_F)}{Q_0^2}\right)\right] + \right.$$
$$\left. + \frac{2}{3}\int_{x_F}^1 dz\ \frac{\wp(x_F/z)z^2 - \wp(x_F)}{1-z} \right\}$$

Lam-Tung relation from GBW

$\sqrt{s} = 14$ TeV

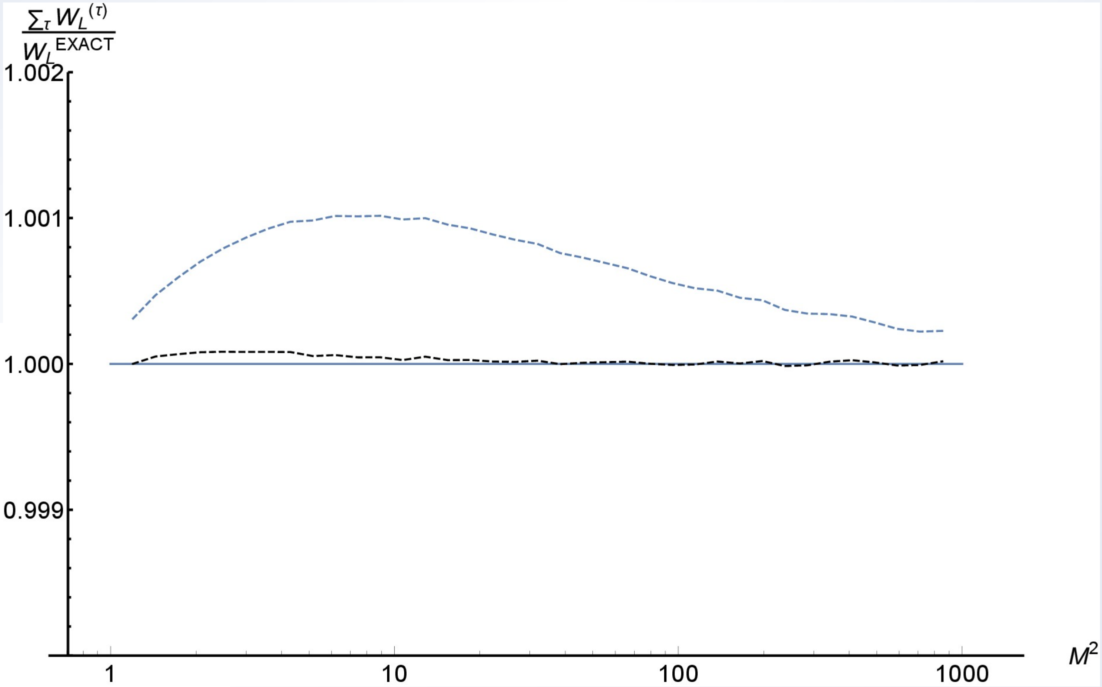
- Significant effects for $M^2 > 10$ GeV^2

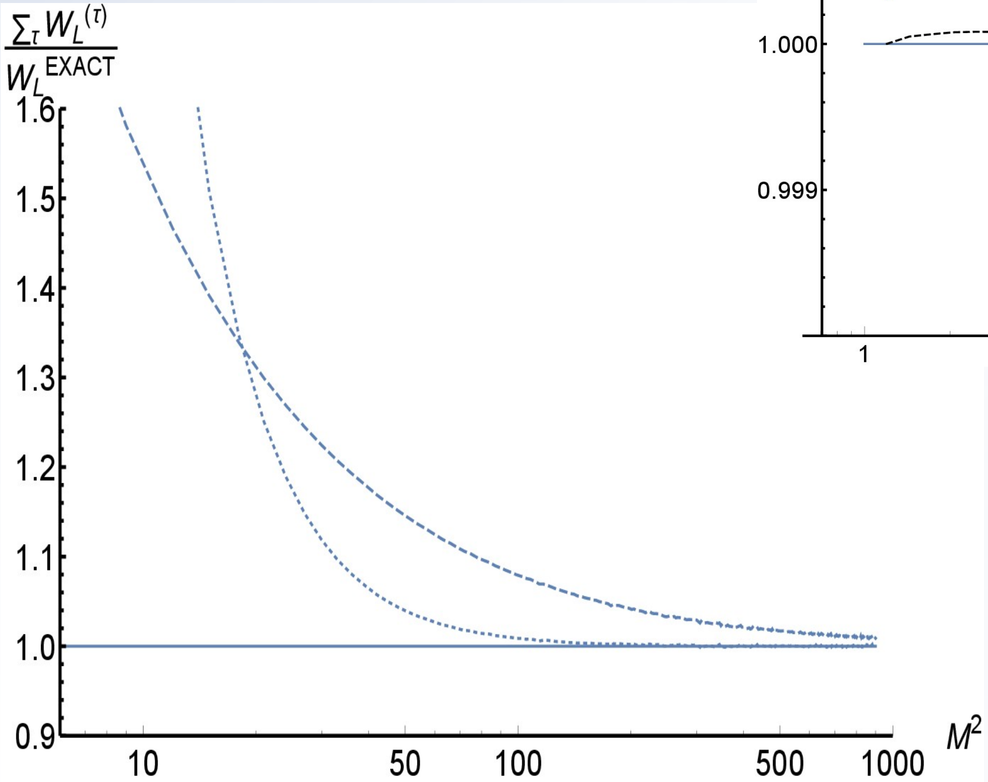


GBW vs BFKL:

$\sqrt{s} = 14\text{ TeV}$

- Higher twists in W_L : ratio of twist 2,4 to all twists

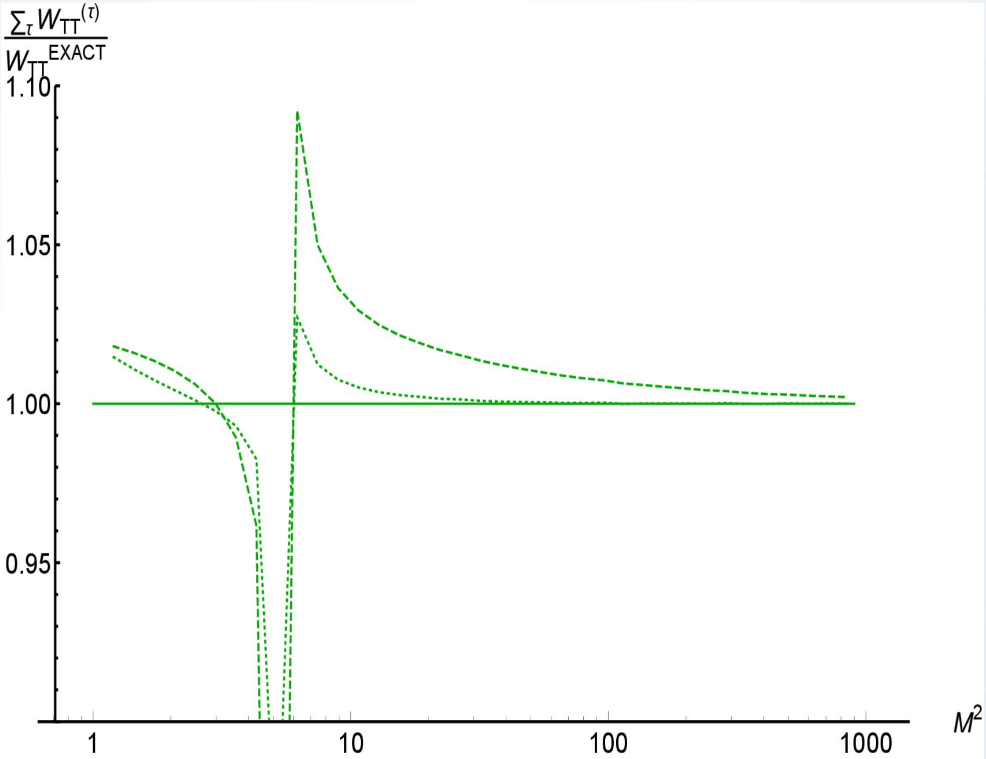


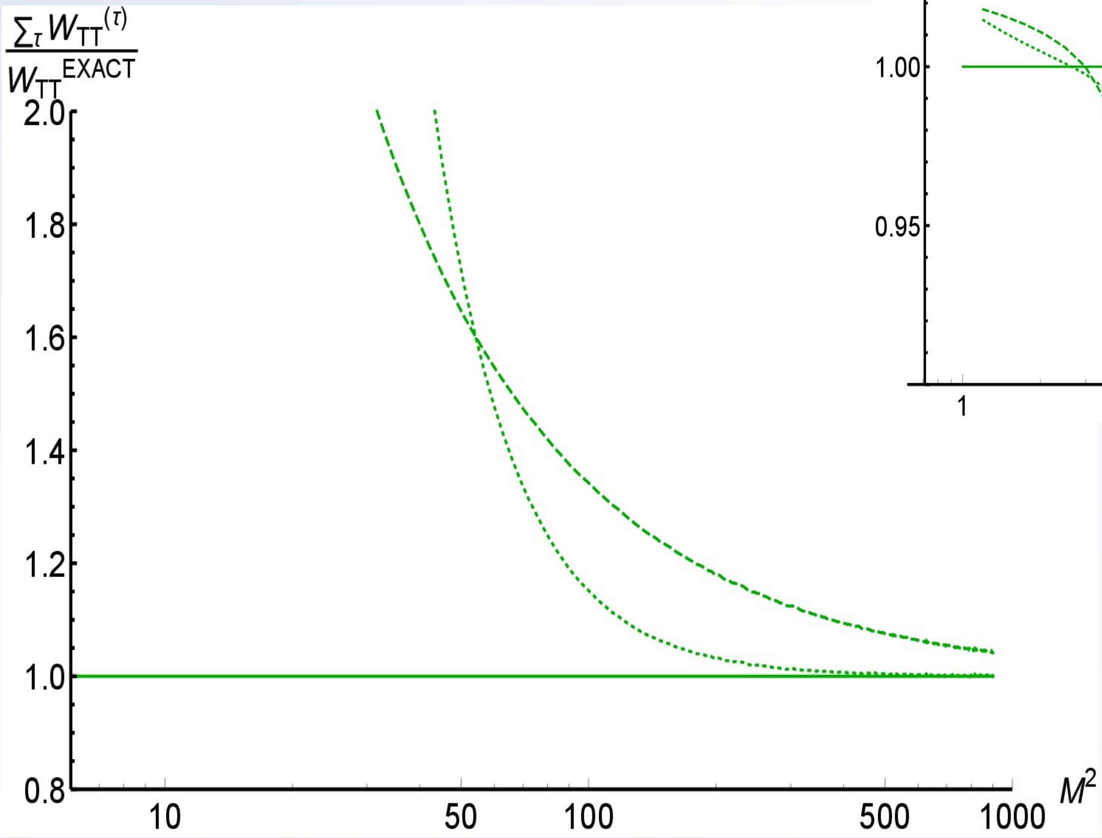


GBW vs BFKL:

$\sqrt{s} = 14\ \text{TeV}$

- Higher twists in W_{TT} : ratio of twist 2,4 to all twists





Conclusions

- We applied Mellin representation of forward Drell-Yan impact factors to to analysis of forward Drell-Yan structure functions
- Models were tested against LHC data: good description of angular averaged cross-sections was found
- Assuming saturation model / BFKL picture - explicit form was found of twist expansion of forward DY structure functions: differential and integrated
- Lam-Tung relation preserved at twist-2, broken beyond → Lam-Tung combination of DY structure functions may be used to measure higher twist terms
- Essentially different predictions for higher twists from GBW and BFKL
- Ongoing investigation of BFKL effects → stay tuned