Exact Solutions for an Expanding and Rotating, Rehadronizing Fireball

- with lattice QCD Equation of State -

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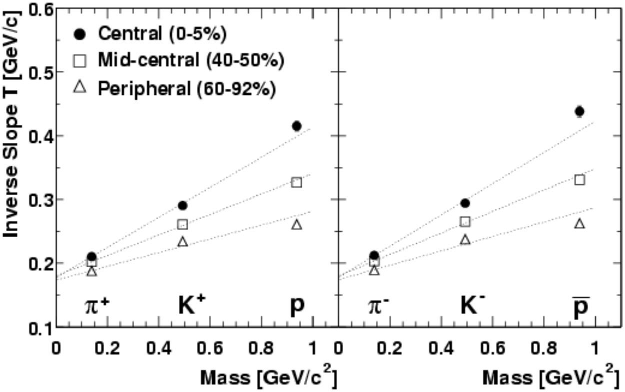
I. Introduction

Motivation

- Deeper understanding of rehadronization
- More accurate description of the fireball evolution
- Previous analytic solutions are single-component
- A multi-component scenario is realistic
- First create a simplified, non-relativistic model
- Relativistic generalizations on progress

I. Introduction



$$T = T_f + m\langle u_t\rangle^2 \Longrightarrow T_i = T_f + m_i\langle u_t\rangle^2 \quad (1)$$

II. Basic equations

Basic equations of non-relativistic hydrodynamics

$$\frac{\partial n}{\partial t} + \nabla \left(n\mathbf{v} \right) = 0, \tag{2}$$

$$\frac{\partial \varepsilon}{\partial t} + \nabla \left(\varepsilon\mathbf{v} \right) = -p\nabla\mathbf{v}, \tag{3}$$

$$mn \left(\frac{\partial}{\partial t} + \mathbf{v}\nabla \right) \mathbf{v} = -\nabla p. \tag{4}$$

Basic equations of relativistic hydrodynamics

$$\partial_\mu \left(nu^\mu \right) = 0, \tag{5}$$

$$\partial_\nu T^{\mu\nu} = 0. \tag{6}$$

II. Basic equations

Before the rehadronization

There's no particle conservation!

$$\varepsilon = T\sigma - p + \mu n \Longrightarrow \varepsilon = T\sigma - p + \sum_i \mu_i n_i, \tag{7}$$

Since

$$\mu_i = 0, \tag{8}$$

$$\varepsilon + p = T\sigma, \tag{9}$$

$$d\varepsilon = T d\sigma, \tag{10}$$

from the energy conservation:

$$\frac{\partial \sigma}{\partial t} + \nabla(\mathbf{v}\sigma) = 0. \tag{11}$$

II. Basic equations

Euler equation ($\mu = 0$, $n \to 0$, $v \ll c = 1$) [3]:

$$(\varepsilon + p)\,(\partial_t + \mathbf{v}\nabla)\,\mathbf{v} = T\sigma\,(\partial_t + \mathbf{v}\nabla)\,\mathbf{v} = -\nabla p. \qquad (12)$$

Equation of state (from lattice QCD [2]):

$$\varepsilon = \kappa_{QCD}(T)p. \qquad (13)$$

Energy conservation $\Longrightarrow$ diff. eq. of temperature [3]:

$$\frac{1+\kappa}{T}\left[\frac{d}{dT}\frac{\kappa T}{1+\kappa}\right](\partial_t + \mathbf{v}\nabla)\,T + \nabla\mathbf{v} = 0. \qquad (14)$$

II. Basic equations

After the rehadronization

It was known for one component ($T \ll m \Longrightarrow \mu \approx m$):

$$\varepsilon + p = \mu n + T\sigma \approx mn. \tag{15}$$

For the multi-component scenario:

$$mn = \sum_i m_i n_i, \tag{16}$$

$$p = \sum_i p_i. \tag{17}$$

But $p \ll mn$ thus:

$$\varepsilon \approx \sum_i m_i n_i \tag{18}$$

II. Basic equations

Euler equation:

$$\sum_i m_i n_i \left(\partial_t + \mathbf{v}\nabla\right)\mathbf{v} = -\sum_i \nabla p_i . \qquad (19)$$

Equation of state:

$$\varepsilon = \kappa_{HRG}(T)p, \qquad (20)$$

$$\lim_{T\to T_f} \kappa_{HRG}(T) = 3/2 \qquad (21)$$

Diff. equation of temperature [3]:

$$\left[\frac{d}{dT}\kappa T\right]\left(\partial_t + \mathbf{v}\nabla\right)T + T\nabla\mathbf{v} = 0. \qquad (22)$$

III. Equation of state

3 classes of solutions:

- $T(t, \mathbf{r}) = T(t)$
- $\kappa = const.$
- $\frac{d}{dT}(\kappa T) = const.$ (new!)

Differential equations for κ:
For sQGP phase:

$$\frac{d}{dT}\left[\frac{T\kappa(T)}{1+\kappa(T)}\right] = \frac{\kappa_Q}{1+\kappa(T)} \tag{23}$$

For hadron gas phase:

$$\frac{d}{dT}\left[T\kappa(T)\right] = \frac{\kappa_c T_c - \kappa_f T_f}{T_c - T_f} \tag{24}$$

III. Equation of state

Solutions:
For sQGP phase:

$$\kappa_{QM}(T) = \frac{\kappa_Q \left(\frac{T}{T_c}\right)^{1+\kappa_Q} + \frac{\kappa_c - \kappa_Q}{\kappa_c + 1}}{\left(\frac{T}{T_c}\right)^{1+\kappa_Q} - \frac{\kappa_c - \kappa_Q}{\kappa_c + 1}}, \tag{25}$$

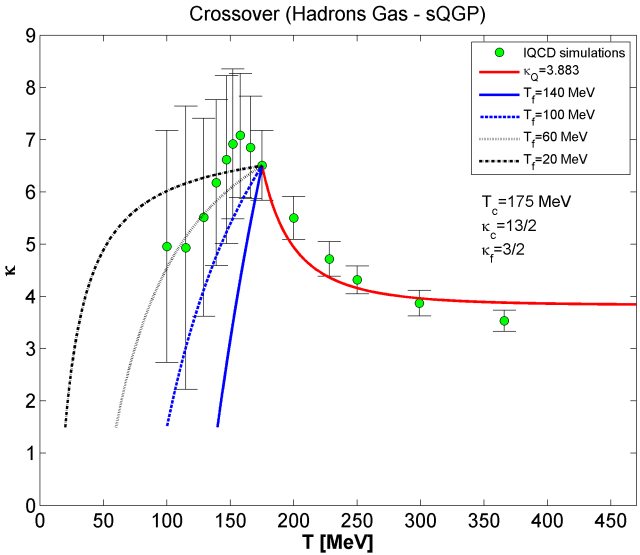
For hadron gas phase:

$$\kappa_{HM}(T) = \frac{\kappa_c T_c - \kappa_f T_f}{T_c - T_f} - \frac{\kappa_c - \kappa_f}{T_c - T_f}\frac{T_c T_f}{T}, \tag{26}$$

where

$$\kappa_f = 3/2. \tag{27}$$

III. Equation of state



III. Equation of state

Curves	χ^2/NDF	CL [%]
$\kappa_Q = 3.833$	6.48/4	16.6
$T_f = 140$ MeV	86.56/6	$1.6 \cdot 10^{-14}$
$T_f = 100$ MeV	7.71/6	26.0
$T_f = 60$ MeV	1.35/6	96.9
$T_f = 20$ MeV	1.22/6	97.6

IV. Crossover

sQGP	Multi-component hadron gas
$\frac{\partial \sigma}{\partial t} + \nabla (\mathbf{v}\sigma) = 0$	$\frac{\partial n_i}{\partial t} + \nabla (\mathbf{v} n_i) = 0, \quad \forall i$
$T\sigma (\partial_t + \mathbf{v}\nabla)\mathbf{v} = -\nabla p$	$\sum_i m_i n_i (\partial_t + \mathbf{v}\nabla)\mathbf{v} = -\sum_i \nabla p_i$
$\frac{1+\kappa}{T}\left[\frac{d}{dT}\frac{\kappa T}{1+\kappa}\right](\partial_t + \mathbf{v}\nabla) T = -\nabla \mathbf{v}$	$\frac{1}{T}\left[\frac{d}{dT}\kappa T\right](\partial_t + \mathbf{v}\nabla) T = -\nabla \mathbf{v}$
$\kappa = \kappa_{QM}(T)$	$\kappa = \kappa_{HM}(T)$

Boundary conditions (B=before, A=after)
t_r: the estimated "moment" of the rehadronization

$$T_B(t_r, \not{r}) = T_A(t_r, \not{r}) \quad (28)$$

$$\mathbf{v}_B(t_r) = \mathbf{v}_A(t_r) \quad (29)$$

$$\kappa_{QGP}(T_B(t_r)) = \kappa_{HG}(T_A(t_r)) \quad (30)$$

$$\{X_B(t_r), Y_B(t_r), Z_B(t_r)\} = \{X_A(t_r), Y_A(t_r), Z_A(t_r)\} \quad (31)$$

IV. Crossover

Ansatz

We are looking for a solution, which allows us to use the same scaling for each component of the hadron gas, therefore the gas expands collectively.

$$\{X_i, Y_i, Z_i\} = \{X, Y, Z\}, \quad \forall i. \tag{32}$$

Ideal gas approximation:

$$p = \sum_i p_i = T \sum_i n_i, \tag{33}$$

replace it to the Euler-equation:

$$\sum_i m_i n_i \left(\partial_t + \mathbf{v}\nabla\right)\mathbf{v} = -T \sum_i \nabla n_i. \tag{34}$$

IV. Crossover

Multi-component solution

The expression of entropy density [3]:

$$\sigma(\mathbf{r},t) = \sigma_r \frac{V_r}{V} e^{-s/2} = \frac{\sigma_r}{n_{i,r}} n_i(\mathbf{r},t) \tag{35}$$

Let's follow Landau's argument:

$$\frac{\sigma(\mathbf{r},t)}{\sigma_r} = \frac{n_i(\mathbf{r},t)}{n_{i,r}} \Longrightarrow \sigma \sim \sigma_r \tag{36}$$

$$n_i(\mathbf{r},t) = n_{i,r} \frac{V_r}{V} e^{-s/2} = n_{i,r} \left(\frac{X_r Y_r Z_r}{XYZ}\right) e^{-s/2} \tag{37}$$

where

$$s = \frac{r_x^2}{X^2} + \frac{r_y^2}{Y^2} + \frac{r_z^2}{Z^2}. \tag{38}$$

IV. Crossover

The velocity field without and with rotation:

$$v_x = \frac{\dot{X}(t)}{X(t)} r_x, \quad v_y = \frac{\dot{Y}(t)}{Y(t)} r_y, \quad v_z = \frac{\dot{Z}(t)}{Z(t)} r_z, \qquad (39)$$

$$v_x = \frac{\dot{R}(t)}{R(t)} r_x - \omega r_y, \quad v_y = \frac{\dot{R}(t)}{R(t)} r_y + \omega r_x, \quad v_z = \frac{\dot{Z}(t)}{Z(t)} r_z. \qquad (40)$$

In the rotational case we use $X(t) = Y(t) = R(t)$ symmetry, and

$$\omega = \omega_0 \left(R_0 / R \right)^2 . \qquad (41)$$

The temperature profile has spatial homogenity:

$$T_A = T(t),$$

$$T_r = T_A(t_r) = T_B(t_r) \approx T_c \approx 175\ MeV.$$

IV. Crossover

From the Euler equation we gain the differential equations of the expanding fireball:

HG solutions	**Single-component**	**Multi-component**
Without rot.	$X\ddot{X} = Y\ddot{Y} = Z\ddot{Z} = \frac{T}{m}$	$X\ddot{X} = Y\ddot{Y} = Z\ddot{Z} = \frac{T}{\langle m \rangle}$
With rot.	$R\ddot{R} - R^2\omega^2 = Z\ddot{Z} = \frac{T}{m}$	$R\ddot{R} - R^2\omega^2 = Z\ddot{Z} = \frac{T}{\langle m \rangle}$

In one component case there's one difference: $m \Longleftrightarrow \langle m \rangle$:

$$\langle m \rangle = \frac{\sum_i m_i n_{i,r}}{\sum_i n_{i,r}} \approx 280\, MeV. \tag{42}$$

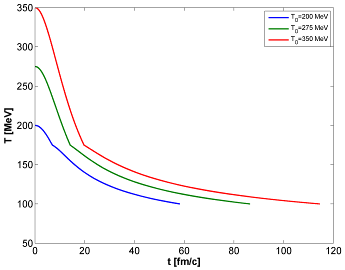
Conclusion

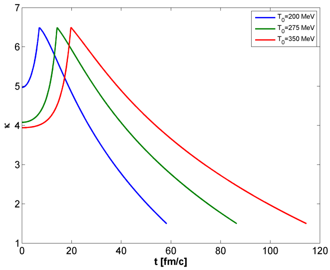
The X, Y and Z scales are independent of the type of particles!

IV. Crossover

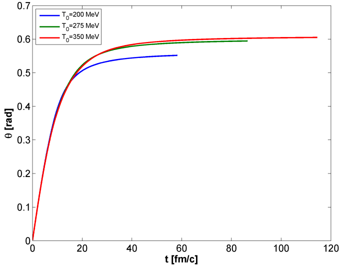
Time dependence

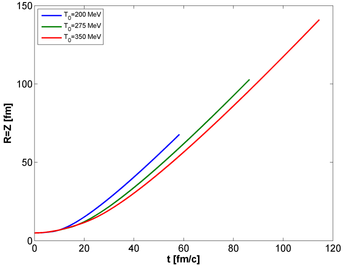
$$R_0 = Z_0 = 5\ fm$$
$$\dot{R}_0 = \dot{Z}_0 = 0$$
$$\theta_0 = 0,\ \ \omega_0 = 0.05\ c/fm$$
$$T_f = 100\ MeV,\ \ \langle m \rangle = 280\ MeV$$





IV. Crossover





At the chemical freezeout temperature:

$$\frac{1}{1+\kappa_c} < \frac{T_c}{\langle m \rangle} \tag{43}$$

The medium has a second "explosion", that starts just after the conversion to the hadron gas!

Conclusion

Non-relativistic approximation breaks down when $\dot{R}$ and $\dot{Z}$ becomes too large, search for relativistic generalization started!

V. Observables

Inverse slope	**Single-component**	**Multi-component**
Without rot.[4]	$T_x = T_f + m\dot{X}_f^{\,2}$ $T_y = T_f + m\dot{Y}_f^{\,2}$ $T_z = T_f + m\dot{Z}_f^{\,2}$	$T_{x,i} = T_f + m_i\dot{X}_f^{\,2}$ $T_{y,i} = T_f + m_i\dot{Y}_f^{\,2}$ $T_{z,i} = T_f + m_i\dot{Z}_f^{\,2}$
With rot. [5]	$T_x = T_f + m\left(\dot{R}_f^{\,2} + \omega_f^2 R_f^2\right)$ $T_y = T_f + m\left(\dot{R}_f^{\,2} + \omega_f^2 R_f^2\right)$ $T_z = T_f + m\dot{Z}_f^{\,2}$	$T_{x,i} = T_f + m_i\left(\dot{R}_f^{\,2} + \omega_f^2 R_f^2\right)$ $T_{y,i} = T_f + m_i\left(\dot{R}_f^{\,2} + \omega_f^2 R_f^2\right)$ $T_{z,i} = T_f + m_i\dot{Z}_f^{\,2}$

Simple method:

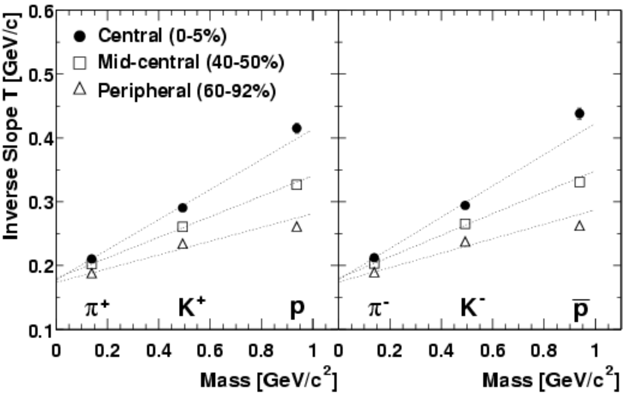
$$m \rightarrow m_i$$

$$T_j \rightarrow T_{j,i}$$

Linear m_i dependence:

$$T_{j,i} = k_1 \cdot m_i + k_2 \qquad (44)$$

V. Observables



$$T_{j,i} = k_1 \cdot m_i + k_2 \tag{45}$$

V. Observables

HBT-radii	**Single-component**	**Multi-component**
Without rot. [4]	$R_x^2 = \frac{X_f^2}{1+\frac{m}{T_f}\dot{X}_f^2}$ $R_y^2 = \frac{Y_f^2}{1+\frac{m}{T_f}\dot{Y}_f^2}$ $R_z^2 = \frac{Z_f^2}{1+\frac{m}{T_f}\dot{Z}_f^2}$	$R_{x,i}^2 = \frac{X_f^2}{1+\frac{m_i}{T_f}\dot{X}_f^2}$ $R_{y,i}^2 = \frac{Y_f^2}{1+\frac{m_i}{T_f}\dot{Y}_f^2}$ $R_{z,i}^2 = \frac{Z_f^2}{1+\frac{m_i}{T_f}\dot{Z}_f^2}$
With rot. [5]	$R_x^2 = \frac{R_f^2}{1+\frac{m}{T_f}\left(\dot{R}_f^2+R^2\omega^2\right)}$ $R_y^2 = \frac{R_f^2}{1+\frac{m}{T_f}\left(\dot{R}_f^2+R^2\omega^2\right)}$ $R_z^2 = \frac{Z_f^2}{1+\frac{m}{T_f}\dot{Z}_f^2}$	$R_{x,i}^2 = \frac{R_f^2}{1+\frac{m_i}{T_f}\left(\dot{R}_f^2+R^2\omega^2\right)}$ $R_{y,i}^2 = \frac{R_f^2}{1+\frac{m_i}{T_f}\left(\dot{R}_f^2+R^2\omega^2\right)}$ $R_{z,i}^2 = \frac{Z_f^2}{1+\frac{m_i}{T_f}\dot{Z}_f^2}$

Simple method:

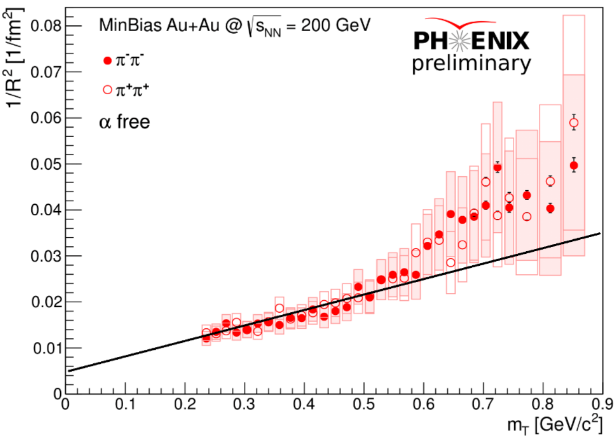
$$m \rightarrow m_i$$

$$R_j \rightarrow R_{j,i}$$

Linear m_i dependence:

$$R_{j,i}^{-2} = c_1 \cdot m_i + c_2 \qquad (46)$$

V. Observables



$$R_{j,i}^{-2} = c_1 \cdot m_i + c_2 \tag{47}$$

VI. Summary

- Rehadronization: crossover $\Longrightarrow$ simple boundary conditions
- Take into consideration the multi-component scenario
- Introduce scales independently from the type of particles
- Gain a similar dynamical equation to the one-component case
- The multi-component scenario does not complicate the description
- Difference: mean mass weighted by the number of particles
- The hadron gas has an exploding dynamics
- We need the relativistic formalism
- $\theta_f(T_0)$ is on progress
- Inverse slope parameters: $T \longrightarrow T_i$
- HBT-radii: $R \longrightarrow R_i$

VII. Bibliography

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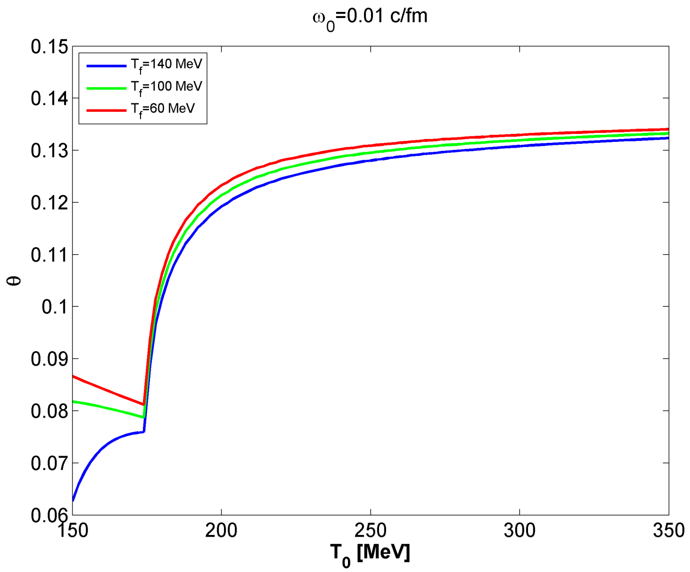
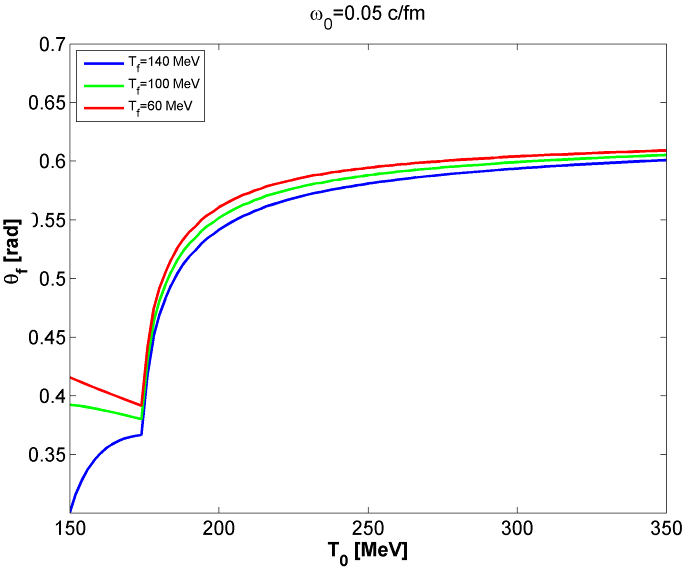

ω0=0.01 c/fm
Tf=140 MeV
Tf=100 MeV
Tf=60 MeV
θ
T0 [MeV]
0.15
0.14
0.13
0.12
0.11
0.1
0.09
0.08
0.07
0.06
150
200
250
300
350



ω0=0.05 c/fm
Tf=140 MeV
Tf=100 MeV
Tf=60 MeV
θf [rad]
T0 [MeV]
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0.6
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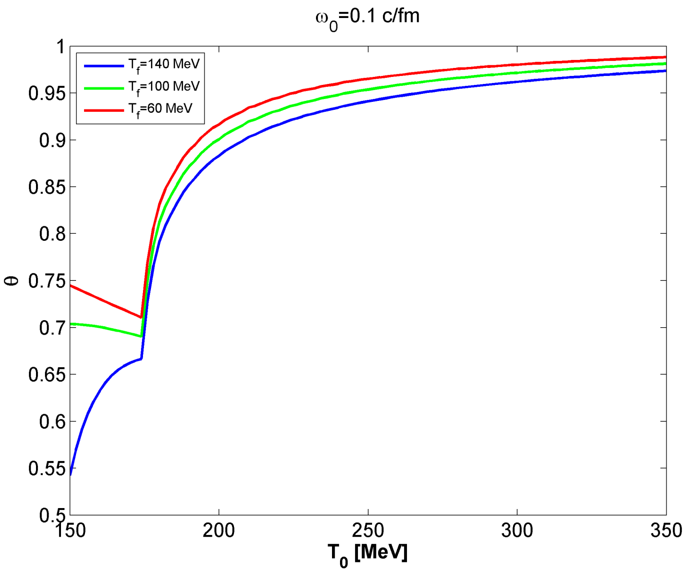
0.5
0.45
0.4
0.35
150
200
250
300
350


ω_0=0.1 c/fm
T_f=140 MeV
T_f=100 MeV
T_f=60 MeV
θ
T_0 [MeV]
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
150
200
250
300
350