Drell-Yan production at forward rapidities: a hybrid factorization approach

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Outline

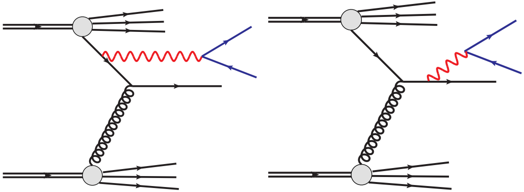
Forward Drell-Yan production as a break up of a quark: $qg^* \to q\gamma^*$

Numerical results, comparison to LHCb & ATLAS data

Conclusions

W. S. and Antoni Szczurek, arXiv:1602.06740,
Phys. Rev. **D93** 074014 (2016).

Forward production of dilepton pairs



- production in the forward region: highly asymmetric parton kinematics: a large-x quark of the right moving proton scatters off the small x gluon field of the left moving proton.
- two contributions: radiation before and after the interaction which interfere destructively.
- quarks move at fixed impact parameters, which differ in the two amplitudes. Emergence of a color dipole cross section after amplitudes are squared.
- dipole models: Golec-Biernat et al., Basso et al., Motyka et al.

Drell-Yan structure functions

Strong interaction dynamics encoded in hadronic tensor $W_{\mu\nu}$.

$$(2\pi)^4 \frac{d\sigma(pp \to l^+(k_+)l^-(k_-)X)}{d^4q} = \frac{(4\pi\alpha_{\rm em})^2}{2SM^4} \cdot W_{\mu\nu} L^{\mu\nu} \cdot d\Phi(q, k_+, k_-)\,.$$

$$W_{\mu\nu} = (\hat{x}_\mu \hat{x}_\nu + \hat{y}_\mu \hat{y}_\nu) W_T + \hat{z}_\mu \hat{z}_\nu W_L + (\hat{y}_\mu \hat{y}_\nu - \hat{x}_\mu \hat{x}_\nu) W_{\Delta\Delta} - (\hat{x}_\mu \hat{z}_\nu + \hat{z}_\mu \hat{x}_\nu) W_\Delta\,,$$

$$\begin{aligned} W_T &= W^{\mu\nu} \epsilon_\mu^{(+)} \epsilon_\nu^{(+)*},\; W_L = W^{\mu\nu} \epsilon_\mu^{(0)} \epsilon_\nu^{(0)}\,, \\ W_\Delta &= W^{\mu\nu} (\epsilon_\mu^{(+)} \epsilon_\nu^{(0)} + \epsilon_\mu^{(0)} \epsilon_\nu^{(+)*}) \frac{1}{\sqrt{2}}\,, W_{\Delta\Delta} = W^{\mu\nu} \epsilon_\mu^{(+)} \epsilon_\nu^{(-)*}\,. \end{aligned}$$

Helicity density matrix of the virtual photon

$$\rho_{\lambda\lambda'} \frac{d\sigma(pp \to \gamma^*(M^2)X)}{dx_F\, d^2\boldsymbol{q}} = \frac{1}{x_F} \frac{\alpha_{\rm em}}{8\pi^2 S} W_{\mu\nu} \epsilon_\mu^{(\lambda)} \epsilon_\nu^{(\lambda')*}\,.$$

Or

$$\rho_{\lambda\lambda'} = \frac{W_{\mu\nu} \epsilon_\mu^{(\lambda)} \epsilon_\nu^{(\lambda')*}}{2W_T + W_L}\,, \; \rho_{++} + \rho_{--} + \rho_{00} = 1\,.$$

The $qp \to \gamma^* qX$ process

Exploit the high energy limit to arrive at the color dipole representation:

$$\hat{\rho}_{\lambda\lambda'} \frac{d\hat{\sigma}(qp \to \gamma^*(z, \boldsymbol{q})X)}{dz d^2\boldsymbol{q}} = \frac{1}{2(2\pi)^2} \overline{\sum_{\sigma,\sigma'}} \int d^2\boldsymbol{r} d^2\boldsymbol{r}' \exp[-i\boldsymbol{q}(\boldsymbol{r}-\boldsymbol{r}')] \psi^{(\lambda)}_{\sigma\sigma'}(z,\boldsymbol{r}) \psi^{(\lambda')*}_{\sigma\sigma'}(z,\boldsymbol{r}')$$
$$\times \Big(\sigma(x_2, z\boldsymbol{r}) + \sigma(x_2, z\boldsymbol{r}') - \sigma(x_2, z(\boldsymbol{r}-\boldsymbol{r}')) \Big) .$$

Go to momentum space representation ("half of a" k_T-factorization !):

$$\psi^{(\lambda)}_{\sigma\sigma'}(z,\boldsymbol{r}) = \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} \exp[-i\boldsymbol{r}\boldsymbol{q}] \psi^{(\lambda)}_{\sigma\sigma'}(z,\boldsymbol{q})$$

$$\sigma(x,\boldsymbol{r}) = \frac{1}{2} \int d^2\boldsymbol{\kappa}\, f(x,\boldsymbol{\kappa})(1-\exp[i\boldsymbol{\kappa}\boldsymbol{r}])(1-\exp[-i\boldsymbol{\kappa}\boldsymbol{r}]) .$$

$$f(x,\boldsymbol{\kappa}) = \frac{4\pi\alpha_S}{N_c} \frac{1}{\boldsymbol{\kappa}^4} \frac{\partial xg(x,\boldsymbol{\kappa}^2)}{\partial \log \boldsymbol{\kappa}^2} = \frac{4\pi\alpha_S}{N_c} \frac{1}{\boldsymbol{\kappa}^4} \mathcal{F}(x,\boldsymbol{\kappa}) .$$

Impact factor representation for $qp \to \gamma^* qX$

$$\hat{\Sigma}_i(z, \boldsymbol{q}, M^2) = \hat{\rho}_i \frac{d\hat{\sigma}(qp \to \gamma^*(z, \boldsymbol{q})X)}{dz d^2\boldsymbol{q}} = \frac{e_q^2 \alpha_{\rm em}}{2N_c} \int \frac{d^2\boldsymbol{\kappa}}{\pi \boldsymbol{\kappa}^4} \alpha_{\rm S}(\bar{q}^2) \mathcal{F}(x_2, \boldsymbol{\kappa}^2)\, I_i(z, \boldsymbol{q}, \boldsymbol{\kappa}),$$

with

$$I_T(z, \boldsymbol{q}, \boldsymbol{\kappa}) = \frac{1 + (1-z)^2}{z} |\boldsymbol{\Phi}|^2 + z^3 m_q^2 \Phi_0^2,$$

$$I_L(z, \boldsymbol{q}, \boldsymbol{\kappa}) = \frac{4(1-z)^2 M^2}{z} \Phi_0^2,$$

$$I_\Delta(z, \boldsymbol{q}, \boldsymbol{\kappa}) = \frac{2(2-z)(1-z)M}{z} \left(\frac{\boldsymbol{q}}{|\boldsymbol{q}|} \cdot \boldsymbol{\Phi} \right) \Phi_0,$$

$$I_{\Delta\Delta}(z, \boldsymbol{q}, \boldsymbol{\kappa}) = \frac{2(1-z)}{z} \left(|\boldsymbol{\Phi}|^2 - 2\left(\frac{\boldsymbol{q}}{|\boldsymbol{q}|} \cdot \boldsymbol{\Phi} \right)^2 \right).$$

$$\boldsymbol{\Phi}(z, \boldsymbol{q}, \boldsymbol{\kappa}) = \frac{\boldsymbol{q}}{\boldsymbol{q}^2 + \varepsilon^2} - \frac{\boldsymbol{q} - z\boldsymbol{\kappa}}{(\boldsymbol{q} - z\boldsymbol{\kappa})^2 + \varepsilon^2}, \; \Phi_0(z, \boldsymbol{q}, \boldsymbol{\kappa}) = \frac{1}{\boldsymbol{q}^2 + \varepsilon^2} - \frac{1}{(\boldsymbol{q} - z\boldsymbol{\kappa})^2 + \varepsilon^2}, \; \varepsilon^2 = (1-z)M^2 + z^2 m_q^2.$$

Hadron level density matrix of production

To go to the hadron level, we assume the collinear factorization on the quark side , choosing a factorization scale $\mu^2 \sim \boldsymbol{q}^2 + \varepsilon^2$.
Note: we do include transverse momentum of the gluon:

$$\begin{aligned}
\Sigma_i(x_F, \boldsymbol{q}, M) &= \sum_f \int dx_1 dz\, \delta(x_F - z x_1) \Big[q_f(x_1, \mu^2) + \bar{q}_f(x_1, \mu^2) \Big] \hat{\Sigma}_i(z, \boldsymbol{q}, M^2)\,. \\
&= \sum_f \frac{e_f^2 \alpha_{\rm em}}{2N_c} \int_{x_F}^1 dx_1 \Big[q_f(x_1, \mu^2) + \bar{q}_f(x_1, \mu^2) \Big] \\
&\times \int \frac{d^2\boldsymbol{\kappa}}{\pi \boldsymbol{\kappa}^4} \mathcal{F}(x_2, \boldsymbol{\kappa}^2) \alpha_S(\bar{q}^2) I_i\Big(\frac{x_F}{x_1}, \boldsymbol{q}, \boldsymbol{\kappa}\Big)\,.
\end{aligned}$$

Inclusive lepton pair production

Inclusive dilepton cross section is a convolution of the density matrices for production and decay

$$\frac{d\sigma(pp \to l^+ l^- X)}{dx_+ dx_- d^2\boldsymbol{k}_+ d^2\boldsymbol{k}_-} = \frac{\alpha_{\rm em}}{(2\pi)^2 M^2} \frac{x_F}{x_+ x_-} \Big\{ \Sigma_T(x_F, \boldsymbol{q}, M^2) D_T\Big(\frac{x_+}{x_F}\Big)$$
$$+ \Sigma_L(x_F, \boldsymbol{q}, M^2) D_L\Big(\frac{x_+}{x_F}\Big)$$
$$+ \Sigma_\Delta(x_F, \boldsymbol{q}, M^2) D_\Delta\Big(\frac{x_+}{x_F}\Big)\Big(\frac{\boldsymbol{l}}{|\boldsymbol{l}|}\cdot\frac{\boldsymbol{q}}{|\boldsymbol{q}|}\Big)$$
$$+ \Sigma_{\Delta\Delta}(x_F, \boldsymbol{q}, M^2) D_{\Delta\Delta}\Big(\frac{x_+}{x_F}\Big)\Big(2\Big(\frac{\boldsymbol{l}}{|\boldsymbol{l}|}\cdot\frac{\boldsymbol{q}}{|\boldsymbol{q}|}\Big)^2 - 1\Big)\Big\} \, .$$

$$x_F = x_+ + x_-, \boldsymbol{q} = \boldsymbol{k}_+ + \boldsymbol{k}_- \, .$$

light-cone relative transverse momentum:

$$\boldsymbol{l} = \frac{x_+}{x_F}\boldsymbol{k}_- - \frac{x_-}{x_F}\boldsymbol{k}_+ \, .$$

Kinematics

Rapidities are obtained as:

$$y_i = \log\left(\frac{x_i\sqrt{S}}{\sqrt{\boldsymbol{k}_i^2}}\right) \leftrightarrow x_i = \sqrt{\frac{\boldsymbol{k}_i^2}{S}} \cdot e^{y_i}\,, i = +, -, J\,. \tag{1}$$

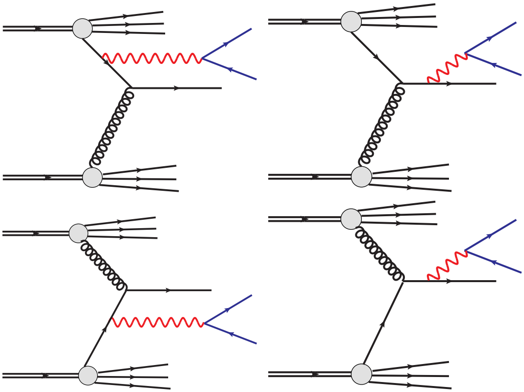
The longitudinal momentum fractions x_1, x_2 entering the quark and gluon distributions are

$$\begin{aligned} x_1 &= \sqrt{\frac{\boldsymbol{k}_+^2}{S}} e^{y_+} + \sqrt{\frac{\boldsymbol{k}_-^2}{S}} e^{y_-} + \sqrt{\frac{\boldsymbol{k}_J^2}{S}} e^{y_J}\,, \\ x_2 &= \sqrt{\frac{\boldsymbol{k}_+^2}{S}} e^{-y_+} + \sqrt{\frac{\boldsymbol{k}_-^2}{S}} e^{-y_-} + \sqrt{\frac{\boldsymbol{k}_J^2}{S}} e^{-y_J}\,. \end{aligned}$$

The invariant mass of the dilepton system is

$$M^2 = m_{\perp+}^2 + m_{\perp-}^2 + 2m_{\perp+}m_{\perp-}\cosh(y_+ - y_-) - \boldsymbol{q}^2\,,\ \ m_{\perp\pm} = \sqrt{\boldsymbol{k}_\pm^2 + m_\pm^2}\,.$$

Both sides now



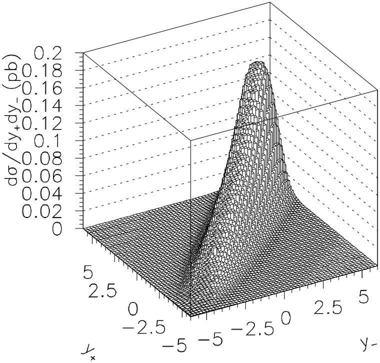
- emission from the right-moving and left-moving quarks: how well are they separated?
- a number of dipole model calculations seem to include only emission from one side...

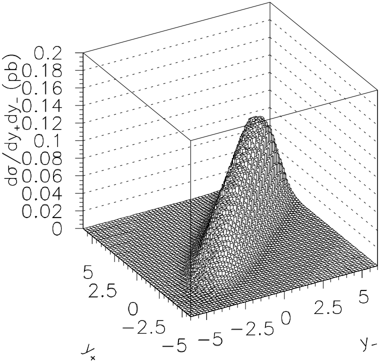
Input to numerical evaluations:

We use the following UGDFs/parton distributions:

- Kimber-Martin-Ryskin UGDF,
  transverse momentum in the last step of evolution
- Kutak-Staśto UGDF,
  includes nonlinear effects
- Albacete, Armesto, Milhano, Salgado
  - solving BK evolution equation and Fourier transform.
- Golec-Biernat UGDF,
  saturation inspired parametrization of photon-nucleon cross section.
- for the quark and antiquark distributions we use MSTW08 leading-order distributions.

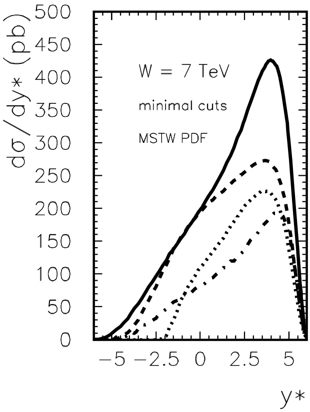
Full rapidity range





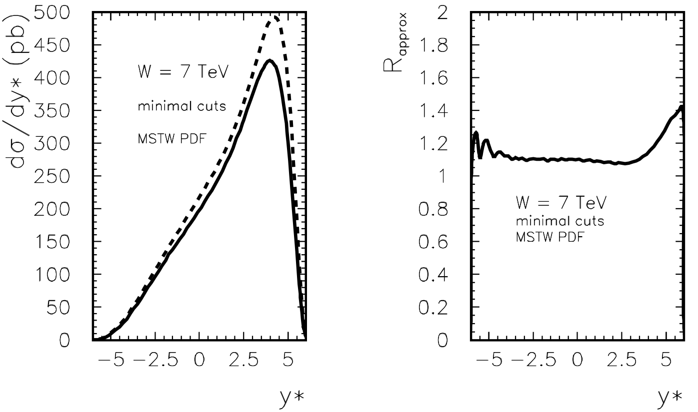
- Two-dimensional (y_+, y_-) distribution for $\sqrt{s} = 7$ TeV and $k_{T+}, k_{T-} > 3$ GeV for MSTW08 PDF and KMR (left) and KS (right) UGDFs.
- The rapidities of both leptons are strongly correlated i.e. $y_+ \approx y_-$.

Full rapidity range



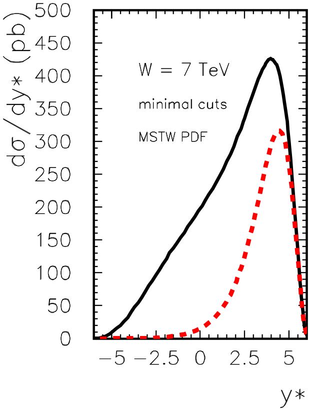
- Distribution in rapidity of the dileptons for $\sqrt{s} = 7$ TeV and $k_{T+}, k_{T-} > 3$ GeV for MSTW08 PDF and different UGDFs: KMR (solid), KS (dashed), AAMS (dotted) and GBW (dash-dotted).

Full rapidity range



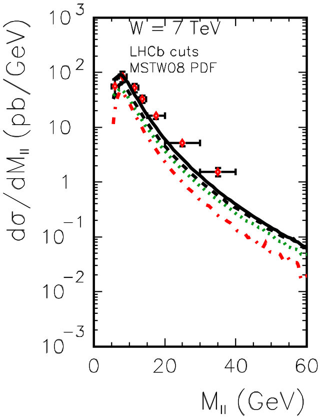
- Distribution in y_* for exact (solid) and approximate (dashed) formula for calculating x_1 and x_2 for $\sqrt{s} = 7$ TeV and $k_{T+}, k_{T-} > 3$ GeV for MSTW08 PDF and KMR UGDF.

Full rapidity range



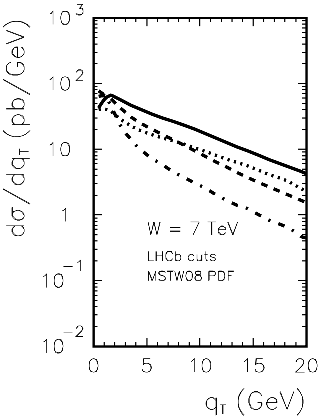
- Distribution in rapidity of the dileptons for $\sqrt{s} = 7$ TeV and $k_{T+}, k_{T-} > 3$ GeV for MSTW08 valence quark distributions and KMR UGDFs. The dashed line is the contribution from valence quarks only.

Forward region - LHCb



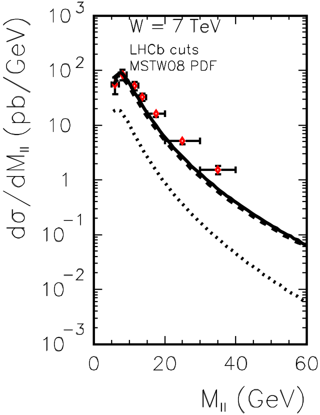
- Invariant mass distribution (only the dominant component) for different UGDFs: KMR (solid), Kutak-Stasto (dashed), AAMS (dotted) and GBW (dash-dotted).

Forward region - LHCb



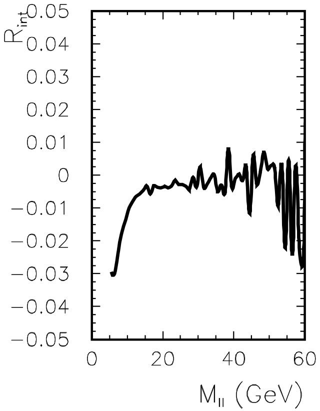
- Dilepton transverse momentum distribution (only the dominant component) for different UGDFs: KMR (solid), Kutak-Stasto (dashed), AAMS (dotted) and GBW (dash-dotted).

Forward region - LHCb



- The T and L contributions to the dilepton invariant mass distribution KMR UGDF was used here.

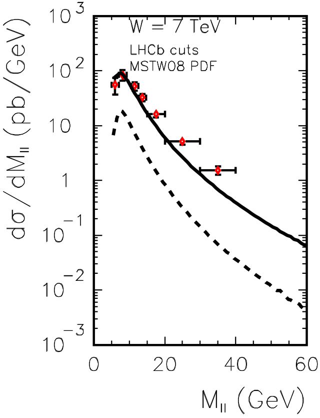
Forward region - LHCb



$$R_{int} = \frac{d\sigma_{all} - d\sigma_{T+L}}{d\sigma_{all}} \; .$$

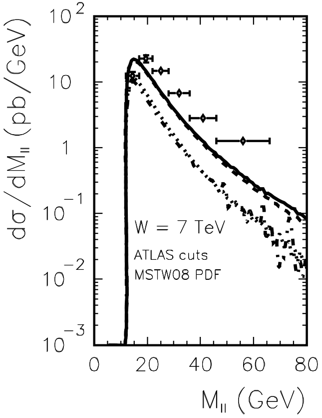
- The R_{int} as a function of M_{ll} for the LHCb kinematics: $2 < y_+, y_- < 4.5$, $k_{T+}, k_{T-} > 3$ GeV. KMR UGDF was used here. The fluctuations are due to insufficient statistics of our Monte Carlo calculation.

Forward region - LHCb



- Contributions of the second-side component for the LHCb kinematics: 2 $< y_+, y_- <$ 4.5, $k_{T+}, k_{T-} >$ 3 GeV. KMR UGDF was used here.

Midrapidity region - ATLAS



- in the central rapidity region k_T of the (anti-)quark ought to be included on an equal footing!

- Invariant dilepton mass distribution for the ATLAS kinematics: -2.4 $< y_+, y_- <$ 2.4, $k_{T+}, k_{T-} >$ 6 GeV. Here both $gq/\bar{q}$ and $q/\bar{q}g$ contributions have been included.

Conclusions

- Forward production of low-mass dileptons: bresmstrahlung of a heavy photon by a fast quark in the small-x gluon field of the "target".
- a "hybrid" factorization, which treats fast (anti-)quarks as collinear partons and includes k_T of the small-x gluon has been employed.
- We do calculate directly lepton distributions. Correct treatment of kinematics important for "x of the gluon".
- The saturation inspired UGDFs fail to describe LHCb data – perhaps expectedly, as $M_{ll} > m_\Upsilon$. Unintegrated glue constructed by the KMR procedure does a good job.
- "Spillover" from emission of the second proton is not negligible in the LHCb kinematics. Neither do valence quarks dominate at LHCb-kinematics.
- ATLAS (and CMS) data around midrapidity are beyond the realm of the hybrid factorization approach.
- The extension to proton-nucleus collisions is straightforward: the photon does not interact, and all rescatterings are included in the correlation function of two Wilson lines (dipole cross section!).