Four-jet production in k_t-factorization

Antoni Szczurek

Institute of Nuclear Physics (PAN), Kraków, Poland

Rzeszów University, Rzeszów, Poland

Low-x 2016

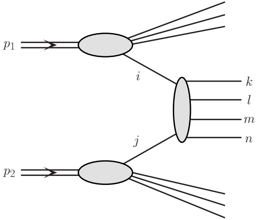
Gyöngyös, Hungary, June 6-11, 2016

Outline

1. k_t-factorization approach for SPS and DPS production of four jets
2. Comparison with ATLAS and CMS data
3. Optimal conditions for DPS, predictions for LHC Run2
4. Summary

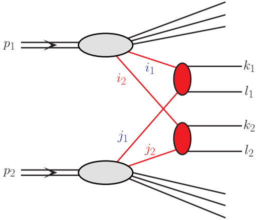
Based on:
Maciuła, Szczurek, Phys. Lett. B 749 (2015) 57-62
(optimal conditions for DPS in collinear approximation, symmetric cuts)
Kutak, Maciuła, Serino, Szczurek, Hameren, JHEP **04** (2016) 1.
(k_t-factorization formalism and first results)
Kutak, Maciuła, Serino, Szczurek, Hameren, arXiv:1605.08240
(optimal conditions for DPS in k_T-factorization)

Four-jet production: Mechanisms under consideration



Single-Parton Scattering (SPS $2 \to 4$)

- Kutak, Maciuła, Serino, Szczurek, Hameren, arXiv:1602.06814 (hep-ph) (published in JHEP)
  AVHLIB (A. van Hameren): https://bitbucket.org/hameren/avhlib
- High-Energy-Fact. (HEF): LO k_T-factorization ($2 \to 4$)
- first time: off-shell initial state partons



Double-Parton Scattering (DPS $4 \to 4$)

- Factorized Ansatz with experimental setup of σ_{eff}
- LO k_T-factorization approach ($2 \to 2 \otimes 2 \to 2$)
- more precise studies of kinematical characteristics and correlation observables

extension of our previous studies based on LO collinear approach (ALPGEN):
Maciuła, Szczurek, Phys. Lett. B 749 (2015) 57-62

Single-parton scattering production of four jets

The collinear factorization formula for the calculation of the inclusive partonic 4-jet cross section at the Born level reads:

$$\sigma^{B}_{4-jets} = \sum_{i,j} \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} \, x_1 f_i(x_1, \mu_F) \, x_2 f_j(x_2, \mu_F) \times \frac{1}{2\hat{s}} \prod_{l=i}^{4} \frac{d^3 k_l}{(2\pi)^3 2E_l} \Theta_{4-jet} \, (2\pi)^4 \, \delta\left(x_1 P_1 + x_2 P_2 - \sum_{l=1}^{4} k_l \right) \overline{\left| \mathcal{M}(i,j \to 4\,\mathrm{part.}) \right|^2} \, . \tag{1}$$

Here $x_{1,2} f_i(x_{1,2}, \mu_F)$ are the collinear PDFs for the $i - th$ parton, carrying $x_{1,2}$ momentum fractions of the proton and evaluated at the factorization scale μ_F; the index l runs over the four partons in the final state, the partonic center of mass energy squared is $\hat{s} = 2\, x_1 x_2 \, P_i \cdot P_j$; the function Θ_{4-jet} takes into account the kinematic cuts applied and $\mathcal{M}$ is the partonic on-shell matrix element, which includes symmetrization effects due to identity of particles in the final state.

Single-parton scattering production of four jets

For HEF (k_t-factorization):

$$\sigma^B_{4-jets} = \sum_{i,j} \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} d^2 k_{T1} d^2 k_{T2} \mathcal{F}_i(x_1, k_{T1}, \mu_F) \mathcal{F}_j(x_2, k_{T2}, \mu_F)$$
$$\times \frac{1}{2\hat{s}} \prod_{l=i}^{4} \frac{d^3 k_l}{(2\pi)^3 2E_l} \Theta_{4-jet} \, (2\pi)^4 \, \delta\left(x_1 P_1 + x_2 P_2 + \vec{k}_{T\,1} + \vec{k}_{T\,2} - \sum_{l=1}^{4} k_l \right)$$
$$\overline{\left|\mathcal{M}(i^*, j^* \to 4\,\text{part.})\right|^2} \,. \qquad (2)$$

Here $\mathcal{F}_i(x_k, k_{Tk}, \mu_F)$ is a transverse momentum dependent (TMD) distribution function for a given type of parton. Similarly as in the collinear factorization case, x_k is the longitudinal momentum fraction, μ_F is a factorization scale. The new degrees of freedom are introduced via $\vec{k}_{Tk}$, which are the parton's transverse momenta, i.e. the momenta perpendicular to the collision axis.

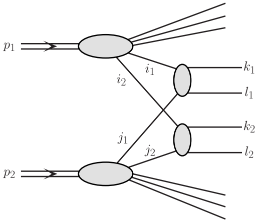
Off-shell matrix elements

- The matrix elements for two off-shell partons calculated with automated code of van Hameren.
- Exact LO matrix elements and full phase space integration (similar to HELAC).
- Major part of NLO corrections automatically included.
- The amplitudes are gauge invariant.
- Off-shell amplitudes are obtained by emdedding them into more complicated on shell processes.
- Dyson-Schwinger recursion relations are used to calculate helicity amplitudes.
- Monte Carlo calculations.
- The method was tested for dijets and the results coincide with parton reggeization method.
- The method was used for dijets and $c\bar{c}c\bar{c}$ production (see R. Maciuła talk).

Four-jet production in double-parton scattering (DPS)

Factorized ansatz (pocket-formula)

In a simple probabilistic picture:



process initiated by two simultaneous hard parton-parton scatterings in one proton-proton interaction $\Rightarrow$

$$\sigma^{DPS}(pp \to 4\text{jets}X) = \frac{C}{\sigma_{eff}} \cdot \sigma^{SPS}(pp \to \text{dijet}X_1) \cdot \sigma^{SPS}(pp \to \text{dijet}X_2)$$

two subprocesses are not correlated and do not interfere

analogy: frequently considered mechanisms of double charm, double gauge boson production and double Drell-Yan anihillation

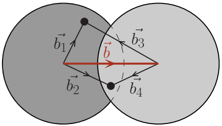
valid also differentially:

$$\frac{d\sigma^{DPS}(pp \to 4\text{jets}\ X)}{d\xi_1 d\xi_2} = \sum_{\substack{i_1,j_1,k_1,l_1 \\ i_2,j_2,k_2,l_2}} \frac{C}{\sigma_{eff}} \frac{d\sigma(i_1 j_1 \to k_1 l_1)}{d\xi_1} \frac{d\sigma(i_2 j_2 \to k_2 l_2)}{d\xi_2},$$

where $C = \left\{ \begin{array}{ll} \frac{1}{2} & \text{if } \ i_1 j_1 = i_2 j_2 \wedge k_1 l_1 = k_2 l_2 \\ 1 & \text{if } \ i_1 j_1 \neq i_2 j_2 \vee k_1 l_1 \neq k_2 l_2 \end{array} \right\}$ and $i, j, k, l = g, u, d, s, c, \bar{u}, \bar{d}, \bar{s}, \bar{c}$.

- combinatorial factors C include identity of the two subprocesses

Factorized ansatz and double-parton distributions (DPDFs)



DPDF - emission of parton i with assumption that second parton j is also emitted:

$$\Gamma_{i,j}(b, x_1, x_2; \mu_1^2, \mu_2^2) = F_i(x_1, \mu_1^2)\; F_j(x_2, \mu_2^2)\; F(b; x_1, x_2, \mu_1^2, \mu_2^2)$$

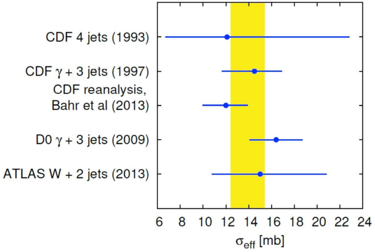
- correlations between two partons
  C. Flensburg et al., JHEP 06, 066 (2011)

in general:

$$\sigma_{eff}(x_1, x_2, x_1', x_2', \mu_1^2, \mu_2^2) = \left(\int d^2b\; F(b; x_1, x_2, \mu_1^2, \mu_2^2)\; F(b; x_1', x_2', \mu_1^2, \mu_2^2)\right)^{-1}$$

Factorized ansatz:

- DPDF in multiplicative form: $F_{ij}(b; x_1, x_2, \mu_1^2, \mu_2^2) = F_i(x_1, \mu_1^2) F_j(x_2, \mu_2^2) F(b)$
- $\sigma_{eff} = \left[\int d^2b\; (F(b))^2\right]^{-1}$, $F(b)$ - energy and process independent



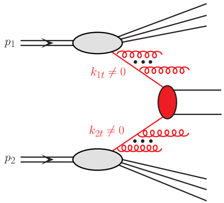
phenomenology: $\sigma_{eff} \Rightarrow$ nonperturbative quantity with a dimension of cross section, connected with transverse size of proton

$\sigma_{eff} \approx$ **15 mb** ($p_\perp$-independent)

a detailed analysis of σ_{eff}:
Seymour, Siódmok, JHEP 10, 113 (2013)

- additional limitations: $x_1 + x_2 < 1$ oraz $x_1' + x_2' < 1$

SPS dijet production: k_t-factorization (semihard) approach



k_t-factorization $\longrightarrow \kappa_{1,t}, \kappa_{2,t} \neq 0$ Collins-Ellis, Nucl. Phys. B360 (1991) 3;
Catani-Ciafaloni-Hautmann, Nucl. Phys. B366 (1991) 135; Ball-Ellis, JHEP 05 (2001) 053

$\Rightarrow$ efficient approach for jet-jet or $Q\bar{Q}$ correlations

- multi-differential cross section

$$\frac{d\sigma}{dy_1 dy_2 d^2p_{1,t} d^2p_{2,t}} = \sum_{i,j} \int \frac{d^2\kappa_{1,t}}{\pi} \frac{d^2\kappa_{2,t}}{\pi} \frac{1}{16\pi^2 (x_1 x_2 s)^2} \overline{|\mathcal{M}_{i^* j^* \to kl}|^2}$$
$$\times \; \delta^2\left(\vec{\kappa}_{1,t} + \vec{\kappa}_{2,t} - \vec{p}_{1,t} - \vec{p}_{2,t}\right) \mathcal{F}_i(x_1, \kappa_{1,t}^2)\, \mathcal{F}_j(x_2, \kappa_{2,t}^2)$$

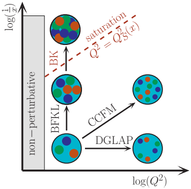
- $\mathcal{F}_i(x_1, \kappa_{1,t}^2)$, $\mathcal{F}_j(x_2, \kappa_{2,t}^2)$ - unintegrated (k_t-dependent) PDFs
- **LO off-shell** $\overline{|\mathcal{M}_{i^* j^* \to kl}|^2} \Rightarrow$ calculated numerically in AVHLIB

  analytical form: Nefedov, Saleev, Shipilova, Phys. Rev. D87, 094030 (2013)
  Quasi Multi Regge Kinematics (QMRK) with effective BFKL NLL vertices

  $$\sqrt{s} \gg p_T, M \gg \Lambda_{QCD} \text{ and } x \ll 1$$

  Parton-Reggeization Approach (k_T-factorization with Reggeized initial partons): an effective way to take into account amount part of radiative corrections at high energy Regge kinematics
- some part of higher-order corrections may be effectively included depending on UPDF model $\Rightarrow$ possible emission of extra (hard) gluons

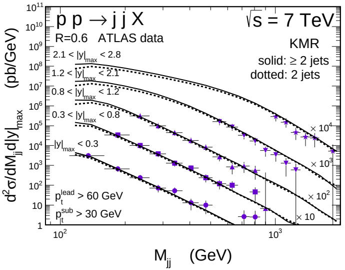
Unintegrated parton distribution functions (UPDFs)

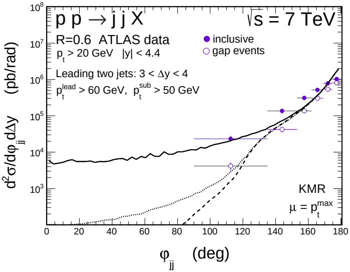


Different evolution equations (or their combinations):

- Kwieciński, Jung (CCFM, wide range of x)
- Kimber-Martin-Ryskin (DGLAP-BFKL, wide range of x)
- Kwieciński-Martin-Staśto (BFKL-DGLAP, small x-values)
- Kutak-Staśto (BK, saturation, only small x-values)

Lessons from inclusive dijet production at the LHC:





- KMR UPDFs work well for jet-jet correlation observables in dijet production

DPS in the framework of k_T-factorization

DPS production of four-jet system within k_T-factorization approach, assuming factorization of the DPS model:

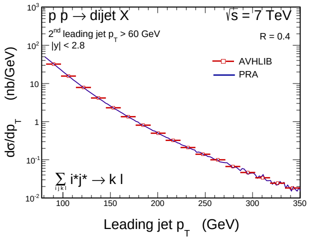
$$\frac{d\sigma^{DPS}(pp \to 4\mathrm{jets}X)}{dy_1 dy_2 d^2p_{1,t} d^2p_{2,t} dy_3 dy_4 d^2p_{3,t} d^2p_{4,t}} = \frac{C}{\sigma_{eff}} \cdot \frac{d\sigma^{SPS}(pp \to j\, j\, X_1)}{dy_1 dy_2 d^2p_{1,t} d^2p_{2,t}} \cdot \frac{d\sigma^{SPS}(pp \to j\, j\, X_2)}{dy_3 dy_4 d^2p_{3,t} d^2p_{4,t}}$$

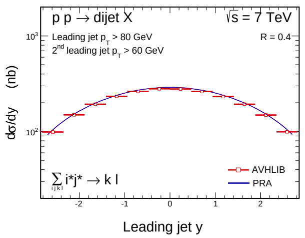
Each step of DPS (each individual scattering):

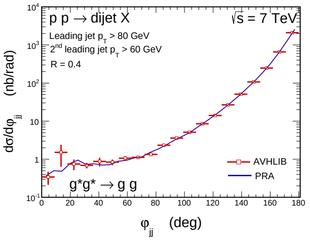
$$\frac{d\sigma^{SPS}(pp \to j\, j\, X)}{dy_1 dy_2 d^2p_{1,t} d^2p_{2,t}} = \frac{1}{16\pi^2 \hat{s}^2} \int \frac{d^2k_{1t}}{\pi} \frac{d^2k_{2t}}{\pi} \overline{|\mathcal{M}_{i^* k^* \to j\, j}|^2}$$
$$\times\ \delta^2\left(\vec{k}_{1t} + \vec{k}_{2t} - \vec{p}_{1t} - \vec{p}_{2t}\right) \mathcal{F}(x_1, k_{1t}^2, \mu^2) \mathcal{F}(x_2, k_{2t}^2, \mu^2)$$

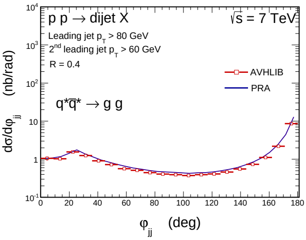
- 9 channels from the $2 \to 2$ SPS $\Rightarrow$ 45 channels for the $4 \to 4$ DPS but only 14 contribute to $\geq 95\%$ of the cross section
- KMR UPDFs from CT10 NLO collinear PDFs
- $n_F = 4$ flavour scheme, running α_s@NLO from MSTW08 package
- scales: $\mu = \mu_R = \mu_F = \frac{1}{2}\sum_i p_T^i$ (sum over final state particles)
- all details the same for $2 \to 4$ SPS and $4 \to 4$ DPS calculations

Automated calculations vs parton reggeizaition approch for two-jets









Parton combinations for 4-jet production, SPS case

There are 19 different channels contributing to the cross section at the parton-level:

$$gg \to 4g\,, gg \to q\bar{q}\,2g\,, qg \to q\,3g\,, q\bar{q} \to q\bar{q}\,2g\,, qq \to qq\,2g\,, qq' \to qq'\,2g\,,$$
$$gg \to q\bar{q}q\bar{q}\,, gg \to q\bar{q}q'\bar{q}'\,, qg \to qgq\bar{q}\,, qg \to qgq'\bar{q}'\,,$$
$$q\bar{q} \to 4g\,, q\bar{q} \to q'\bar{q}'\,2g\,, q\bar{q} \to q\bar{q}q\bar{q}\,, q\bar{q} \to q\bar{q}q'\bar{q}'\,, q\bar{q} \to q'\bar{q}'q'\bar{q}'\,, q\bar{q} \to q$$
$$qq \to qqq\bar{q}\,, qq \to qqq'\bar{q}'\,, qq' \to qq'q\bar{q}\,,$$

The processes in the first line are the dominant channels, contributing together to $\sim 93\%$ of the total cross section. This stays true in the HEF framework as well.

Parton combinations for 4-jet production, DPS case

We have to include all the possible 45 channels which can be obtained by coupling in all possible distinct ways the 8 channels for the $2 \to 2$ SPS process, i.e.

$$\begin{aligned}
\#1 &= gg \to gg\,, \quad \#5 = q\bar{q} \to q'\bar{q}'\,,\\
\#2 &= gg \to q\bar{q}\,, \quad \#6 = q\bar{q} \to gg\,,\\
\#3 &= qg \to qg\,, \quad \#7 = qq \to qq\,,\\
\#4 &= q\bar{q} \to q\bar{q}\,, \quad \#8 = qq' \to qq'\,.
\end{aligned}$$

We find that the pairs $(1,1)$, $(1,2)$, $(1,3)$, $(1,7)$, $(1,8)$, $(3,3)$ $(3,7)$, $(3,8)$ account for more than 95 % of the total cross section for all the sets of cuts considered in this paper.

ATLAS four jets

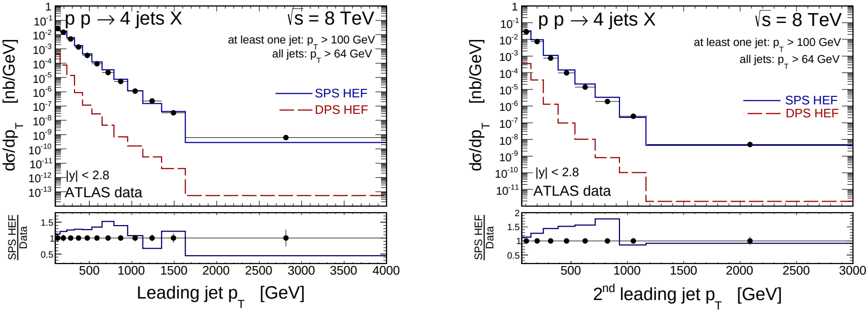


Figure: HEF prediction of the differential cross sections for the transverse momenta of the first two leading jets compared to the ATLAS data. In addition we show the ratio of the SPS HEF result to the ATLAS data.

ATLAS four jets

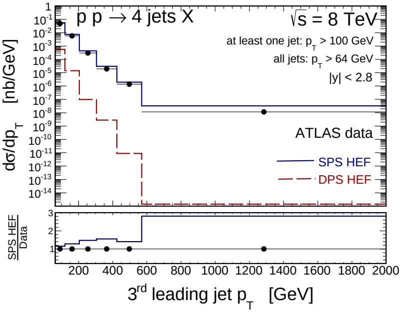


Figure: HEF prediction of the differential cross sections for the transverse momenta of the 3rd and 4th leading jets compared to the ATLAS data. In addition we show the ratio of the SPS HEF result to the ATLAS data.

Symmetric cuts in k_t factorization

In our previous search for optimal conditions for DPS in LO collinear approach (Maciuła-Szczurek) mostly symmetric cuts were used.

However ...

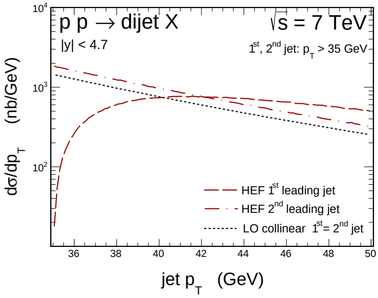


Figure: The transverse momentum distribution of the leading (long dashed line) and subleading (long dashed-dotted line) jet for the dijet production in HEF and LO collinear approaches. The LO collinear approach (short dashed line) in which case both jets give the same distribution.

CMS four jets

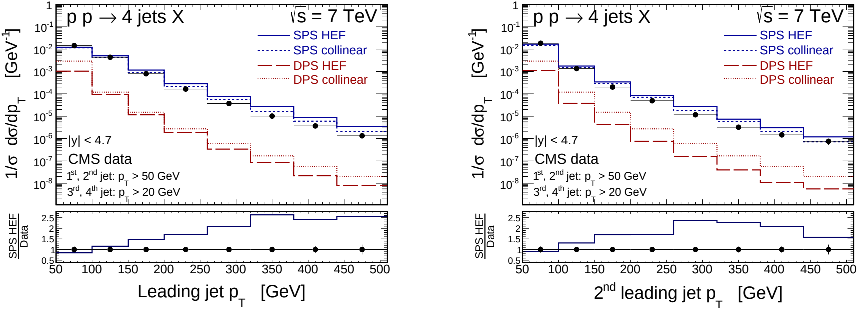


Figure: Comparison of the LO collinear and HEF predictions to the CMS data for the 1st and 2nd leading jets. In addition we show the ratio of the SPS HEF result to the CMS data.

CMS four jets

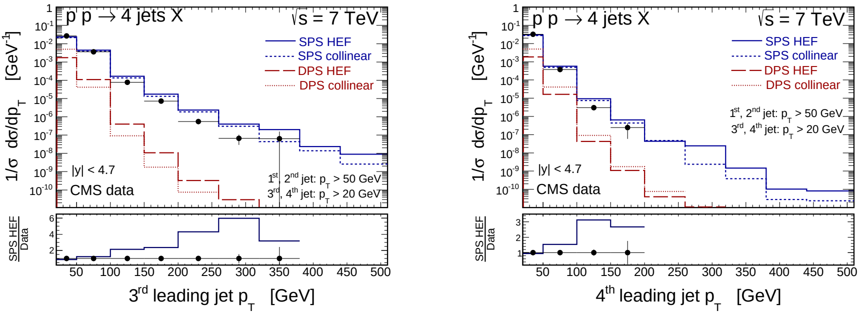


Figure: Comparison of the LO collinear and HEF predictions for the CMS data for the 3rd and 4th leading jets. In addition we show the ratio of the SPS HEF result to the CMS data.

A more complicated correlation variable

$$\Delta S = \arccos\left(\frac{\vec{p}_T(j_1^{\text{hard}}, j_2^{\text{hard}}) \cdot \vec{p}_T(j_1^{\text{soft}}, j_2^{\text{soft}})}{|\vec{p}_T(j_1^{\text{hard}}, j_2^{\text{hard}})| \cdot |\vec{p}_T(j_1^{\text{soft}}, j_2^{\text{soft}})|}\right), \quad (3)$$

where $\vec{p}_T(j_i, j_k)$ stands for the sum of the transverse momenta of the two jets in arguments.

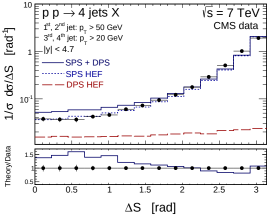


Figure: Comparison of the HEF predictions to the CMS data for ΔS spectrum. In addition we show the ratio of the (SPS+DPS) HEF result to the CMS data.

A more complicated correlation variable

TMD toy model with the Gaussian smearing of the collinear parton distribution:

$$\mathcal{F}_p(x, k_T^2, \mu^2) = G(k_T^2; \sigma) x p(x, \mu^2). \quad (4)$$

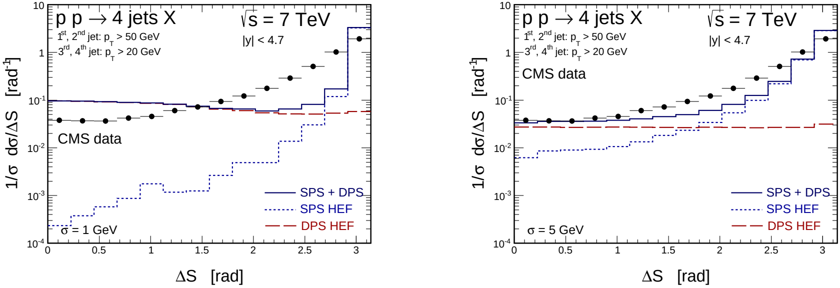


Figure: Distribution in ΔS for the toy Gaussian model of TMDs with σ = 1 GeV (left) and σ = 5 GeV (right).

Asymmetric cuts

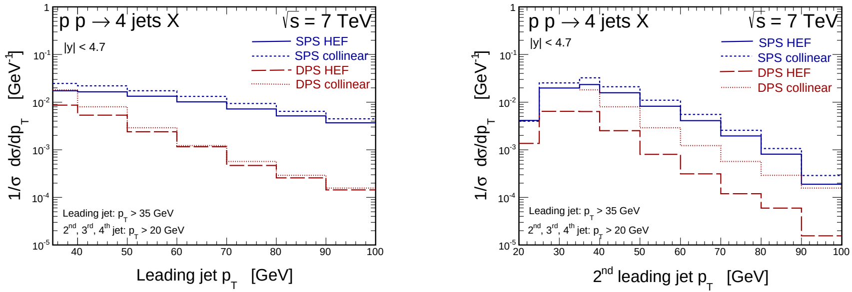


Figure: LO collinear and HEF predictions for the 1st and 2nd leading jets with the asymmetric cuts.

Asymmetric cuts

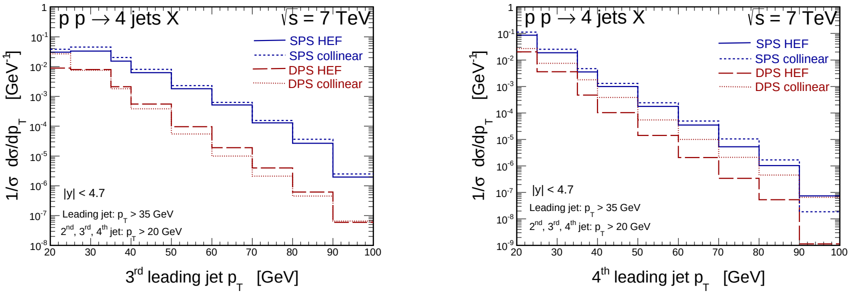


Figure: LO collinear and HEF predictions for the 3rd and 4th leading jets with asymmetric cuts.

Asymmetric cuts

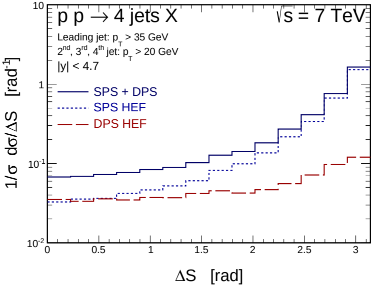
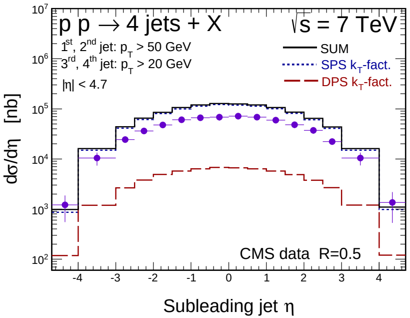
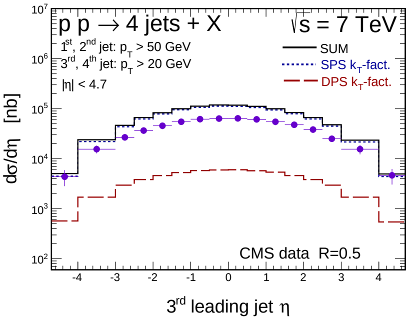


Figure: HEF prediction for ΔS with asymmetric cuts.

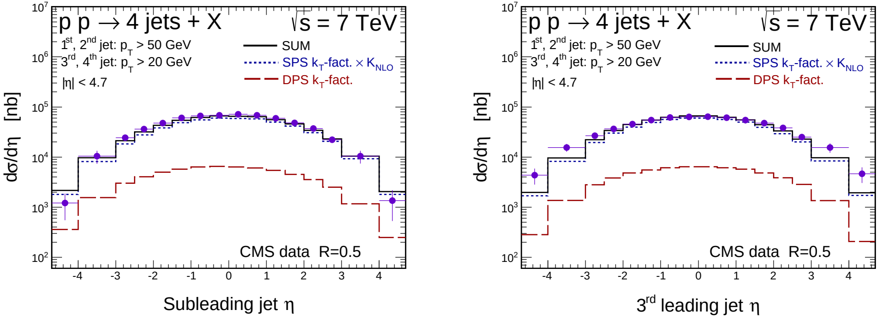
CMS four-jets: SPS + DPS in the k_T-factorization





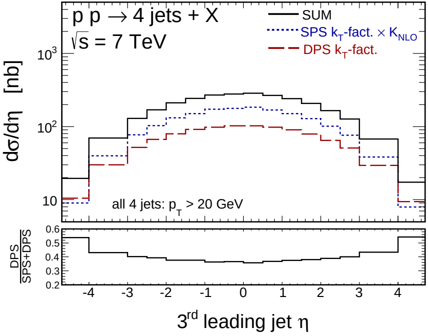
- the SPS component above the data ⇒ the same problem with the ALPGEN code (LO collinear approach) ⇒ exact SPS NLO calculations needed?
- first full SPS NLO (collinear) four-jets: Z. Bern et al., Phys. Rev. Lett. 109, 042001 (2012)
  NLO corrections ⇒ damping of the cross section ⇒ $K_{NLO} \approx 0.5$
- SPS $2 \to 2$: $K_{NLO} \approx 1.1 - 1.2$ ⇒ much less important for DPS
- much better description of exp. data for harder p_T cuts

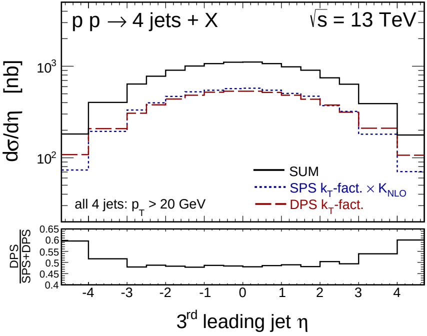
CMS four-jets: SPS + DPS in the k_T-factorization



- Now: quite good description of the CMS data
- 3rd leading jet (softer): forward/backward region slightly underestimated
- very small DPS contribution ⇒ unsupportive CMS cuts: $1^{st}, 2^{nd}$ jet $p_T > 50$ GeV
- **DPS favoured**: small p_T region

DPS effects in four-jet sample: lowering p_T cuts

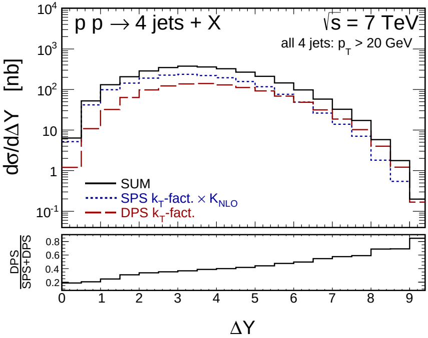


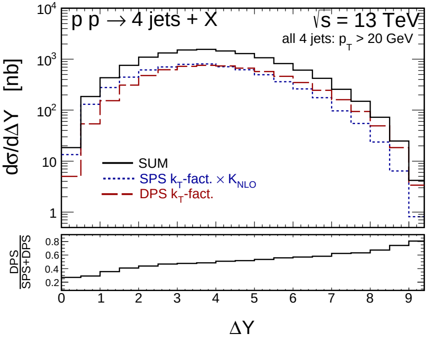


- all 4 jets: $20 < p_T < 100 GeV$
- 13 TeV: DPS contribution $\geq 50\%$
- **DPS favoured**: forward/backward rapidity region

DPS effects in four-jet sample: large rapidity distance

Rapidity difference between jets most remote in rapidity

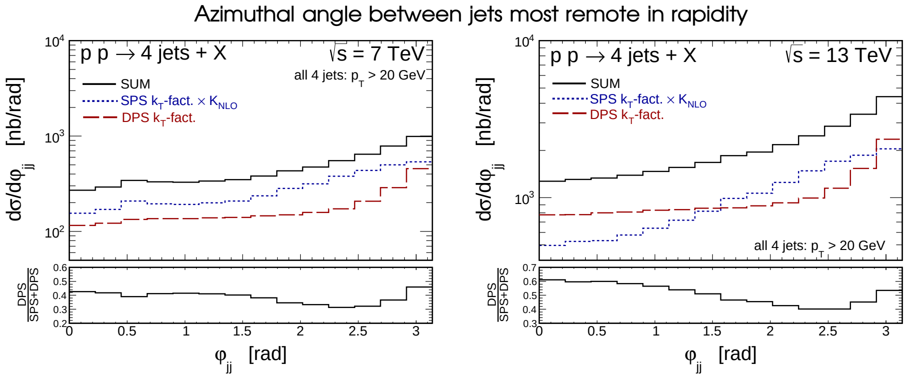




- 13 TeV:

  $\Delta Y > 6 \Rightarrow$ four-jet sample dominated by DPS

DPS effects in four-jet sample: large rapidity distance



- 13 TeV:

  $\varphi_{jj} < \frac{\pi}{2} \Rightarrow$ four-jet sample dominated by DPS

Another DPS/SPS discriminating variable

Define the variable (used by ATLAS):

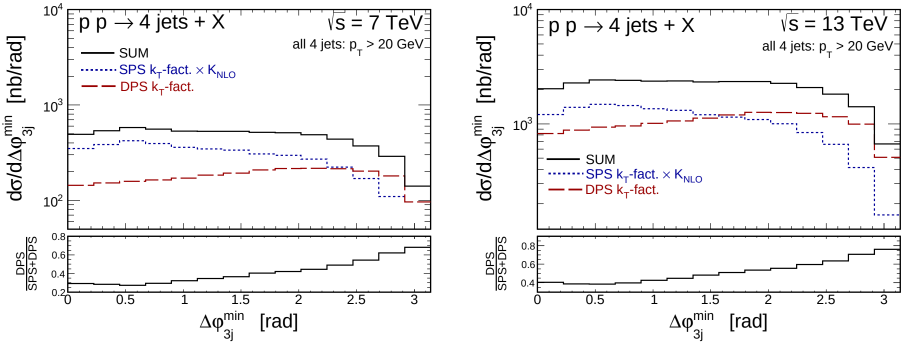
$$\Delta\varphi_{3j}^{min} \equiv min_{\substack{i,j,k\in\{1,2,3,4\} \\ i\neq j\neq k}} \left(|\varphi_i - \varphi_j| + |\varphi_j - \varphi_k|\right) . \quad (5)$$

Three out of four azimuthal angles enter.

Configurations with one jet recoiling against the other three are characterised by lower values of $\Delta\varphi_{3j}^{min}$ with respect to the two-against-two configurations. A minimum, in fact, will be obtained in the first case for the three i, j, k jets in the same half hemisphere, whereas it is not possible for the second configuration. Obviously, the first one would be allowed only by SPS in a collinear framework, whereas the second one would be enhanced by DPS. In k_t-factorization approach this situation is smeared out by the presence of transverse momenta of the initial state partons.

DPS effects in four-jet sample: special angular correlation

Minimum azimuthal separation between any three jets



- variable proposed by ATLAS analysis: JHEP 12, 105 (2015)
- distinguishes events with two-against-two jets (large $\Delta\varphi_{3j}^{min}$) from the recoil of three jets against one jet (small $\Delta\varphi_{3j}^{min}$)
- 13 TeV:

$$\Delta\varphi_{3j}^{min} > \frac{\pi}{2} \Rightarrow \text{four-jet sample dominated by DPS}$$

Conclusions

- A formalism how to calculate SPS four jet production within k_t-factorization with off-shell partons has been developed and proposed.
- Corresponding machinery for SPS has been constructed.
- A similar machinery for DPS four-jet production has been constructed.
- We have performed first calculations and compared our results with ATLAS and CMS data.
- The difference of the role of cuts on results of collinear and k_t-factorization results for DPS was discussed.
- A recipe for DPS dominated four-jet sample at $\sqrt{s} = 13$ TeV has been discussed.

Conclusions

How to maximize DPS in k_t-factorization?
(similar to LO collinear)

1. crucial: lower transverse momentum cuts for all 4 jets: $p_T > 20$ GeV
   - asymmetric configuration also acceptable:
     leading jet $p_T > 35$ GeV; $2^{nd}, 3^{rd}, 4^{th}$ jet $p_T > 20$ GeV
     however any further increasing of the p_T cuts leads to significant damping of the DPS contribution
2. concentrate on large jet-jet rapidity separations: $\Delta Y > 6$
3. useful angular jet-jet correlations: $\varphi_{jj} < \frac{\pi}{2}$, $\Delta\varphi_{3j}^{min} > \frac{\pi}{2}$

Thank you for your attention!