

High energy effects in multi-jet production at LHC

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in collaboration with

F. Caporale, G. Chachamis, D. Gordo Gómez, B. Murdaca, A. Sabio Vera

based on

Phys. Rev. Lett. 116 (2016) no.1, 012001 [[arXiv:1508.07711](https://arxiv.org/abs/1508.07711)]

Eur. Phys. J. C 76 (2016) no.3, 165 [[arXiv:1512.03364](https://arxiv.org/abs/1512.03364)]

[arXiv:1603.07785](https://arxiv.org/abs/1603.07785); [arXiv:1606.00574](https://arxiv.org/abs/1606.00574)

Low-x Meeting 2016
June 6th - 11th, 2016
Károly Róbert Főiskola, Gyöngyös (Hungary)



Outline

- 1 Introduction
 - Motivation
- 2 Three-jet production
 - A new way to probe BFKL
 - Three-jet phenomenology: hadronic level analysis
- 3 Four-jet production
 - Four-jet BFKL azimuthal profile
 - Four-jet phenomenology: hadronic level predictions
- 4 Conclusions & Outlook

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Motivation

So far, search for BFKL effects had these general drawbacks:

- ◊ too low \sqrt{s} or rapidity intervals among tagged particles in the final state
- ◊ too inclusive observables, other approaches can fit them

Advent of LHC:

- higher energies \leftrightarrow larger rapidity gaps
- unique opportunity to **test pQCD in the high-energy limit**
- disentangle applicability region of energy-log resummation (**BFKL approach**)

[V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975, 1976, 1977)]
[Y.Y. Balitskii, L.N. Lipatov (1978)]

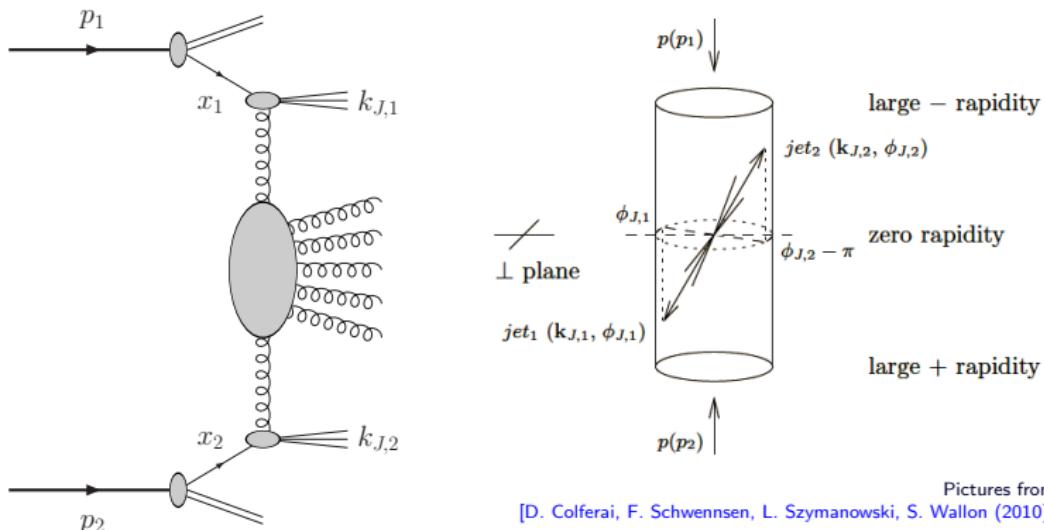
Last years:

hadroproduction of two jets featuring high transverse momenta and well separated in rapidity, so called **Mueller–Navelet jets**...

- ◊ ...possibility to define *infrared-safe* observables...
- ◊ ...and constrain the PDFs...
- ◊ ...theory vs experiment

[B. Ducloué, L. Szymanowski, S. Wallon (2014)]
[F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2014)]

Mueller–Navelet jets



Pictures from
[D. Colferai, F. Schwennsen, L. Szymanowski, S. Wallon (2010)]

...large jet transverse momenta: $\vec{k}_{J,1}^2 \sim \vec{k}_{J,2}^2 \gg \Lambda_{\text{QCD}}^2$

...large rapidity gap between jets (high energies) $\Rightarrow \Delta y = \ln \frac{x_{J,1}x_{J,2}s}{|\vec{k}_{J,1}||\vec{k}_{J,2}|}$

How could we further and deeply probe BFKL?

1. Study a less inclusive two-body final state...

Di-hadron production

[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016)]

- ◊ inclusive production of a pair of charged light hadrons well separated in rapidity
- ◊ hadrons can be detected at the LHC at much smaller values of the transverse momentum than jets
- ◊ possibility to constrain not only the PDFs, but also the FFs!

2. Study three- and four-body final state processes...

Multi-jet production

- ◊ demand the tagging of one or/and two further jets in more central regions of the detectors with a relative separation in rapidity from each other
- ◊ definition of new, **suitable BFKL observables...**
- ◊ ...in order to further investigate the azimuthal distribution of the final state

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- ◊ ...in order to further investigate the azimuthal distribution of the final state

Looking for new observables...

- ! We would like to study observables for which the k_T (any k_T along the BFKL ladder) enters the game...
 - ◊ ...to probe not only the general properties of the BFKL ladder, but also “to peek into the interior” ...
 - ◊ ...by studying azimuthal decorrelations where the k_T of extra particles introduces a new dependence...

...multi-jet production!

[R. Maciula, A. Szczurek (2014, 2015)]
[K. Kutak, R. Maciula, M. Serino, A. Szczurek, A. van Hameren (2016, 2016)]

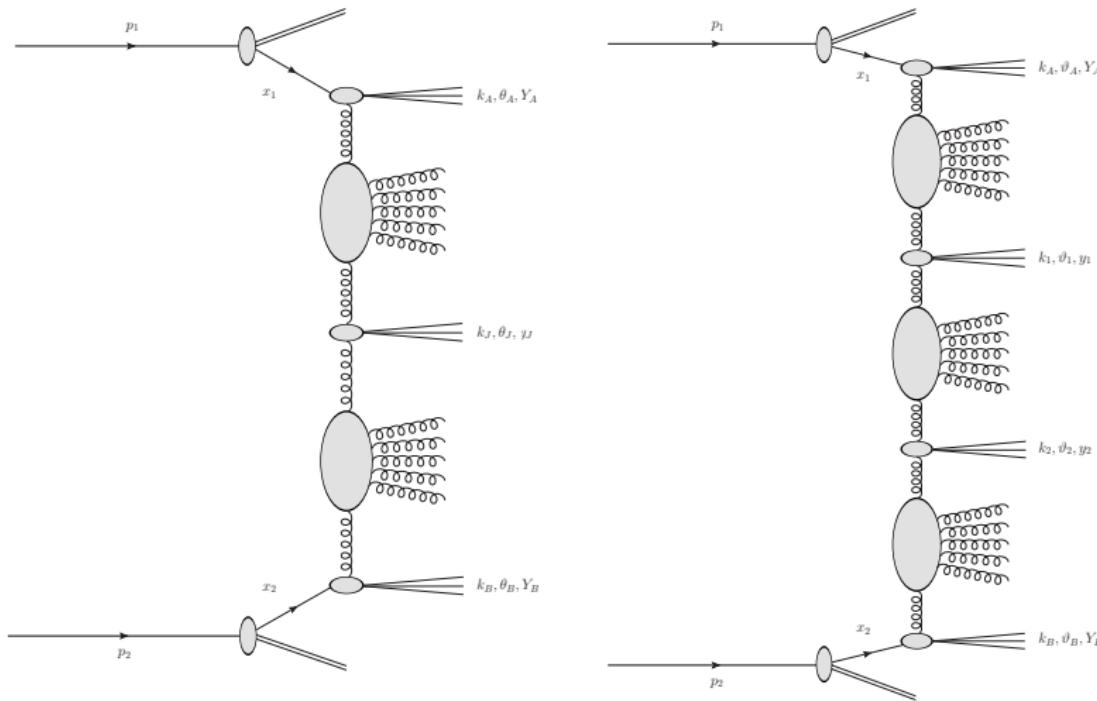
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Three- and four-jet production



[F. Caporale, G. Chachamis, B. Murdaca, A. Sabio Vera (2015)]

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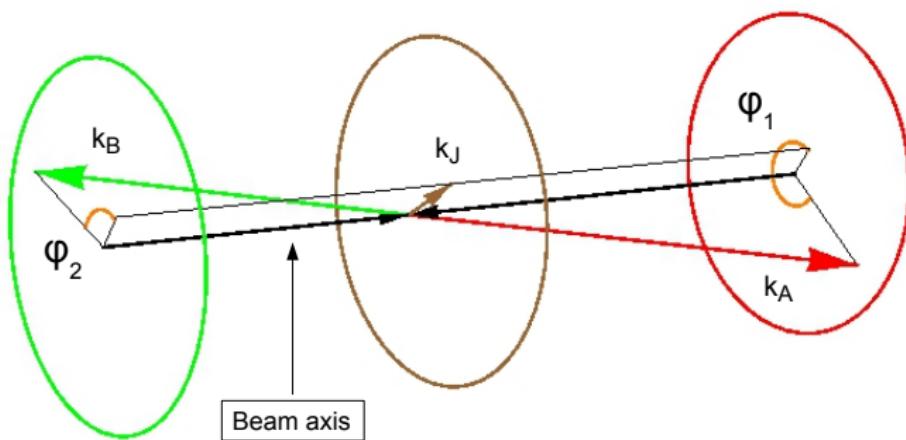
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An event with three tagged jets

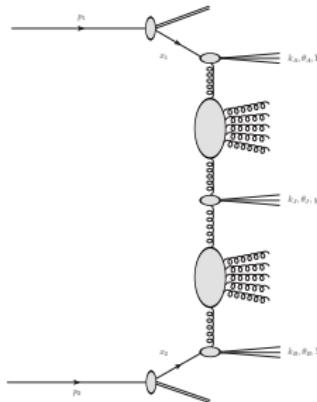


$$Y_B < y_J < Y_A$$

The three-jet partonic cross section

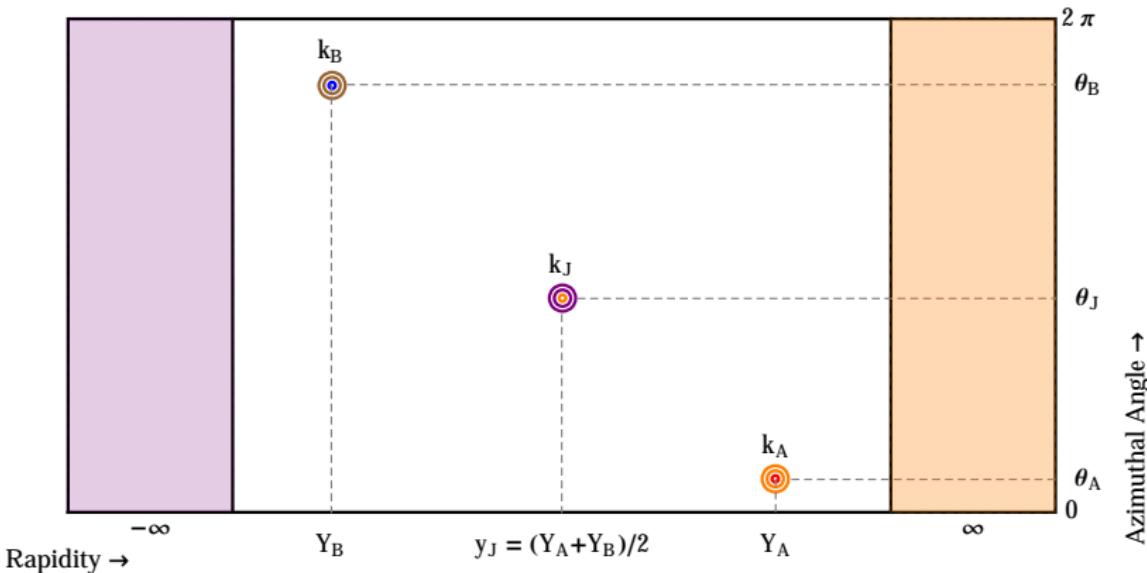
Starting point: differential partonic cross-section (no PDFs)

$$\frac{d^3\hat{\sigma}^{\text{3-jet}}}{dk_J d\theta_J dy_J} = \frac{\bar{\alpha}_s}{\pi k_J} \int d^2\vec{p}_A \int d^2\vec{p}_B \delta^{(2)}(\vec{p}_A + \vec{k}_J - \vec{p}_B) \\ \times \varphi(\vec{k}_A, \vec{p}_A, Y_A - y_J) \varphi(\vec{p}_B, \vec{k}_B, y_J - Y_B)$$



- Multi-Regge kinematics rapidity ordering: $Y_B < y_J < Y_A$
- k_J lie above the experimental resolution scale
- φ is the LO BFKL gluon Green function
- $\bar{\alpha}_s = \alpha_s N_c / \pi$

A three-jet primitive lego-plot



Generalized azimuthal correlations - partonic level

Prescription: integrate over all angles after using the projections on the two azimuthal angle differences indicated below...

→ ...to define:

$$\begin{aligned} & \int_0^{2\pi} d\theta_A \int_0^{2\pi} d\theta_B \int_0^{2\pi} d\theta_J \cos(M(\theta_A - \theta_J - \pi)) \cos(N(\theta_J - \theta_B - \pi)) \frac{d^3 \hat{\sigma}^{3\text{-jet}}}{dk_J d\theta_J dy_J} \\ &= \bar{\alpha}_s \sum_{L=0}^N \binom{N}{L} (k_J^2)^{\frac{L-1}{2}} \int_0^\infty dp^2 (p^2)^{\frac{N-L}{2}} \int_0^{2\pi} d\theta \frac{(-1)^{M+N} \cos(M\theta) \cos((N-L)\theta)}{\sqrt{(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2} \cos\theta)^N}} \\ & \times \phi_M(k_A^2, p^2, Y_A - y_J) \phi_N(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2} \cos\theta, k_B^2, y_J - Y_B) \end{aligned}$$

Main observables: **generalized azimuthal correlation momenta**

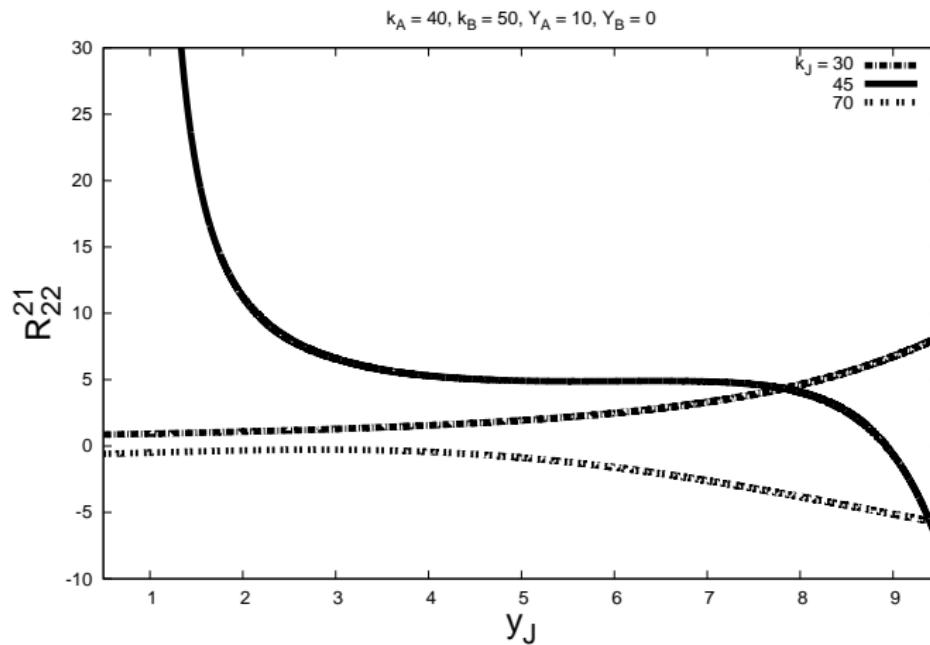
$$\mathcal{R}_{PQ}^{MN} = \frac{\mathcal{C}_{MN}}{\mathcal{C}_{PR}} = \frac{\langle \cos(M(\theta_A - \theta_J - \pi)) \cos(N(\theta_J - \theta_B - \pi)) \rangle}{\langle \cos(P(\theta_A - \theta_J - \pi)) \cos(Q(\theta_J - \theta_B - \pi)) \rangle}$$

Remove the contribution from the zero conformal spin

→ drastically reduce the dependence on collinear configurations

study \mathcal{R}_{PQ}^{MN} with integer $M, N, P, Q > 0$

Partonic prediction of \mathcal{R}_{22}^{21} for $k_J = 30, 45, 70$ GeV



[F. Caporale, G. Chachamis, B. Murdaca, A. Sabio Vera (2015)]

$Y_A - Y_B$ is fixed to 10; y_J varies between 0.5 and 9.5.

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Next step: hadronic level predictions

- Introduce PDFs and running of the strong coupling:

$$\begin{aligned} \frac{d\sigma^{3\text{-jet}}}{dk_A dY_A d\theta_A dk_B dY_B d\theta_B dk_J dy_J d\theta_J} = \\ \frac{8\pi^3 C_F \bar{\alpha}_s(\mu_R)^3}{N_C^3} \frac{x_{J_A} x_{J_B}}{k_A k_B k_J} \int d^2 \vec{p}_A \int d^2 \vec{p}_B \delta^{(2)}(\vec{p}_A + \vec{k}_J - \vec{p}_B) \\ \times \left(\frac{N_C}{C_F} f_g(x_{J_A}, \mu_F) + \sum_{r=q,\bar{q}} f_r(x_{J_A}, \mu_F) \right) \\ \times \left(\frac{N_C}{C_F} f_g(x_{J_B}, \mu_F) + \sum_{s=q,\bar{q}} f_s(x_{J_B}, \mu_F) \right) \\ \times \varphi(\vec{k}_A, \vec{p}_A, Y_A - y_J) \varphi(\vec{p}_B, \vec{k}_B, y_J - Y_B) \end{aligned}$$

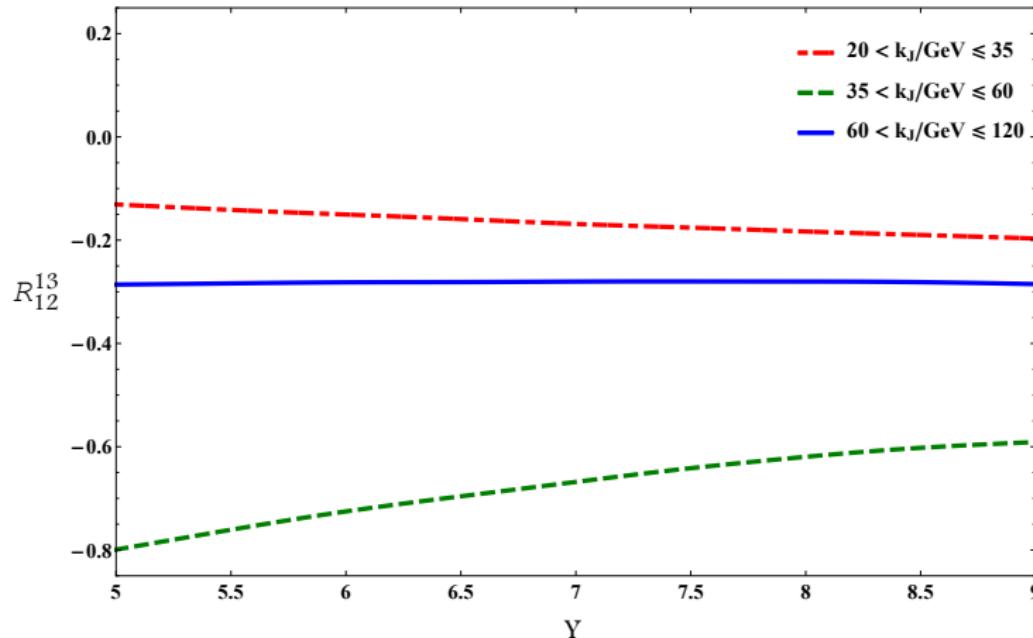
- Match the LHC kinematical cuts (integrate $d\sigma^{3\text{-jet}}$ on k_T and rapidities):
 - ◊ 1. $k_A \geq 35$ GeV; $k_B \geq 35$ GeV; symmetric cuts
 - 2. $k_A \geq 35$ GeV; $k_B \geq 50$ GeV; asymmetric cuts
 - ◊ $Y = Y_A - Y_B$ fixed;
 $y_J = (Y_A + Y_B)/2$
 - ◊ $\sqrt{s} = 7, 13$ TeV

Three-jet phenomenology: hadronic level analysis

R_{12}^{13} vs $Y = Y_A - Y_B$ for three different k_J bins

$k_A^{\min} = 35 \text{ GeV}$, $k_B^{\min} = 35 \text{ GeV}$, $k_A^{\max} = k_B^{\max} = 60 \text{ GeV}$ (symmetric)

$\sqrt{s} = 13 \text{ TeV}$; $k_B^{\min} = 35 \text{ GeV}$



[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2016)]

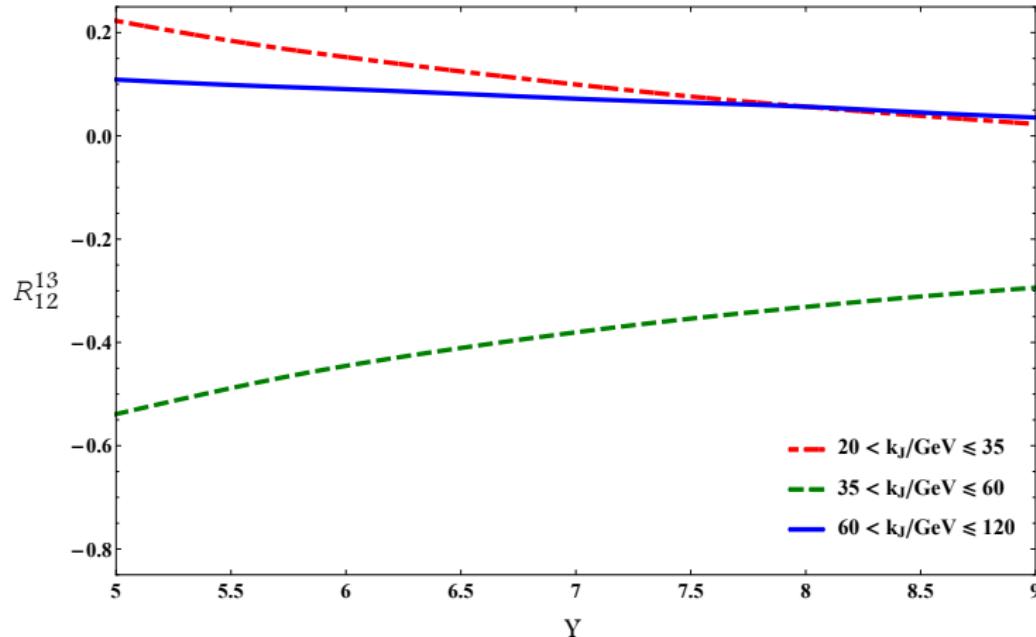
Y is the rapidity difference between the most forward/backward jet; $y_J = \frac{Y_A + Y_B}{2}$.

Three-jet phenomenology: hadronic level analysis

R_{12}^{13} vs $Y = Y_A - Y_B$ for three different k_J bins

$k_A^{\min} = 35 \text{ GeV}$, $k_B^{\min} = 50 \text{ GeV}$, $k_A^{\max} = k_B^{\max} = 60 \text{ GeV}$ (asymmetric)

$\sqrt{s} = 13 \text{ TeV}$; $k_B^{\min} = 50 \text{ GeV}$

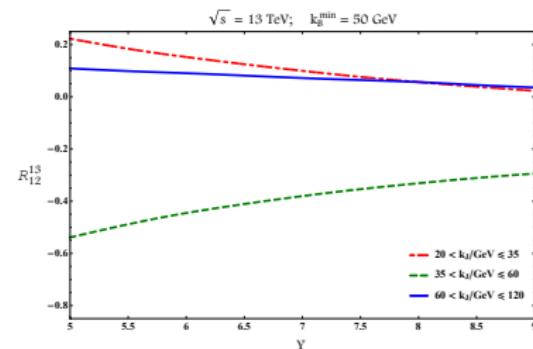
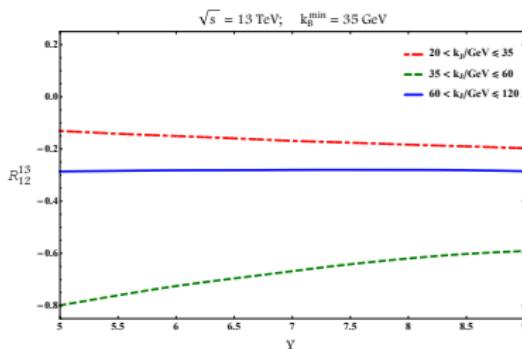
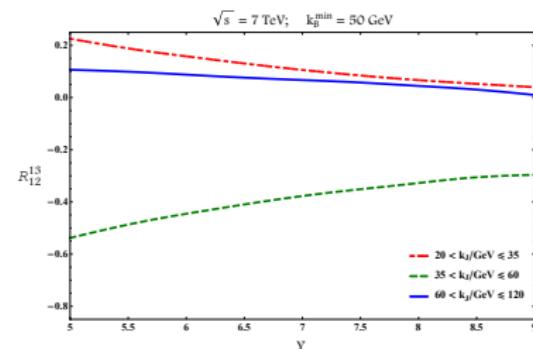
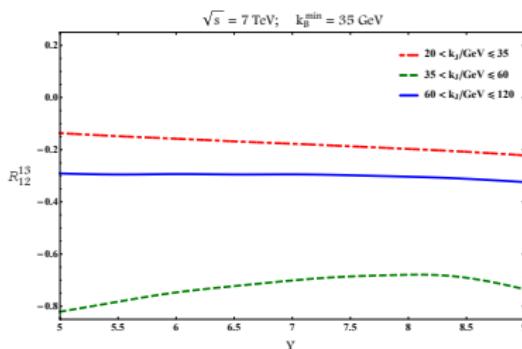


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Y is the rapidity difference between the most forward/backward jet; $y_J = \frac{Y_A + Y_B}{2}$.

Three-jet phenomenology: hadronic level analysis

R_{12}^{13} vs $Y = Y_A - Y_B$, \sqrt{s} and k_B^{\min} for three k_J bins



[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2016)]

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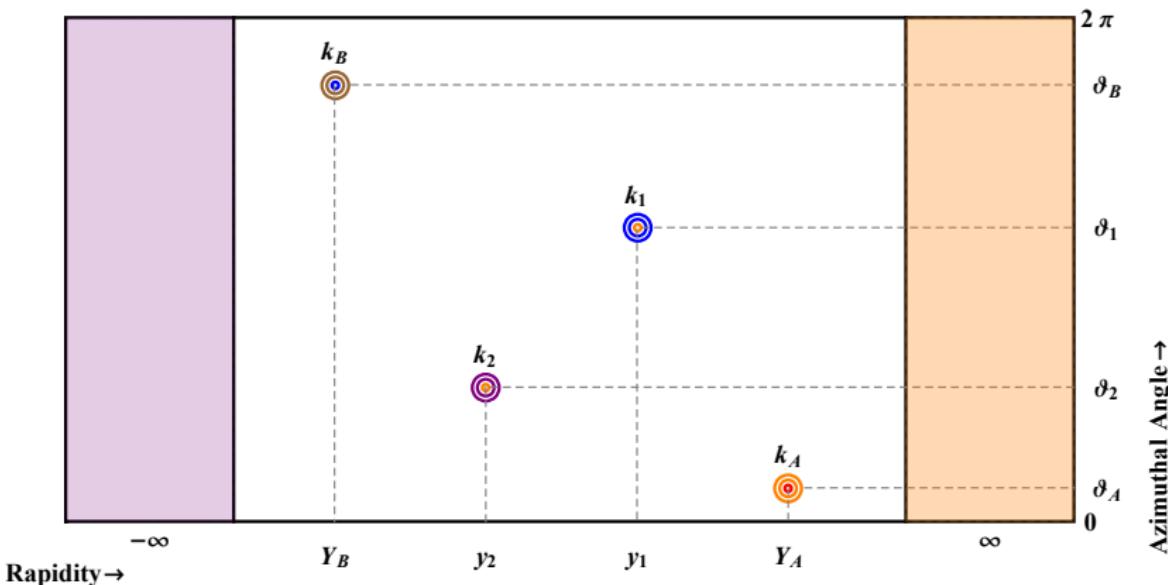
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Generalized azimuthal correlations - partonic level

As for the three-jet case...

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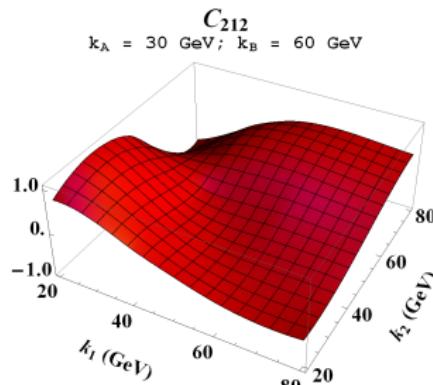
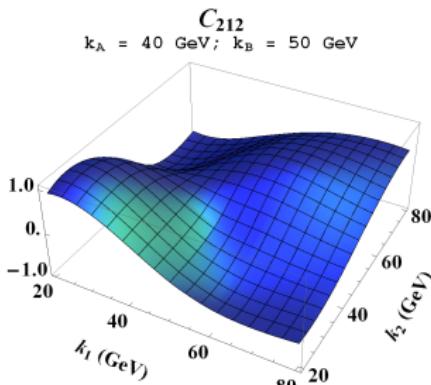
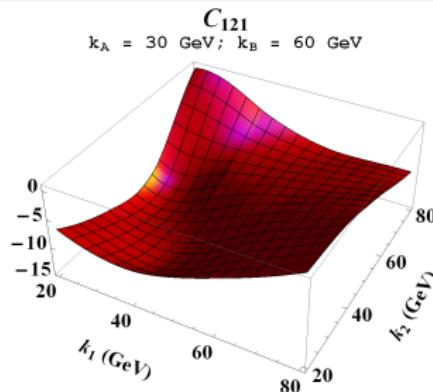
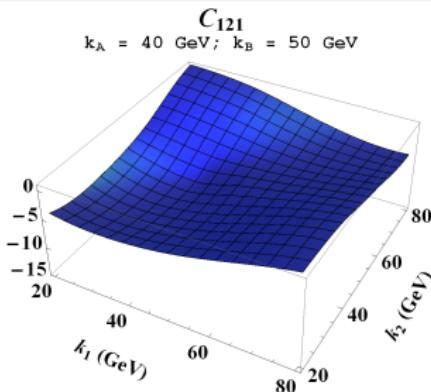
→ ...to define:

$$\mathcal{C}_{MNL} = \int_0^{2\pi} d\vartheta_A \int_0^{2\pi} d\vartheta_B \int_0^{2\pi} d\vartheta_1 \int_0^{2\pi} d\vartheta_2 \cos(M(\vartheta_A - \vartheta_1 - \pi)) \cos(N(\vartheta_1 - \vartheta_2 - \pi)) \cos(L(\vartheta_2 - \vartheta_B - \pi)) \frac{d^6\hat{\vartheta}^{4\text{-jet}}(\vec{k}_A, \vec{k}_B, Y_A - Y_B)}{dk_1 dy_1 d\vartheta_1 dk_2 d\vartheta_2 dy_2}$$

Main observables: **generalized azimuthal correlation momenta**

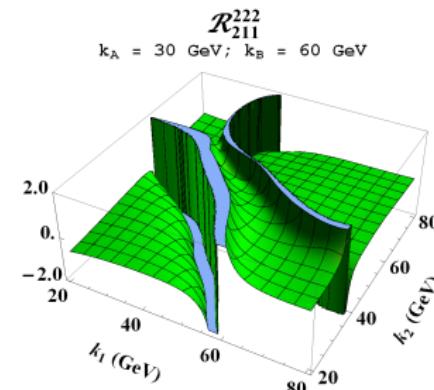
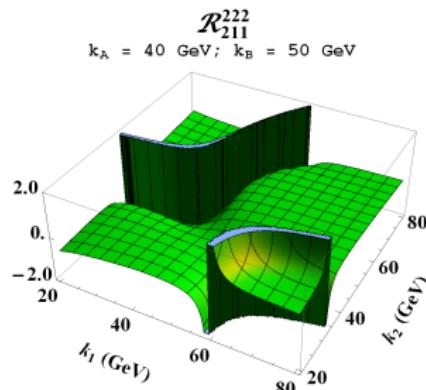
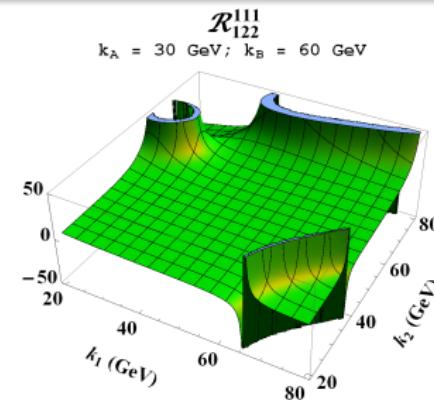
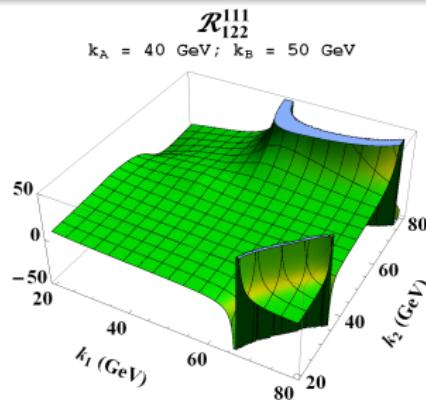
$$\mathcal{R}_{PQR}^{MNL} = \frac{\mathcal{C}_{MNL}}{\mathcal{C}_{PRQ}} = \frac{\langle \cos(M(\vartheta_A - \vartheta_1 - \pi)) \cos(N(\vartheta_1 - \vartheta_2 - \pi)) \cos(L(\vartheta_2 - \vartheta_B - \pi)) \rangle}{\langle \cos(P(\vartheta_A - \vartheta_1 - \pi)) \cos(Q(\vartheta_1 - \vartheta_2 - \pi)) \cos(R(\vartheta_2 - \vartheta_B - \pi)) \rangle}$$

Partonic prediction of \mathcal{C}_{MNL} vs $k_{1,2}$



[F. Caporale, F.G. C., G. Chachamis, A. Sabio Vera (2016)]

Partonic prediction of \mathcal{R}_{PQR}^{MNL} vs $k_{1,2}$



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Next step: hadronic level predictions

- Introduce PDFs and running of the strong coupling

- Use realistic LHC kinematical cuts:

- ◊ 1. $k_A^{\min} = 35 \text{ GeV}$, $k_A^{\max} = 60 \text{ GeV}$

$$k_B^{\min} = 45 \text{ GeV}, k_B^{\max} = 60 \text{ GeV}$$

$$k_1^{\min} = 20 \text{ GeV}, k_1^{\max} = 35 \text{ GeV}$$

$$k_2^{\min} = 60 \text{ GeV}, k_2^{\max} = 90 \text{ GeV}$$

- 2. $k_A^{\min} = 35 \text{ GeV}$, $k_A^{\max} = 60 \text{ GeV}$

$$k_B^{\min} = 45 \text{ GeV}, k_B^{\max} = 60 \text{ GeV}$$

$$k_1^{\min} = 25 \text{ GeV}, k_1^{\max} = 50 \text{ GeV}$$

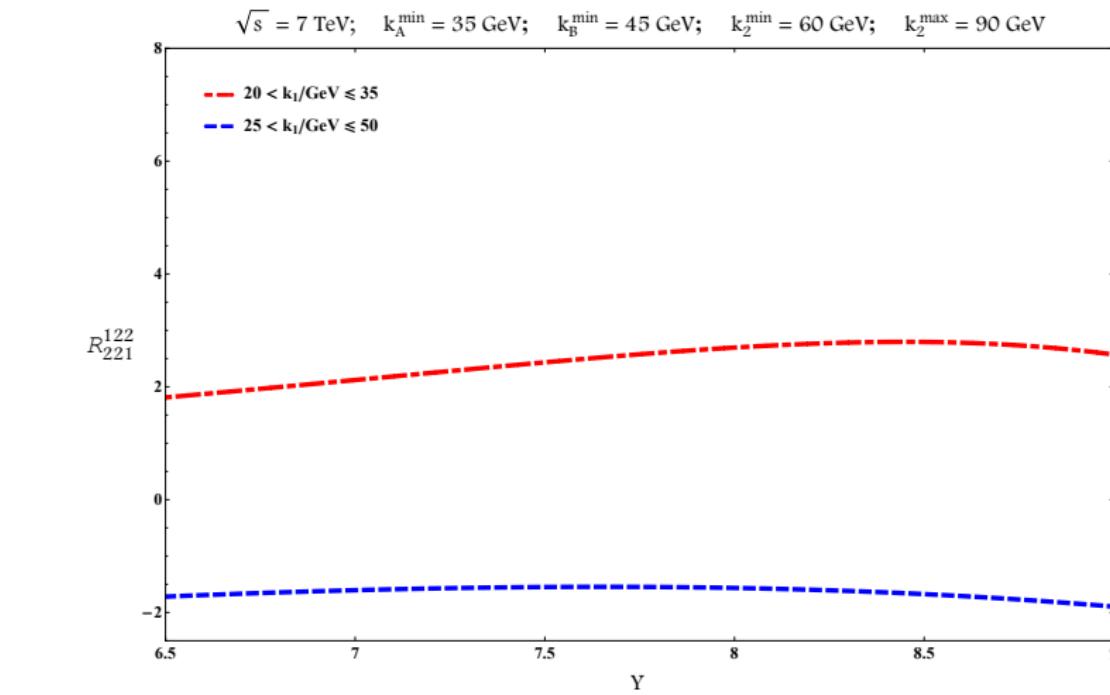
$$k_2^{\min} = 60 \text{ GeV}, k_2^{\max} = 90 \text{ GeV}$$

- ◊ $Y = Y_A - Y_B$ fixed;

$$Y_A - y_1 = y_1 - y_2 = y_2 - Y_B = Y/3$$

- ◊ $\sqrt{s} = 7, 13 \text{ TeV}$

R_{221}^{122} at $\sqrt{s} = 7$ TeV vs $Y = Y_A - Y_B$ for two k_1 bins

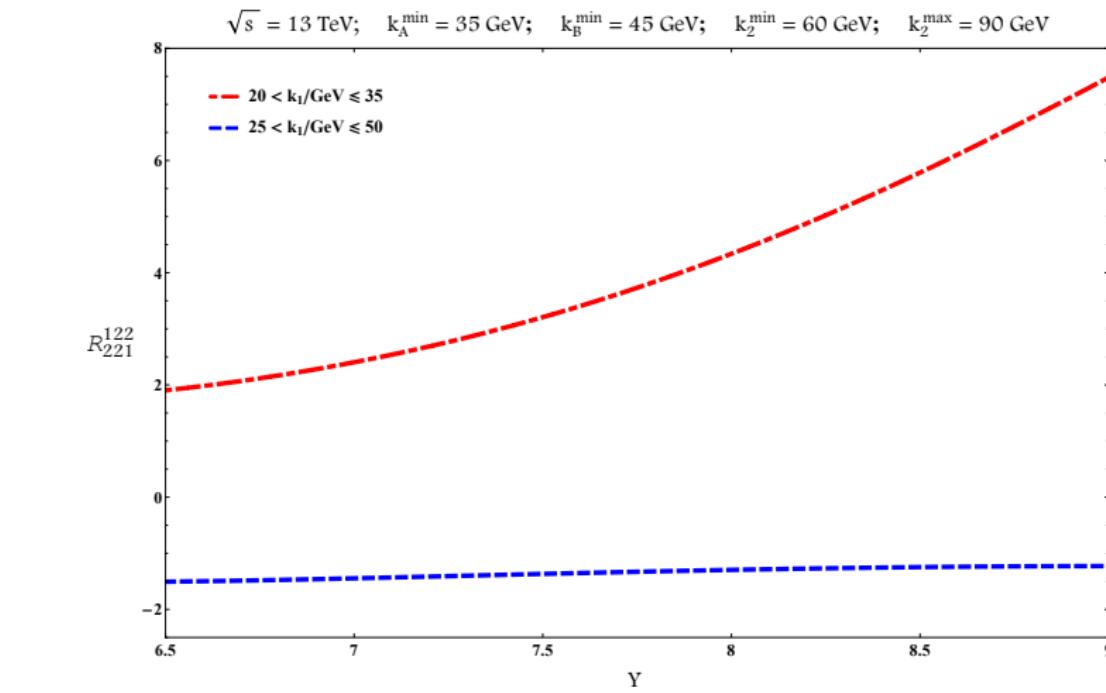


[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2016)]

Y is the rapidity difference between the most forward/backward jet;

$$Y_A - y_1 = y_1 - y_2 = y_2 - Y_B = Y/3.$$

R_{221}^{122} at $\sqrt{s} = 13$ TeV vs $Y = Y_A - Y_B$ for two k_1 bins

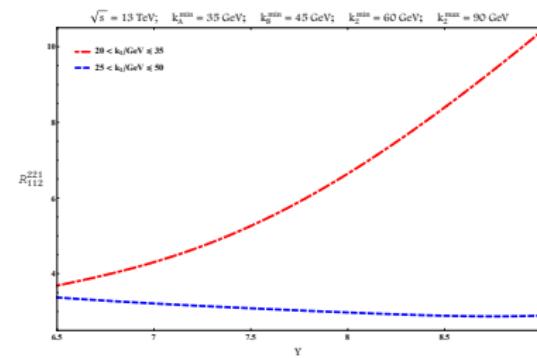
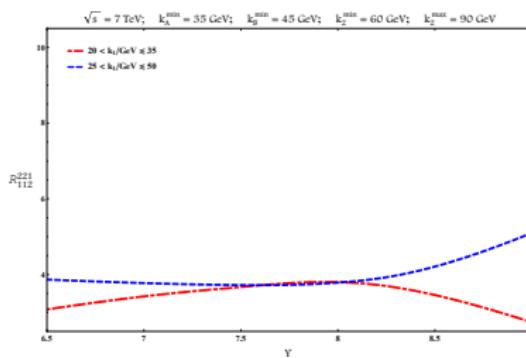
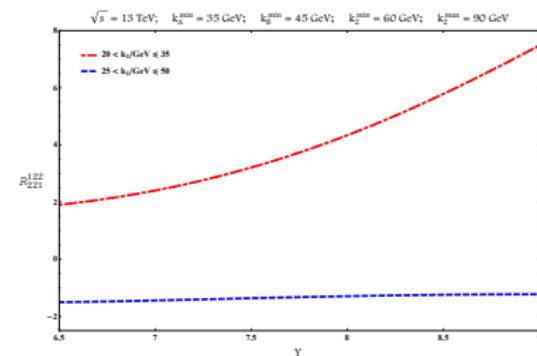
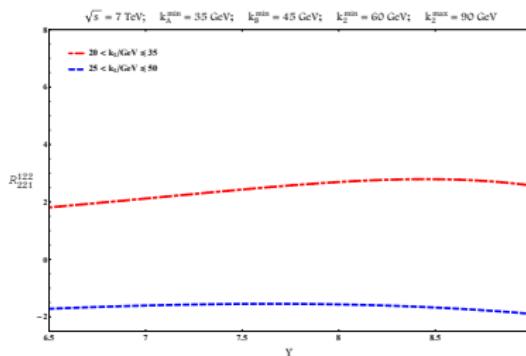


[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2016)]

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R_{221}^{122} and R_{112}^{221} vs $Y = Y_A - Y_B$ and \sqrt{s} for two k_1 bins



[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2016)]

Conclusions...

- Study of processes with **three** and **four** tagged jets to propose new, more exclusive, BFKL observables: **generalized azimuthal correlation momenta**
- Ratios of correlation functions used to minimize the influence of higher order corrections
- Comparison with experimental data suggested and needed

...Outlook

- ◊ Dependence on rapidity bins (asymmetric configurations for the central jet(s))
[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (in progress)]
- ◊ Three- and four-jets in the NLA accuracy: improved kernel(s), scale optimization
[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (in progress)]
- ◊ Comparison with analyses where the four-jet predictions stem from two independent gluon ladders
[R. Maciula, A. Szczurek (2014, 2015)]
[K. Kutak, R. Maciula, M. Serino, A. Szczurek, A. van Hameren (2016, 2016)]
- ◊ Probe BFKL through other processes...
 - ▶ Di-hadron production in the full NLA
 - ▶ Heavy quark pair production[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (in progress)]

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To be continued...

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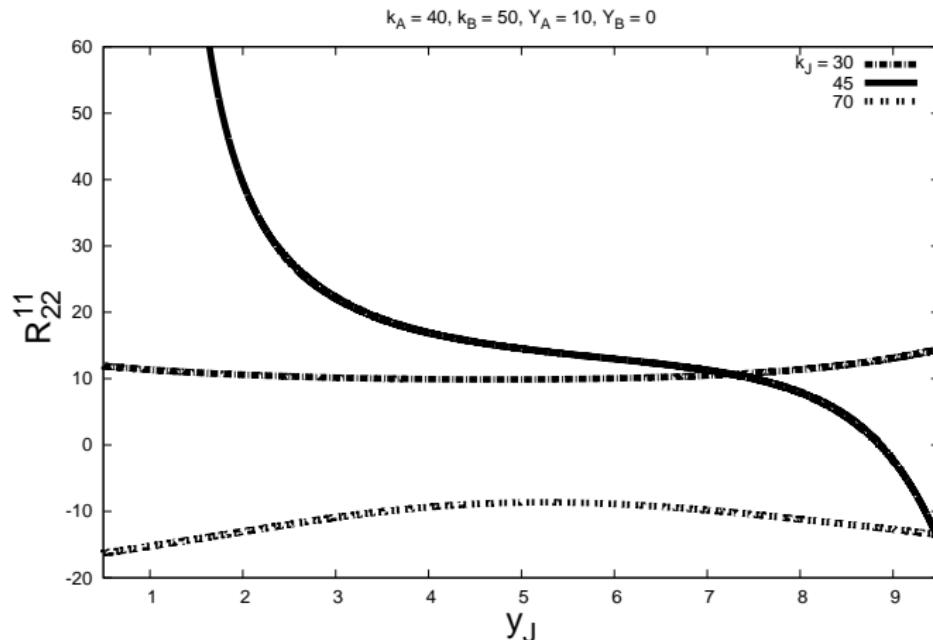
R. Bertelli INFN, Frascati

Thanks for your
attention!!

BACKUP slides

BACKUP slides

Partonic prediction of \mathcal{R}_{22}^{11} for $k_J = 30, 45, 70$ GeV



[F. Caporale, G. Chachamis, B. Murdaca, A. Sabio Vera (2015)]

$Y_A - Y_B$ is fixed to 10; y_J varies between 0.5 and 9.5.

BACKUP slides

Four-jets: generalized azimuthal coefficients - partonic level

$$\begin{aligned}
 C_{MNL} &= \int_0^{2\pi} d\vartheta_A \int_0^{2\pi} d\vartheta_B \int_0^{2\pi} d\vartheta_1 \int_0^{2\pi} d\vartheta_2 \cos(M(\vartheta_A - \vartheta_1 - \pi)) \\
 &\quad \cos(N(\vartheta_1 - \vartheta_2 - \pi)) \cos(L(\vartheta_2 - \vartheta_B - \pi)) \frac{d^6 \sigma^{\text{4-jet}}(\vec{k}_A, \vec{k}_B, Y_A - Y_B)}{dk_1 dy_1 d\vartheta_1 dk_2 d\vartheta_2 dy_2} \\
 &= \frac{2\pi^2 \bar{\alpha}_s (\mu_R)^2}{k_1 k_2} (-1)^{M+N+L} (\tilde{\Omega}_{M,N,L} + \tilde{\Omega}_{M,N,-L} + \tilde{\Omega}_{M,-N,L} \\
 &\quad + \tilde{\Omega}_{M,-N,-L} + \tilde{\Omega}_{-M,N,L} + \tilde{\Omega}_{-M,N,-L} + \tilde{\Omega}_{-M,-N,L} + \tilde{\Omega}_{-M,-N,-L})
 \end{aligned}$$

with

$$\begin{aligned}
 \tilde{\Omega}_{m,n,l} &= \int_0^{+\infty} dp_A p_A \int_0^{+\infty} dp_B p_B \int_0^{2\pi} d\phi_A \int_0^{2\pi} d\phi_B \\
 &\quad \frac{e^{-im\phi_A} e^{il\phi_B} (p_A e^{i\phi_A} + k_1)^n (p_B e^{-i\phi_B} - k_2)^n}{\sqrt{(p_A^2 + k_1^2 + 2p_A k_1 \cos \phi_A)^n} \sqrt{(p_B^2 + k_2^2 - 2p_B k_2 \cos \phi_B)^n}} \\
 &\quad \varphi_m(|\vec{k}_A|, |\vec{p}_A|, Y_A - y_1) \varphi_l(|\vec{p}_B|, |\vec{k}_B|, y_2 - Y_B) \\
 &\quad \varphi_n\left(\sqrt{p_A^2 + k_1^2 + 2p_A k_1 \cos \phi_A}, \sqrt{p_B^2 + k_2^2 - 2p_B k_2 \cos \phi_B}, y_1 - y_2\right)
 \end{aligned}$$

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Four-jets: generalized azimuthal coefficients - partonic level

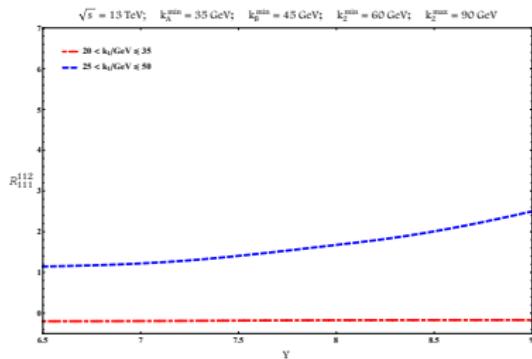
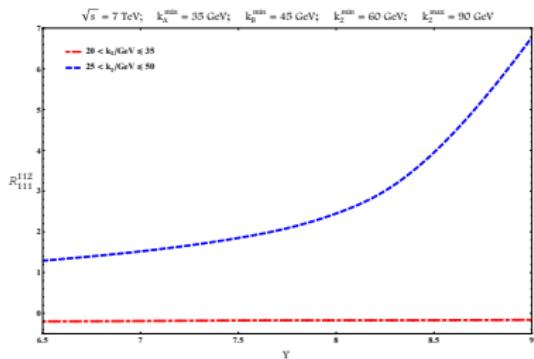
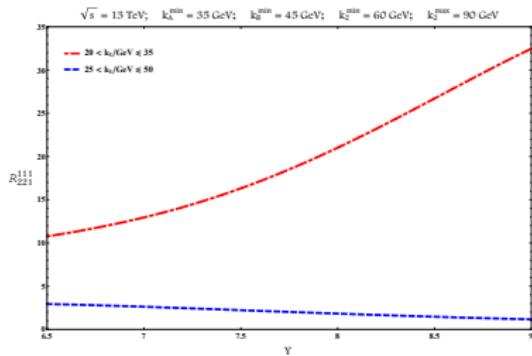
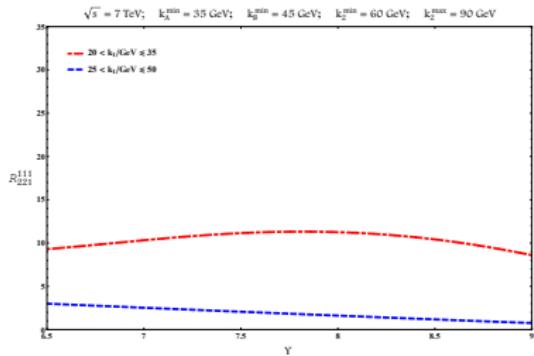
$$C_{MNL} = \int_0^{2\pi} d\vartheta_A \int_0^{2\pi} d\vartheta_B \int_0^{2\pi} d\vartheta_1 \int_0^{2\pi} d\vartheta_2 \cos(M(\vartheta_A - \vartheta_1 - \pi)) \cos(N(\vartheta_1 - \vartheta_2 - \pi)) \cos(L(\vartheta_2 - \vartheta_B - \pi)) \frac{d^6 \sigma^{\text{4-jet}}(\vec{k}_A, \vec{k}_B, Y_A - Y_B)}{dk_1 dy_1 d\vartheta_1 dk_2 d\vartheta_2 dy_2}$$

Main observables: **generalized azimuthal correlation momenta**

$$\mathcal{R}_{PQR}^{MNL} = \frac{C_{MNL}}{C_{PRQ}} = \frac{\langle \cos(M(\vartheta_A - \vartheta_1 - \pi)) \cos(N(\vartheta_1 - \vartheta_2 - \pi)) \cos(L(\vartheta_2 - \vartheta_B - \pi)) \rangle}{\langle \cos(P(\vartheta_A - \vartheta_1 - \pi)) \cos(Q(\vartheta_1 - \vartheta_2 - \pi)) \cos(R(\vartheta_2 - \vartheta_B - \pi)) \rangle}$$

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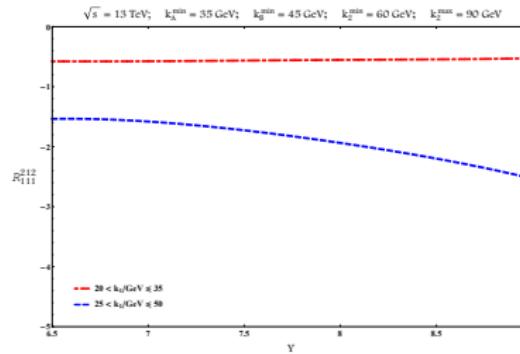
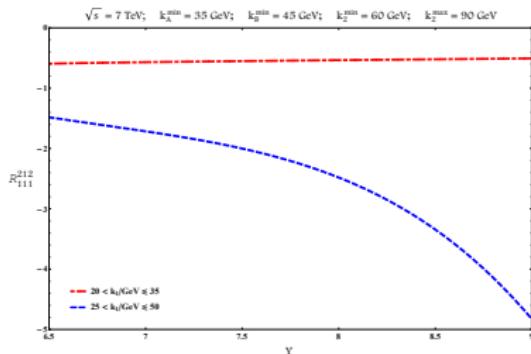
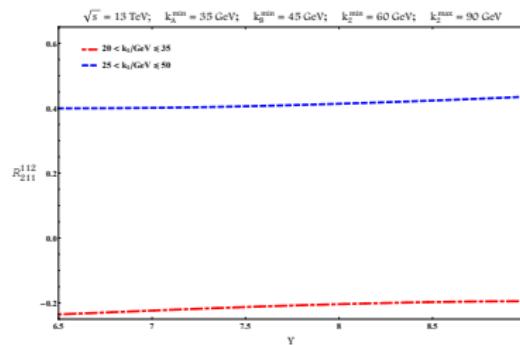
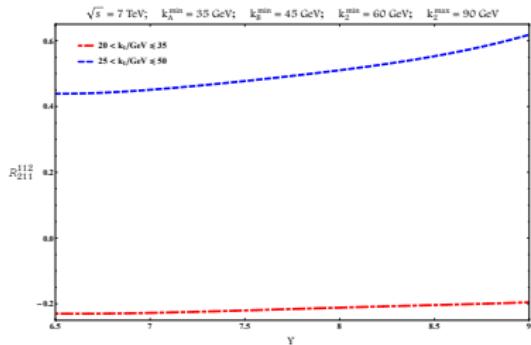
R_{221}^{111} and R_{111}^{112} vs $Y = Y_A - Y_B$ and \sqrt{s} for two k_1 bins



[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2016)]

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R_{211}^{112} and R_{111}^{212} vs $Y = Y_A - Y_B$ and \sqrt{s} for two k_1 bins



[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2016)]