Custodial Symmetry Violation in the Georgi-Machacek model

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Outline

• Introduction and Motivation
• The Georgi-Machacek Model (GM)
• Incorporating Custodial Violation
• Results
The Standard Model

- SM has a scalar SU(2)$_L$ doublet $\phi = \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix}$
  - Isospin $\frac{1}{2}$, hypercharge 1

- SM Lagrangian

$$\mathcal{L}_H = |D_\mu \phi|^2 - \left[ -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \right] - [y_f f_R \phi^\dagger F_L + \text{h. c.}]$$

- $\mu^2 < 0$: spontaneous symmetry breaking
  - non-zero vev; fixed by minimizing Higgs potential
  - Goldstones are gauged away (eaten by W and Z)

$$\nu^2 = \frac{\mu^2}{\lambda} \quad m_H = 2\lambda \nu^2 \quad \phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}} (\nu + h + iG^0) \end{pmatrix}$$
The Standard Model (Cont.)

- Higgs Couplings are proportional to generated mass

\[ \mathcal{L}_H = |D_\mu \phi|^2 - \left[-\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2\right] - [y_f \bar{f}_R \phi^\dagger F_L + h.c] \]

- \( W \) and \( Z \):

\[ M_W^2 = \frac{g^2 v^2}{4} \quad \text{hWW:} \quad 2i \frac{M_W^2}{v^2} g^{\mu\nu} \]

\[ M_Z^2 = \frac{g^2 v^2}{4 \cos^2(\theta_W)} \quad \text{hZZ:} \quad \frac{2iM_Z^2}{v^2} g^{\mu\nu} \]

- Fermions:

\[ m_f = \frac{y_f v}{\sqrt{2}} \quad \text{h} \bar{f} f: \quad \frac{i m_f}{v} \]
Beyond the SM: Why Isospin triplets

• Gauge invariance -> Fermion masses only generated by doublet; W and Z can be generated by any isospin multiplet
  – Can use triplet models to limit exotic fraction of $M_Z$ and $M_W$

• Can enhance $hVV$ couplings compared to SM
  – Could mask non-SM contribution to measured Higgs branching ratios
Problems with Higher Isospin

• The $\rho$ parameter: measure of relative strength of charged and neutral weak currents

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2(\theta_w)} = \sum_k 2 \left[ T_k (T_k + 1) - \frac{Y_k^2}{4} \right] v_k^2$$

PDG 2014: $\rho = 1.00040 \pm 0.00024$

• With SM doublet $\phi$, real triplet $\xi$ ($Y=0$) and complex triplet $\chi$ ($Y=1$) $\rho$ becomes:

$$\rho = \frac{v_{\phi}^2 + 4v_{\xi}^2 + 4v_{\chi}^2}{v_{\phi}^2 + 8v_{\chi}^2}$$
Georgi-Machacek Model (GM)

Georgi & Machacek (1985); Chanowitz & Golden (1985)

• Impose global $SU(2)_L \times SU(R)_2$ symmetry on scalar potential (9 parameters)
  – EWSB: Breaks to custodial SU(2); fixes $\rho = 1$ at tree level

• GM: $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$, $\chi = \begin{pmatrix} \chi^{++} \\ \chi^+ \\ \chi^0 \end{pmatrix}$, $\xi = \begin{pmatrix} \xi^+ \\ \xi^0 \\ \xi^- \end{pmatrix}$
  – Where the neutral components decompose:
    $\phi^0 \to v_\phi + \frac{\phi^{0,r} + i\phi^{0,i}}{\sqrt{2}}$, $\chi^0 \to v_\chi + \frac{\chi^{0,r} + i\chi^{0,i}}{\sqrt{2}}$, $\xi^0 \to v_\xi + \xi^0$
  – The vevs are parameterized by $c_H = \frac{v_\phi}{v}$, $s_H = \frac{2\sqrt{2}v_\chi}{v}$

\[ v_\phi^2 + 8v_\chi^2 = v^2 \equiv \frac{1}{\sqrt{2}G_F} \]
Georgi-Machacek Model (GM)

- Model gives 13 fields classified based their SU(2)\(_c\) transformation properties
  - A 5-plet (H\(_5^{±±}\), H\(_5^0\), H\(_5^{±}\)) with mass m\(_5\)
  - A triplet (H\(_3^{±}\), H\(_3^0\)) with mass m\(_3\)
  - 2 singlets (h\(_0\), H\(_0\)) with mass m\(_h\), m\(_H\)
  - 3 Goldstones that get eaten by W and Z

Search for H\(_5^{±}\) in H\(_5^{±}\) → W\(^±\) Z → q\_1q\_2

ATLAS 1503.04233
Custodial Violation in GM

• Has been known for many years that custodial symmetry is broken by the hypercharge loop effects (Gunion, Vega & Wudka 1991)

• Hypercharge loop diagrams -> parameters of the scalar potential run away from the custodial preserving relation (V now has 16 parameters)
Custodial Violation in GM

• Running of the parameters are described by the renormalization group equations (RGEs)

• Assume Custodial Symmetry holds exactly at some cutoff scale $\Lambda$ use RGEs to run down to weak scale (parameterized by $v$)

• In general RGEs of the form:

$$\frac{d}{dt} \tilde{\lambda}(t) = \beta(\tilde{\lambda}) \quad t = \log \left( \frac{p}{\Lambda} \right)$$
Custodial Violation in GM

• For small ratio of scales $\delta(t) \equiv t - t_0 = \log(v/\Lambda) < 0$ we have:

$$\tilde{X}(t) = X(t) + \frac{\beta_X^Y \delta t}{16\pi^2} \equiv X(t) + \delta \tilde{X}$$

• i.e. Consider the custodial violating states as perturbations of the custodial symmetric states

$$\begin{pmatrix} H_5^+ \\ H_3^+ \\ G^+ \end{pmatrix} : \begin{pmatrix} m_5^2 & 0 & 0 \\ 0 & m_3^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} m_5^2 + O(\delta) & 0(\delta) & 0(\delta) \\ 0(\delta) & m_3^2 + O(\delta) & 0(\delta) \\ 0(\delta) & 0(\delta) & 0 \end{pmatrix}$$
Running of the Rho Parameter

• The running of rho parameter limits the size of the allowed cut-off scale
• For given test point can find largest possible cut-off scale that still preserves rho parameter within 2σ of measured value
• Running causes mixing between states and splits degenerate multiplets
Running of Rho Parameter

Test Pt: $\lambda_i = 0.1$, $M_i = 100$ GeV, $\mu_2^2 = -8462$ GeV$^2$, $\mu_3^2 = 90000$ GeV$^2$, $V_\chi = 17$ GeV
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$V_\chi=17$ GeV
Some Contours

Bench Mark Pt: \( m_h = 125 \text{ GeV} \)

\( \lambda_2 = 0.4 \left( \frac{m_5}{1000 \text{ GeV}} \right) \)

\( \lambda_3 = -0.1 \)

\( \lambda_4 = 0.2 \)

\( M_1 = \sqrt{2} \, s_H \left( m_5^2 + v^2 \right) / v \)

\( M_2 = M_1 / 6 \)

Search for \( H^\pm \) in \( H^\pm \rightarrow W^\pm Z \rightarrow qqll \)

ATLAS 1503.04233C
Coupling to Vector Bosons

• Can parameterize BSM couplings as:

\[ hWW: 2i \frac{M_W^2}{v^2} \kappa_W^h g^{\mu\nu} \quad hZZ: \frac{2iM_Z^2}{v^2} \kappa_Z^h g^{\mu\nu} \]

• Can use these to define:

\[ \lambda_{WZ}^h = \frac{\kappa_W^h}{\kappa_Z^h} \]

– Measure of BSM physics
– \( \lambda_{WZ}^h = 1 \) in GM but not with custodial violation
– Due to \( H_5^0 \) mixing and difference between \( v_\xi \) and \( v_\chi \)

• \( \lambda_{WZ}^h = 0.9 \pm 1 \) at 1 sigma (ATLAS and CMS, 1606.02266)
Some Contours

Bench Mark Pt: $m_h = 125$ GeV

$\lambda_2 = 0.4(m_5/1000\text{GeV})$

$\lambda_3 = -0.1$

$\lambda_4 = 0.2$

$M_1 = \sqrt{2} s_H (m_5^2 + v^2)/v$

$M_2 = M_1/6$

Search for $H^\pm$ in $H^\pm \rightarrow W^\pm Z \rightarrow q\bar{q}l\bar{l}$

ATLAS 1503.04233C
Conclusions

• GM interesting benchmark of higher isospins
  – Custodial symmetry imposed at tree level

• Hypercharge gauge interaction violates custodial symmetry
  – Custodial symmetric scale can be quite large

• Running leads custodial violation in SM-like Higgs couplings to VBs
  – $\lambda^h_{WZ}$ becomes greater than 1
GM Potential

- Write in terms of bidoublet and bitriplet

\[
V(\Phi, \chi) = \frac{\mu_2^2}{2} Tr(\Phi^\dagger \Phi) + \frac{\mu_3^2}{2} Tr(X^\dagger X) + \lambda_1 [Tr(\Phi^\dagger \Phi)]^2 + \lambda_2 [Tr(\Phi^\dagger \Phi) Tr(X^\dagger X)] + \lambda_3 Tr(X^\dagger XX^\dagger X) \\
\quad + \lambda_4 [Tr(X^\dagger X)]^2 - \lambda_5 Tr(\Phi^\dagger \tau^a \Phi \tau^b) Tr(X^\dagger t^a X t^b) - M_1 Tr(\Phi^\dagger \tau^a \Phi \tau^b) (UXU^\dagger)_{ab} \\
\quad - M_2 Tr(X^\dagger t^a X t^b) (UXU^\dagger)_{ab}
\]

\[
\Phi = \begin{pmatrix}
\phi^{0*} & \phi^+ \\
-\phi^{++} & \phi^0
\end{pmatrix}
\]

\[
\begin{align*}
\tilde{\lambda}_1 &= 4\lambda_1 \\
\tilde{\lambda}_2 &= 2\lambda_3 \\
\tilde{\lambda}_3 &= -2\lambda_5 \\
\tilde{\lambda}_4 &= -\sqrt{2}\lambda_5 \\
\tilde{\lambda}_5 &= 4\lambda_2 \\
\tilde{\lambda}_6 &= 2\lambda_2 \\
\tilde{\lambda}_7 &= 2\lambda_3 + 4\lambda_4 \\
\tilde{\lambda}_8 &= \lambda_3 + \lambda_4 \\
\tilde{\lambda}_9 &= 4\lambda_3 \\
\tilde{\lambda}_{10} &= 4\lambda_4 \\
\tilde{M}_1^1 &= M_1 \\
\tilde{M}_1 &= M_1 \\
\tilde{M}_2 &= M_2
\end{align*}
\]

\[
\chi = \begin{pmatrix}
\chi^{0*} & \xi^* & \chi^{++} \\
-\chi^{++} & \xi^0 & \chi^+ \\
\chi^{+++} & -\xi^{++} & \chi^0
\end{pmatrix}
\]
GM BenchMark Point

• parameterize \((\mu^2_1, \mu^2_2, \lambda_1, \lambda_5)\) as \((G_F, m_5, m_h, s_H)\)

• \(m_5, s_H\) parameters most related to direct searches for \(H_5\)

• maximum possible theoretical allowed parameter space

• \(m_3 > m_5\) so no \(H_5 \rightarrow H_3\) VB i.e \(BR(H_5 \rightarrow VBS) = 1\) at tree level