



Custodial Symmetry Violation in the Georgi- Machacek model

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Outline

- Introduction and Motivation
- The Georgi-Machacek Model (GM)
- Incorporating Custodial Violation
- Results



The Standard Model

- SM has a scalar $SU(2)_L$ doublet $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$
 - Isospin $\frac{1}{2}$, hypercharge 1
- SM Lagrangian

$$\mathcal{L}_H = |D_\mu \phi|^2 - \left[-\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \right] - [y_f \bar{f}_R \phi^\dagger F_L + h.c.]$$

- $\mu^2 < 0$: spontaneous symmetry breaking
 - non-zero vev; fixed by minimizing Higgs potential
 - Goldstones are gauged away (eaten by W and Z)

$$v^2 = \frac{\mu^2}{\lambda} \quad m_H = 2\lambda v^2 \quad \phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h + iG^0) \end{pmatrix}$$



The Standard Model (Cont.)

- Higgs Couplings are proportional to generated mass

$$\mathcal{L}_H = |D_\mu \phi|^2 - \left[-\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \right] - [y_f \bar{f}_R \phi^\dagger F_L + h.c.]$$

– W and Z:

$$M_W^2 = \frac{g^2 v^2}{4}$$

$$hWW: 2i \frac{M_W^2}{v^2} g^{\mu\nu}$$

$$M_Z^2 = \frac{g^2 v^2}{4 \cos^2(\theta_w)}$$

$$hZZ: \frac{2iM_Z^2}{v^2} g^{\mu\nu}$$

– Fermions:

$$m_f = \frac{y_f v}{\sqrt{2}}$$

$$hf\bar{f}: \frac{im_f}{v}$$



Beyond the SM: Why Isospin triplets

- Gauge invariance \rightarrow Fermion masses only generated by doublet; W and Z can be generated by any isospin multiplet
 - Can use triplet models to limit exotic fraction of M_Z and M_W
- Can enhance hVV couplings compared to SM
 - Could mask non-SM contribution to measured Higgs branching ratios



Problems with Higher Isospin

- The ρ parameter: measure of relative strength of charged and neutral weak currents

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2(\theta_w)} = \sum_k \frac{2 \left[T_k(T_k + 1) - \frac{Y_k^2}{4} \right] v_k^2}{\sum_k Y_k^2 v_k^2}$$

PDG 2014: $\rho = 1.00040 \pm 0.00024$

- With SM doublet ϕ , real triplet ξ ($Y=0$) and complex triplet χ ($Y=1$) ρ becomes:

$$\rho = \frac{v_\phi^2 + 4v_\xi^2 + 4v_\chi^2}{v_\phi^2 + 8v_\chi^2}$$



Georgi-Machacek Model (GM)

Georgi & Machacek (1985); Chanowitz & Golden (1985)

- Impose global $SU(2)_L \times SU(R)_2$ symmetry on scalar potential (9 parameters)
 - EWSB: Breaks to custodial $SU(2)$; fixes $\rho=1$ at tree level

- GM: $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ $\chi = \begin{pmatrix} \chi^{++} \\ \chi^+ \\ \chi^0 \end{pmatrix}$ $\xi = \begin{pmatrix} \xi^+ \\ \xi^0 \\ \xi^- \end{pmatrix}$

- Where the neutral components decompose:

$$\phi^0 \rightarrow \frac{v_\phi}{\sqrt{2}} + \frac{\phi^{0,r} + i\phi^{0,i}}{\sqrt{2}} \quad \chi^0 \rightarrow v_\chi + \frac{\chi^{0,r} + i\chi^{0,i}}{\sqrt{2}} \quad \xi^0 \rightarrow v_\xi + \xi^0$$

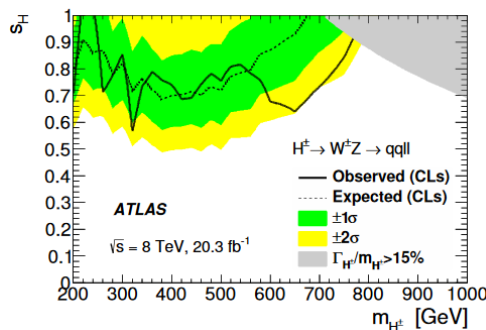
- The vevs are parameterized by $c_H = \frac{v_\phi}{v}$ $s_H = \frac{2\sqrt{2}v_\chi}{v}$

$$v_\phi^2 + 8v_\chi^2 = v^2 \equiv \frac{1}{\sqrt{2}G_F}$$



Georgi-Machacek Model (GM)

- Model gives 13 fields classified based their $SU(2)_C$ transformation properties
 - A 5-plet ($H_5^{\pm\pm}, H_5^0, H_5^\pm$) with mass m_5
 - A triplet (H_3^\pm, H_3^0) with mass m_3
 - 2 singlets (h^0, H^0) with mass m_h, m_H
 - 3 Goldstones that get eaten by W and Z

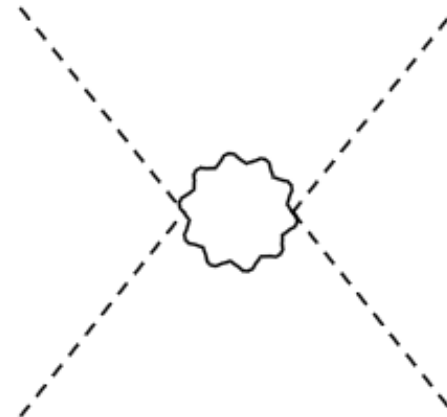


Search for H_5^\pm in $H_5^\pm \rightarrow W^\pm Z \rightarrow qqll$
ATLAS 1503.04233



Custodial Violation in GM

- Has been known for many years that custodial symmetry is broken by the hypercharge loop effects (Gunion, Vega & Wudka 1991)
- Hypercharge loop diagrams \rightarrow parameters of the scalar potential run away from the custodial preserving relation (V now has 16 parameters)





Custodial Violation in GM

- Running of the parameters are described by the renormalization group equations (RGEs)
- Assume Custodial Symmetry holds exactly at some cutoff scale Λ use RGEs to run down to weak scale (parameterized by v)
- In general RGEs of the form:

$$\frac{d}{dt} \tilde{\lambda}(t) = \beta(\tilde{\lambda}) \quad t = \log\left(\frac{p}{\Lambda}\right)$$



Custodial Violation in GM

- For small ratio of scales $\delta(t) \equiv t - t_0 = \log(v/\Lambda) < 0$ we have:

$$\tilde{X}(t) = X(t) + \frac{\beta_X^Y \delta t}{16\pi^2} \equiv X(t) + \delta_{\tilde{X}}$$

- i.e. Consider the custodial violating states as perturbations of the custodial symmetric states

$$\begin{pmatrix} H_5^+ \\ H_3^+ \\ G^+ \end{pmatrix} : \begin{pmatrix} m_5^2 & 0 & 0 \\ 0 & m_3^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} m_5^2 + O(\delta) & O(\delta) & O(\delta) \\ O(\delta) & m_3^2 + O(\delta) & O(\delta) \\ O(\delta) & O(\delta) & 0 \end{pmatrix}$$

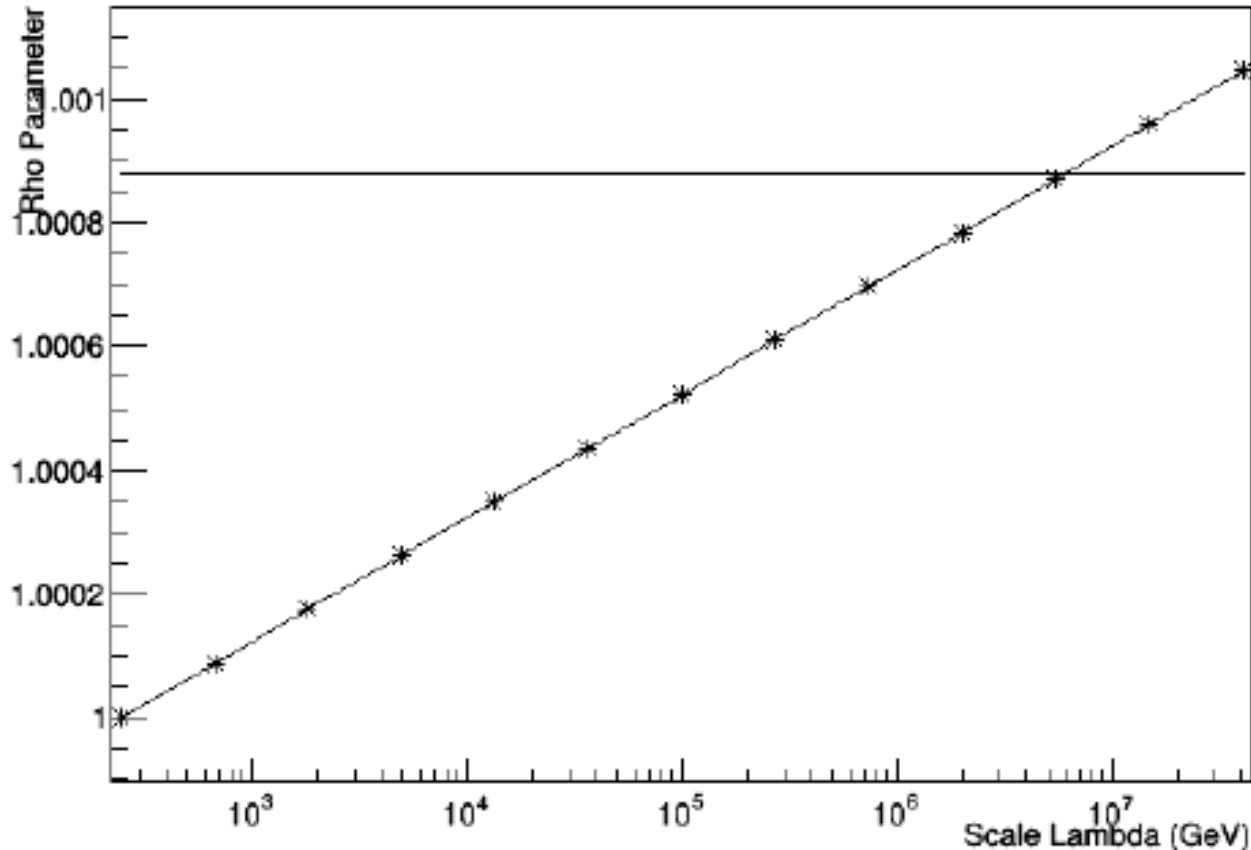


Running of the Rho Parameter

- The running of rho parameter limits the size of the allowed cut-off scale
- For given test point can find largest possible cut-off scale that still preserves rho parameter within 2σ of measured value
- Running causes mixing between states and splits degenerate multiplets



Running of Rho Parameter

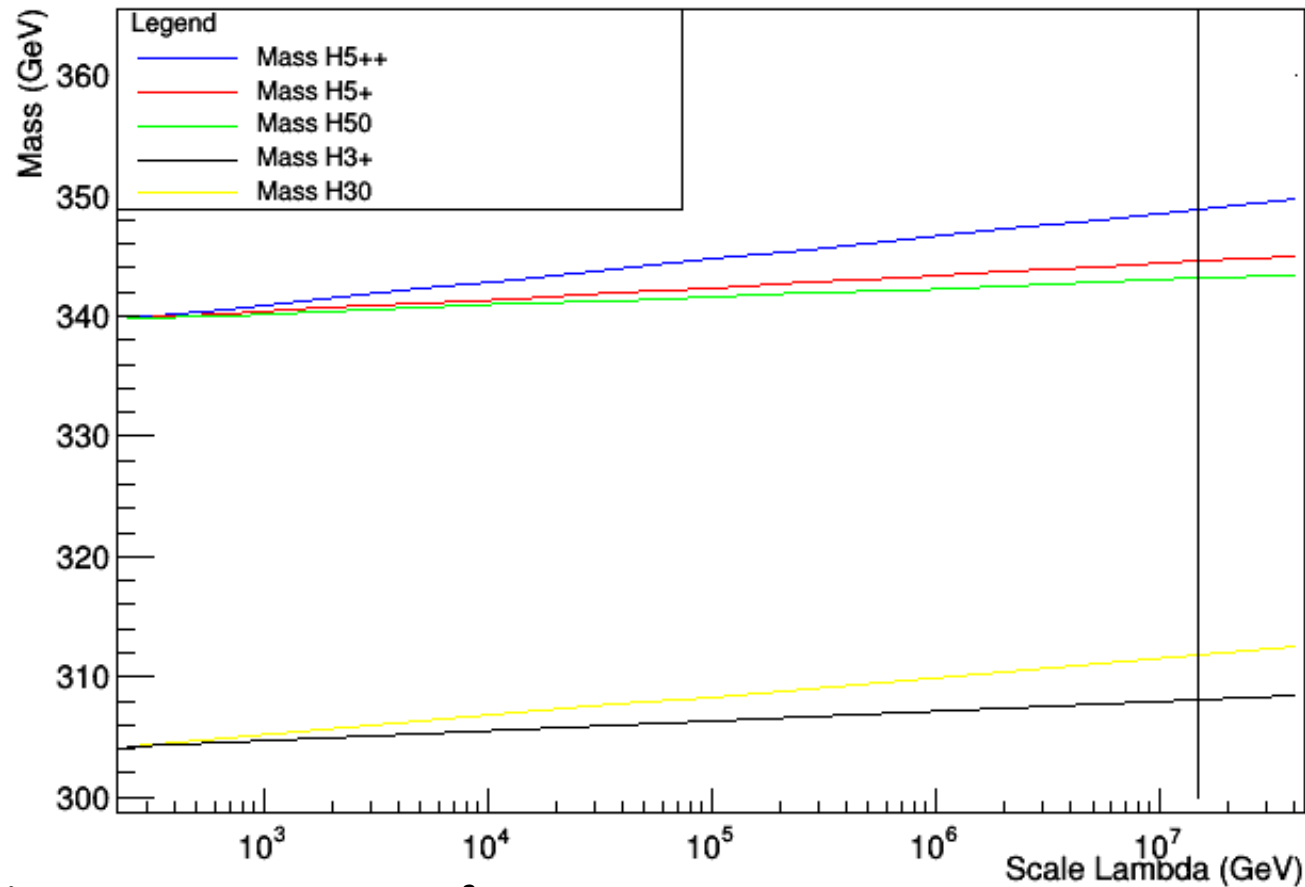


Test Pt: $\lambda_1=0.1$, $M_1=100$ GeV, $\mu_2^2=-$
 8462 GeV², $\mu_3^2=90000$, GeV²
 $V_\chi=17$ GeV

FIG. 1. ρ Parameter



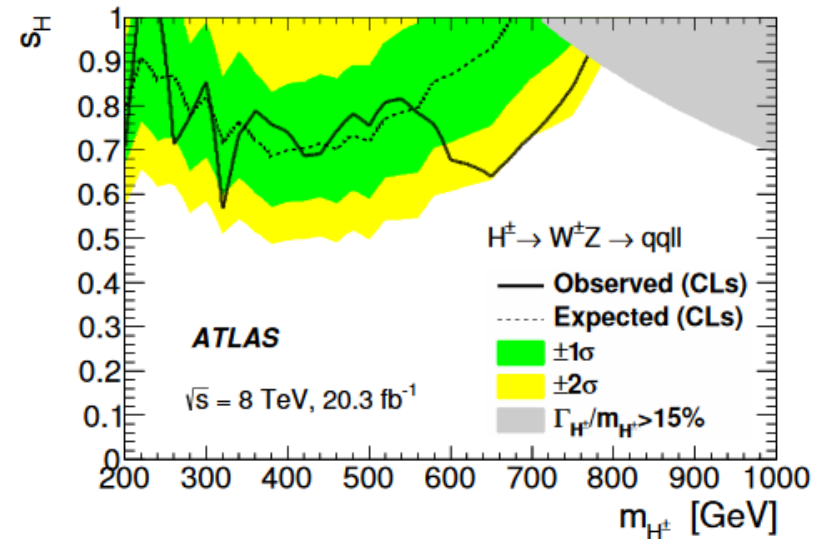
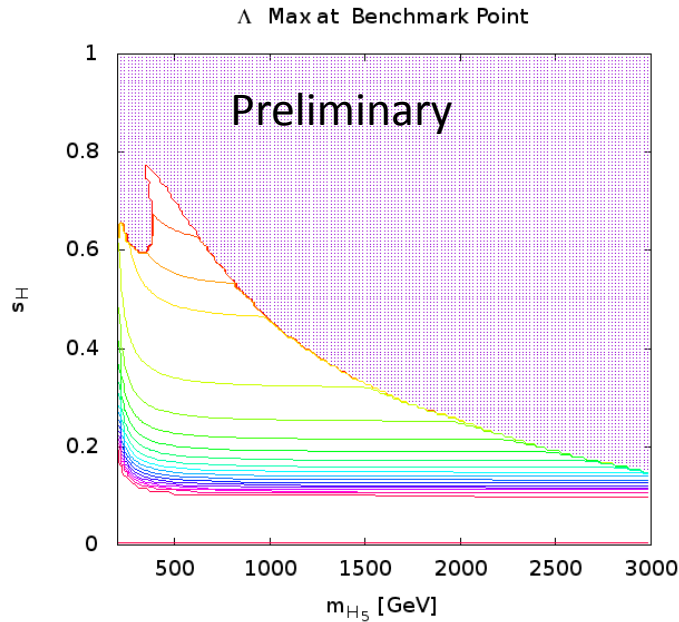
Mass Splitting



Test Pt: $\lambda_i=0.1$, $M_i=100$ GeV, $\mu_2^2=-$
 8462 GeV², $\mu_3^2=90000$, GeV²
 $V_\chi=17$ GeV



Some Contours



Bench Mark Pt: $m_h = 125 \text{ GeV}$

$$\lambda_2 = 0.4(m_5/1000\text{GeV})$$

$$\lambda_3 = -0.1$$

$$\lambda_4 = 0.2$$

$$M_1 = \sqrt{2} s_H (m_5^2 + v^2)/v$$

$$M_2 = M_1/6$$

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 ATLAS 1503.04233C



Coupling to Vector Bosons

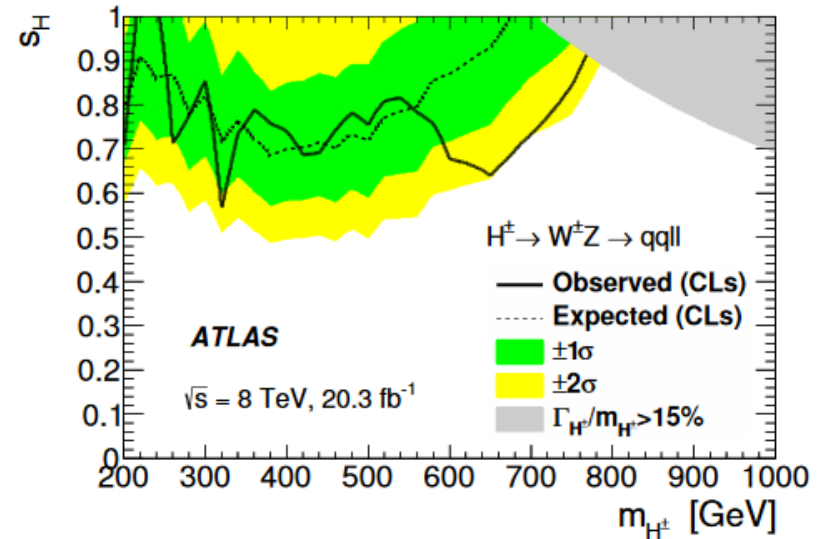
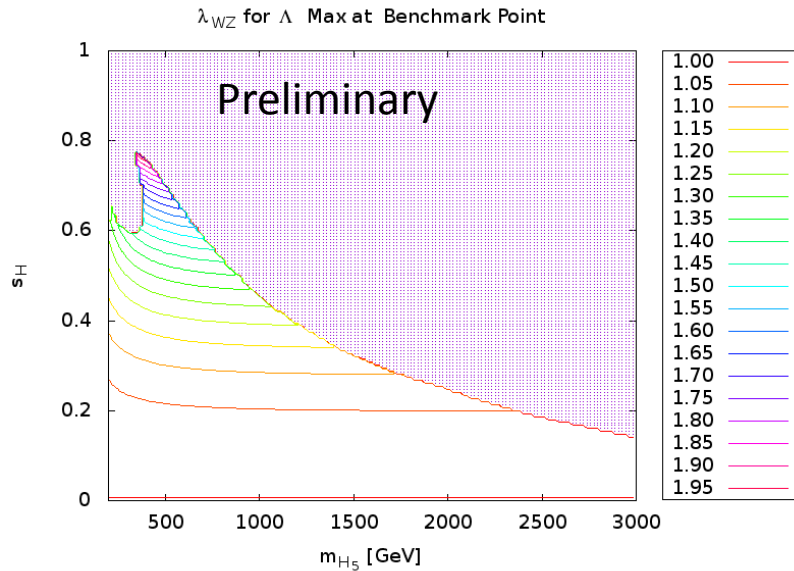
- Can parameterize BSM couplings as:

$$h_{WW}: 2i \frac{M_W^2}{v^2} \kappa_W^h g^{\mu\nu} \quad h_{ZZ}: \frac{2iM_Z^2}{v^2} \kappa_Z^h g^{\mu\nu}$$

- Can use these to define: $\lambda_{WZ}^h = \frac{\kappa_W^h}{\kappa_Z^h}$
 - Measure of BSM physics
 - $\lambda_{WZ}^h = 1$ in GM but not with custodial violation
 - Due to H_5^0 mixing and difference between v_ξ and v_χ
- $\lambda_{WZ}^h = 0.9 \pm 1$ at 1 sigma (ATLAS and CMS, 1606.02266)



Some Contours



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ATLAS 1503.04233C



Conclusions

- GM interesting benchmark of higher isospins
 - Custodial symmetry imposed at tree level
- Hypercharge gauge interaction violates custodial symmetry
 - Custodial symmetric scale can be quite large
- Running leads custodial violation in SM-like Higgs couplings to VBs
 - λ_{WZ}^h becomes greater than 1



Backup Slides



GM Potential

- Write in terms of bidoublet and bitriplet

$$V(\Phi, X) = \frac{\mu_2^2}{2} \text{Tr}(\Phi^\dagger \Phi) + \frac{\mu_3^2}{2} \text{Tr}(X^\dagger X) + \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 + \lambda_2 [\text{Tr}(\Phi^\dagger \Phi) \text{Tr}(X^\dagger X)] + \lambda_3 \text{Tr}(X^\dagger X X^\dagger X) \\ + \lambda_4 [\text{Tr}(X^\dagger X)]^2 - \lambda_5 \text{Tr}(\Phi^\dagger \tau^a \Phi \tau^b) \text{Tr}(X^\dagger t^a X t^b) - M_1 \text{Tr}(\Phi^\dagger \tau^a \Phi \tau^b) (UXU^\dagger)_{ab} \\ - M_2 \text{Tr}(X^\dagger t^a X t^b) (UXU^\dagger)_{ab}$$

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^{+*} & \phi^0 \end{pmatrix}$$

$$X = \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ -\chi^{+*} & \xi^0 & \chi^+ \\ \chi^{++*} & -\xi^{+*} & \chi^0 \end{pmatrix}$$

$$\begin{aligned} \tilde{\lambda}_1 &= 4\lambda_1 \\ \tilde{\lambda}_2 &= 2\lambda_3 \\ \tilde{\lambda}_3 &= -2\lambda_5 \\ \tilde{\lambda}_4 &= -\sqrt{2}\lambda_5 \\ \tilde{\lambda}_5 &= 4\lambda_2 \\ \tilde{\lambda}_6 &= 2\lambda_2 \\ \tilde{\lambda}_7 &= 2\lambda_3 + 4\lambda_4 \\ \tilde{\lambda}_8 &= \lambda_3 + \lambda_4 \\ \tilde{\lambda}_9 &= 4\lambda_3 \\ \tilde{\lambda}_{10} &= 4\lambda_4 \\ \tilde{M}_1^1 &= M_1 \\ \tilde{M}_1 &= M_1 \end{aligned}$$



GM BenchMark Point

- parameterize $(\mu^2_1, \mu^2_2, \lambda_1, \lambda_5)$ as (G_F, m_5, m_h, s_H)
- m_5, s_H parameters most related to direct searches for H_5
- maximum possible theoretical allowed parameter space
- $m_3 > m_5$ so no $H_5 \rightarrow H_3 VB$ i.e $BR(H_5 \rightarrow VBs) = 1$ at tree level