Spatio-temporal correlations after a quantum quench in the Bose-Hubbard model

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Outline

Motivation

- Review of recent work on BHM and related experiments
- Why is it important to study correlations?

Theoretical formalism

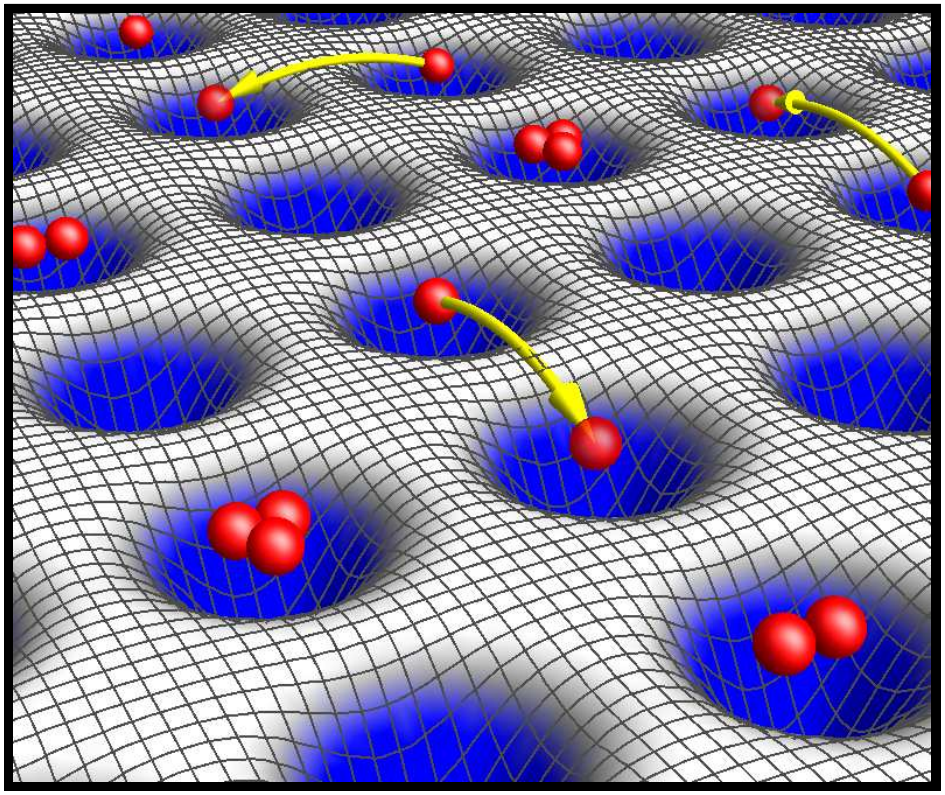
- Schwinger-Keldysh technique $\rightarrow$ to describe out-of-equilibrium phenomena
- 2PI effective action approach $\rightarrow$ to calculate Green's functions
- Effective theory of BHM
- Low-frequency approximation

Numerical results

- Focus on quenches within MI regime
- Quasi-momentum distribution after quench
- Light-cone spreading of correlations

Bosons in an optical lattice

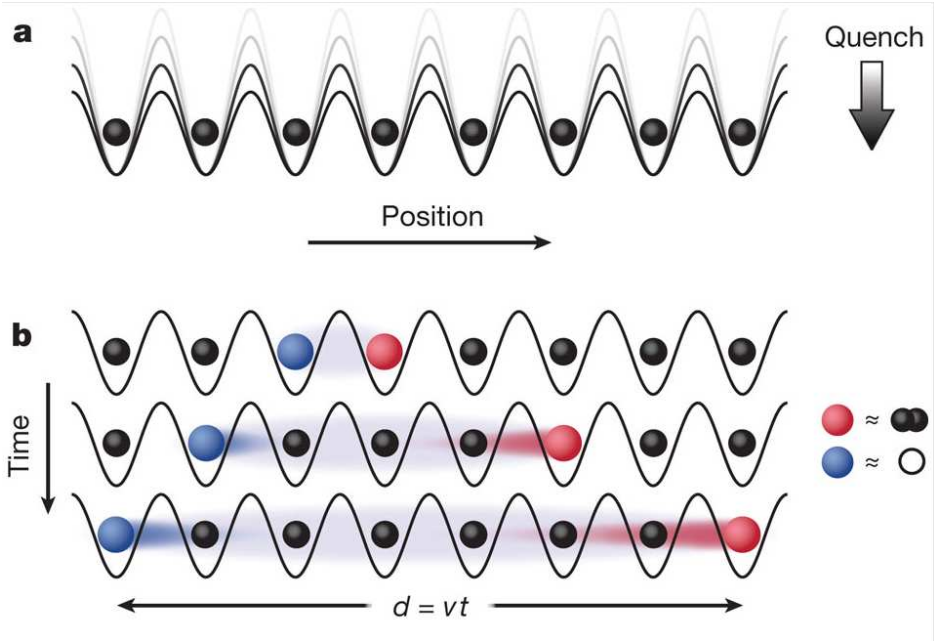
- Can be described by the BHM [1]
  - Simplest model for interacting bosons on a lattice
- Optical lattices are versatile systems
  - Tune parameters in real-time [2,3]
  - Dynamically traverse quantum phase transitions
- Transition between superfluid and Mott-insulator observed via quench [4]
  - SF: Phase-coherent state
  - MI: Localised state
- Example of out-of-equilibrium dynamics in interacting quantum system



[1] Jaksch *et al.*, PRL **81**, 3108 (1998)
[2] Bloch, Nature Physics **1**, 23 (2005)
[3] Jaksch & Zoller, Annals of Physics **315**, 52 (2005)
[4] Greiner *et al.* Nature **415**, 39 (2002)

Correlation functions

Cheneau *et al.* (2012)



Quenches from SF → MI by changing lattice potential (varies hopping in BHM)

- Slow relaxation times after quench from SF → MI [1],[2]
- Presence of harmonic trap → MI regions impede relaxation via mass transport [3]

Quenches within MI regime:

- Light-cone-like spreading of correlations in 1D systems [4],[5]
- Similar predictions for higher dimensions [6]

Suggests further study of correlations in BHM:

- Numerical approaches limited to 1D systems → require analytical approach
- Formalism needs to be accurate in both phase regimes

[1] PRA **84**, 033620 (2011)
[2] PRL **104**, 160403 (2010)
[3] PRL **106**, 125301 (2011)
[4] PRA **85**, 053625 (2012)
[5] Nature **481**, 484 (2012)
[6] PRA **87**, 063616 (2013)

Blue - Theoretical studies
Red - Experimental studies

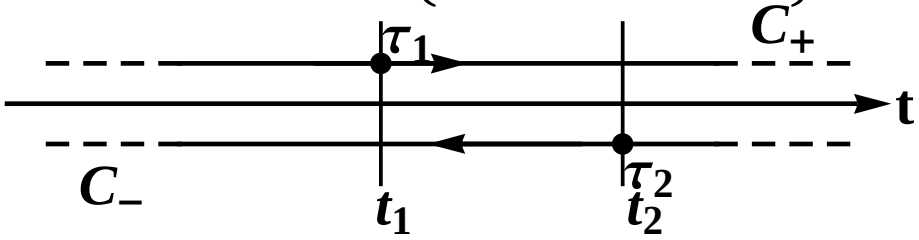
2PI Formalism

We want to calculate the equations of motion of the two-point Green's function $G_{ij}(t_1, t_2) \equiv -i\langle \hat{\Phi}_i(t_1)\hat{\Phi}_j^\dagger(t_2)\rangle$

- Standard approach: 2PI-formalism
- Advantage: treats the calculation of G and the mean field $\phi_i(t) \equiv \langle \hat{\Phi}_i(t)\rangle$ (i.e. superfluid order parameter) on equal footing

Apply the Schwinger-Keldysh technique in conjunction with the 2PI approach

- Schwinger-Keldysh: Allows for the description of out-of-equilibrium phenomena
- 2PI formalism: Used to calculate equations of motion for G and ϕ
- Price to pay - doubling of fields $\left\{\hat{\Phi}_{i+}^{(\dagger)}(t), \hat{\Phi}_{i-}^{(\dagger)}(t)\right\}$ in theory



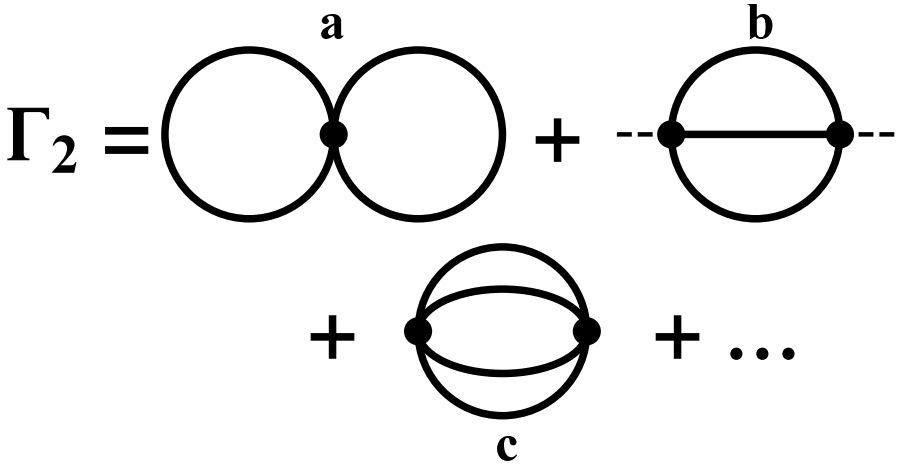
2PI Equations of motion

Cornwall *et al.* [1] showed that G and ϕ satisfy the following equations of motion

$$\frac{\delta S[\phi]}{\delta \phi_a} + \frac{1}{2}\left[i\frac{\delta [D^{-1}]_{bc}}{\delta \phi_a} G_{bc}\right] + \frac{\delta \Gamma_2}{\delta \phi_a} = 0$$

$$i\,[G]^{-1}_{ab} = i\,[D]^{-1}_{ab} - i\Sigma_{ab} \qquad \text{(Dyson's Equation)}$$

- Action (describing theory): $S[\Phi]$
- Inverse "bare" propagator: $iD^{-1}_{ab} \equiv \frac{\delta^2 S[\phi]}{\delta \phi_a \delta \phi_b}$
- Self energy: $\Sigma_{ab} \equiv 2i\frac{\delta \Gamma_2}{\delta G_{ab}}$



- Dots: interaction vertices in $S[\Phi]$
- Solid lines: G
- Dotted lines: ϕ

[1] Cornwall *et al.*, PRL **10**, 2428 (1974)

Real-time strong-coupling approach

- Generalize imaginary time strong-coupling approach to obtain an effective theory of BHM

$$S[\Phi,\Phi^*] = \int dt_1 dt_2 \sum_{ij} \Phi^*_{i\alpha}(t_1)\left(\delta_{ij}\left[\mathcal{G}\right]^{-1}_{i\alpha\beta}(t_1,t_2) + 2\delta(t_1-t_2)J_{ij}(t_1)\sigma^1_{\alpha\beta}\right)\Phi_{j\beta}(t_2)$$

$$+\frac{1}{4}\int dt_1 dt_2 dt_3 dt_4 \sum_i u_{i,\alpha_1\alpha_2\alpha_3\alpha_4}(t_1,t_2,t_3,t_4)\Phi^*_{i\alpha_1}(t_1)\Phi^*_{i\alpha_2}(t_2)\Phi_{i\alpha_3}(t_3)\Phi_{i\alpha_4}(t_4)$$

  - -Imaginary time: K. Sengupta and N. Dupuis, PRA **71**, 033629 (2005)
    -Real time: M.P. Kennett and D. Dalidovich, PRA **84**, 033620 (2011)
  - Expected to provide accurate description of quench dynamics in both SF & MI regimes

- $\mathcal{G}$: Local ($J=0$) 2-point Green's function
- $u_{i,\alpha_1\alpha_2\alpha_3\alpha_4}(t_1,t_2,t_3,t_4)$: ($J=0$) 4-point vertex
  - Local in space – non-local in time

- Our work: apply 2PI formalism to this effective theory of BHM

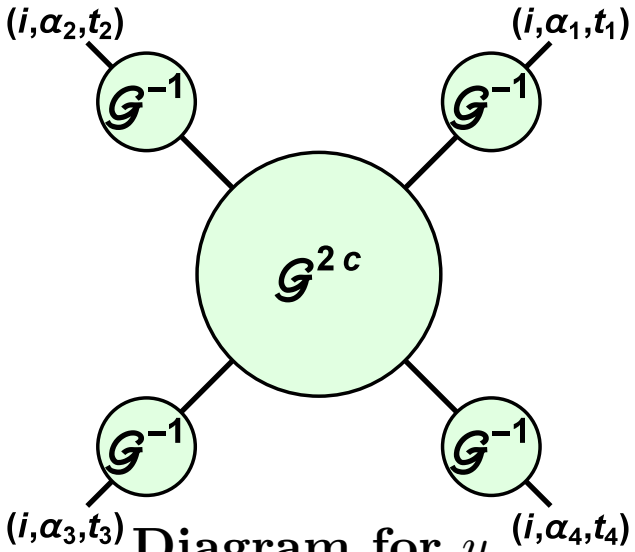


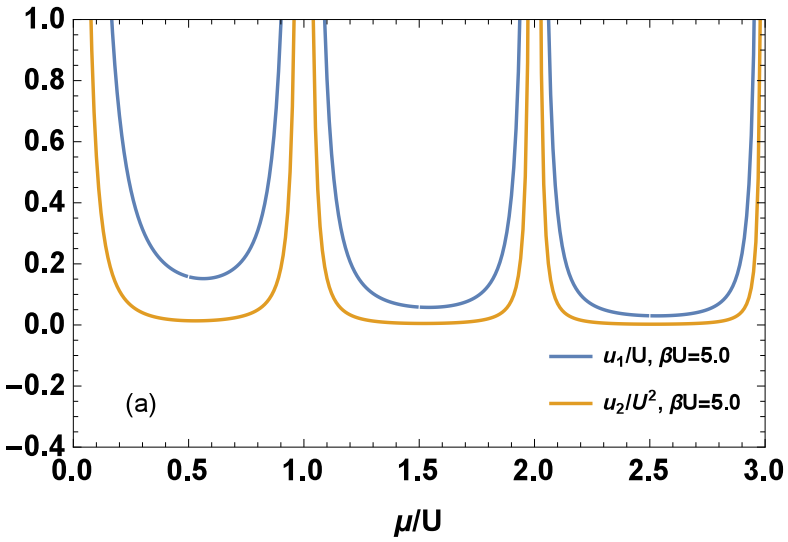
Diagram for u

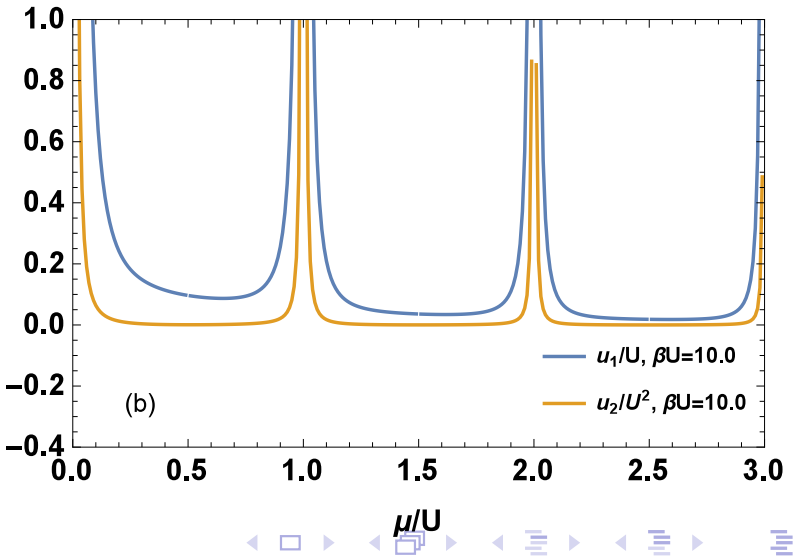
Low-frequency approximation

- To second order in u, equations of motion contain 7 time-integrals
  - Require further approximations to be amenable to numerics

Low-frequency approximation

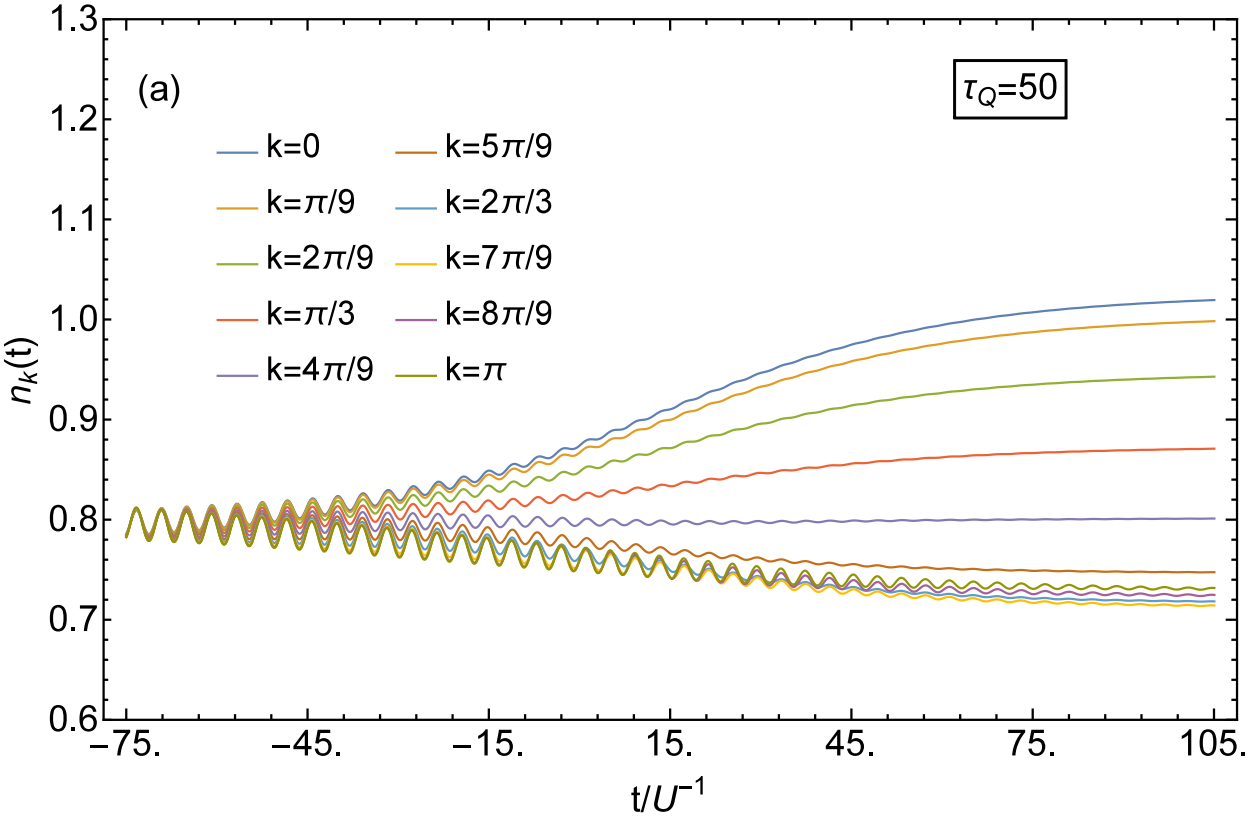
- Focus on low-frequency dynamics
- Ignore $\mu/U \approx$ integer, i.e. away from degenerate MI states
- Low-frequency limit of vertices: $u_{i,\alpha_1\alpha_2\alpha_3\alpha_4}(\omega_1,\omega_2,\omega_3,\omega_4) \to u_1$ or u_2, depending on Keldysh indices $\{\alpha\}$
- Net result: 7 time-integrals $\to$ 1 time-integral



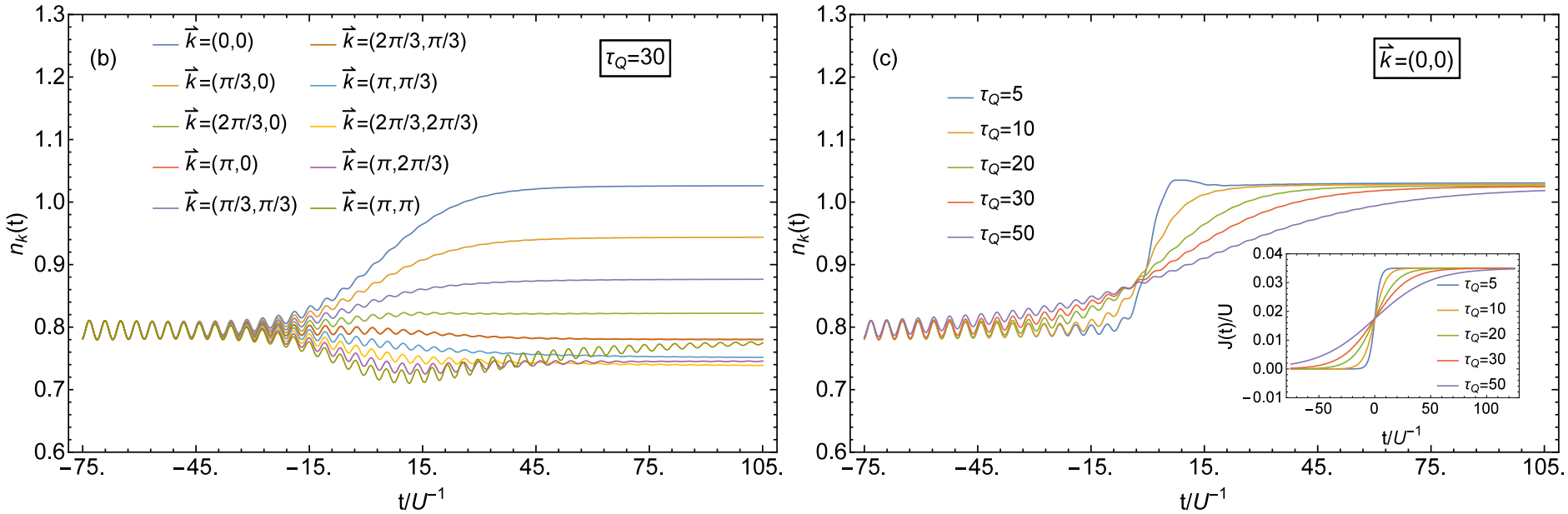


Numerical Results

- Focus on special case of quenches within MI regime
  - $\left\langle \hat{\Phi}^{(\dagger)}_{r_1}(t_1) \right\rangle = \left\langle \hat{\Phi}_{r_1,\alpha_1}(t_1)\hat{\Phi}_{r_2,\alpha_2}(t_2) \right\rangle = \left\langle \hat{\Phi}^{(\dagger)}_{r_1,\alpha_1}(t_1)\hat{\Phi}^{(\dagger)}_{r_2,\alpha_2}(t_2) \right\rangle = 0$



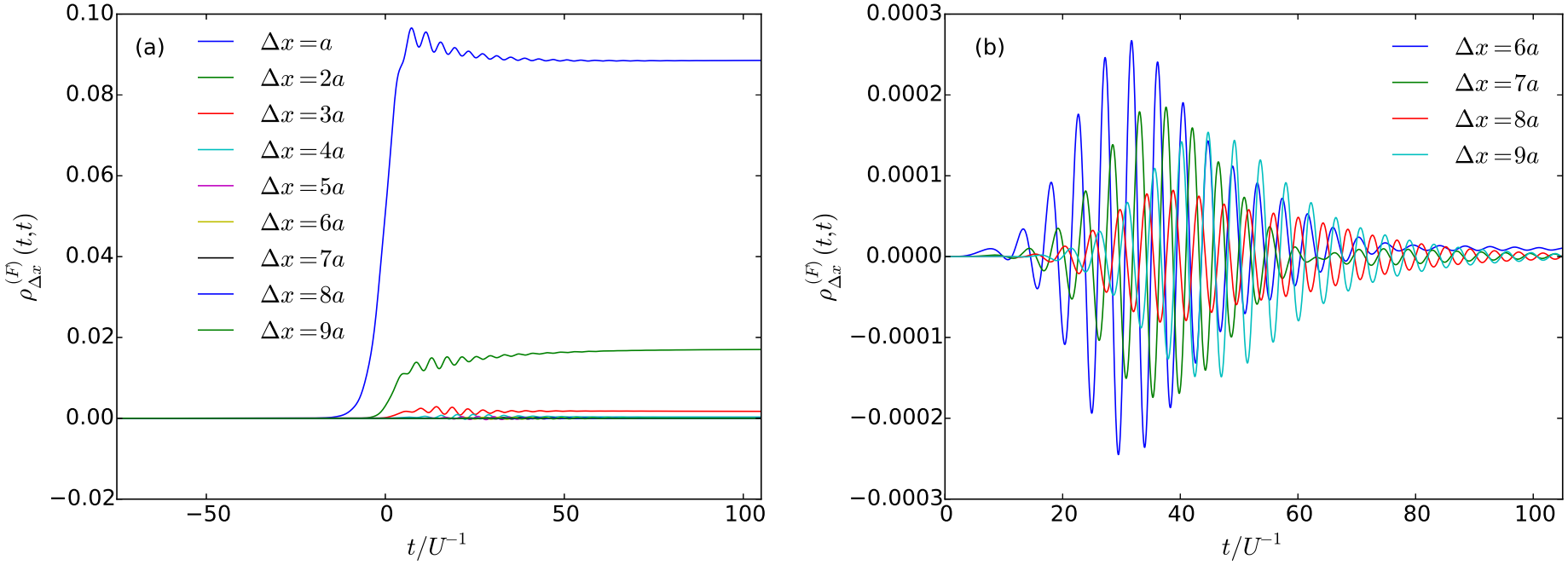
- Quasi-momentum distribution $n_{\vec{k}}(t) = \left\langle \hat{\Phi}^{\dagger}_{\vec{k}}(t)\hat{\Phi}_{\vec{k}}(t) \right\rangle$
- (a) 18 site chain; varied $\vec{k}$; fixed quench timescale τ_Q
  - $\mu = 0.25, T = 0, (J/U)_i = 0, (J/U)_f = 0.07$



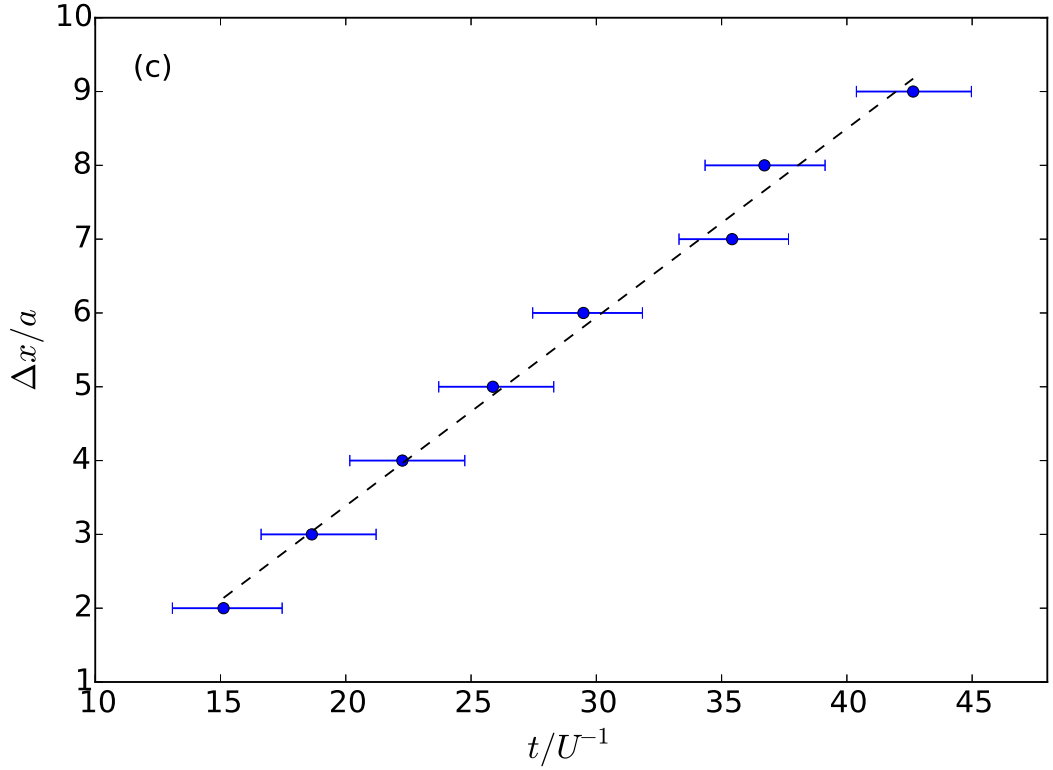
- (b-c) (6×6) square system
  - (b) Fixed τ_Q; varied $\vec{k}$
  - (c) Fixed $\vec{k}$; varied τ_Q
  - $\mu = 0.25, T = 0, (J/U)_i = 0, (J/U)_f = 0.035$

- General remark: $\vec{k} = 0$ grows the most after quench
  - Small depletion in higher momentum states
  - Consistent with a growing correlation length as SF phase is approached

- Single-particle correlations in real-space $\rho^{(F)}_{\Delta x}(t,t) = \left\langle \hat{\Phi}^{\dagger}_{\Delta x}(t)\hat{\Phi}_0(t) \right\rangle$

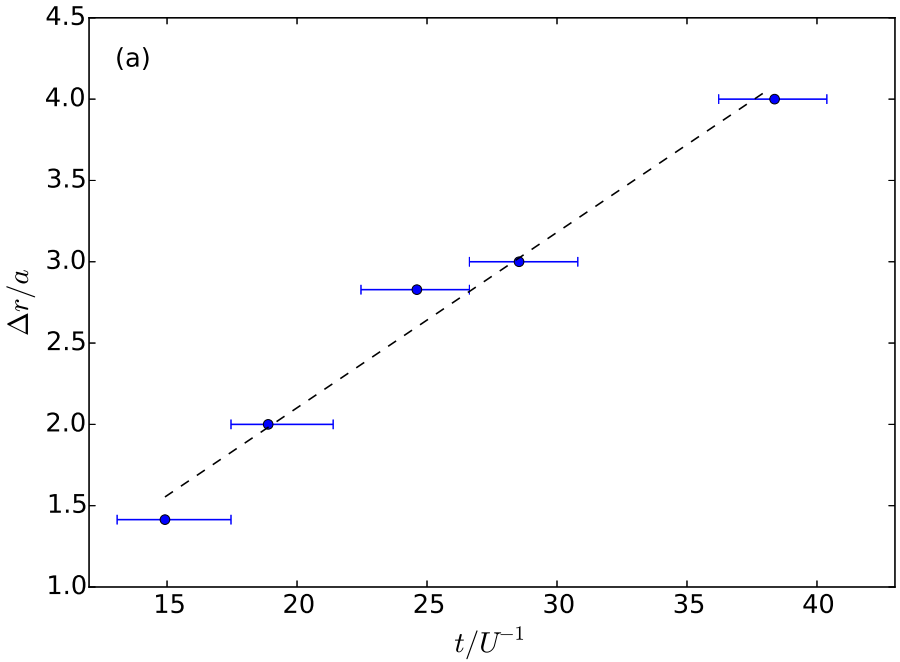


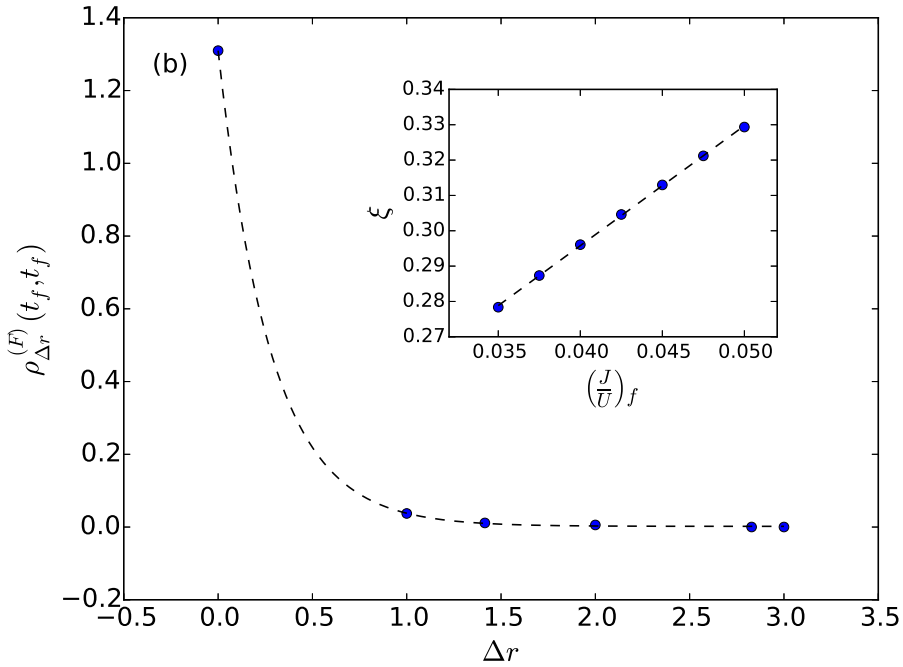
- (a) $\rho^{(F)}_{\Delta x}(t,t)$; 18 site chain
  - $\mu = 0.25, T = 0, \tau_Q = 5, (J/U)_i = 0, (J/U)_f = 0.07$
- (b) Zoom-in of (a)
  - Propagation of correlation front, i.e. moving correlation "wave-packet"



- (c) Position of correlation front vs. time
  - Light-cone spreading of correlations
  - Propagation speed: $v_{1D} \simeq 3.7 \frac{Ja}{\hbar}$
- Compared to literature:
  - Barmettler *et al.* [1]: $v_{1D} = 6 \frac{Ja}{\hbar}$ for $U = \infty$,
    $v_{1D} = 4 \frac{Ja}{\hbar}$ for $U = 0$ (exact results)
  - Natu and Mueller [2]: $v_{1D} = 3.7 \frac{Ja}{\hbar}$ for weak interactions

[1] PRA **85**, 053625 (2012)
[2] PRA **87**, 063616 (2013)





- (8×8) square system
  - $\mu = 0.25, T = 0, \tau_Q = 5, (J/U)_i = 0, (J/U)_f = 0.03725$
- (a) Light-cone effect persists; $v_{2D} = 2.9\frac{Ja}{\hbar}$
  - Navez and Schützhold result [1]: $6.2\frac{Ja}{\hbar}$
  - Our result expected to be more accurate with increasing dimensionality
- (b) Long-time single-particle correlations $\rho^{(F)}_{\Delta r}(t_\infty, t_\infty)$
  - Exponential decay with increasing separation distance Δr
  - Inset: Correlation length ξ grows with final hopping ratio $(J/U)_f$

[1] PRA **82**, 063616 (2010)

Conclusion

- Derived 2PI Schwinger-Keldysh action for the Bose-Hubbard model at strong coupling allowing for time dependent hopping
  - Obtained 2PI equations of motion for correlations and the order parameter during a quantum quench
- Observed light-cone spreading of correlations in MI quenches
  - We observed a 20% decrease in v_d from $d = 1 \to 2$
  - Potentially large enough to be verified by experiment
- Future work and challenges:
  - Study other types of quenches, e.g. SF $\to$ MI
  - Extend to multi-component BHMs
  - Equations of motion for quenches involving SF phase can be quite stiff, numerical stability an issue

Equations of motion in MI regime

First equation of motion:

$$
\begin{aligned}
0 = &\sum_{r'} \left\{ \delta_{r_1 r'} \left(\upsilon - i\lambda \frac{\partial}{\partial t_1} - \kappa^2 \frac{\partial^2}{\partial t_1^2} \right) + 2J_{r_1 r'}(t_1) \right\} \rho^{(\rho)}_{r' r_2}(t_1, t_2) \\
&- 2iu_1 \rho^{(F)}_{r_1 r_1}(t_1, t_1) \rho^{(\rho)}_{r_1 r_2}(t_1, t_2) - u_2 \int_{t_2}^{t_1} dt' \rho^{(\rho)}_{r_1 r_1}(t_1, t') \rho^{(\rho)}_{r_1 r_1}(t', t_2) \\
&- 2iu_1^2 \int_{t_2}^{t_1} dt' \sum_{r'} \Big\{ 8\rho^{(F)}_{r_1 r'}(t_1, t') \rho^{(\rho)}_{r_1 r'}(t_1, t') \rho^{(F)}_{r' r_1}(t', t_1) \\
&\qquad + \left(4 \left[\rho^{(F)}_{r_1 r'}(t_1, t') \right]^2 - \left[\rho^{(\rho)}_{r_1 r'}(t_1, t') \right]^2 \right) \rho^{(\rho)}_{r' r_1}(t', t_1) \Big\} \rho^{(\rho)}_{r' r_2}(t', t_2)
\end{aligned}
$$

- Spectral function: $\rho^{(\rho)}_{r_1 r_2}(t_1, t_2) \equiv i \left(\langle \Phi_{r_1}(t_1) \Phi^\dagger_{r_2}(t_2) \rangle - \langle \Phi_{r_2}(t_2) \Phi^\dagger_{r_1}(t_1) \rangle \right)$
- Statistical propagator: $\rho^{(F)}_{r_1 r_2}(t_1, t_2) \equiv \frac{1}{2} \left(\langle \Phi_{r_1}(t_1) \Phi^\dagger_{r_2}(t_2) \rangle + \langle \Phi_{r_2}(t_2) \Phi^\dagger_{r_1}(t_1) \rangle \right)$
- υ, λ, κ, u_1, u_2 are nontrivial functions of U and μ

Second equation of motion:

$$
\begin{aligned}
0 = &\sum_{r'} \left\{ \delta_{r_1 r'} \left(\upsilon - i\lambda \frac{\partial}{\partial t_1} - \kappa^2 \frac{\partial^2}{\partial t_1^2} \right) + 2J_{r_1 r'}(t_1) \right\} \rho^{(F)}_{r' r_2}(t_1, t_2) \\
&- 2iu_1 \rho^{(F)}_{r_1 r_1}(t_1, t_1) \rho^{(F)}_{r_1 r_2}(t_1, t_2) \\
&- u_2 \left(\int_{-\infty}^{t_1} dt' \rho^{(\rho)}_{r_1 r_1}(t_1, t_2') \rho^{(F)}_{r_1 r_1}(t', t_2) + \int_{-\infty}^{t_2} dt' \rho^{(F)}_{r_1 r_1}(t_1, t') \rho^{(\rho)}_{r_1 r_1}(t', t_2) \right) \\
&- \frac{1}{2} i u_1^2 \int_{-\infty}^{t_1} dt' \sum_{r'} \left\{ 8 \rho^{(F)}_{r_1 r'}(t_1, t') \rho^{(\rho)}_{r_1 r'}(t_1, t') \rho^{(F)}_{r' r_1}(t', t_1) \right. \\
&\qquad \left. + \left(4 \left[\rho^{(F)}_{r_1 r'}(t_1, t') \right]^2 - \left[\rho^{(\rho)}_{r_1 r'}(t_1, t') \right]^2 \right) \rho^{(\rho)}_{r' r_1}(t', t_1) \right\} \rho^{(F)}_{r' r_2}(t', t_2) \\
&- \frac{1}{2} i u_1^2 \int_{-\infty}^{t_2} dt' \sum_{r'} \left\{ 2 \rho^{(F)}_{r_1 r'}(t_1, t') \rho^{(\rho)}_{r_1 r'}(t_1, t') \rho^{(\rho)}_{r' r_1}(t', t_1) \right. \\
&\qquad \left. + \left(4 \left[\rho^{(F)}_{r_1 r'}(t_1, t') \right]^2 - \left[\rho^{(\rho)}_{r_1 r'}(t_1, t') \right]^2 \right) \rho^{(F)}_{r' r_1}(t', t_1) \right\} \rho^{(\rho)}_{r' r_2}(t', t_2)
\end{aligned}
$$