Scalar fields in a shell

The response of an Unruh-DeWitt detector inside, and what it means for us outside

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Can we detect global structure?

The Einstein Equivalence Principle states:

"The outcome of any local non-gravitational experiment in a freely falling laboratory is independent of the velocity of the laboratory and its location in spacetime."

However, the local quantum vacuum state depends on global structure. Therefore, we investigate whether a detector can probe the global structure of spacetime using the local vacuum.

Our model detector is an Unruh-DeWitt detector, a simple model capturing most of the features of the photon interaction. It is a two-level system, with a gap Ω, interacting with a scalar field via

$$H_I = \lambda\chi(\tau)\mu(\tau)\phi(x(\tau)).$$

Previous works have shown that such a detector can detect a local gravitational field (e.g. [1,2]). However, the inside of a shell is flat. Can a detector detect a transparent shell from inside, by probing the local vacuum alone?

The response function

Let us write the mode corresponding to ω, *l*, m as

$$\Psi_{\omega lm} = \frac{1}{\sqrt{4\pi\omega}} e^{-i\omega t} Y_{lm}(\theta,\phi)\frac{1}{r}\rho_{\omega l}(r).$$

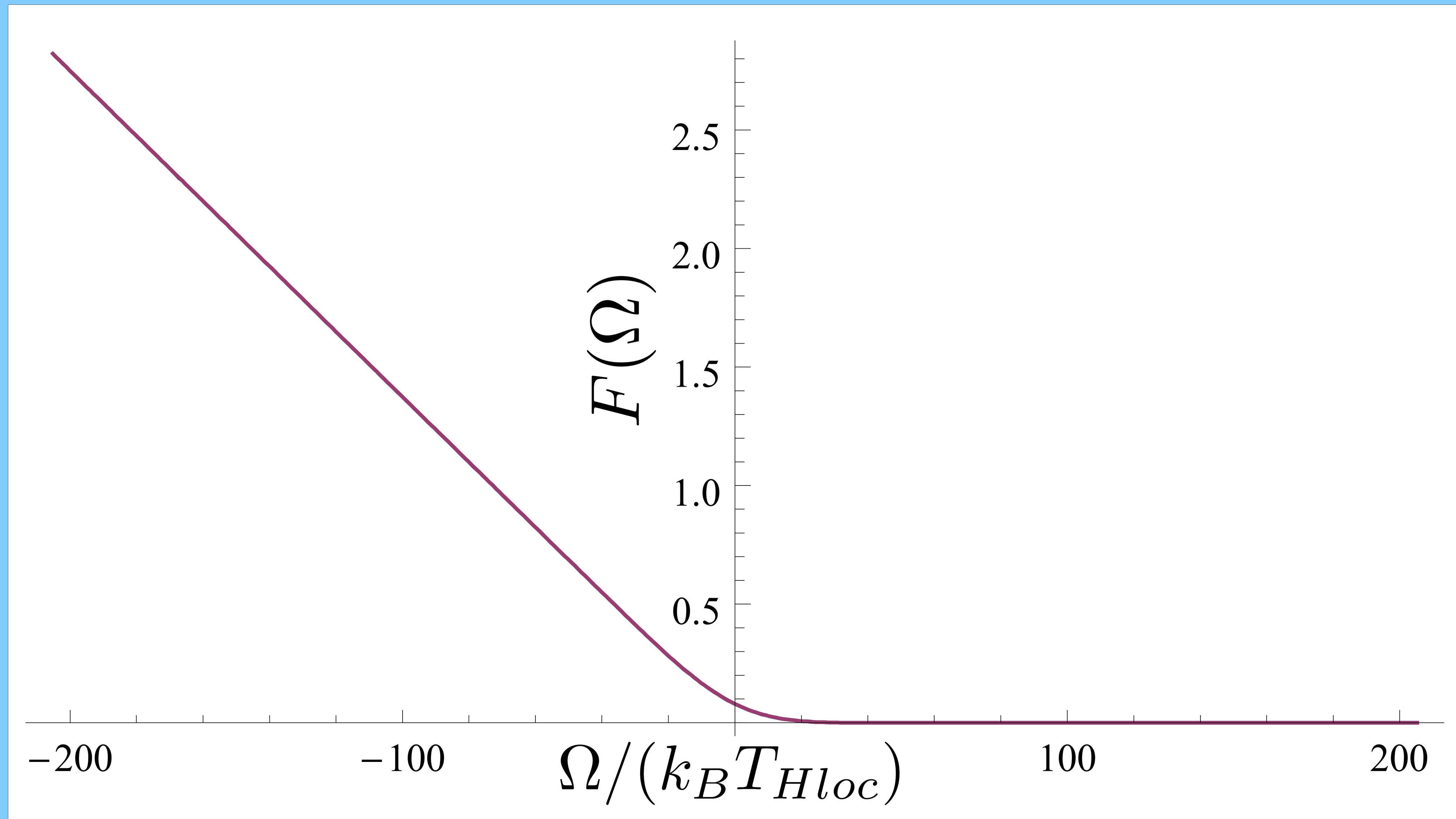
We showed that the response function of the detector depends on the radial part of the modes, ρ, and the Fourier-transformed switching function via

$$F(\Omega) = \int_0^\infty \frac{d\tilde{\omega}}{8\pi\tilde{\omega}r^2}|\tilde{\chi}(\tilde{\omega}+\Omega)|^2\rho_{\omega,l=0}(r=0)^2.$$

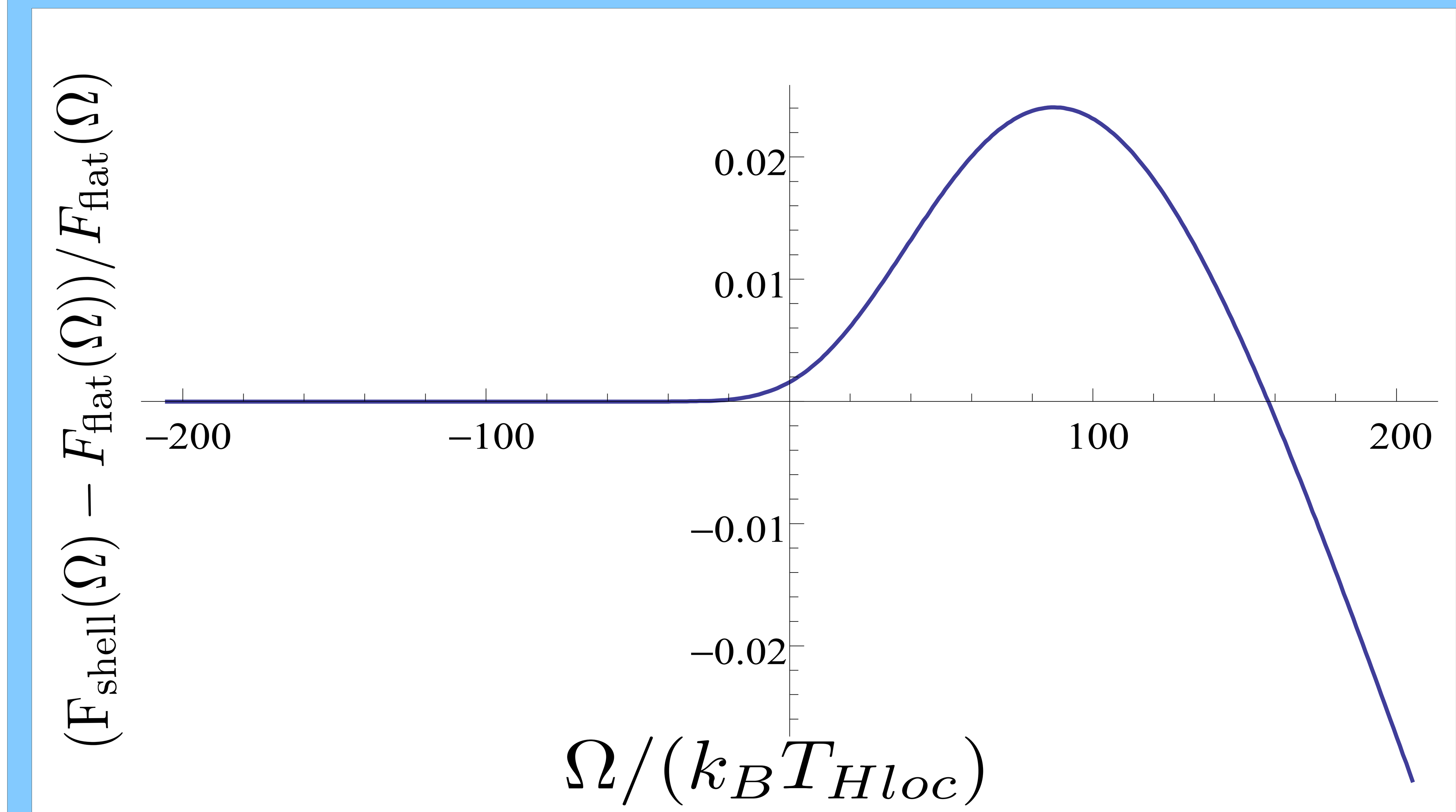
We integrate over all local frequencies $\tilde{\omega}$; thus, in principle, a detector can detect *any* change in the radial mode functions, and thus any change in the metric, no matter how far away.

To demonstrate this numerically, we calculated the response function for an R=3 shell and flat space, for switching time σ=0.5, and plotted the response function, as well as the relative difference.

The response F(Ω) vs. detector gap Ω

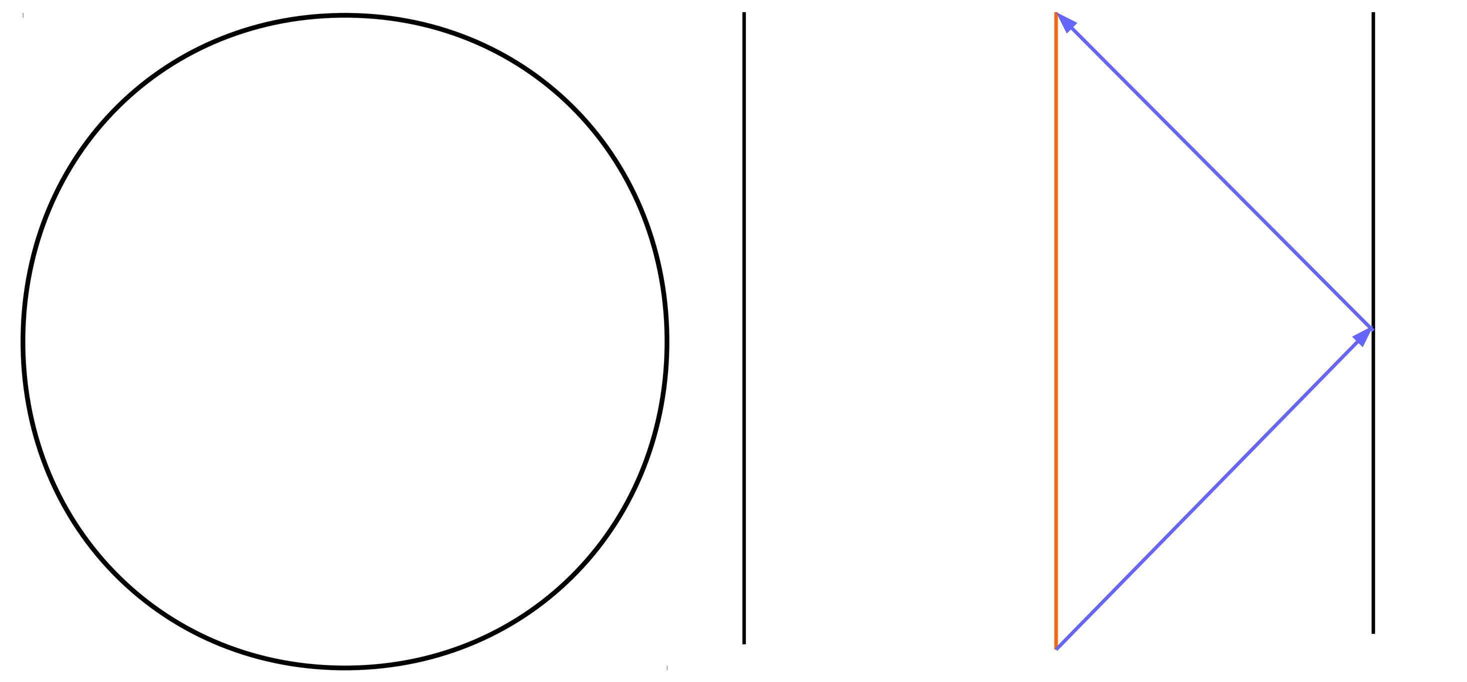


The relative difference in responses vs. Ω



The shell

Consider a massive transparent shell. The interior is flat, while outside is Schwarzschild with R_S=1. Therefore, the only difference between the shell's metric and the Minkowski space is outside the shell. If we switch via χ(τ) quickly enough, we can ensure the detector interacts only with the interior.



$$ds^2 = \begin{cases} -(1-1/r_+)dt^2 + (1-1/r_+)^{-1}dr_+^2 + r_+^2 d\Omega_2^2, & r_+ > R \\ -(1-1/R)dt^2 + dr_-^2 + r_-^2 d\Omega_2^2, & r_- < R \end{cases}$$

We solve for the modes of the Klein-Gordon equation, and express the vacuum state in terms of the modes.

Results

While both the response functions decay rapidly, we found that there is an order 0.1-1% difference in response between them. The relative response is strongest at large positive gap, where the absolute response is smallest; an experiment will likely have to compromise to detect this effect.

Therefore, we have demonstrated that a detector can detect the global structure of spacetime, even when switched on for times much shorter than the characteristic scale of the global structure, by probing the local vacuum.

It is possible that an array of detectors may be able to deduce more detail about global structure, perhaps even in a time-dependent space. Further research may clarify this further.

References
1) L. Hodgkinson, J. Louko, A.C. Ottewill. Phys. Rev. D **89**, 104002 (2014)
2) K.K. Ng, L. Hodgkinson, J. Louko, R.B. Mann, E. Martin-Martinez. Phys. Rev. D **90**, 064003 (2014)
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