# Turnaround radius in an accelerated universe for Einstein and for modified gravity

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work with A. Prain, M. Lapierre-Léonard *JCAP* 10, 013 (2015); *Phys. Dark Univ.* 11, 11 (2016)

## Outline

## Turnaround radius with Hawking mass

- Part of a larger program aiming at applying quasilocal mass in cosmology. Already used to test whether Newtonian *N*-Body simulations of large scale structures are reliable (VF, Prain & Lapierre-Léonard, PRD 2015).
- Consider present accelerated era of the universe and the largest bound objects in the sky. The turnaround radius was suggested as a possible way to test dark energy (Roupas *et al.* 2014, PRD 89, 083002; Pavlidou & Tomaras 2014, JCAP 09, 020; Pavlidou, Tetradis, Tomaras 2014, JCAP 05, 017)
- but the concept of TR is older (Souriau 1981; Stuchlik 1983; Stuchlik *et al.* 1989-2005; Mizony & Lachiéze-Rey 2005; Blau & Rollier 2008, ...)
- Consider an accelerated FLRW universe with one spherical inhomogeneity; massive test particles with zero radial initial velocity cannot collapse if they are outside a critical radius $R_c$ (*turnaround radius*), but can only expand.

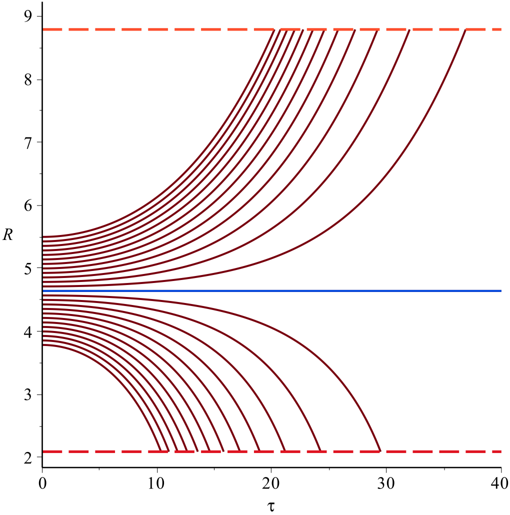
For $R < R_c$, outer layers of dust reach zero radial acceleration and collapse under self-gravity. If you cross outside $R_c$ in geodesic motion, you will never fall back.

TR studied in Schwarzschild-de Sitter, Lemaître-Tolman-Bondi, and McVittie spacetimes.

**SdS** (heuristic)**:**

$$ds^2 = -\left(1 - \frac{2M}{R} - H^2 R^2\right) dt^2 + \frac{dR^2}{1-\frac{2M}{R}-H^2R^2} + R^2 d\Omega_{(2)}^2$$

$$H = \sqrt{\Lambda/3}\,, \qquad R_c = \left(\frac{3GM}{\Lambda}\right)^{1/3}$$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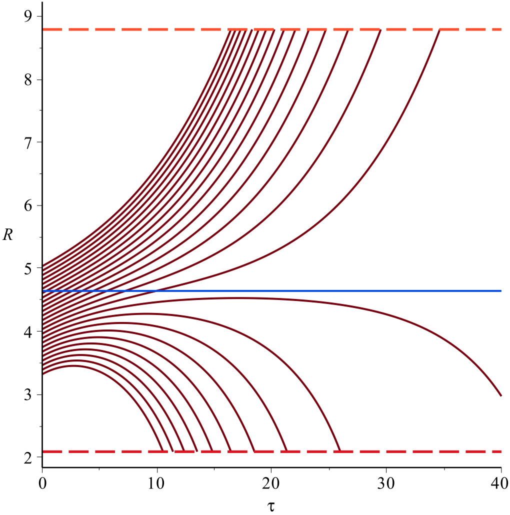
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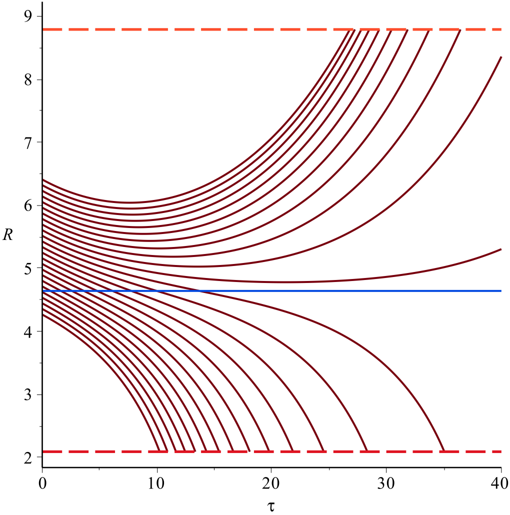
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Radial timelike geodesics obey $R(\tau) = \left(R^3 - R_c^3\right) H^2 / R^2$
LTB models (dust) Pavlidou, Tetradis & Tomaras 2014 have

$$ds^2 = -dt^2 + \frac{R'(t,r)}{1+f(r)}dr^2 + R^2(t,r)d\Omega_{(2)}^2$$

with $' \equiv d/dr$, $f(r)$ related to initial density profile. Radial timelike geodesics obey

$$\ddot{R} = -\frac{G\mathcal{M}(r)}{R^2} + \frac{\Lambda R}{3}$$

and the turnaround radius is $R_c = \left(\frac{3G\mathcal{M}(r_c)}{\Lambda}\right)^{1/3}$ where $\mathcal{M}(r) = \int_0^R dR\, R^2 \rho$ Lemaître mass.

More realistic: post-FLRW space (1st order)

$$ds^2 = a^2(\eta)\left[-(1+2\phi)\,d\eta^2 + (1-2\phi)\left(dr^2 + r^2 d\Omega^2_{(2)}\right)\right]$$

Pavlidou, Tetradis & Tomaras find timelike radial geodesics obey

$$\ddot{R} = -\frac{4\pi}{3}\left(\rho_{\rm DE} + 3P_{\rm DE}\right)R - \frac{G\mathcal{M}(r)}{R^2} = \frac{\ddot{a}}{a} - \frac{G\mathcal{M}(r)}{R^2}$$

where it is suggested (but not written down)

$$\mathcal{M}(r) = \int_0^R dR\, R^2 \rho_{\rm total}$$

$$\rightarrow \quad R_c = \left(\frac{3\mathcal{M}}{4(3w+1)\pi\rho_{\rm DE}}\right)^{1/3}$$

(reduces to SdS expression for $w = -1$).

**Questions:** (not answered, nor posed)

- gauge-invariance;
- what is the "mass in a sphere of radius $R$"? Should it include $\rho_{\mathrm{DE}}$? If not, why? Should it include only $\rho_{\mathrm{perturbation}}$? Why?

Use Hawking-Hayward quasilocal energy (reduces to Misner-Sharp-Hernandez mass in spherical symmetry) and a new splitting of it. Assumptions:

- GR is valid
- 1st order in metric perturbations; spherical symmetry $\phi = \phi(r)$ (consequences of $\phi \neq \phi(r)$ discussed in Barrow & Saich 1993, MNRAS 262, 717)
- FLRW background, spatially flat, accelerated by DE with $\rho_{DE}, P_{DE} = w\rho_{DE}$

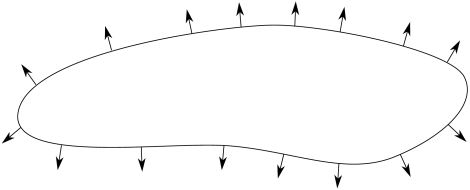
Physical mass is the Hawking quasilocal energy[1]

Idea: total mass in a region bounded by a surface $S$ is measured by behaviour of null geodesics at $S$

$S =$ closed spacelike orientable 2-surface

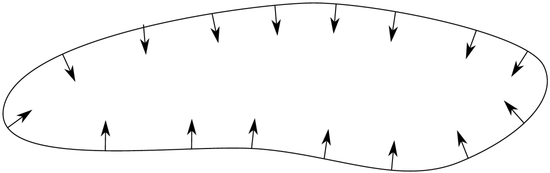
$\mathcal{R} =$ induced Ricci scalar on $S$

$\theta_{(\pm)} =$ expansions of outgoing/ingoing null geodesic congruences from $S$



[1] S.W. Hawking 1968, *J. Math. Phys.* 9, 568; S.A. Hayward 1994, *Phys. Rev. D* 49, 831

# General perturbations of FLRW



$\sigma^{(\pm)}_{ab}$ = shear tensors of null congruences
$\omega^a$ = projection on $S$ of the commutator of null normal vectors to $S$ (anholonomicity)

$\mu$ = volume 2-form on $S$

$A$ = area of $S$

$$M_{\mathrm{HH}} \equiv \frac{1}{8\pi}\sqrt{\frac{A}{16\pi}} \int_S \mu \left( \mathcal{R} + \theta_{(+)}\theta_{(-)} - \frac{1}{2}\sigma^{(+)}_{ab}\,\sigma^{ab}_{(-)} - 2\omega_a\,\omega^a \right)$$

Compute for

$$d\tilde{s}^2 = a^2(\eta) \underbrace{\left[ -\left(1+2\phi_N\right) d\eta^2 + \left(1-2\phi_N\right) \left( dr^2 + r^2 d\Omega_{(2)}^2 \right) \right]}_{\text{post-Newtonian}}$$

and attempt to decompose as $M_{\text{HH}} =$ (local) +(cosmological) to first order (general perts.)

Final result (with two methods) is

$$\tilde{M}_{\mathrm{H}} = \underbrace{\Omega M_{\mathrm{H}} - \frac{R\Omega_{,\eta}}{4\pi}\frac{\Omega_{,\eta}}{\Omega}\int_S \mu\,\phi_N}_{\text{local}} + \underbrace{\frac{R^3}{2}\frac{\Omega_{,\eta}^2}{\Omega}}_{\text{cosmological}}$$

Prain, Vitagliano, VF & Lapierre-Léonard, *Class. Quantum Grav.*, in press

Now adapt to spherical symmetry $\rightarrow$

$$M_{\rm H} = ma + \frac{H^2 R^3}{2}(1-\phi) \simeq= ma + \frac{H^2 R^3}{2}$$

with $m = \int d^3\vec{x}\, \nabla^2 \phi$ Newtonian mass $\sim$ comoving length scale $ma \sim$ physical length scale. Criterion for a system on the verge of breaking down is now

local part $\quad ma = \frac{H^2 R^3}{2} \quad$ cosmological part $\rightarrow$

$$R_c(t) = \left(\frac{2ma}{H^2}\right)^{1/3}$$

Now $H^2 = 8\pi G \rho_{\rm DE}/3 \rightarrow \; R_c(t) = \left(\frac{3ma}{4\pi\rho_{\rm DE}}\right)^{1/3}$ and, if

$w = $ const., $\quad R_c = \left(\frac{3ma}{4\pi\rho_0}\right)^{1/3} a^{\frac{3w+4}{3}}$

Compare with Pavlidou, Tetradis & Tomaras

$$\frac{R_c}{R_c^{(PTT)}} = \left(\frac{|3w+1|}{2}\right)^{1/3} \approx 1 \qquad \text{if} \;\; w \approx -1$$

but now

- no ambiguities in “mass inside a sphere of radius $R_c$”;
- rigorous derivation of turnaround radius $R_c$
- important if you want to constrain $w$

Can express $R_c = R_c(z)$ and invert to obtain

$$\int dz \, \frac{w(z)+1}{z+1} = \ln\left[\left(\frac{3ma}{4\pi\rho}\right)^{1/3} \frac{1}{R(z)}\right]$$

If $w =$ const. reduces to

$$w(z) = -1 + \frac{\ln\left[\left(\frac{3ma}{4\pi\rho_0}\right)^{1/3} \frac{1}{R_c(z)}\right]}{\ln(z+1)}$$

constrain $w$ if $ma$ and $R_c$ are known.

## Turnaround radius in ST gravity

$$ds^2 = a^2(\eta) \left[ -(1+2\psi)\, d\eta^2 + (1-2\phi) \left( dr^2 + r^2 d\Omega^2_{(2)} \right) \right] ,$$

$\phi = \phi(r), \psi = \psi(r)$. Massive test particles follow timelike geodesics

$$\frac{du^a}{d\tau} + \Gamma^a_{bc} u^b u^c = 0 ,$$

$u_c u^c = -1$ and the geodesic eq. give

$$\frac{du^0}{d\tau} + \frac{a_\eta}{a} (u^0)^2 + 2\psi' u^0 u^1 + \frac{a_\eta}{a} (1 - 2\phi - 2\psi)(u^1)^2 = 0$$
$$\frac{du^1}{d\tau} + \psi' (u^0)^2 + \frac{2a_\eta}{a} u^0 u^1 - \phi' (u^1)^2 = 0$$

Areal radius is $R(t,r) = ar\sqrt{1-2\phi} \simeq ar\,(1-\phi)$, further manipulation yields

$$\frac{d^2R}{dt^2} = \left[\ddot{a}r + \frac{\dot{a}u^1}{au^0} + \frac{1}{au^0}\frac{d}{d\tau}\left(\frac{u^1}{u^0}\right)\right](1-\phi)$$

Criterion locating the (unique) turnaround radius is $d^2R/dt^2 = 0$, which becomes

$$\ddot{a}r - \frac{\psi'}{a} = 0$$

In terms of the *areal* turnaround radius,

$$R_c = a(t) r_c \left[1 - \phi(r_c)\right]$$

or, using the gravitational slip $\xi \equiv \left(\phi - \psi\right)/\phi$,

$$\ddot{a}\, R_c \left(1 + \phi_c\right) - \phi_c' \left(1 - \xi_c\right) + \phi_c \xi_c' = 0$$

## Conclusions

- Turnaround radius is an opportunity to test gravity and the $\Lambda$CDM model.
- Split $M_{\rm H}$ for spherical perturbations of FLRW $\rightarrow$ rigorous derivation of $R_c$, small correction, much needed clarification of "mass".
- In modified gravity, no accepted $M_{\rm H}$, use criterion $\ddot{R} = 0 \rightarrow$ eq. for $R_c$ in ST gravity.
- Is it important? Astronomers claim that the upper bound on $R_c$ in GR is exceeded by far in two galaxy groups (Lee *et al.* 2015, *Astrophys. J.* 815, 43; Lee, arXiv:1603.06672).
  Wait and see!

# THANK YOU