Generating Einstein gravity, cosmological constant and Higgs mass from restricted Weyl invariance

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Restricted Weyl invariant action

Consider the action that includes R^2 gravity non-minimally coupled to the Higgs boson field Φ (with no explicit mass),

$$S_0 = \int d^4x\sqrt{-g}\left(\alpha R^2 - \xi R|\Phi|^2 - (\partial_\mu\bar{\Phi}\partial^\mu\Phi) - \lambda|\Phi|^4\right) .$$

- The above dimensionless action in 4D is not Weyl invariant but has a symmetry that is larger than scale invariance.
- It is invariant under the transformations $g_{\mu\nu} \to \Omega^2(x) g_{\mu\nu}$, $\Phi \to \Phi/\Omega(x)$ with $\Box\Omega(x) \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu\Omega(x) = 0$. This was dubbed *restricted Weyl invariance* (A. Edery and Y. Nakayama, Phys.Rev. D90 (2014) 043007).

Equivalent form of the action

We introduce the auxiliary field φ and rewrite the action into the equivalent form

$$S_1 = \int d^4x\sqrt{-g}\Big[-\alpha\left(c_1\varphi + R + \frac{c_2}{\alpha}|\Phi|^2\right)^2 + \\ \alpha R^2 - \xi R|\Phi|^2 - (\partial_\mu\bar{\Phi}\partial^\mu\Phi) - \lambda|\Phi|^4\Big]$$

where c_1 and c_2 are arbitrary constants.

The first term (which is squared) does nothing after performing the Gaussian integral over φ in the path integral i.e.

$$\int \mathcal{D}\varphi e^{-i\alpha\, c_1^2 \int d^4x\sqrt{-g}(\varphi - f(x))^2} = \text{const}$$

Expanding the action

Expanding the squared term in the action yields

$$S_2 = \int d^4x\sqrt{-g}\,\big(-c_1^2\alpha\varphi^2 - 2\alpha c_1\varphi R$$
$$-(\xi + 2c_2)R|\Phi|^2 - (\partial_\mu\bar{\Phi}\partial^\mu\Phi) - 2c_1c_2\varphi|\Phi|^2 - (\alpha^{-1}c_2^2 + \lambda)|\Phi|^4\big)\ .$$

The above action maintains restricted Weyl invariance if the auxiliary field φ transforms as $\varphi \to \varphi/\Omega^2$

i.e. the action is invariant under the transformations $g_{\mu\nu} \to \Omega^2 g_{\mu\nu}$, $\Phi \to \Phi/\Omega$ and $\varphi \to \varphi/\Omega^2$ with $\Box\Omega = 0$.

Weyl transformation

We now perform the Weyl transformation

$$g_{\mu\nu} \to \varphi^{-1} g_{\mu\nu}$$

$$\sqrt{-g} \to \varphi^{-2}\sqrt{-g}$$

$$R \to \varphi R - 6\varphi^{3/2}\Box\varphi^{-1/2}$$

$$\Phi \to \varphi^{1/2}\Phi\ .$$

Einstein-Hilbert action with cosmological constant and Higgs mass

In this new frame, the action becomes

$$S_3 = \int d^4x\sqrt{-g}\,\Big(-\alpha c_1^2 - 2\,\alpha c_1 R - \partial_\mu\bar{\Phi}\partial^\mu\Phi - 2\,c_1 c_2|\Phi|^2 - (\alpha^{-1}c_2^2+\lambda)|\Phi|^4 - (\xi+2c_2)R\,|\Phi|^2 + 3\,\alpha c_1\frac{1}{\varphi^2}\partial_\mu\varphi\,\partial^\mu\varphi + (6(\xi+2c_2)-1)\varphi^{1/2}\Box\varphi^{-1/2}|\Phi|^2\Big)\,.$$

- $-2\,\alpha c_1 R$ is the Einstein-Hilbert term with $-2\alpha c_1$ determining Newton's constant (in our convention αc_1 is negative)
- $-\alpha\, c_1^2$ determines the cosmological constant and we are free to choose its value by setting the value of c_1
- The physical Higgs mass is determined by $-2\,c_1\,c_2$ and can be adjusted freely by choosing an appropriate value for c_2.

The scalar field φ

The auxiliary field φ couples to the Higgs field and gravity. If its coupling to the Higgs field is experimentally small, we are free to eliminate it by choosing $\xi + 2c_2$ to be $1/6$.

The action then simplifies to

$$S_4 = \int d^4x\sqrt{-g}\,\Big(- \alpha c_1^2 - 2\,\alpha c_1 R - \partial_\mu\bar{\Phi}\partial^\mu\Phi - 2\,c_1 c_2|\Phi|^2 - (\alpha^{-1}c_2^2 + \lambda)|\Phi|^4 - \frac{1}{6}R\,|\Phi|^2 + 3\,\alpha c_1\frac{1}{\varphi^2}\partial_\mu\varphi\,\partial^\mu\varphi\Big)\,.$$

Eliminating the coupling between the auxiliary field φ and the Higgs field Φ *fixes* the coefficient for the nonminimal coupling between gravity and the Higgs field i.e. the coefficient of $R\Phi^2$ is no longer arbitrary but equal to $-1/6$.

Finally, the kinetic term for φ can be made canonical by defining the scalar field $\psi \equiv \sqrt{-3\alpha c_1}\,\ln\varphi$ so that $3\,\alpha c_1 \dfrac{1}{\varphi^2}\partial_\mu\varphi\,\partial^\mu\varphi = -\partial_\mu\psi\,\partial^\mu\psi$. The action then reduces to

$$S_5 = \int d^4x\sqrt{-g}\,\Big(-\alpha c_1^2 - 2\,\alpha c_1 R - \partial_\mu\bar{\Phi}\partial^\mu\Phi - 2\,c_1 c_2|\Phi|^2 - (\alpha^{-1}c_2^2+\lambda)|\Phi|^4 - \frac{1}{6}R\,|\Phi|^2 - \partial_\mu\psi\,\partial^\mu\psi\Big)\,.$$

Our model therefore contains an extra massless minimally coupled scalar field ψ besides the Higgs field.

Thank You

Symmetry of final action

The restricted Weyl symmetry of our original action manifests itself in the final action through a symmetry under a transformation of the auxiliary field φ only. The action is invariant under

$\varphi \to \varphi/\Omega^2$, $g_{\mu\nu} \to g_{\mu\nu}$, $\Phi \to \Phi$ with condition $\Box\Omega - \partial_\mu(\ln\varphi)\partial^\mu\Omega = 0$.

This can be understood as follows. We replaced $g_{\mu\nu}$ with $\hat{g}_{\mu\nu} = \varphi^{-1} g_{\mu\nu}$. Originally, the restricted Weyl symmetry acts as $\hat{g}_{\mu\nu} \to \hat{g}_{\mu\nu}\Omega^2$, $\varphi \to \varphi/\Omega^2$ and $\Phi \to \Phi/\Omega$ with $\hat{\Box}\Omega = 0$. After the replacement, it acts as $g_{\mu\nu} \to g_{\mu\nu}$, $\Phi \to \Phi$, $\varphi \to \varphi/\Omega^2$ with the condition $\hat{\Box}\Omega = 0$. In terms of $g_{\mu\nu}$, the condition $\hat{\Box}\Omega = 0$ is $\Box\Omega - \partial_\mu(\ln\varphi)\partial^\mu\Omega = 0$.

Inclusion of other Standard Model fields

We now briefly discuss the inclusion of the other standard model fields into the original action. They are collectively given by gauge fields A_μ with gauge group $G = U(1) \times SU(2) \times SU(3)$ and (Weyl) fermions Ψ under various representations of G. We may consider the (restricted) Weyl invariant action schematically given by

$$-\int d^4x\sqrt{-g}\Big(\frac{1}{2g^2}\,\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} + \theta\,\mathrm{Tr}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} + \bar{\Psi}D_\mu\gamma^\mu\Psi + \lambda(\Psi\Psi\Phi) + \bar{\lambda}(\bar{\Psi}\bar{\Psi}\bar{\Phi}) + \text{Higgs}\Big).$$

The Higgs sector is essentially given in the previous action by replacing the derivative ∂_μ with the gauge covariant one. The gauge coupling constant g, the theta angle θ, and Yukawa coupling λ are all dimensionless and it is easy to see that the above action does not change under a restricted Weyl transformation supplemented by $\Psi \to \varphi^{3/4}\Psi$. Note that there is no direct coupling to the extra massless scalar field φ (except for the Higgs sector we have already discussed).