The spontaneous $\mathbb{Z}_2$ breaking Twin Higgs

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Outline

1. The Higgs mass, the hierarchy problem, and the pursuit of naturalness

2. Beyond the Standard Model (BSM) solutions including *neutral naturalness*

3. The original Twin Higgs

4. The spontaneous $\mathbb{Z}_2$ breaking Twin Higgs
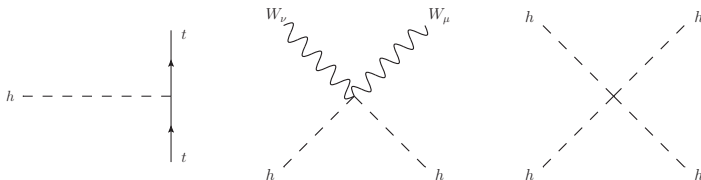
5. Conclusion

The Higgs mass

The Standard Model (SM) Lagrangian contains a Higgs mass term
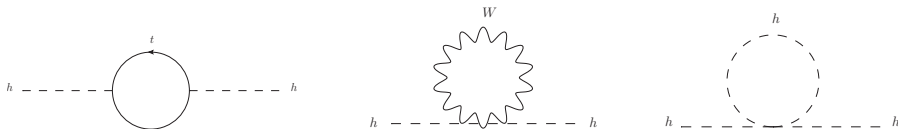
$$\mathcal{L}_{SM} \supset -\frac{1}{2} m_h^2 h^2$$

and also some Higgs couplings

$$\mathcal{L}_{SM} \supset -\frac{y_t}{\sqrt{2}} h \bar{t} t + \frac{g^2}{4} h^2 W^{\mu+} W^-_\mu - \frac{1}{4} \lambda h^4.$$

We can use these couplings to draw the following diagrams

each of which contribute to the mass of the Higgs boson

$$\delta m_h^2 \sim \int^\Lambda \frac{d^4 k}{k^2} \sim \Lambda^2.$$

The Higgs mass is quadratically sensitive to the cutoff Λ!

The hierarchy problem

Imagine a world where the SM is all that there is up to the scale where quantum gravity becomes important. In that case we have that $\Lambda \approx M_P \approx 10^{18}$ GeV. This leads to a tuning of one part in the

$$\frac{M_P^2}{m_h^2} \approx \frac{(10^{18})^2}{(10^2)^2} = 10^{32}.$$

Are we to believe this? Is Nature this tuned?

The pursuit of naturalness

Field	Symmetry as $m \to 0$	Implications
Spin 1/2 $-m\bar{\Psi}\Psi$	$\Psi \to e^{i\alpha\gamma_5}\Psi$ (chiral symmetry)	$\delta m \propto m$ natural
Spin 1 $\frac{1}{2}m^2 A_\mu A^\mu$	$A_\mu \to A_\mu - \frac{1}{e}\partial_\mu\alpha$ (gauge invariance)	$\delta m \propto m$ natural
Spin 0 $-\frac{1}{2}m^2\phi^2$	None	$\delta m \propto \Lambda$ unnatural

"Perhaps this is the reason why there seem to be no elementary scalar fields in Nature." - An Introduction to Quantum Field Theory, Peskin and Schroeder

The most well known solution: Supersymmetry

Supersymmetry is a symmetry linking bosons to fermions capable of solving the hierarchy problem. Every SM particle has a partner particle. Most importantly, quadratic contributions to scalar masses cancel between supersymmetric partner particles.
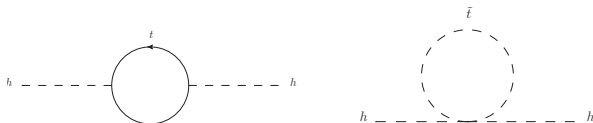
However, natural Supersymmetry appears to be on thin ice.

ATLAS SUSY Searches* - 95% CL Lower Limits

Status: March 2016

ATLAS Preliminary

$\sqrt{s} = 7, 8, 13$ TeV

| Model	e, μ, τ, γ	Jets	E_T^{miss}	$\int \mathcal{L}\, dt\,[\text{fb}^{-1}]$	Mass limit	Reference
Inclusive Searches						
MSUGRA/CMSSM	0-3 e, μ/1-2 τ	2-10 jets/3 b	Yes	20.3	$\tilde{q}, \tilde{g}$ 1.85 TeV $m(\tilde{q})=m(\tilde{g})$	1507.05525
$\tilde{q}\tilde{q}, \tilde{q}\to q\tilde{\chi}_1^0$	0	2-6 jets	Yes	3.2	$\tilde{q}$ 980 GeV $m(\tilde{\chi}_1^0)=0$ GeV, $m(1^{st} \text{gen.} \tilde{q})=m(2^{nd} \text{gen.} \tilde{q})$	ATLAS-CONF-2015-062
$\tilde{q}\tilde{q}, \tilde{q}\to q\tilde{\chi}_1^0$ (compressed)	mono-jet	1-3 jets	Yes	3.2	$\tilde{q}$ 608 GeV $m(\tilde{q})-m(\tilde{\chi}_1^0)<5$ GeV	Te:approx
$\tilde{g}\tilde{g}, \tilde{g}\to q\bar{q}\tilde{\chi}_1^0$	2 e, μ (off-Z)	2 jets	Yes	20.3	$\tilde{g}$ 820 GeV	1503.03290
$\tilde{g}\tilde{g}, \tilde{g}\to q\bar{q}\tilde{\chi}_1^0$	0	2-6 jets	Yes	3.2	$\tilde{g}$ 1.52 TeV $m(\tilde{\chi}_1^0)=0$ GeV	ATLAS-CONF-2015-062
$\tilde{g}\tilde{g}, \tilde{g}\to q\bar{q}\tilde{\chi}_1^\pm \to q\bar{q}W^\pm \tilde{\chi}_1^0$	1 e, μ	2-6 jets	Yes	3.3	$\tilde{g}$ 1.6 TeV $m(\tilde{\chi}_1^0)<200$ GeV, $m(\tilde{\chi}^\pm)=0.5(m(\tilde{\chi}_1^0)+m(\tilde{g}))$	ATLAS-CONF-2015-076
$\tilde{g}\tilde{g}, \tilde{g}\to q\bar{q}(\ell\ell/\nu\nu)\tilde{\chi}_1^0$	2 e, μ	0-3 jets	Yes	20	$\tilde{g}$ 1.38 TeV $m(\tilde{\chi}_1^0)=0$ GeV	1501.03555
$\tilde{g}\tilde{g}, \tilde{g}\to q\bar{q}WZ\tilde{\chi}_1^0$	0	7-10 jets	Yes	20.3	$\tilde{g}$ 1.4 TeV $m(\tilde{\chi}_1^0)<100$ GeV	1602.06194
GMSB ($\tilde{\ell}$ NLSP)	1-2 τ + 0-1 ℓ	0-2 jets	Yes	20.3	$\tilde{g}$ 1.63 TeV $\tan\beta>20$	1407.0603
GGM (bino NLSP)	2 γ		Yes	20.3	$\tilde{g}$ 1.34 TeV $c\tau(\text{NLSP})<0.1$ mm	1507.05493
GGM (higgsino-bino NLSP)	γ	1 b	Yes	20.3	$\tilde{g}$ 1.37 TeV $m(\tilde{\chi}_1^0)=950$ GeV, $c\tau(\text{NLSP})<0.1$ mm, $\mu<0$	1507.05493
GGM (higgsino-bino NLSP)	γ	2 jets	Yes	20.3	$\tilde{g}$ 1.3 TeV $m(\tilde{\chi}_1^0)=950$ GeV, $c\tau(\text{NLSP})<0.1$ mm, $\mu>0$	1507.05493
GGM (higgsino NLSP)	2 e, μ (Z)	2 jets	Yes	20.3	$\tilde{g}$ 900 GeV $m(\text{NLSP})=430$ GeV	1503.03290
Gravitino LSP	0	mono-jet	Yes	20.3	$F^{1/2}$ scale 865 GeV $m(\tilde{G})>1.8\times10^{-4}$ eV, $m(\tilde{g})=m(\tilde{q})=1.5$ TeV	1502.01518
3rd gen. $\tilde{g}$ med.						
$\tilde{g}\tilde{g}, \tilde{g}\to b\bar{b}\tilde{\chi}_1^0$	0	3 b	Yes	3.3	$\tilde{g}$ 1.78 TeV $m(\tilde{\chi}_1^0)<600$ GeV	ATLAS-CONF-2015-067
$\tilde{g}\tilde{g}, \tilde{g}\to t\bar{t}\tilde{\chi}_1^0$	0-1 e, μ	3 b	Yes	3.3	$\tilde{g}$ 1.76 TeV $m(\tilde{\chi}_1^0)<600$ GeV	Te:approx
$\tilde{g}\tilde{g}, \tilde{g}\to b\bar{t}\tilde{\chi}_1^+$	0-1 e, μ	3 b	Yes	20.1	$\tilde{g}$ 1.37 TeV $m(\tilde{\chi}_1^0)<300$ GeV	1407.0600
3rd gen. squarks direct production						
$\tilde{b}_1\tilde{b}_1, \tilde{b}_1\to b\tilde{\chi}_1^0$	0	2 b	Yes	3.2	$\tilde{b}_1$ 840 GeV $m(\tilde{\chi}_1^0)<100$ GeV	ATLAS-CONF-2015-066
$\tilde{b}_1\tilde{b}_1, \tilde{b}_1\to t\tilde{\chi}_1^\pm$	2 e, μ (SS)	0-3 b	Yes	3.2	$\tilde{b}_1$ 325-540 GeV $m(\tilde{\chi}_1^\pm)=50$ GeV, $m(\tilde{\chi}_1^\pm)=m(\tilde{\chi}_1^0)+55$ GeV	1602.09058
$\tilde{t}_1\tilde{t}_1, \tilde{t}_1\to b\tilde{\chi}_1^\pm$	1-2 e, μ	1-2 b	Yes	4.7/20.3	$\tilde{t}_1$ 117-170 GeV 200-500 GeV $m(\tilde{\chi}_1^0)=2m(\tilde{\chi}_1^\pm), m(\tilde{\chi}_1^\pm)=55$ GeV	1209.2102, 1407.0583
$\tilde{t}_1\tilde{t}_1, \tilde{t}_1\to W b\tilde{\chi}_1^0$ or $t\tilde{\chi}_1^0$	0-2 e, μ	0-2 jets/1-2 b	Yes	20.3	$\tilde{t}_1$ 90-198 GeV 205-715 GeV 745-785 GeV $m(\tilde{\chi}_1^0)=1$ GeV	1506.08616, ATLAS-CONF-2016-007
$\tilde{t}_1\tilde{t}_1, \tilde{t}_1\to c\tilde{\chi}_1^0$	0	mono-jet/c-tag	Yes	20.3	$\tilde{t}_1$ 90-245 GeV $m(\tilde{t}_1)-m(\tilde{\chi}_1^0)<85$ GeV	1407.0608
$\tilde{t}_1\tilde{t}_1$ (natural GMSB)	2 e, μ (Z)	1 b	Yes	20.3	$\tilde{t}_2$ 150-600 GeV $m(\tilde{\chi}_1^0)>150$ GeV	1403.5222
$\tilde{t}_2\tilde{t}_2, \tilde{t}_2\to \tilde{t}_1 + Z$	3 e, μ (Z)	1 b	Yes	20.3	$\tilde{t}_2$ 290-610 GeV $m(\tilde{\chi}_1^0)=200$ GeV	1403.5222
$\tilde{t}_2\tilde{t}_2, \tilde{t}_2\to \tilde{t}_1 + h$	1 e, μ	6 jets + 2 b	Yes	20.3	$\tilde{t}_2$ 320-620 GeV $m(\tilde{\chi}_1^0)=0$ GeV	1506.08616
EW direct						
$\tilde{\ell}_{L,R}\tilde{\ell}_{L,R}, \tilde{\ell}\to \ell\tilde{\chi}_1^0$	2 e, μ	0	Yes	20.3	$\tilde{\ell}$ 90-335 GeV $m(\tilde{\chi}_1^0)=0$ GeV	1403.5294
$\tilde{\chi}_1^+\tilde{\chi}_1^-, \tilde{\chi}_1^+\to \tilde{\ell}\nu(\ell\tilde{\nu})$	2 e, μ	0	Yes	20.3	$\tilde{\chi}_1^\pm$ 140-475 GeV $m(\tilde{\chi}_1^0)=0$ GeV, $m(\tilde{\ell}, \tilde{\nu})=0.5(m(\tilde{\chi}_1^\pm)+m(\tilde{\chi}_1^0))$	1403.5294
$\tilde{\chi}_1^+\tilde{\chi}_1^-, \tilde{\chi}_1^+\to \tilde{\tau}\nu(\tau\tilde{\nu})$	2 τ		Yes	20.3	$\tilde{\chi}_1^\pm$ 355 GeV $m(\tilde{\chi}_1^0)=0$ GeV, $m(\tilde{\tau}, \tilde{\nu})=0.5(m(\tilde{\chi}_1^\pm)+m(\tilde{\chi}_1^0))$	1407.0350
$\tilde{\chi}_1^\pm\tilde{\chi}_2^0\to \tilde{\ell}_L\nu\tilde{\ell}_L\ell(\tilde{\nu}\nu), \ell\tilde{\nu}\tilde{\ell}_L\ell(\tilde{\nu}\nu)$	3 e, μ	0	Yes	20.3	$\tilde{\chi}_1^\pm, \tilde{\chi}_2^0$ 715 GeV $m(\tilde{\chi}_1^\pm)=m(\tilde{\chi}_2^0), m(\tilde{\chi}_1^0)=0$, sleptons decoupled	1402.7029
$\tilde{\chi}_1^\pm\tilde{\chi}_2^0\to W\tilde{\chi}_1^0 Z\tilde{\chi}_1^0$	2-3 e, μ	0-2 jets	Yes	20.3	$\tilde{\chi}_1^\pm, \tilde{\chi}_2^0$ 425 GeV $m(\tilde{\chi}_1^\pm)=m(\tilde{\chi}_2^0), m(\tilde{\chi}_1^0)=0$, sleptons decoupled	1403.5294, 1402.7029
$\tilde{\chi}_2^0\tilde{\chi}_3^0, \tilde{\chi}_{2,3}^0\to \tilde{\ell}_R\ell$	4 e, μ	0	Yes	20.3	$\tilde{\chi}_2^0, \tilde{\chi}_3^0$ 635 GeV $m(\tilde{\chi}_2^0)=m(\tilde{\chi}_3^0), m(\tilde{\chi}_1^0)=0, m(\tilde{\ell}, \tilde{\nu})=0.5(m(\tilde{\chi}_2^0)+m(\tilde{\chi}_1^0))$	1405.5086
GGM (wino NLSP) weak prod.	1 $e, \mu + \gamma$		Yes	20.3	$\tilde{W}$ 115-370 GeV $c\tau<1$ mm	1507.05493
Long-lived particles						
Direct $\tilde{\chi}_1^+\tilde{\chi}_1^-$ prod., long-lived $\tilde{\chi}_1^\pm$	Disapp. trk	1 jet	Yes	20.3	$\tilde{\chi}_1^\pm$ 270 GeV $m(\tilde{\chi}_1^\pm)-m(\tilde{\chi}_1^0)=160$ MeV, $\tau(\tilde{\chi}_1^\pm)=0.2$ ns	1310.3675
Direct $\tilde{\chi}_1^+\tilde{\chi}_1^-$ prod., long-lived $\tilde{\chi}_1^\pm$	dE/dx trk		Yes	18.4	$\tilde{\chi}_1^\pm$ 495 GeV $m(\tilde{\chi}_1^\pm)-m(\tilde{\chi}_1^0)=160$ MeV, $\tau(\tilde{\chi}_1^\pm)<15$ ns	1506.05332
Stable, stopped $\tilde{g}$ R-hadron	0	1-5 jets	Yes	27.9	$\tilde{g}$ 850 GeV $m(\tilde{\chi}_1^0)=100$ GeV, 10 μs$<c\tau(\tilde{g})<1000$ s	1310.6584
Metastable $\tilde{g}$ R-hadron				3.2	$\tilde{g}$ 1.54 TeV $m(\tilde{\chi}_1^0)=100$ GeV	Te:approx
GMSB, stable $\tilde{\tau}, \tilde{\chi}_1^0\to \tilde{\tau}(\tilde{e}, \tilde{\mu})+\tau(e, \mu)$	1-2 μ			19.1	$\tilde{\chi}_1^0$ 537 GeV 10<tanβ<50	1411.6795
GMSB, $\tilde{\chi}_1^0\to \gamma\tilde{G}$, long-lived $\tilde{\chi}_1^0$	2 γ		Yes	20.3	$\tilde{\chi}_1^0$ 440 GeV $1<\tau(\tilde{\chi}_1^0)<3$ ns, SPS8 model	1409.5542
$\tilde{g}\tilde{g}, \tilde{\chi}_1^0\to e e\nu/e\mu\nu/\mu\mu\nu$	displ. ee/eμ/μμ			20.3	$\tilde{\chi}_1^0$ 1.0 TeV $7<c\tau(\tilde{\chi}_1^0)<740$ mm, $m(\tilde{g})=1.3$ TeV	1504.05162
GGM $\tilde{g}\tilde{g}, \tilde{\chi}_1^0\to ZG$	displ. vtx + jets			20.3	$\tilde{\chi}_1^0$ 1.0 TeV $6<c\tau(\tilde{\chi}_1^0)<480$ mm, $m(\tilde{g})=1.1$ TeV	1504.05162
RPV						
LFV $pp\to \tilde{\nu}_\tau + X, \tilde{\nu}_\tau\to e\mu/e\tau/\mu\tau$	$e\mu, e\tau, \mu\tau$			20.3	$\tilde{\nu}_\tau$ 1.7 TeV $\lambda'_{311}=0.11, \lambda_{132/133/233}=0.07$	1503.04430
Bilinear RPV CMSSM	2 e, μ (SS)	0-3 b	Yes	20.3	$\tilde{q}, \tilde{g}$ 1.45 TeV $m(\tilde{q})=m(\tilde{g}), c\tau_{LSP}<1$ mm	1404.2500
$\tilde{\chi}_1^+\tilde{\chi}_1^-, \tilde{\chi}_1^+\to W\tilde{\chi}_1^0, \tilde{\chi}_1^0\to ee\tilde{\nu}_\mu, e\mu\tilde{\nu}_e$	4 e, μ		Yes	20.3	$\tilde{\chi}_1^\pm$ 760 GeV $m(\tilde{\chi}_1^0)>0.2\times m(\tilde{\chi}_1^\pm), \lambda_{121}\neq 0$	1405.5086
$\tilde{\chi}_1^+\tilde{\chi}_1^-, \tilde{\chi}_1^+\to W\tilde{\chi}_1^0, \tilde{\chi}_1^0\to \tau\tau\tilde{\nu}_e, e\tau\tilde{\nu}_\tau$	3 $e, \mu + \tau$		Yes	20.3	$\tilde{\chi}_1^\pm$ 450 GeV $m(\tilde{\chi}_1^0)>0.2\times m(\tilde{\chi}_1^\pm), \lambda_{133}\neq 0$	1405.5086
$\tilde{g}\tilde{g}, \tilde{g}\to qqq$	0	6-7 jets		20.3	$\tilde{g}$ 917 GeV BRt=BR$_b$=BR$_c$=0%	1502.05686
$\tilde{g}\tilde{g}, \tilde{g}\to q\bar{q}\tilde{\chi}_1^0, \tilde{\chi}_1^0\to qqq$	0	6-7 jets		20.3	$\tilde{g}$ 980 GeV $m(\tilde{\chi}_1^0)=600$ GeV	1502.05686
$\tilde{g}\tilde{g}, \tilde{g}\to \tilde{t}_1 t, \tilde{t}_1\to bs$	2 e, μ (SS)	0-3 b	Yes	20.3	$\tilde{g}$ 880 GeV	1404.250
$\tilde{t}_1\tilde{t}_1, \tilde{t}_1\to bd$	0	2 jets + 2 b		20.3	$\tilde{t}_1$ 320 GeV	1601.07453
$\tilde{t}_1\tilde{t}_1, \tilde{t}_1\to b\ell$	2 e, μ	2 b		20.3	$\tilde{t}_1$ 0.4-1.0 TeV BR($\tilde{t}_1\to be/\mu$)>20%	ATLAS-CONF-2015-015
Other	Scalar charm, $\tilde{c}\to c\tilde{\chi}_1^0$	0	2 c	Yes	20.3	$\tilde{c}$ 510 GeV $m(\tilde{\chi}_1^0)<200$ GeV	1501.01325

*Only a selection of the available mass limits on new states or phenomena is shown.

Mass scale [TeV]

10^{-1} 1

Generally, the constraints on Supersymmetry are severe because the superpartners are charged under the SM gauge groups. This leads to large production cross sections at the LHC.
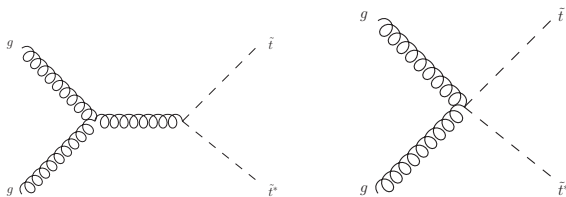
Neutral Naturalness

If we can somehow construct a model where the partner particles are neutral under the SM gauge groups then we could avoid experimental constraints. This is the idea of neutral naturalness. These types of theories only tend to solve the "little" hierarchy problem: they keep the Higgs mass natural only up to the highest scales probed by the LHC.

The Twin Higgs

Start with a global $SU(4)$ symmetry and consider a Higgs field H transforming as a fundamental under it. Next, write the $SU(4)$ symmetric potential

$$V_{SU(4)}(H) = -\mu^2 H^\dagger H + \lambda(H^\dagger H)^2$$

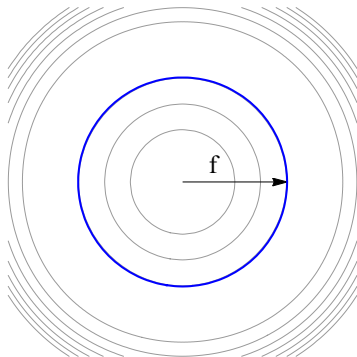where $\mu^2 > 0$. This is the famous "Mexican hat" potential.
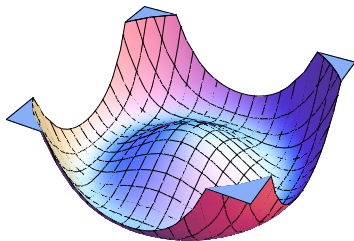
The potential is minimized by a non-zero vev

$$\langle H \rangle \equiv f = \frac{\mu}{\sqrt{2\lambda}}$$

and spontaneous symmetry breaking occurs. Here $SU(4)$ is broken to $SU(3)$ which gives 7 Goldstone bosons. As we will see, the Higgs will ultimately be identified as one of these Goldstone bosons.

Think about the Higgs as an excitation about the bottom the trough.

Right now the bottom of the trough is flat so the Higgs is (more generally Goldstone bosons are) massless.

To give the Higgs a mass, explicitly break the $SU(4)$ by gauging a $SU(2)_A \times SU(2)_B$ subgroup. This divides the Higgs field H in two

$$H = \begin{pmatrix} H_A \\ H_B \end{pmatrix}.$$

The A sector will be identified with the SM while the B sector is a "mirror" sector. Six of the 7 Goldstone bosons are eaten by gauge fields leaving only one left: the Higgs!

This gauging gives a quadratically divergent contribution to the potential

$$\Delta V(H) = \frac{9g_A^2\Lambda^2}{64\pi^2}H_A^\dagger H_A + \frac{9g_B^2\Lambda^2}{64\pi^2}H_B^\dagger H_B$$

where g_A and g_B are the coupling constants for $SU(2)_A$ and $SU(2)_B$ respectively and Λ is the cutoff. Now, enforce a discrete $\mathbb{Z}_2$ between the A and the B sectors fixing $g_A = g_B = g$. Then

$$\begin{aligned}
\Delta V(H) &= \frac{9g^2\Lambda^2}{64\pi^2}(H_A^\dagger H_A + H_B^\dagger H_B) \\
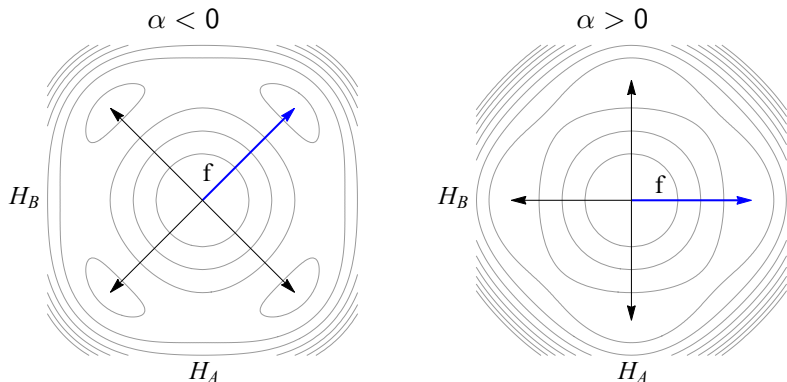&= \frac{9g^2\Lambda^2}{64\pi^2}(H^\dagger H).
\end{aligned}$$

The potential accidentally preserves the original $SU(4)$. The Higgs does not receive a quadratically divergent contribution to its mass!

However, sub-leading terms will give a $SU(4)$ breaking contribution to the potential

$$\Delta V_{\cancel{SU(4)}}(H) = \alpha H_A^\dagger H_A H_B^\dagger H_B$$

where α is naturally small.

The details of the vev structure depends on sign of α.

In either case, the bottom of the trough is now a tiny bit "wavy". The Higgs acquires a small mass and is now identified as a pseudo-Goldstone boson of an approximate $SU(4)$ symmetry.
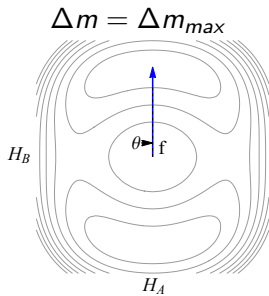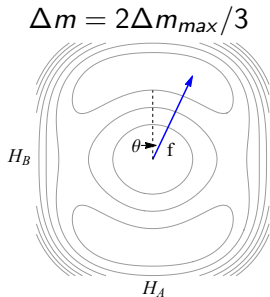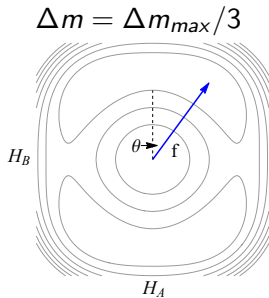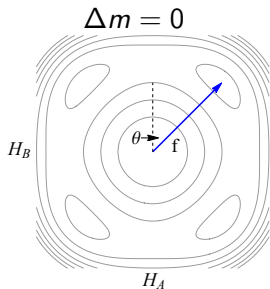
The $\alpha < 0$ minimum preserves the $\mathbb{Z}_2$ symmetry. However this scenario is problematic because

- it is incompatible with Higgs signal strength measurements,
- the energy scale $\sim 4\pi f$, at which new physics needs to appear to avoid fine-tuning, is then not much larger than in the SM.

From here on we set $\langle H_A \rangle = v \approx 174$ GeV and attempt to maximize the ratio f/v. To do this, we need to introduce an explicit $\mathbb{Z}_2$ breaking term in the potential

$$V_{\not{\mathbb{Z}_2}}(H) = \Delta m^2 H_A^\dagger H_A.$$

Increasing Δm^2 pushes the vev f towards the B sector.

$\Delta m = 0$

H_B, θ, f, H_A

$\Delta m = \Delta m_{max}/3$

H_B, θ, f, H_A

$\Delta m = 2\Delta m_{max}/3$

H_B, θ, f, H_A

$\Delta m = \Delta m_{max}$

H_B, θ, f, H_A

As shown above, there is a maximum value of Δm^2 after which $\langle H_A \rangle = 0$. By minimizing the potential, one can show that

$$\Delta m^2_{max} = -\frac{\alpha \mu^2}{2\lambda}.$$

Another interesting relation is

$$\sin^2 \theta = \frac{v^2}{f^2} = \frac{1}{2}\left(1 - \frac{\Delta m^2}{(-\alpha f^2)}\right) \approx \frac{1}{2}\left(1 - \frac{\Delta m^2}{\Delta m^2_{max}}\right).$$

To achieve a large f/v ratio, Δm^2 needs to be tuned close to Δm^2_{max}.

If $\alpha > 0$, the vevs fall in one sector only and the minimum breaks the $\mathbb{Z}_2$ symmetry. However this scenario is inviable because

- soft potential terms cannot remove the zero vev from the axis,
- thus the vev must fall in the SM sector,
- which results with a massless mirror sector,
- and this is incompatible with cosmology.

Recap:

- For $\alpha < 0$, we need to tune Δm^2 to achieve a large ratio of vevs.
- For $\alpha > 0$, one of the vevs was stuck on an axis.

Both problems are related to the shortcomings of

$$V_{\cancel{\mathbb{Z}_2}}(H) = \Delta m^2 H_A^\dagger H_A.$$

What would happen if we had terms linear in H_A or H_B instead? Could we alleviate these problems?

Let's find out!

The spontaneous $\mathbb{Z}_2$ breaking Twin Higgs

Start with an approximate global $SU(4)$ symmetry and consider two Higgs fields H_1 and H_2 each transforming as fundamentals under it. Next write the potentials
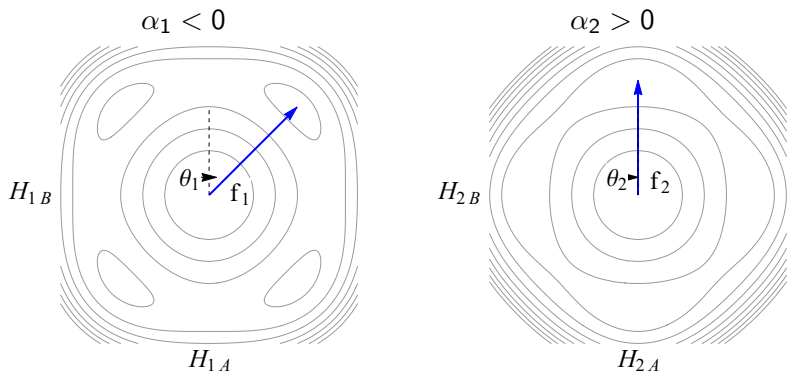
$$V_{H_1}(H_1) = -\mu_1^2 H_1^\dagger H_1 + \lambda_1 (H_1^\dagger H_1)^2 + \alpha_1 H_{1A}^\dagger H_{1A} H_{1B}^\dagger H_{1B}$$

and

$$V_{H_2}(H_2) = -\mu_2^2 H_2^\dagger H_2 + \lambda_2 (H_2^\dagger H_2)^2 + \alpha_2 H_{2A}^\dagger H_{2A} H_{2B}^\dagger H_{2B}$$

where $\mu_1^2 > 0$, $\mu_2^2 > 0$, $\alpha_1 < 0$, and $\alpha_2 > 0$.

At the moment, the vev structure looks like

where H_1 preserves the $\mathbb{Z}_2$ symmetry while H_2 breaks it. Without loss of generality, we assign the vev of H_2 to fall in the B sector.

The next step is to introduce a term that connects the two Higges. This is given by

$$
\begin{aligned}
V_{H_1 H_2}(H_1, H_2) &= -B_\mu H_1^\dagger H_2 + \text{h.c.} \\
&= -B_\mu (H_{1A}^\dagger H_{2A} + H_{1B}^\dagger H_{2B}) + \text{h.c..}
\end{aligned}
$$

This term transmits the $\mathbb{Z}_2$ breaking effects from the broken to the unbroken sector. For example, setting H_{2B} to its vev results with the term
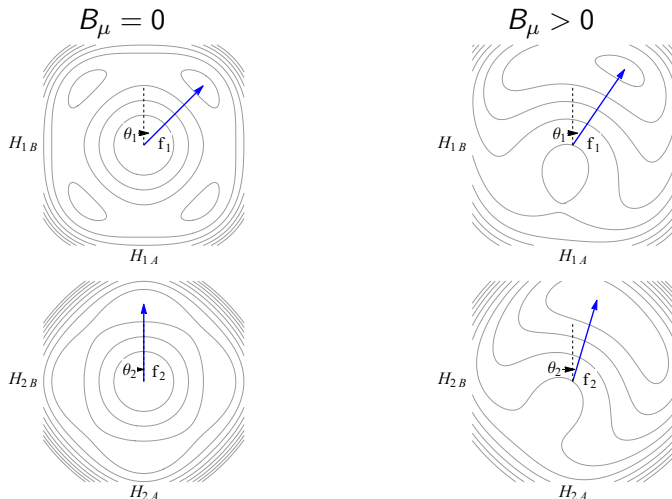
$$
H_{1B}^\dagger \langle H_{2B} \rangle + \text{h.c.}
$$

which is an effective tadpole for H_{1B}, driving the H_1 vev towards the B sector. Additionally, setting H_{1A} to its vev results with the term

$$
H_{2A}^\dagger \langle H_{1A} \rangle + \text{h.c.}
$$

which is an effective tadpole for H_{2A}, lifting its vev off the axis.

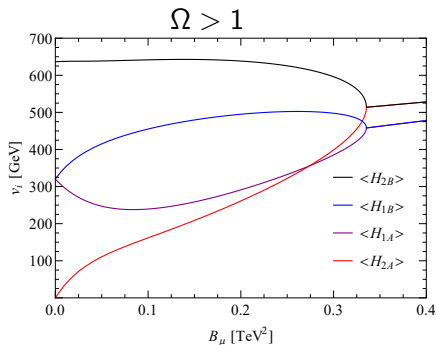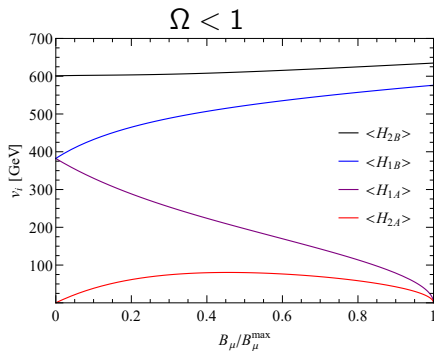The effects of these tadpole terms can be seen below.

Contours in the H_{1A}, H_{1B} plane drawn with H_{2A}, H_{2B} set to their vevs, and vice versa.

In general, the vev structure is more complicated in this model than in the original Twin Higgs. Defining

$$\Omega \equiv -\frac{\alpha_1}{\alpha_2}\left(\frac{f_1}{f_2}\right)^4$$

we get two possible vev structures.

We consider the case $\Omega < 1$.

Analogous to the Twin Higgs, there is a maximum value of B_μ after which $\langle H_{1A} \rangle = \langle H_{2A} \rangle = 0$. By minimizing the potential, one can show

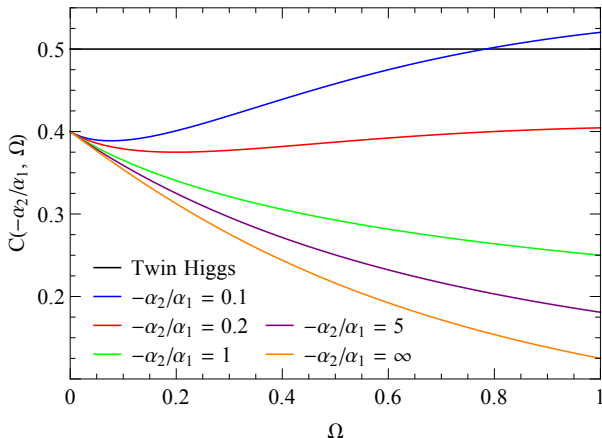$$B_\mu^{max} \approx -\frac{\alpha_1 f_1^3}{f_2(1-\Omega)}.$$

Another interesting result is that in the small angles approximation the ratio of vevs v^2/f_1^2 can be computed

$$\frac{v^2}{f_1^2} \approx \frac{3}{8(1+g(\Omega))} \left(1 + \left(-\frac{\alpha_2}{\alpha_1}\right)^{-1/2} \Omega^{3/2}\right) \left(1 - \frac{B_\mu}{B_\mu^{max}}\right)$$

$$\equiv C(-\alpha_2/\alpha_1, \Omega) \left(1 - \frac{B_\mu}{B_\mu^{max}}\right)$$

where

$$g(\Omega) = \frac{1}{16}(15\Omega^2 + 18\Omega - 1).$$

We can compare this ratio of vevs with the Twin Higgs result.

For most of the parameter space, if the two models have the same ratio of vevs, then the spontaneously $\mathbb{Z}_2$ breaking Twin Higgs is less tuned.

To show this concretely, we compute the tuning in a more systematic fashion.

For the original Twin Higgs

- Four parameters: μ^2, λ, α, and Δm^2
- Set $\lambda = 1$ and use μ^2, α, and Δm^2 to get correct Higgs mass, SM vev, and to set the ratio f/v to a given value.

For the spontaneous $\mathbb{Z}_2$ breaking Twin Higgs

- Seven parameters: μ_1^2, μ_2^2, λ_1, λ_2, α_1, α_2, and B_μ
- Set $\lambda_1 = \lambda_2 = 1$ and use μ_1^2, α_1, and B_μ to get correct Higgs mass, SM vev, and to set the ratio f_1/v to a given value.
- Two free parameters left: μ_2^2 and α_2. We scan the parameter space in terms of μ_2^2/μ_1^2 and $-\alpha_2/\alpha_1$.

The tuning can be computed in the following way.

For the Twin Higgs
- Define

$$\Delta_{TH} = \left| \frac{\partial(v^2/f^2)}{\partial \ln \Delta m^2} \right|.$$

- The tuning is then Δ_{TH}^{-1}.
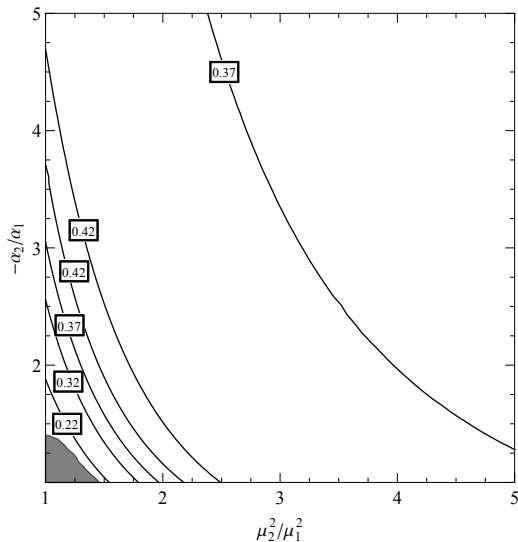- Setting $f/v = 3$ gives a benchmark tuning of 27.7%.

For the spontaneous $\mathbb{Z}_2$ breaking Twin Higgs
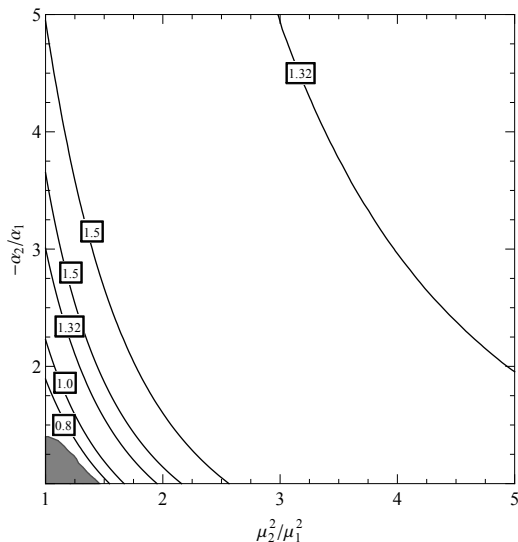- Define

$$\Delta_{Spontaneous} = \text{Max} \left\{ \left| \frac{\partial(v^2/f_1^2)}{\partial \ln B_\mu} \right|, \left| \frac{\partial(v^2/f_1^2)}{\partial \ln \mu_2^2} \right|, \left| \frac{\partial(v^2/f_1^2)}{\partial \ln \lambda_2} \right|, \left| \frac{\partial(v^2/f_1^2)}{\partial \ln \alpha_2} \right| \right\}.$$

- The tuning is then $\Delta_{Spontaneous}^{-1}$.

Setting $f_1/v = 3$ gives a tuning in our model of

Comparing this to the Twin Higgs gives a ratio of tunings of

Conclusion

- If Nature were only the SM, then it would be fine-tuned.
- However, BSM physics can potentially reduce this tuning. A prime candidate theory is Supersymmetry.
- But current experimental searches are placing strong limits on supersymmetric partner particles.
- Naturalness can still be achieved with partner particles not charged under the SM gauge groups.
- The Twin Higgs is perhaps the most famous example of this.
- The spontaneously $\mathbb{Z}_2$ breaking Twin Higgs attempts to improve the tuning even further.

Back up slides

The resulting 125 GeV Higgs boson turns out to be more "A"-like in the spontaneous $\mathbb{Z}_2$ breaking Twin Higgs than in the original Twin Higgs. To see this we decompose the Higgs as

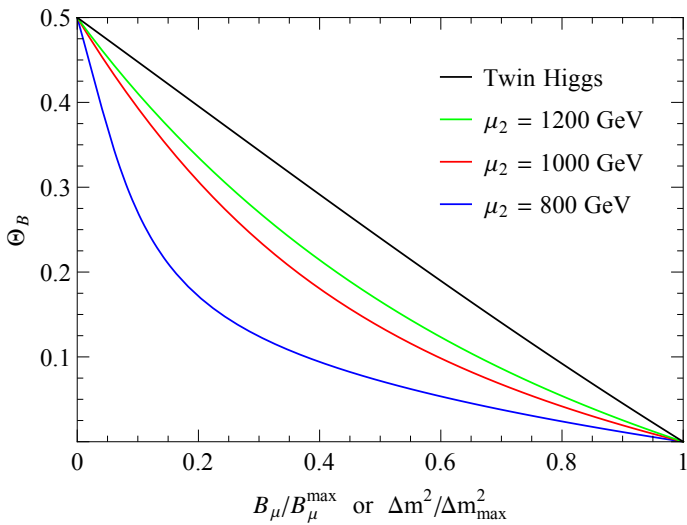$$h = ah_{1A} + bh_{2A} + ch_{1B} + dh_{2B}$$

where h_{1A} is defined as

$$H_{1A}^0 = (v_{1A} + (h_{1A} + iA_{1A})/\sqrt{2})$$

and identically for the other h_i's. We then define the parameter

$$\Theta_B \equiv c^2 + d^2$$

which measures how much the Higgs is "B"-like.

We have already discussed how quadratically divergent contributions to the Higgs mass from the gauge bosons cancel. However, we have not examined the Yukawa sector, and, in particular, the top quark. One possible way to couple the top quark to the Higgs in the Twin Higgs is

$$\mathcal{L}_{top} = -y_t(\bar{q}_A \tilde{H}_A t_A^c + \bar{q}_B \tilde{H}_B t_B^c) + \text{h.c.}$$

where q_B and t_B^c are mirror sector fermions. Notice that this term is $\mathbb{Z}_2$ symmetric; this is enough to ensure the cancellation of quadratic divergences.
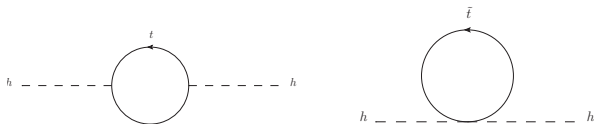
In our model, we choose the top to couple to H_1 only and to follow the same structure as above.

The one-loop leading radiative corrections for the original Twin Higgs are

$$\delta\mu^2 = \frac{1}{16\pi^2} \left(6y_t^2 - \frac{9}{4}g^2 - \frac{3}{4}g'^2 - 10\lambda - 2\alpha \right) \Lambda^2,$$

$$\delta\lambda = \frac{1}{16\pi^2} \left(6y_t^4 - \frac{9}{8}g^4 - \frac{3}{4}g^2g'^2 - \frac{3}{8}g'^4 - 32\lambda^2 - 8\lambda\alpha - 2\alpha^2 \right) \ln\frac{\Lambda}{f},$$

$$\delta\alpha = \frac{1}{16\pi^2} \left(-12y_t^4 + \frac{9}{4}g^4 + \frac{3}{2}g^2g'^2 + \frac{3}{4}g'^4 - 24\lambda\alpha \right) \ln\frac{\Lambda}{f},$$

$$\delta\Delta m^2 = \frac{1}{16\pi^2} \left(-4\lambda + 4\alpha \right) \Delta m^2 \ln\frac{\Lambda}{f}.$$

In our case, the one-loop leading radiative corrections are

$$\delta\mu_1^2 = \frac{1}{16\pi^2}\left(6y_t^2 - \frac{9}{4}g^2 - \frac{3}{4}g'^2 - 10\lambda_1 - 2\alpha_1\right)\Lambda^2,$$

$$\delta\lambda_1 = \frac{1}{16\pi^2}\left(6y_t^4 - \frac{9}{8}g^4 - \frac{3}{4}g^2g'^2 - \frac{3}{8}g'^4 - 32\lambda_1^2 - 8\lambda_1\alpha_1 - 2\alpha_1^2\right)\ln\frac{\Lambda}{f_1},$$

$$\delta\alpha_1 = \frac{1}{16\pi^2}\left(-12y_t^4 + \frac{9}{4}g^4 + \frac{3}{2}g^2g'^2 + \frac{3}{4}g'^4 - 24\lambda_1\alpha_1\right)\ln\frac{\Lambda}{f_1},$$

$$\delta\mu_2^2 = \frac{1}{16\pi^2}\left(-\frac{9}{4}g^2 - \frac{3}{4}g'^2 - 10\lambda_2 - 2\alpha_2\right)\Lambda^2,$$

$$\delta\lambda_2 = \frac{1}{16\pi^2}\left(-\frac{9}{8}g^4 - \frac{3}{4}g^2g'^2 - \frac{3}{8}g'^4 - 32\lambda_2^2 - 8\lambda_2\alpha_2 - 2\alpha_2^2\right)\ln\frac{\Lambda}{f_2},$$

$$\delta\alpha_2 = \frac{1}{16\pi^2}\left(\frac{9}{4}g^4 + \frac{3}{2}g^2g'^2 + \frac{3}{4}g'^4 - 24\lambda_2\alpha_2\right)\ln\frac{\Lambda}{f_2},$$
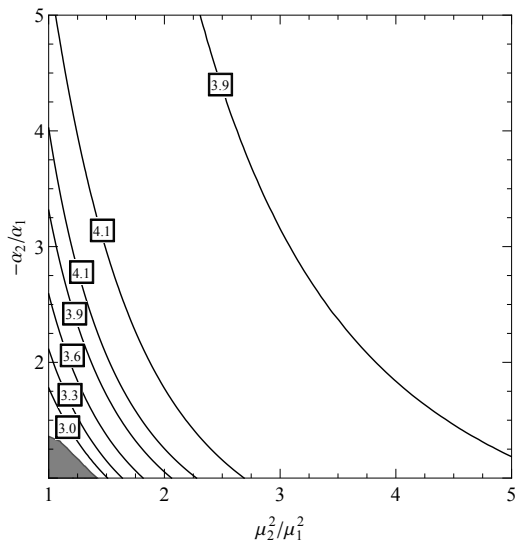
$$\delta B_\mu = 0,$$

$$\delta\kappa = \frac{1}{16\pi^2}\left(-\frac{9}{4}g^4 - \frac{3}{2}g^2g'^2 - \frac{3}{4}g'^4\right)\ln\frac{\Lambda}{f_1}.$$

The parameter κ is the coefficient for the operator

$$-\kappa(H_{1A}^{\dagger}H_{1A}H_{2A}^{\dagger}H_{2A} + H_{1B}^{\dagger}H_{1B}H_{2B}^{\dagger}H_{2B}).$$

Before, we set the ratio of vevs and then found the tuning. But we can also do the opposite. If we set the tuning to 20% then we get $f/v = 3.42$ in the original Twin Higgs.

Setting the tuning to 20% gives f_1/v in our model of

Comparing this to the Twin Higgs gives $(f_1/v)/(f/v)$ of