Precision EW calculations for future experiments

(Hadronic contributions to the muon anomalous magnetic moment and Direct CP violation in Kaon decays from lattice QCD)

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Outline I

The magnetic moment of the muon

Interaction of particle with static magnetic field

$$V(\vec{x}) \;=\; -\vec{\mu} \cdot \vec{B}_{\rm ext}$$

The magnetic moment $\vec{\mu}$ is proportional to its spin ($c = \hbar = 1$)

$$\vec{\mu} \;=\; g \left(\frac{e}{2m}\right) \vec{S}$$

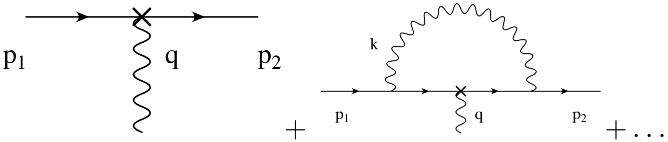
The Landé g-factor is predicted from the free Dirac eq. to be

$$g \;=\; 2$$

for elementary fermions

The magnetic moment of the muon

In interacting quantum (field) theory g gets corrections



$$\langle \mu(p')|J^\mu|\mu(p)\rangle = \bar{u}(p')\left(\gamma^\mu F_1(q^2) + i\,\frac{[\gamma^\mu,\gamma^\nu]\,q^\nu}{2}\,\frac{F_2(q^2)}{2m}\right)u(p)$$

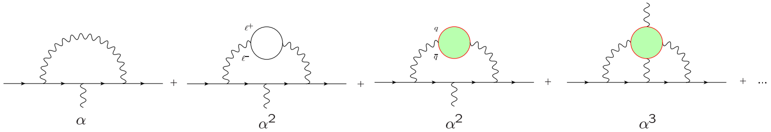
which results from Lorentz invariance and charge conservation when the muon is on-mass-shell and where $q = p' - p$

$$F_2(0) = \frac{g-2}{2} \equiv a_\mu \qquad (F_1(0) = 1)$$

(the anomalous magnetic moment, or anomaly)

Compute these corrections order-by-order in perturbation theory by expanding $\Gamma^{\mu}(q^2)$ in QED coupling constant

$$\alpha \;=\; \frac{e^2}{4\pi} = \frac{1}{137} + \dots$$



Corrections begin at $\mathcal{O}(\alpha)$; Schwinger term $= \frac{\alpha}{2\pi} = 0.0011614\dots$

hadronic contributions $\sim 6 \times 10^{-5}$ smaller, dominate theory error.

Experiment - Standard Model Theory = difference

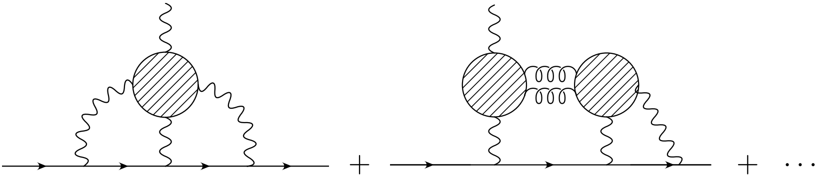
SM Contribution	Value $\pm$ Error ($\times 10^{11}$)	Ref
QED (5 loops)	116584718.951 ± 0.080	[Aoyama et al., 2012]
HVP LO	6923 ± 42	[Davier et al., 2011]
	6949 ± 43	[Hagiwara et al., 2011]
HVP NLO	-98.4 ± 0.7	[Hagiwara et al., 2011]
		[Kurz et al., 2014]
HVP NNLO	12.4 ± 0.1	[Kurz et al., 2014]
HLbL	105 ± 26	[Prades et al., 2009]
HLbL (NLO)	3 ± 2	[Colangelo et al., 2014b]
Weak (2 loops)	153.6 ± 1.0	[Gnendiger et al., 2013]
SM Tot (0.42 ppm)	116591802 ± 49	[Davier et al., 2011]
(0.43 ppm)	116591828 ± 50	[Hagiwara et al., 2011]
(0.51 ppm)	116591840 ± 59	[Aoyama et al., 2012]
Exp (0.54 ppm)	116592089 ± 63	[Bennett et al., 2006]
Diff (Exp – SM)	287 ± 80	[Davier et al., 2011]
	261 ± 78	[Hagiwara et al., 2011]
	249 ± 87	[Aoyama et al., 2012]

New experiments+new theory=new physics

- Fermilab E989 early 2017, aims for 0.14 ppm
- J-PARC E34 late 2010's, aims for 0.3-0.4 ppm
- Today $a_\mu(\mathrm{Expt})$-$a_\mu(\mathrm{SM}) \approx 2.9 - 3.6\sigma$
- If both central values stay the same,
  - E989 ($\sim 4\times$ smaller error) $\rightarrow \sim 5\sigma$
  - E989+new HLBL theory (models+lattice, 10%) $\rightarrow \sim 6\sigma$
  - E989+new HLBL +new HVP (50% reduction) $\rightarrow \sim 8\sigma$
- Good for discriminating models if discovery of BSM at LHC

  [Stckinger, 2013]
- Lattice calculations important to trust theory errors

Hadronic light-by-light (HLbL) scattering



- Models: $(105 \pm 26) \times 10^{-11}$ [Prades et al., 2009, Benayoun et al., 2014]
  $(116 \pm 40) \times 10^{-11}$ [Jegerlehner and Nyffeler, 2009]

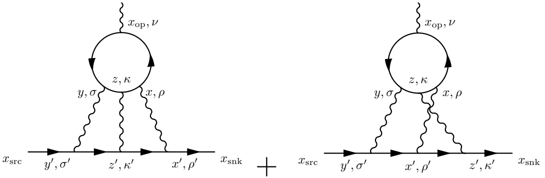
  systematic errors difficult to quantify
- First non-PT QED+QCD calculation [Blum et al., 2015b]
- Very rapid progress with pQED+QCD [Blum et al., 2015c]
- New HLbL scattering calculation by Mainz group [Green et al., 2015]
- Dispersive approach difficult, but progress is being made

  [Colangelo et al., 2014c, Colangelo et al., 2014a, Pauk and Vanderhaeghen, 2014b,

  Pauk and Vanderhaeghen, 2014a, Colangelo et al., 2015a]

Brief aside: Lattice setup

- Compute correlation functions (*e.g.* $\langle j^\mu(x) j^\nu(y)\rangle$, $j^\mu = \bar{\psi}\gamma_\mu\psi$) in Feynman path integral formalism
- 4(5)D hypercubic lattice regularization, non-zero lattice spacing a and finite volume V (extrap $a \to 0$, $V \to \infty$)
- Handle fermion integrals analytically. Propagators inverse of large sparse matrix M, lattice Dirac operator (domain wall, staggered, Wilson, ...). Costliest part of calculation
- Do path integrals over gauge fields stochastically by Monte Carlo importance sampling: generate ensemble of gauge field configurations $\{U(x)\}$ with weight $\det M(U) \exp -S_g$, then $\langle \cdots \rangle$ simple average over ensemble
- Statistical errors $O(1/\sqrt{N_{\mathrm{meas}}})$
- work entirely in Euclidean space time, analytically continue back to Minkowski at the end (usually trivial)

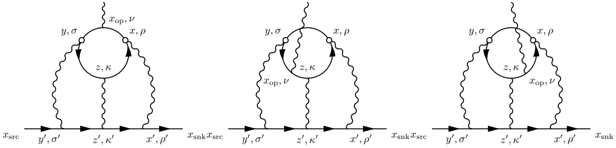


+ 4 more

- Compute quark loop non-perturbatively using lattice QCD
- Photons, muon on lattice, but use (exact) tree-level props
- Work in configuration space
- Key insight: quark loop exponentially suppressed with x and y separation. Concentrate on "short distance" (π Compton λ)
- Do QED two-loop, quark-loop integrals stochastically
- Chiral (DW) fermions at finite lattice spacing: UV properties like in continuum, modified by $O(a^2)$

Point source method (L. Jin) [Blum et al., 2015c]

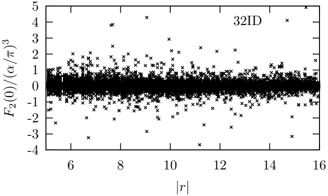
- Importance sampling strategy: focus on "short" distance ($\lesssim$ pion λ_C). Choose two points x, y on quark loop randomly with empirical distribution. Other two points summed over exactly (including external photon vertex)
- WI satisfied on each config (important to control statistical error as $q \to 0$ where correlation function vanishes)

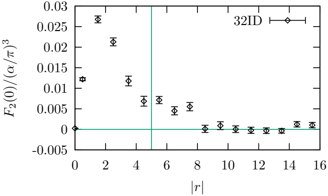


- Moment method ($e^{iq\cdot x} \approx 1 + iq \cdot x$) allows direct $q = 0$ calc
- AMA used for quark propagators. Separate quark propagators into low and high mode parts. Treat low exactly, high approx (cheap). Remove bias with correction (comp infreq).

Point source method (L. Jin) [Blum et al., 2015c]

- First results for connected contribution[Blum et al., 2015c]:





$$|r| = |x - y|$$

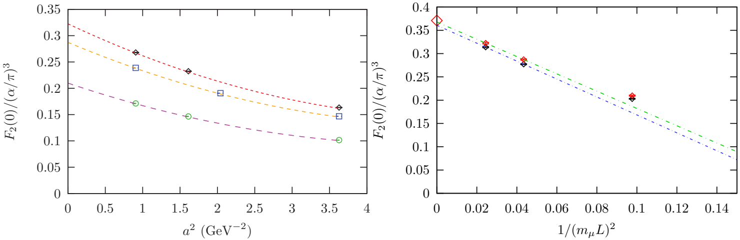
171 MeV pion, $m_\pi L \gtrsim 4$, $m_\mu = 128$ MeV, $L = 4.6$ fm
AMA: 1000 LM, $\gtrsim 6000$ meas/conf, 23 conf

$F_2(0)/(\alpha/\pi)^3 = 0.1054(54)$ (connected contribution)
5% statistical error for nearly physical pion mass!

- 13.2 BG/Q Rack-days (Rack = 1024 nodes = 16384 cores)

Continuum and ∞ volume limits in QED (L. Jin) [Blum et al., 2015c]

QED systematics large, $O(a^4)$, $O(1/L^2)$, but under control

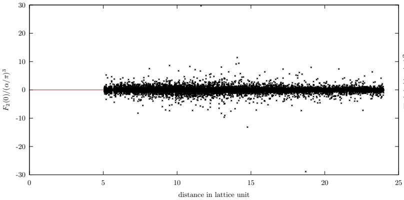


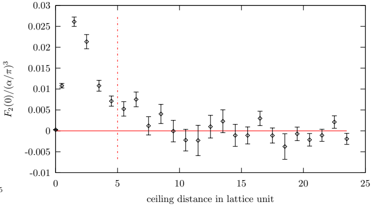
- a set using physical muon mass, *i.e.*, input parameter am_μ
- Limits quite consistent with well known PT result
- Very good check on method/code

Physical point ($m_\pi = 140$ MeV) calculation [Jin et al., 2015]

ALCC award on MIRA (100 PF BG/Q) at ANL ALCF,

- Physical mass 2+1f Möbius DWF ensemble (RBC/UKQCD), $(5.5 \text{ fm})^3$ QCD box, $a = 0.114$ fm ($a^{-1} = 1.7848$ GeV)
- Use AMA with 2000 low-modes, $\sim$ 4500 sloppy props per configuration





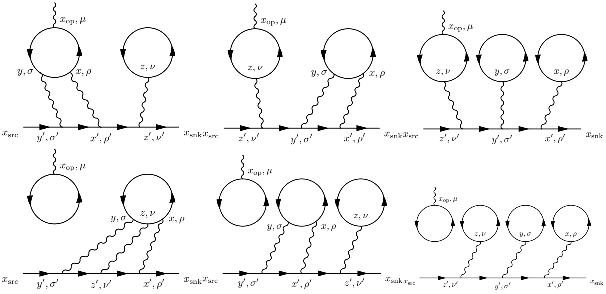
Preliminary: $(0.0854 \pm 0.0098) \times (\alpha/\pi)^3$,

connected HLbL contribution (11% statistical error)

models: $(0.0837 \pm 0.0207) \times (\alpha/\pi)^3$

Expect 25-30% FV, 10-20% non-zero a errors

Disconnected contribution (M. Hayakawa) [Hayakawa et al., 2015]

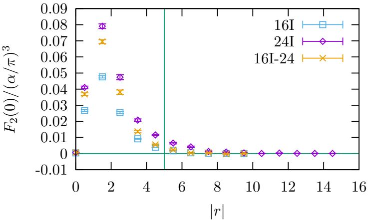


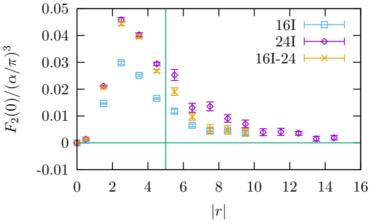
+ perms

- SU(3) Flavor (only 1 survives), Zweig suppressed
- But, needed to cancel 1 π^0 exchange to get η' right
- Requires explicit HVP subtraction when any quark loop with two photons is not connected to others by gluons
- Use same importance sampling as for connected
- Tests underway on small lattice

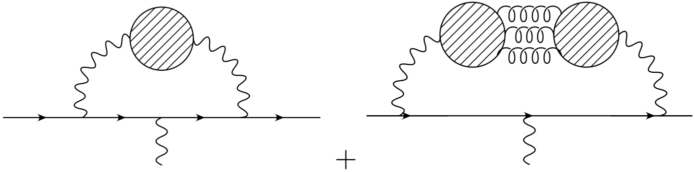
Solving QED FV effects [Jin et al., 2015]

- Integrand exponentially suppressed with distance between any pair of points on the quark loop. FV effect is small.
- Amplitude *not* suppressed with distance between points on muon line and loop. FV effect is large.
- Put QED in larger box, QCD unchanged
- use ∞ volume photon on finite box (Lehner, Lattice 2015)
- Can compute average QCD loop and do muon line once, offline, so free to experiment with size of QED box





Hadronic Vacuum Polarization (HVP) contribution to g-2



Using lattice QCD and continuum, ∞-volume pQED

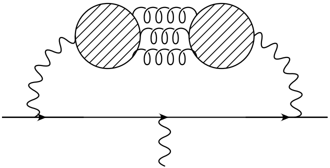
[Blum, 2003, Lautrup et al., 1971]

$$a_\mu^{\rm HVP} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dq^2\, f(q^2)\, \hat{\Pi}(q^2)$$

$f(q^2)$ is known, $\hat{\Pi}(q^2)$ is subtracted HVP, $\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$

$$\begin{aligned}\Pi^{\mu\nu}(q) &= \int e^{iqx} \langle j^\mu(x) j^\nu(0)\rangle \qquad j^\mu(x) = \sum_i Q_i \bar{\psi}(x)\gamma^\mu\psi(x) \\ &= \Pi(q^2)(q^\mu q^\nu - q^2\delta^{\mu\nu})\end{aligned}$$

Disconnected contribution to HVP (C. Lehner) [Blum et al., 2015a]

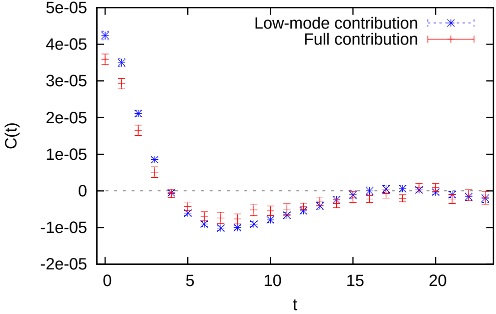


- Expected to be small (vanishes in SU(3) limit)
- Still important to reach (sub-) percent precision
- Physical pion mass Möbius-DWF ensemble RBC/UKQCD
- use All-to-All strategy [Foley et al., 2005]
  - Compute (2000) low-mode contribution exactly, on every point of lattice (enormous gain in statistics)
  - Compute high mode part stochastically using "sparse" grids of random Z_2 noise sources
- (degenerate)light - strange difference computed directly (Mainz Group [Gulpers et al., 2014])
- Use AMA strategy

Disconnected Contribution to HVP (C. Lehner) [Blum et al., 2015a]

- Low mode separation crucial since light- strange don't cancel
- contributions above m_s suppressed
- (sparse) random sources effective for high modes

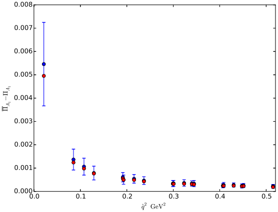
$$\Pi(q^2) - \Pi(0) = \sum_t \left(\frac{\cos(qt) - 1}{q^2} + \frac{1}{2} t^2 \right) C(t)$$

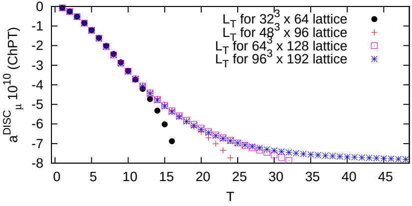


- $-(9.6 \pm 3.3) \times 10^{-10}$ or about 1.5% of total at 3 σ level

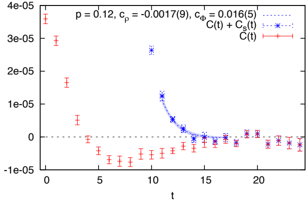
Disconnected Contribution to HVP Systematics

- non-zero lattice spacing: proxy strange-connected 5%
- FV, ChiPT [Aubin et al., 2015, Della Morte and Juttner, 2010]: 14.6%





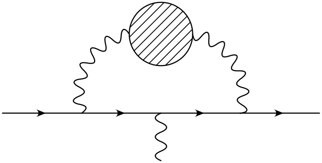
- missing long distance piece 17.7%



$$-(9.6 \pm 3.3 \pm 2.3) \times 10^{-10}$$

Better than 1% accuracy on total HVP!

Quark Connected Contribution to HVP



- Relatively harder: need (sub) percent accuracy
- On going calculations at the physical point by several groups
- Current calculations, $\gtrsim 3\%$ error (aggressive)
- Finite volume effects significant barrier [Aubin et al., 2015]
- lots of activity by many groups

Outline I

CP violated in $K \to \pi\pi_{I=0,2}$ decays (ϵ'), $K - \overline{K}$ mixing (ϵ)

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\right\}$$
$$= 1.66(23) \times 10^{-3} \qquad (\mathrm{exp})$$

[Batley et al., 2002, Alavi-Harati et al., 2003]

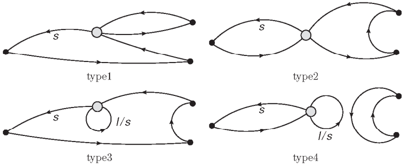
where A_0, A_1 computed in SM from $\langle K|H_W|\pi\pi_{I=0,2}\rangle$,

$$H_W = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \sum_{i=1}^{10} \left[z_i(\mu) + \tau y_i(\mu)\right] Q_i(\mu).$$

where $\tau = -V_{ts}^* V_{td} / V_{us}^* V_{ud}$ encodes the phase of the CKM matrix and y_i, z_i are Wilson coefficients

Lattice Calculation of $K \to \pi\pi$

- Lellouch-Lüscher method: finite volume $\to \infty$ volume
- For I=0 use G-parity boundary conditions (conserves isospin), physical kinematics for ground state [Kim and Christ, 2009]
- I=0 difficult: disconnected diagrams! Use All-to-all strategy



- Lattice operators renormalized non-perturbatively, matched to $\overline{\mathrm{MS}}$ with continuum PT
- A_0: 32^3 (4.6 fm), physical point 2+1 Möbius-DWF (G-parity)
- A_2: 48^3 and 64^3 (5.5 fm), physical point 2+1 Möbius-DWF

Calculation in the SM (C. Kelly, D. Zhang)

[Bai et al., 2015]

i	Re(A_0)(GeV)	Im(A_0)(GeV)
1	$1.02(0.20)(0.07) \times 10^{-7}$	0
2	$3.63(0.91)(0.28) \times 10^{-7}$	0
3	$-1.19(1.58)(1.12) \times 10^{-10}$	$1.54(2.04)(1.45) \times 10^{-12}$
4	$-1.86(0.63)(0.33) \times 10^{-9}$	$1.82(0.62)(0.32) \times 10^{-11}$
5	$-8.72(2.17)(1.80) \times 10^{-10}$	$1.57(0.39)(0.32) \times 10^{-12}$
6	$3.33(0.85)(0.22) \times 10^{-9}$	$-3.57(0.91)(0.24) \times 10^{-11}$
7	$2.40(0.41)(0.00) \times 10^{-11}$	$8.55(1.45)(0.00) \times 10^{-14}$
8	$-1.33(0.04)(0.00) \times 10^{-10}$	$-1.71(0.05)(0.00) \times 10^{-12}$
9	$-7.12(1.90)(0.46) \times 10^{-12}$	$-2.43(0.65)(0.16) \times 10^{-12}$
10	$7.57(2.72)(0.71) \times 10^{-12}$	$-4.74(1.70)(0.44) \times 10^{-13}$
Tot	$4.66(0.96)(0.27) \times 10^{-7}$	$-1.0(1.19)(0.32) \times 10^{-11}$

Description	Error	Description	Error
Finite lattice spacing	8%	Finite volume	7%
Wilson coefficients	12%	Excited states	$\leq$ 5%
Parametric errors	5%	Operator renormalization	15%
Unphysical kinematics	$\leq$ 3%	Lellouch-Lüscher factor	11%
Total (added in quadrature)			26%

Calculation in the SM (T. Janowski)

[Blum et al., 2015d]

ReA_2 dominated by (27,1) operator , ImA_2 (8,8)

ReA_2 systematic errors	48^3	64^3	cont.
NPR (nonperturbative)	0.1%	0.1%	0.1%
NPR (perturbative)	2.9%	2.5%	2.9%
Finite-volume corrections	2.2%	2.4%	2.4%
Unphysical kinematics	1.8%	4.5%	4.5%
Wilson coefficients	6.8%	6.8%	6.8%
Derivative of the phase shift	1.1%	0.6%	1.1%
Total	8%	9%	9%

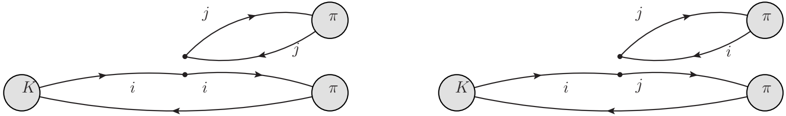
ImA_2 systematic errors	48^3	64^3	cont
NPR (nonperturbative)	0.1%	0.1%	0.1%
NPR (perturbative)	7.0%	6.2%	7.0%
Finite-volume corrections	2.4%	2.6%	2.6%
Unphysical kinematics	0.2%	1.1%	1.1%
Wilson coefficients	10%	8%	10%
Derivative of the phase shift	1.1%	0.6%	1.1%
Total	12%	10%	12%

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = 1.38(5.15)(4.43) \times 10^{-4}, \text{ 2.1 } \sigma \text{ below experiment}$$

Calculation in the SM

- $\mathrm{Re}A_0$, $\mathrm{Re}A_2$ agree with experimental values, good check of our methods
- $\Delta I = 1/2$ rule explanation: strong cancelation between dominant contractions in $\mathrm{Re}A_2$, sum in $\mathrm{Re}A_0$

[Boyle et al., 2013, Blum et al., 2015d]



combined with enhancement from Wilson coefficients

- We find $\delta_0 = 23.8(4.9)(1.2)^\circ$ which is somewhat below the value determined from the Roy equation analysis

[Colangelo et al., 2001, Colangelo et al., 2015b]

Outline I

Summary/Outlook

"physical point" ensembles / improved meas. tech. are powerful:

- 11% stat. errors for connected HLBL contribution to g-2. moving on to disconnected HLBL, non-zero *a*, FV systematics
- 40% errors for disconnected HVP (already below 1 percent). moving on to connected HVP (1% stat, 3% total soon)
- First complete calculation of ϵ'/ϵ (2.1 σ diff with SM). Working now to decrease uncertainty in our result by 2

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- Lattice computations done on
  - Ds cluster at FNAL (USQCD)
  - USQCD BG/Q at BNL
  - UKQCD BG/Q at Edinburgh
  - Mira (BG/Q) at ALCF

Outline I

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