Next-to-next-to-leading order approximation to perturbative QCD at the LHC: the recent progress

Kirill Melnikov

TTP KIT

Aspen Winter Conference ``Particle physics on the verge of discoveries”

Aspen, January 10-16, 2015

Outline

1) Introduction

2) Technical developments

3) Processes computed to NNLO QCD and lessons learned

4) Conclusion

Introduction

Perturbative QCD has become a practical tool for making precise predictions for hard hadron collider processes. Depending on how the LHC program will evolve, such predictions may be needed for discovering physics beyond the Standard Model or they will be needed for understanding the nature of such physics when it is discovered. For this reason, improving theoretical description of hard hadron collisions at the LHC is useful and important.

To make pQCD predictions for hard hadron collider processes, we use the QCD factorization theorem that states

$$\sigma = \sum_{ij} \int \mathrm{d}x_1 \mathrm{d}x_2 f_i(x_1) f_j(x_2) \sigma_{ij}(x_1, x_2) F_J \left(1 + \mathcal{O}(\Lambda_{\rm QCD}/Q)\right).$$

If we look at the various ingredients in this formula, we find that parton distribution functions and partonic cross sections are, in general, known to a precision of about O(10-20) percent. This precision typically corresponds to next-to-leading order or the one-loop approximation.

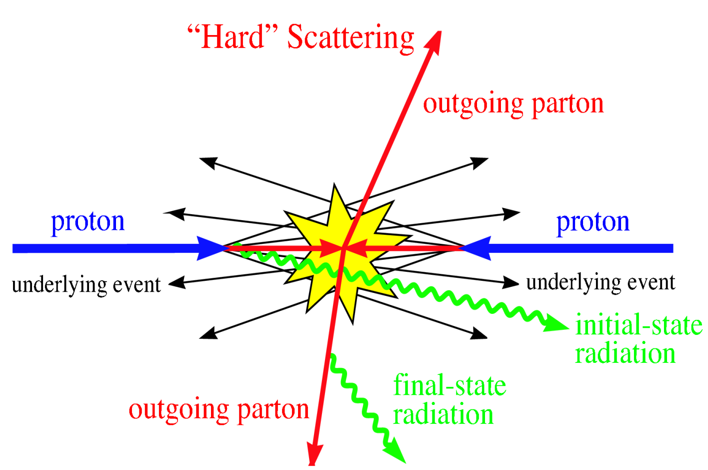
We should contrast this with the expected size of non-perturbative corrections. Indeed, for a typical values of hard scales, the non-perturbative contributions change the predicted LHC cross sections by just a few percent. Therefore, these non-perturbative effects are much smaller than the residual uncertainty of NLO QCD computations.

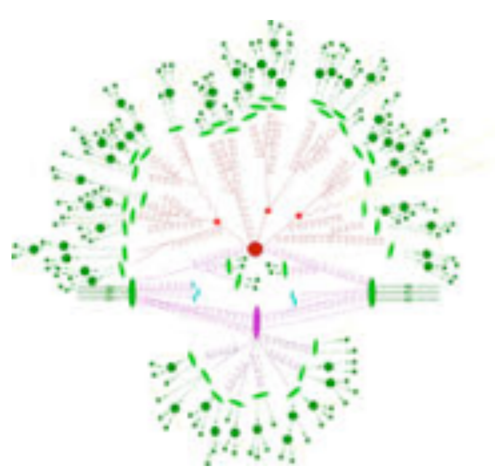
Introduction

Since the theory of non-perturbative corrections does not exist, their magnitude provides an ultimate precision target on the theory side: going beyond it does not make sense unless the theory of non-perturbative corrections is established, but reaching this (few percent) precision is justified. To get there, one needs the NNLO QCD predictions; this is a simple consequence of the numerical value of the strong coupling constant at 100 GeV.

There are many non-trivial issues (mostly of experimental nature) that have to be understood if one wants to benefit from such a high precision but this is a separate issue. On the other hand, to provide maximal benefit for theory/experiment cross-talk, such predictions should be realistic, i.e. they should be performed at a fully differential level and applied to realistic final states.

In recent years, progress towards reaching the NNLO accuracy for large number of LHC processes was very impressive. Paraphrasing what has been said about NLO computations just a few years ago, we are living through the NNLO QCD revolution. This implies that we have large and constantly increasing number of processes that are known to the NNLO QCD accuracy.





Processes currently known through NNLO

dijets	O(3%)	gluon-gluon, gluon-quark	PDFs, strong couplings, BSM
H+0 jet	O(3-5 %)	fully inclusive (N3LO)	Higgs couplings
H+1 jet	O(7%)	fully exclusive; Higgs decays, infinite mass tops	Higgs couplings, Higgs p_t, structure for the ggH vertex.
tT pair	O(4%)	fully exclusive, stable tops	top cross section, mass, p_t, FB asymmetry, PDFs, BSM
single top	O(1%)	fully exclusive, stable tops, t-channel	V_{tb}, width, PDFs
WBF	O(1%)	exclusive, VBF cuts	Higgs couplings
W+j	O(1%)	fully exclusive, decays	PDFs
Z+j	O(1-3%)	decays, off-shell effects	PDFs
ZH	O(3-5 %)	decays to bb at NLO	Higgs couplings (H-> bb)
ZZ	O(4%)	fully exclusive	Trilinear gauge couplings, BSM
WW	O(3%)	fully inclusive	Trilinear gauge couplings, BSM
top decay	O(1-2 %)	exclusive	Top couplings
H -> bb	O(1-2 %)	exclusive, massless	Higgs couplings, boosted

Techniques for NNLO computations

Ingredients for NNLO computations

A NNLO QCD computation is, essentially, a two-loop computation. However, in theories with massless particles, two-loop computations are insufficient for obtaining a physical answer: two-loop computations need to be combined with contributions of higher-multiplicity processes to physical observables.

Suppose we want to compute the NNLO QCD correction to a process pp -> X . To do this, we need:

a) two-loop scattering amplitudes for a process X ;

b) one-loop amplitudes for a process X+g;

d) tree-level amplitudes for a process X+gg, X+qQ etc.

Among these items, computation of two-loop scattering amplitudes is an important challenge.

An established framework based on the parametrization of scattering amplitudes in terms of Lorentz-invariant form factors, processing contributing diagrams and using the integration-by-parts identities with the goal to express large number of integrals through a few master integrals starts to show signs of being inefficient, especially for processes with large number of external legs and/or large number of kinematic invariants. For the time being it still can be used but the question for the future is what will replace it.

Two-loop calculations: amplitudes and integrals

Here are a few things that we learned recently about two-loop computations:

1) Calculation of master integrals using differential equations in kinematic variables is now a method of choice. It has benefited from an understanding of how the bookkeeping in such calculations can be streamlined by choosing appropriate master integrals and working with particular special functions.

Remiddi, Kotikov, Henn, Papadopoulos

2) We are able to successfully study master integrals with up to 4 kinematic invariants and there are indications that even larger number of kinematic invariants can be dealt with.

Gehrmann, Henn, Tancredi, Caola, Smirnov(s), Papadopoulos, Tommasini, Wever

3) Internal masses is a big challenge since they introduce new special functions whose iterative properties are not yet fully understood.

4) There are interesting attempts to understand if two-loop computations can be done using unitarity techniques, that turned out to be so powerful at one-loop. While there was an impressive progress in this field related to classification of integrand residuals based on techniques from algebraic geometry, there are still many outstanding issues.

Badger, Frellesvig, Zhang, Mastrolia, Ita

NNLO calculations: loops and real emissions

An important achievement of the past few years was the development of theoretical methods that allow us to perform NNLO QCD computations for hard hadron collider processes of a sufficiently general nature.

Consider NNLO QCD corrections to a tree process pp -> X. There are three sources of infra-red divergencies that must be considered:

1) two-loop virtual corrections to pp -> X, where all infrared singularities are explicit;

2) one-loop virtual corrections to pp -> X+g, where some infrared singularities are explicit and some appear only after the integration of the final state gluon;

3) process pp -> X+ g+ g where all infra-red singularities appear only after integration over final state gluon(s) is carried out.

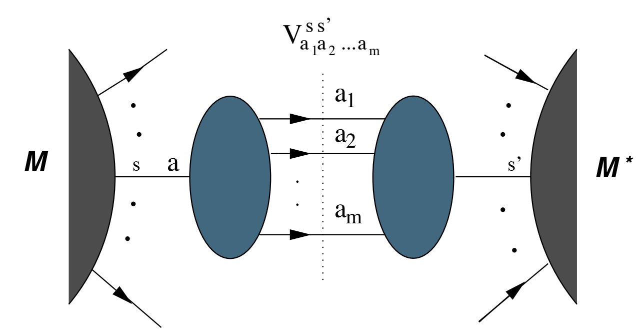
The key problem here is that we would like to achieve the cancellation of infra-red singularities at NNLO without integrating over kinematic variables of those final state particles that are accessible in experiment; but this seems to be impossible given that in real emission processes singularities are produced only after the phase-space integration...

NNLO calculations: loops and real emissions

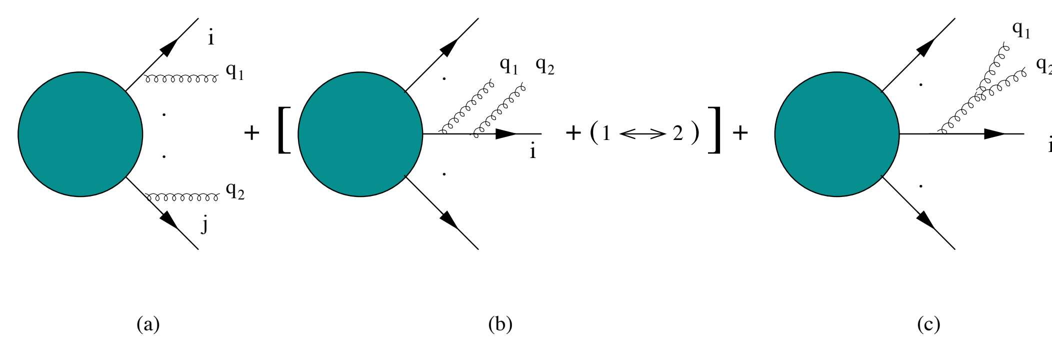
It is easy to recognize that for achieving the cancellation of infra-red and collinear divergences, we only need to integrate over phase-space regions which can generate the singularities.

These are the regions where external particles can become soft and/or collinear to each other and where any measurable differences between final states with different multiplicities become unobservable. In these regions, "singular" matrix elements factorize into universal singular functions and non-singular matrix element of lower multiplicity.

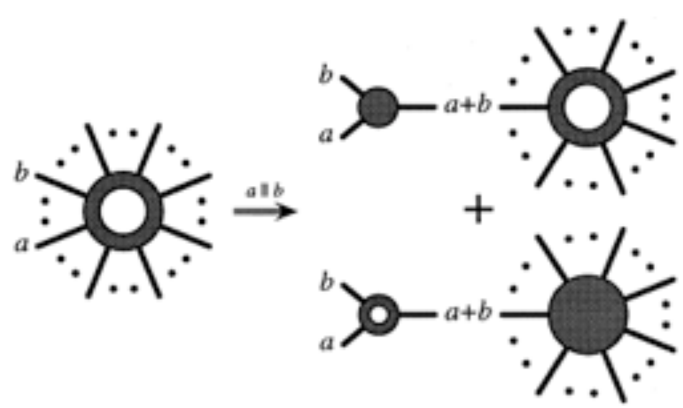
$$\mathcal{M}_{n+i+j} = F_{ij}\mathcal{M}_n$$



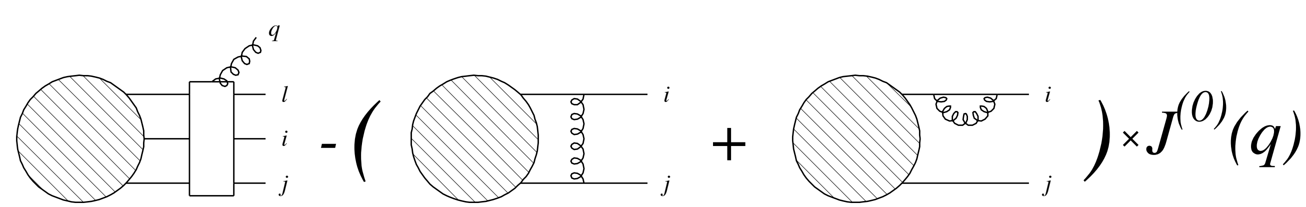
Collinear factorization (Catani, Grazzini)



Soft factorization (Catani, Grazzini)



Collinear factorization at one-loop (Kosower, Uwer)



Soft factorization at one-loop (Catani, Grazzini)

NNLO calculations: loops and real emissions

A universal, simplified form of scattering amplitudes in kinematic regions responsible for the appearance of singularities, together with factorization of multi-particle phase-space, allows us to extract and, eventually, cancel them in a generic, process-independent way.

There are two basic methods familiar from NLO computations: slicing and subtraction.

Slicing methods include: q_t-subtraction and N-jettiness;

$$\int \mathrm{d}\Phi_n |\mathcal{M}|^2 F_J = \int_{\rm regular} \mathrm{d}\Phi_n |\mathcal{M}|^2 F_J + \int_{\rm singular} \mathrm{d}\Phi_n |\mathcal{M}|^2_{\rm approx} \tilde{F}_J$$

Catani, Grazzini; Bougezhal, Focke, Liu, Petriello; Gaunt, Stahlhofen, Tackmann, Walsh.

Subtraction methods include: antenna, improved sector decomposition and projection to Born.

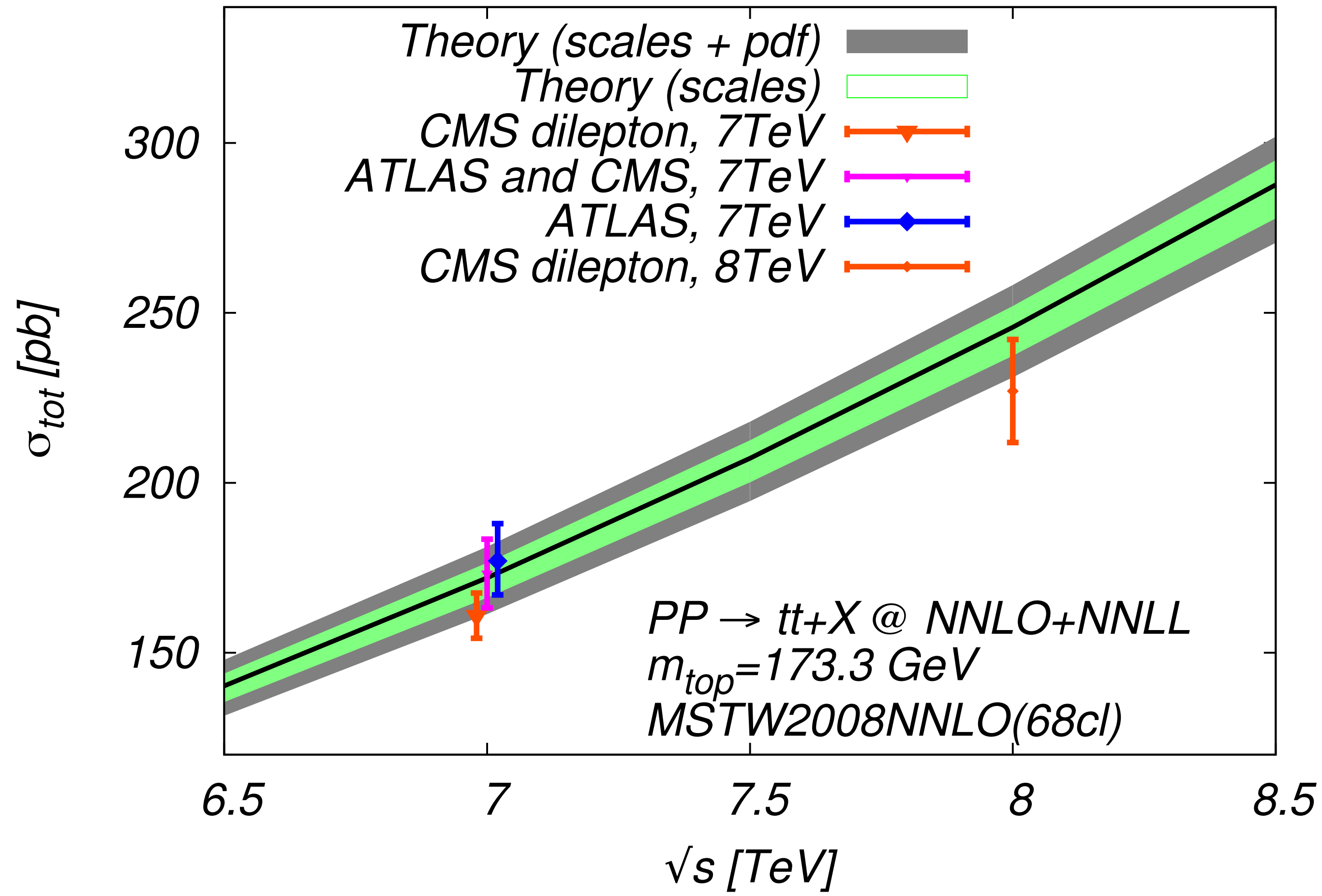
$$\int \mathrm{d}\Phi_n |\mathcal{M}|^2 F_J = \int \mathrm{d}\Phi_n \left(|\mathcal{M}|^2 F_J - |\mathcal{M}|^2_{\rm approx} \tilde{F}_J \right) + \int \mathrm{d}\Phi_n |\mathcal{M}|^2_{\rm approx} \tilde{F}_J$$

Gehrmann-de Ridder, Gehrmann, Glover; Czakon; Bougezhal, Petriello, K.M.
Cacciari, Dreyer, Kalberg, Salam, Zanderighi

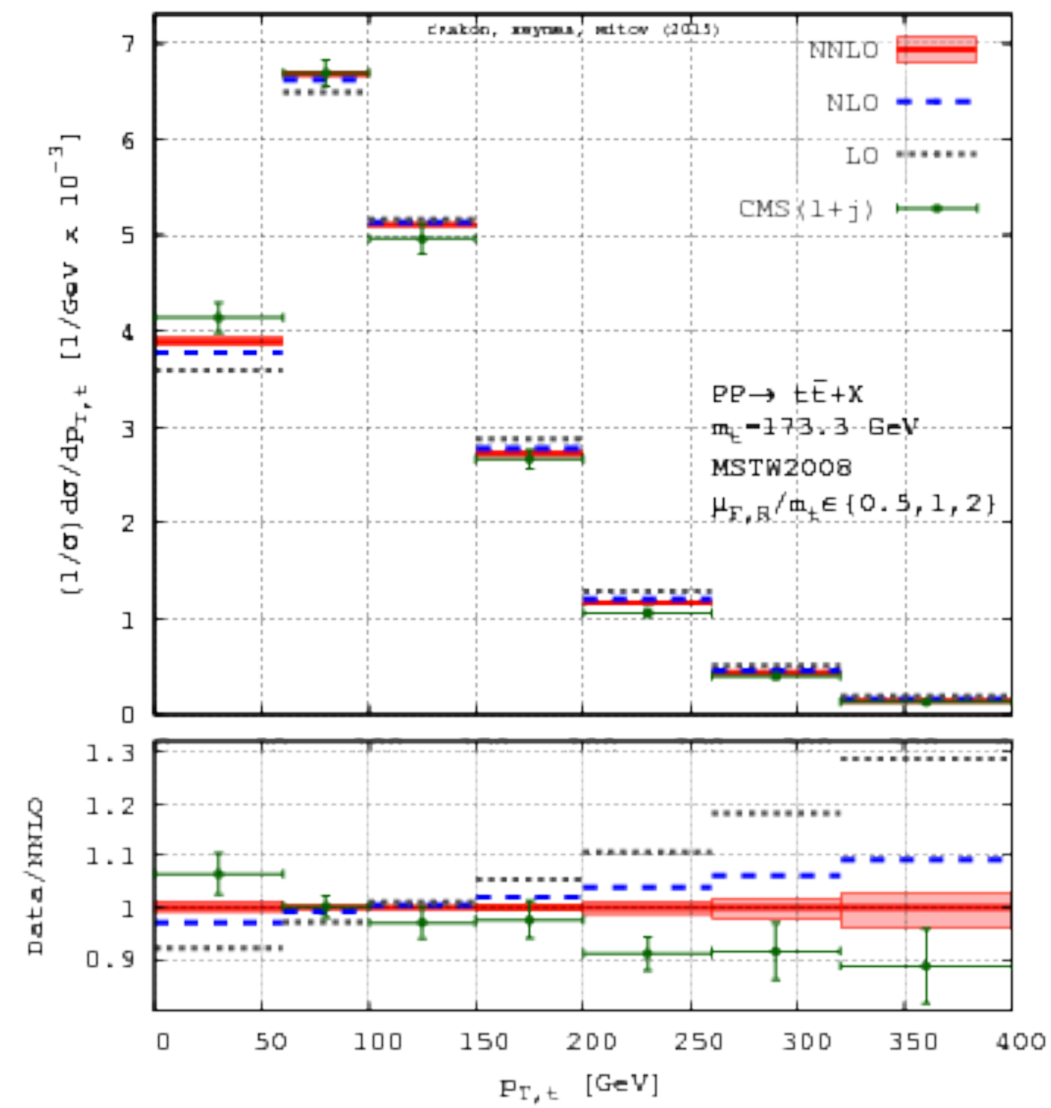
All these methods work and have been used in a large number of recent NNLO QCD computations.

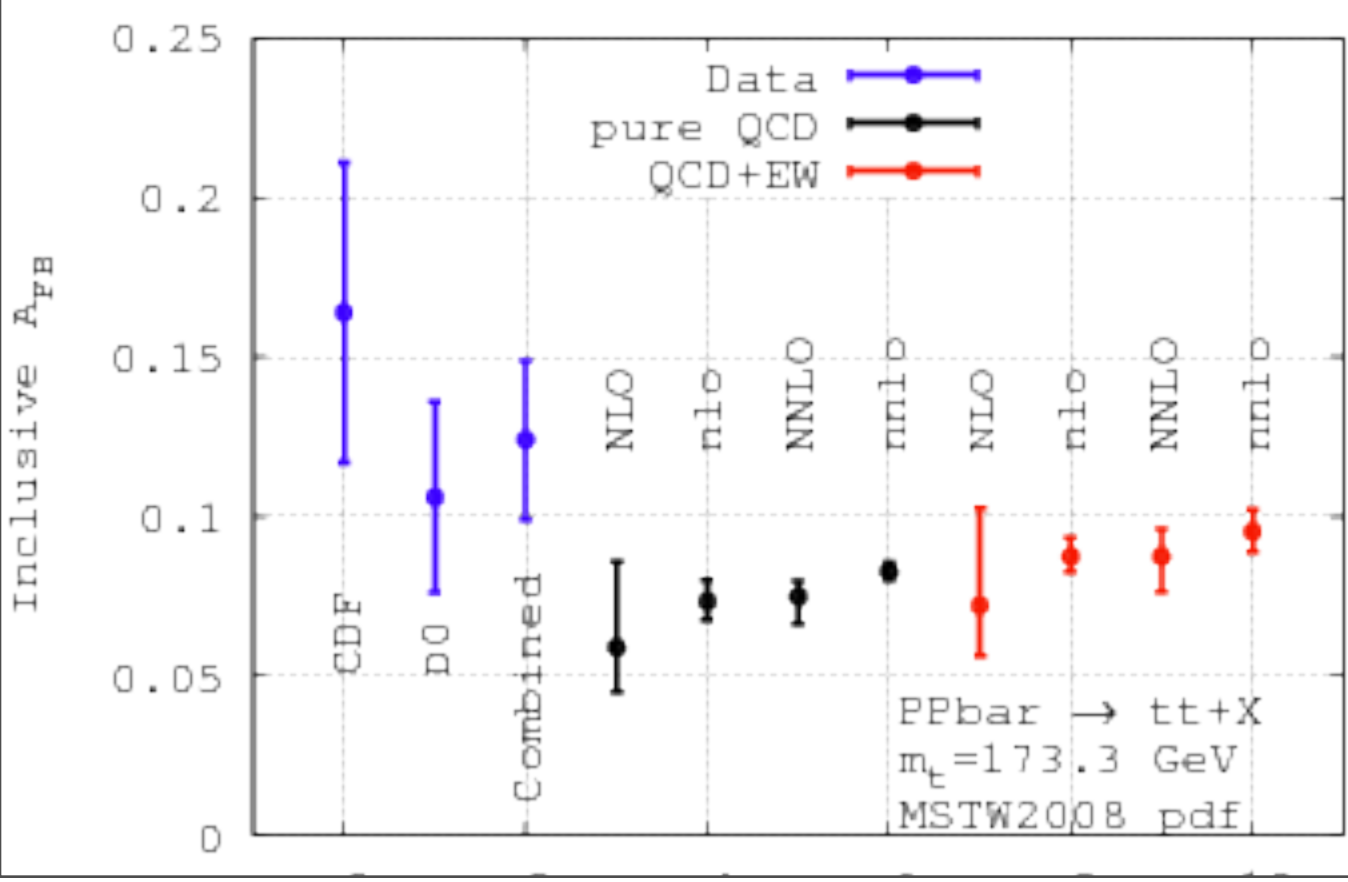
The NNLO QCD physics results

Top pair production



Collider	σ_{tot} [pb]	scales [pb]	pdf [pb]
Tevatron	7.009	+0.259(3.7%) −0.374(5.3%)	+0.169(2.4%) −0.121(1.7%)
LHC 7 TeV	167.0	+6.7(4.0%) −10.7(6.4%)	+4.6(2.8%) −4.7(2.8%)
LHC 8 TeV	239.1	+9.2(3.9%) −14.8(6.2%)	+6.1(2.5%) −6.2(2.6%)
LHC 14 TeV	933.0	+31.8(3.4%) −51.0(5.5%)	+16.1(1.7%) −17.6(1.9%)

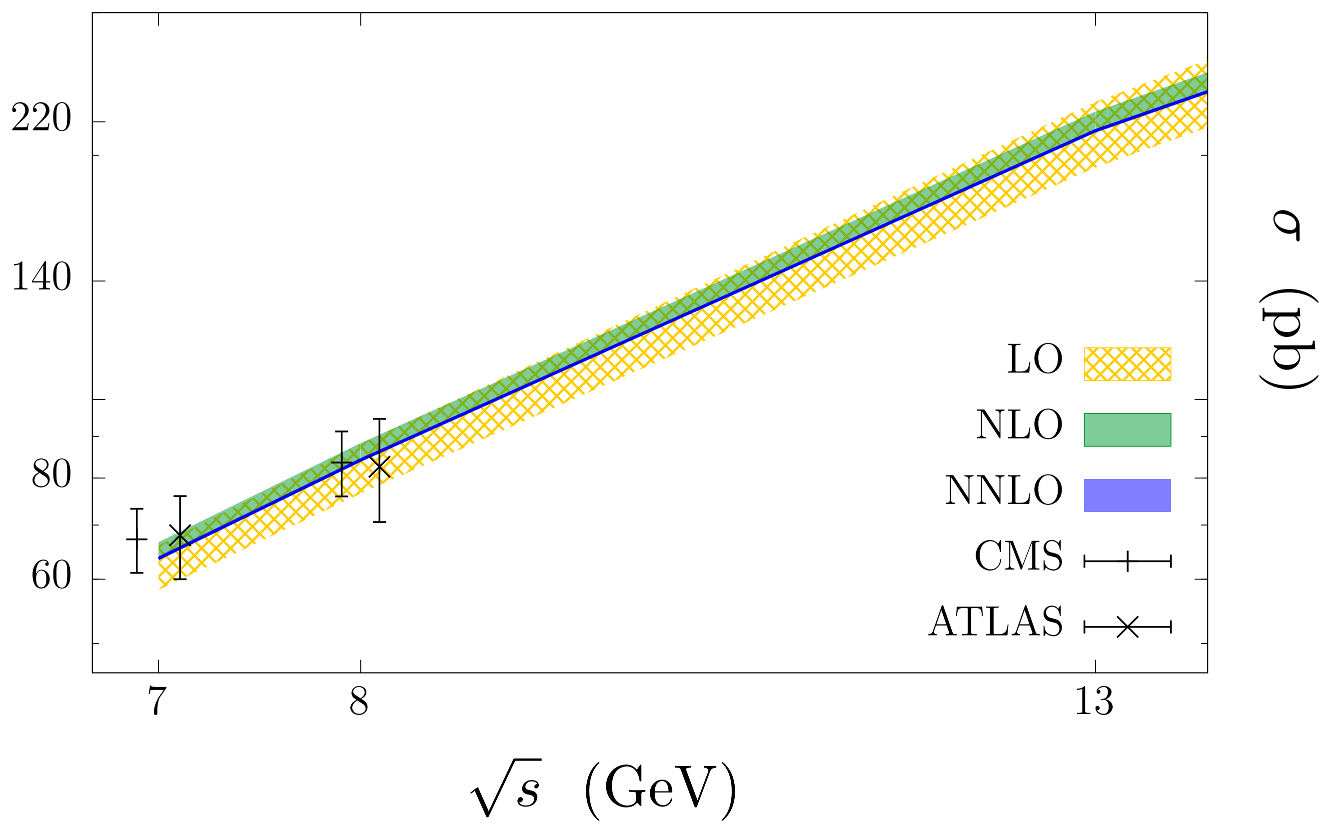


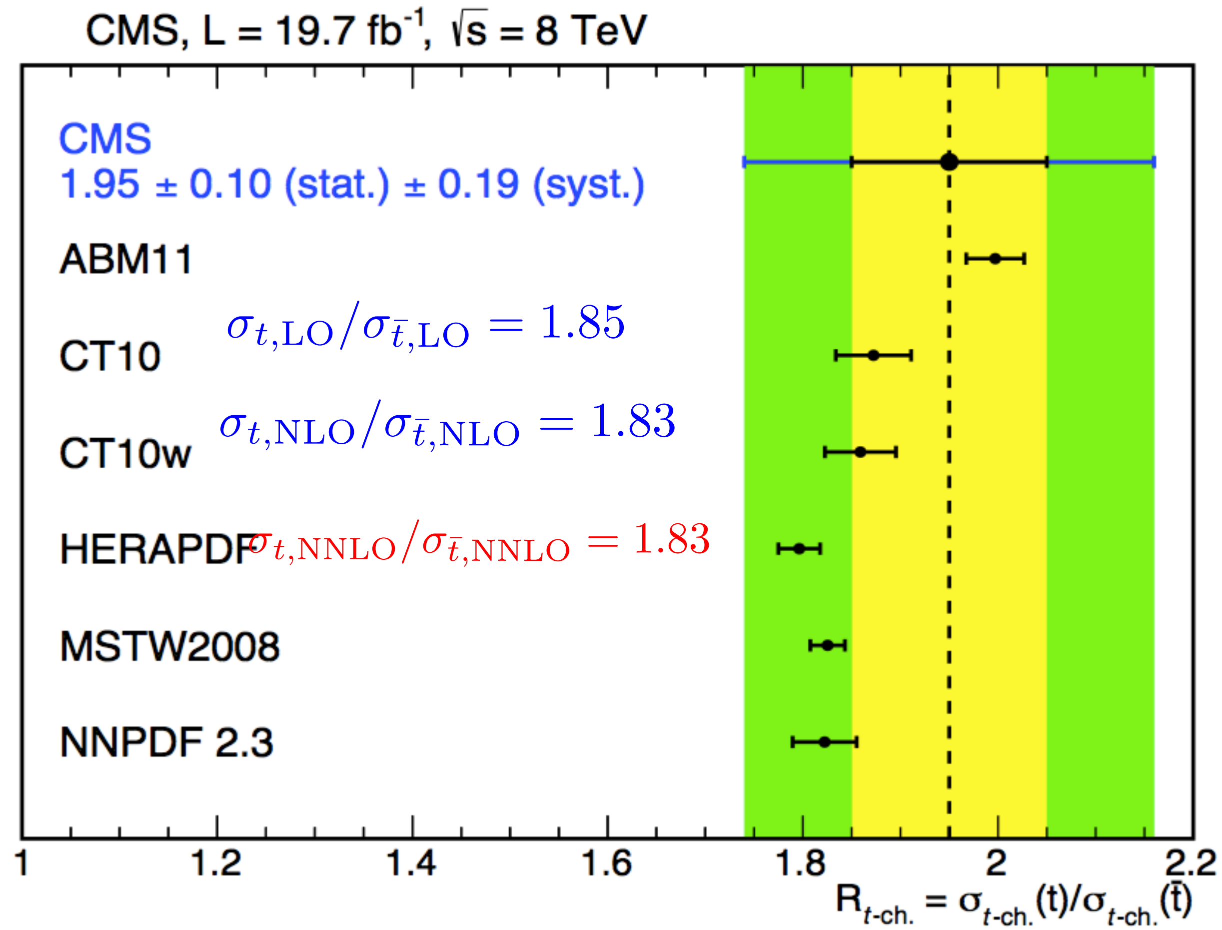


Czakon, Mitov, Fiedler, Heymes

An ongoing effort by Abelof, Gehrmann de Ridder , Pozzorini

Single top production (t-channel)



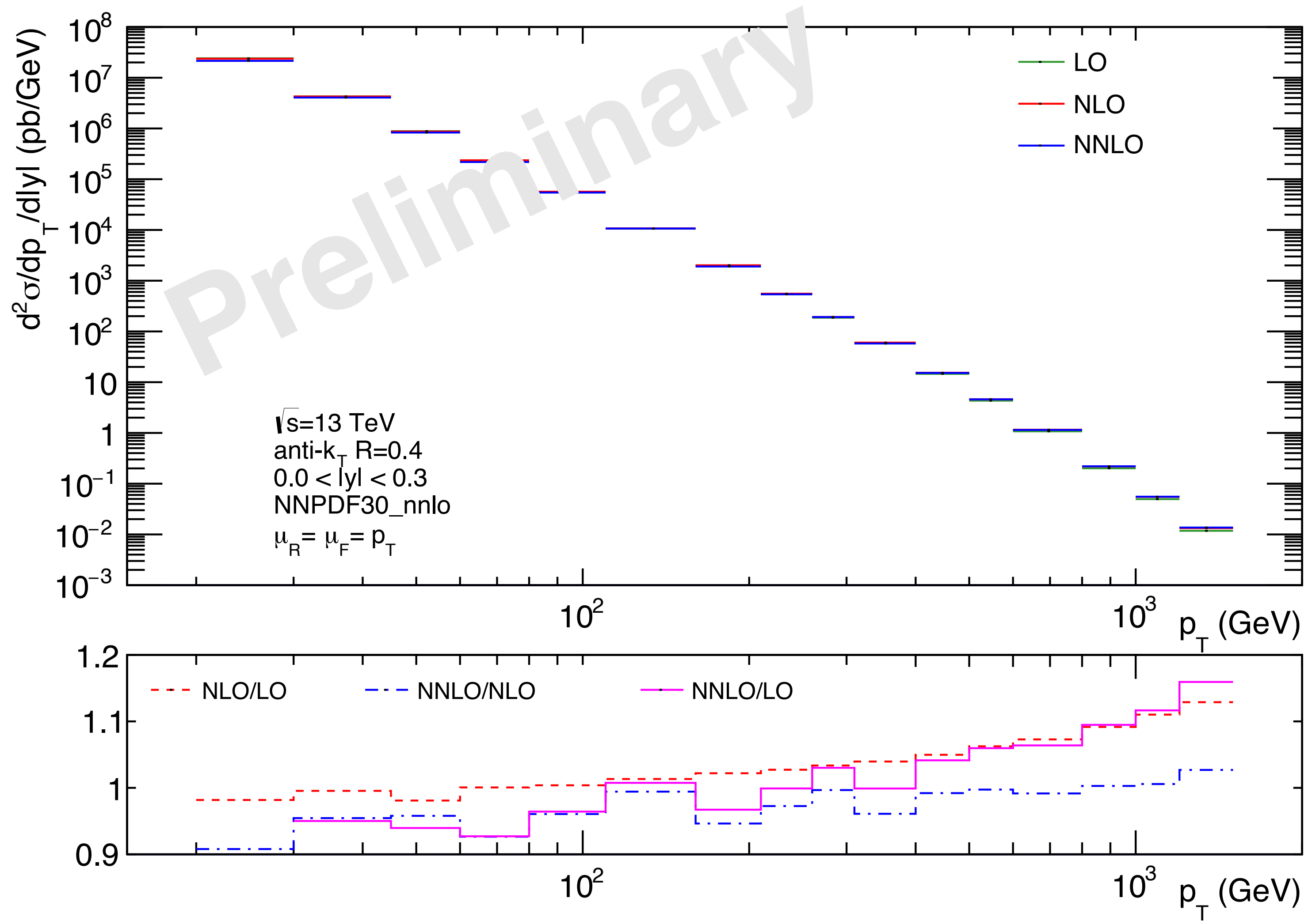


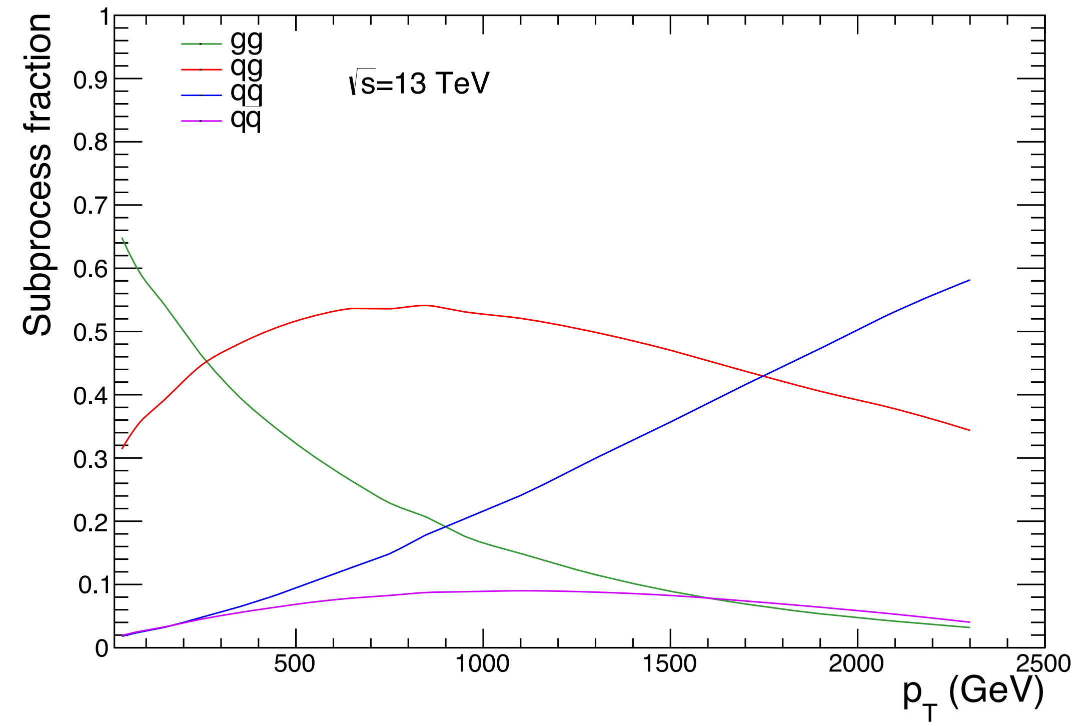
Burcherseifer, Caola, K.M.

$p_\perp$	σ_{LO}, pb	σ_{NLO}, pb	δ_{NLO}	σ_{NNLO}, pb	δ_{NNLO}
0 GeV	$53.8^{+3.0}_{-4.3}$	$55.1^{+1.6}_{-0.9}$	$+2.4\%$	$54.2^{+0.5}_{-0.2}$	-1.6%
20 GeV	$46.6^{+2.5}_{-3.7}$	$48.9^{+1.2}_{-0.5}$	$+4.9\%$	$48.3^{+0.3}_{-0.02}$	-1.2%
40 GeV	$33.4^{+1.7}_{-2.5}$	$36.5^{+0.6}_{-0.03}$	$+9.3\%$	$36.5^{+0.1}_{+0.1}$	-0.1%
60 GeV	$22.0^{+1.0}_{-1.5}$	$25.0^{+0.2}_{+0.3}$	$+13.6\%$	$25.4^{-0.1}_{+0.2}$	$+1.6\%$

The precision on the inclusive cross section is about one percent. Ratio of top and anti-top cross sections is sensitive to parton distribution functions at relatively large values of x and should be used as one of the standard candles for PDF determinations.

Di-jet production



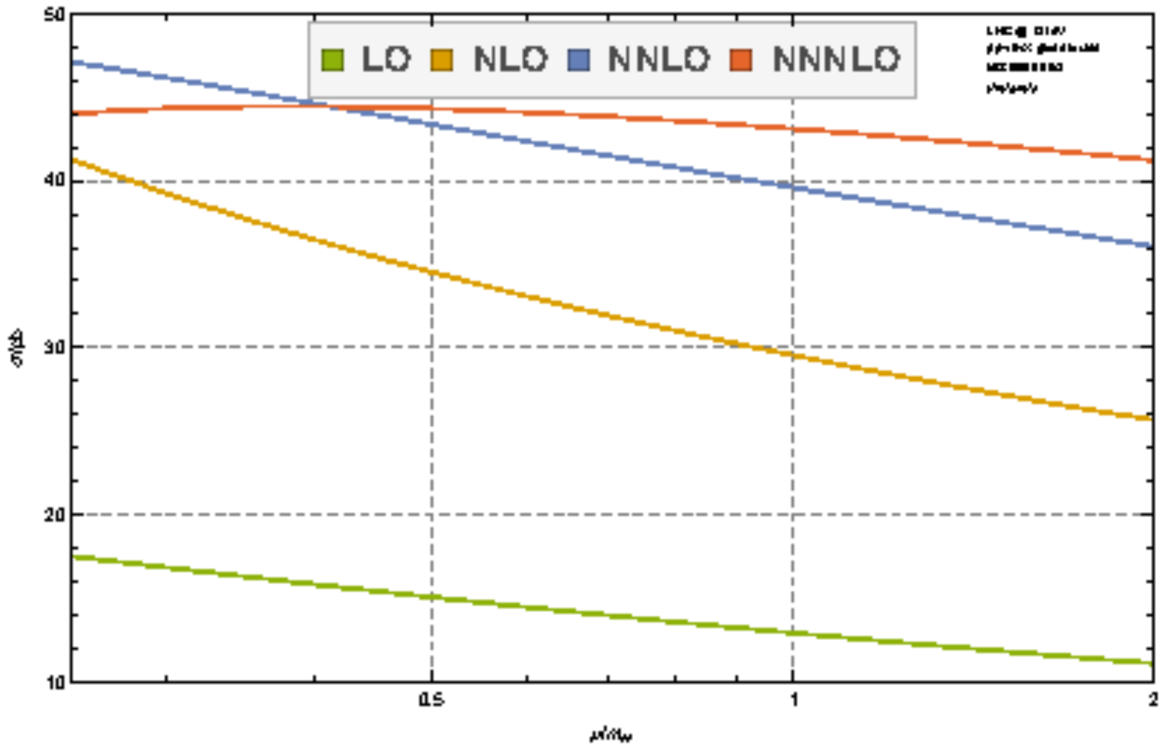


Results are for gluon-gluon and quark-gluon (preliminary) initial states. Not all color factors included for quark-gluon channel. Flat NNLO/NLO K-factors; small corrections (may change if other channels included). Results for various orders obtained with NNLO PDFs.

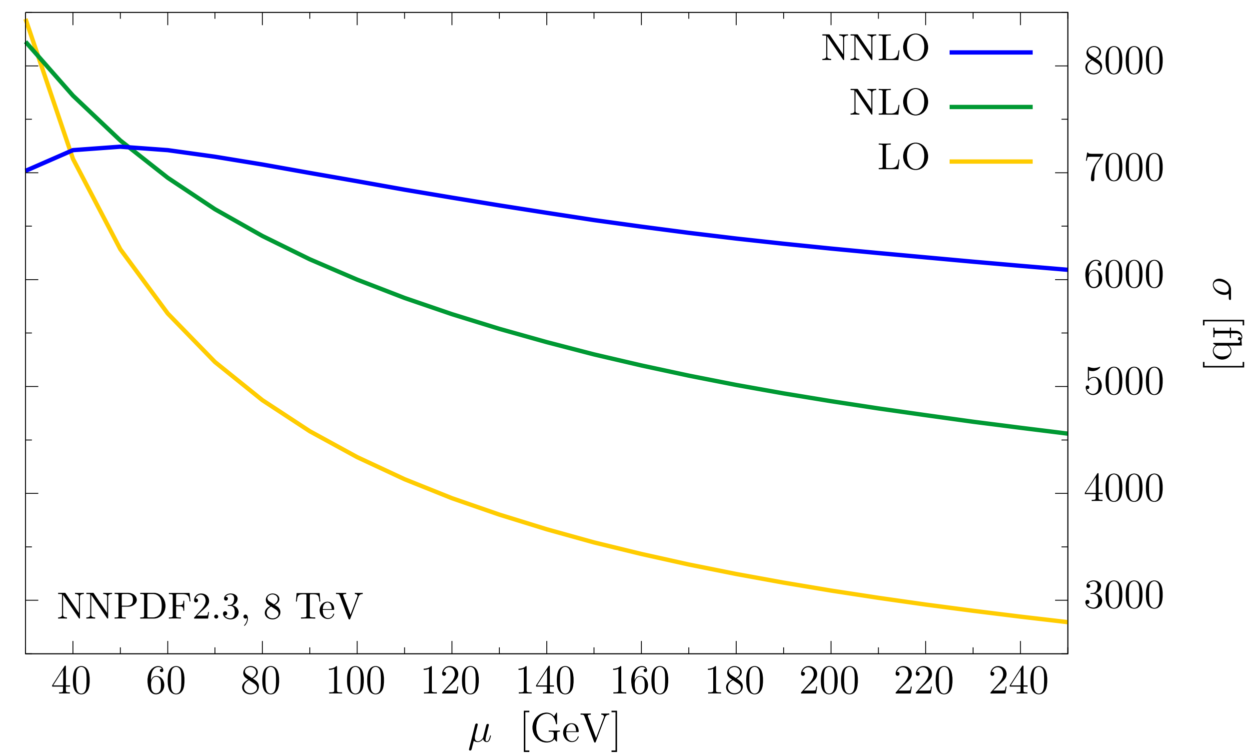
Currie, Gehrmann-de Ridder, Gehrmann, Glover, Pires

Higgs production: efficiencies

Higgs production at N[3]LO and H+jet production at NNLO appear at the same order in perturbation theory. One can combine those results to compute zero-jet cross sections for Higgs production. The results imply that resummations are not particularly relevant for these studies.



Anastasiou, Duhr, Dulat, Furlan, Herzog, Gehrmann, Mistlberger etc.



R. Boughezal, F. Caola, K.M., F. Petriello, M. Schulze

H+j@NNLO was also studied: Chen, Jaquier, Gehrmann, Glover; Boughezal, Focke, Liu, Petriello

LHC 13 TeV	$\epsilon^{\mathrm{N^3LO+NNLL+LL_R}}$	$\Sigma_{\text{0-jet}}^{\mathrm{N^3LO+NNLL+LL_R}}$ [pb]	$\Sigma_{\text{0-jet}}^{\mathrm{N^3LO}}$	$\Sigma_{\text{0-jet}}^{\mathrm{NNLO+NNLL}}$
$p_{t,\mathrm{veto}} = 25\,\mathrm{GeV}$	$0.539^{+0.017}_{-0.008}$	$24.7^{+0.8}_{-1.0}$	$24.3^{+0.5}_{-1.0}$	$24.6^{+2.6}_{-3.8}$
$p_{t,\mathrm{veto}} = 30\,\mathrm{GeV}$	$0.608^{+0.016}_{-0.007}$	$27.9^{+0.7}_{-1.1}$	$27.5^{+0.5}_{-1.1}$	$27.7^{+2.9}_{-4.0}$

A. Banfi, F. Caola, F. Dreyer, P. Monni, G.Salam, G. Zanderighi, F. Dulat

Higgs cross sections: even more fiducial

To go even more fiducial (i.e. realistic), one can let the Higgs decay and compare results with measured cross sections / distributions of the ATLAS collaboration.

Atlas cuts on photons and jets

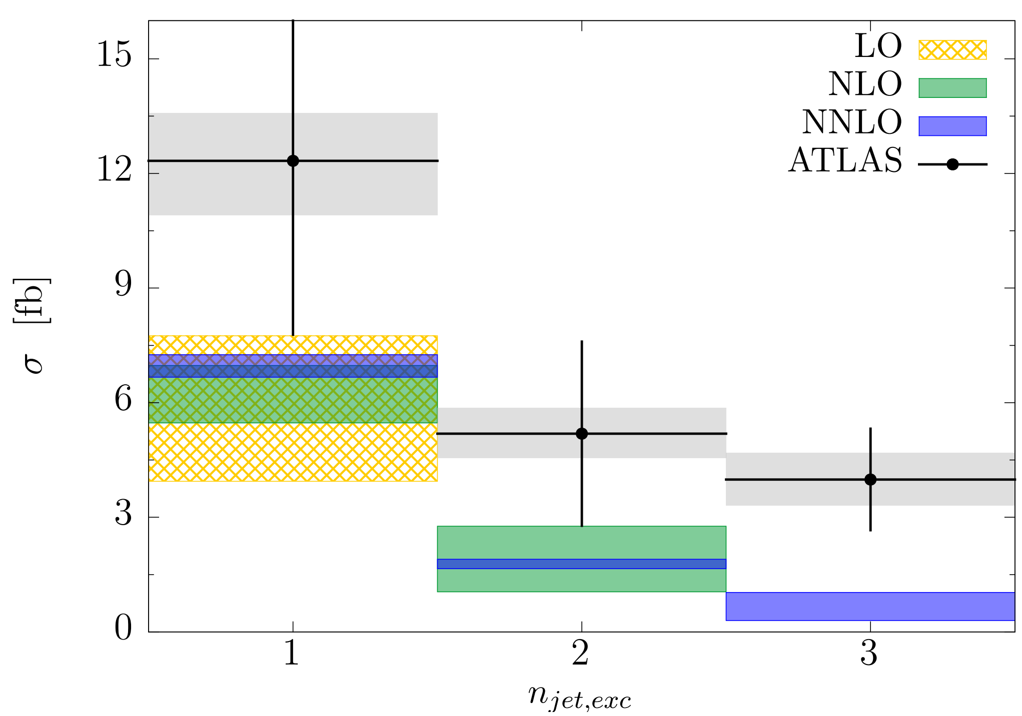
$$\text{anti}-\text{k}_t, \quad \Delta R = 0.4, \quad p_{j\perp} = 30~\text{GeV}, \text{abs}(y_j) < 4.4$$

$$p_{\perp,\gamma_1} > 43.75~\text{GeV}, \quad p_{\perp,\gamma_2} = 31.25~\text{GeV}, \quad \Delta R_{\gamma j} > 0.4$$

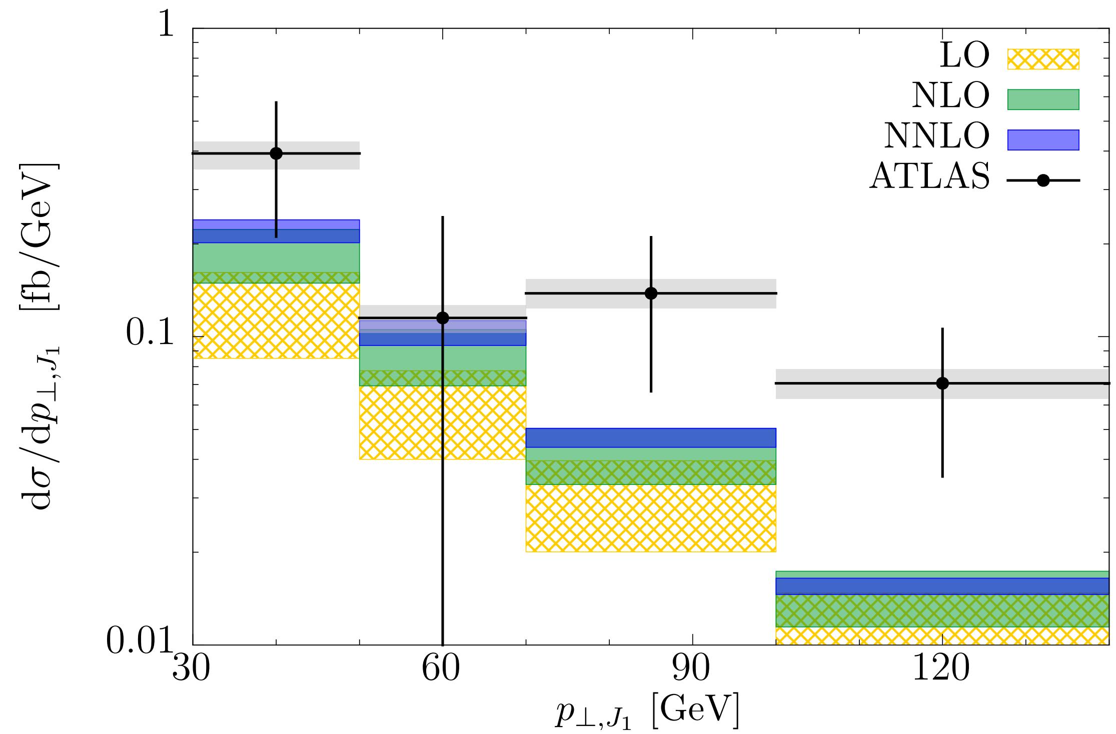
$$\sigma^{\text{fid}}_{1j,\text{ATLAS}} = 21.5 \pm 5.3(\text{stat}) \pm 2.3(\text{syst}) \pm 0.6~\text{lum fb}$$

$$\sigma^{\text{fid}}_{\text{NNLO}} = 9.46^{+0.56}_{-0.84}~\text{fb}$$

F. Caola, K.M., M. Schulze



Exclusive jet cross sections

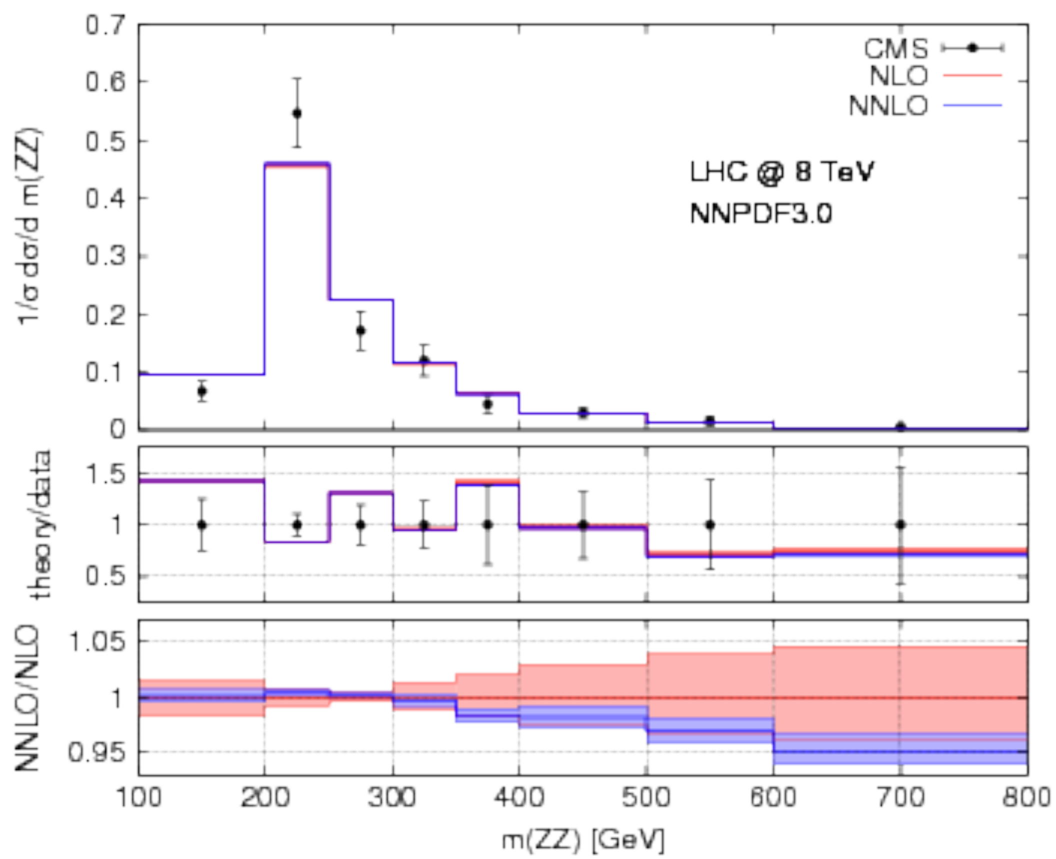


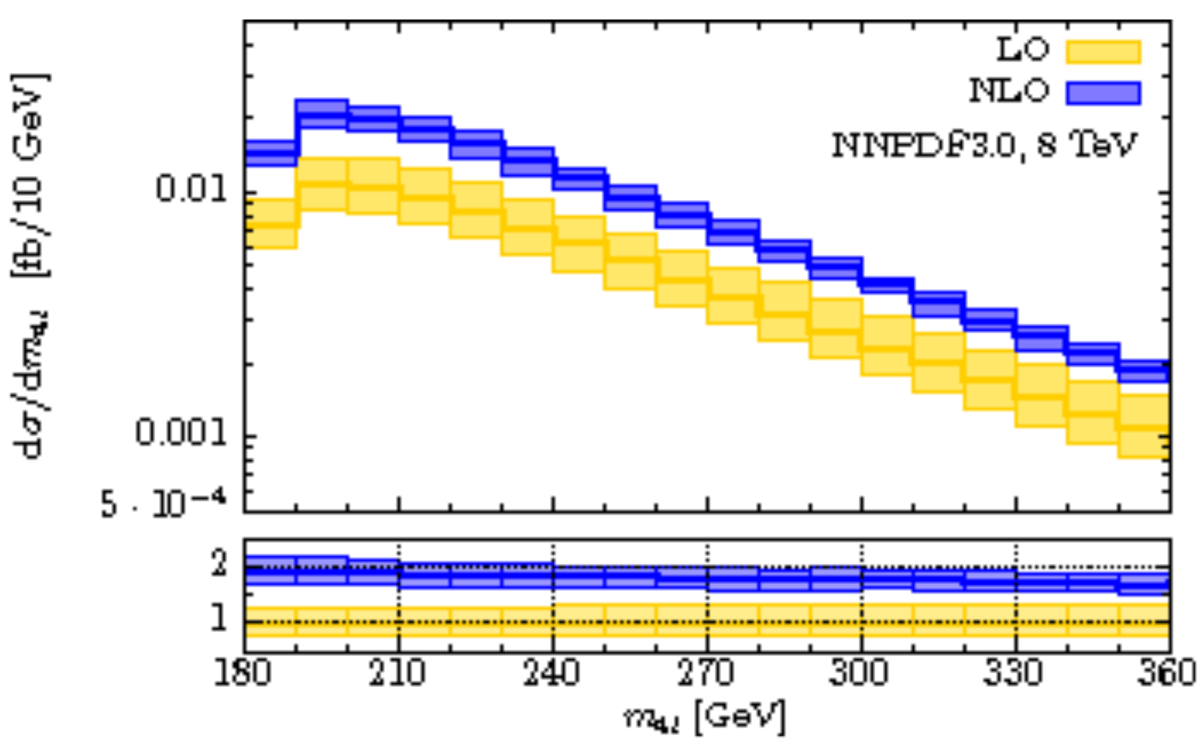
Transverse momentum distribution of a leading jet

Off-shell Higgs: constraining the width and all that

To constrain the Higgs couplings in the off-shell region to O(20%) (and the Higgs width to within a factor 2), O(10%) prediction for qq->ZZ and O(50%) prediction for gg ->ZZ is required. This was a significant challenge but we have almost overcome it (top quark loops) !

$$\sigma_{\rm NNLO}^{q\bar{q}\to 2e2\mu} = 19.6(6)\ {\rm fb} \qquad \sigma_{\rm NNLO}^{gg\to 2e2\mu} = 2.0(2)\ {\rm fb}$$



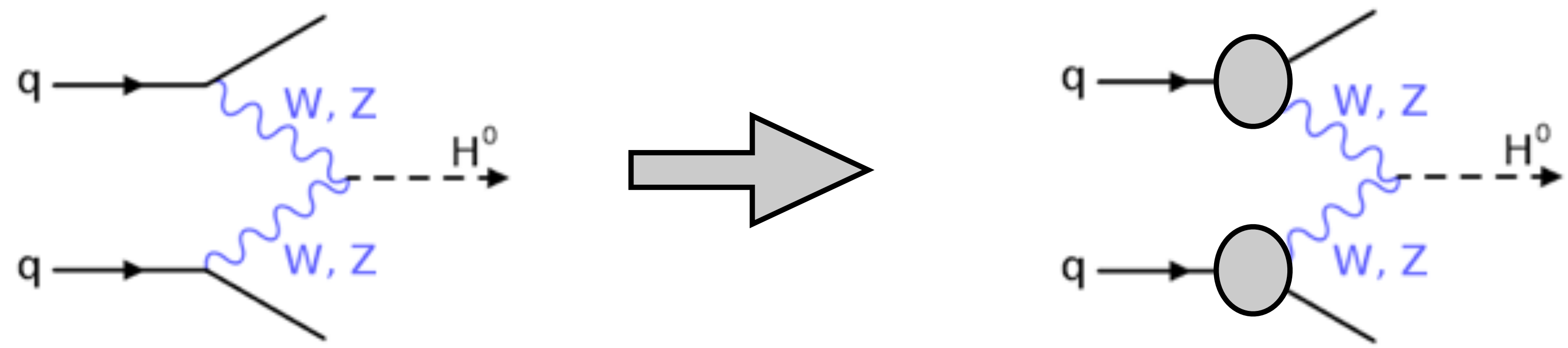


F. Caola, K. Melnikov, R. Rontsch, L. Tancredi

T. Gehrmann, M. Grazzini, S. Kallweit, P. Maierhoefer, A. von Manteuffel, S. Pozzorini, D. Rathlev, L. Tancredi

Higgs boson production in weak boson fusion

Estimating NNLO QCD corrections to WBF fusion by mapping the problem on the inclusive DIS apparently does not work. QCD corrections are different.

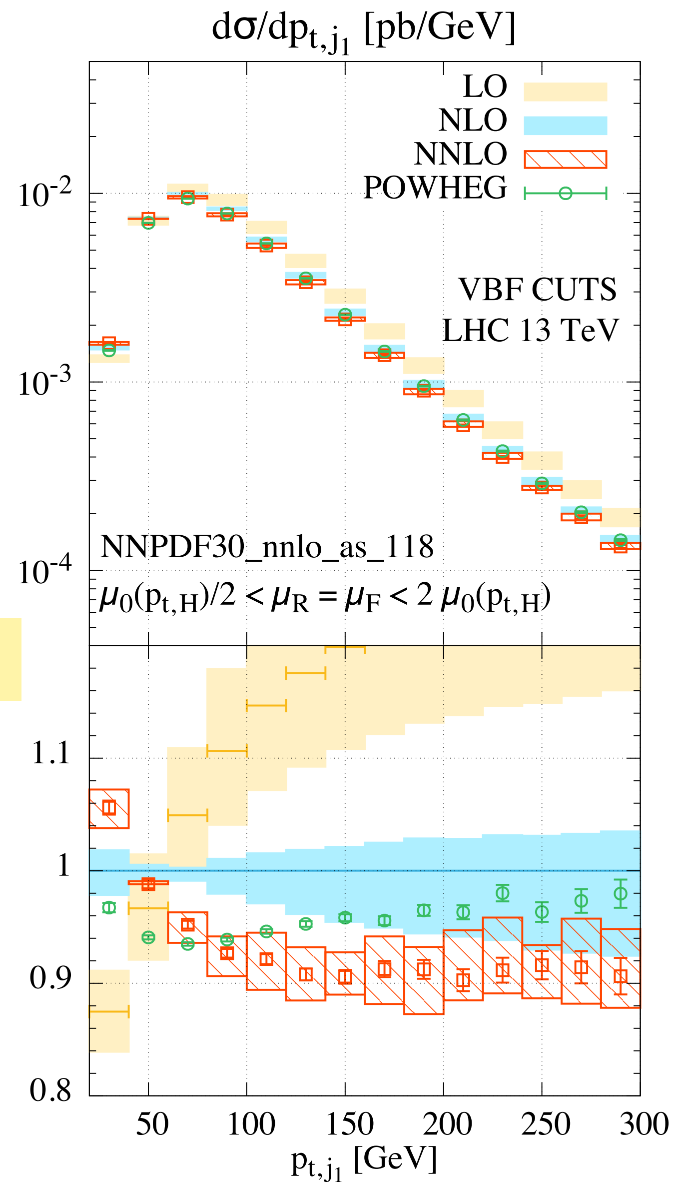


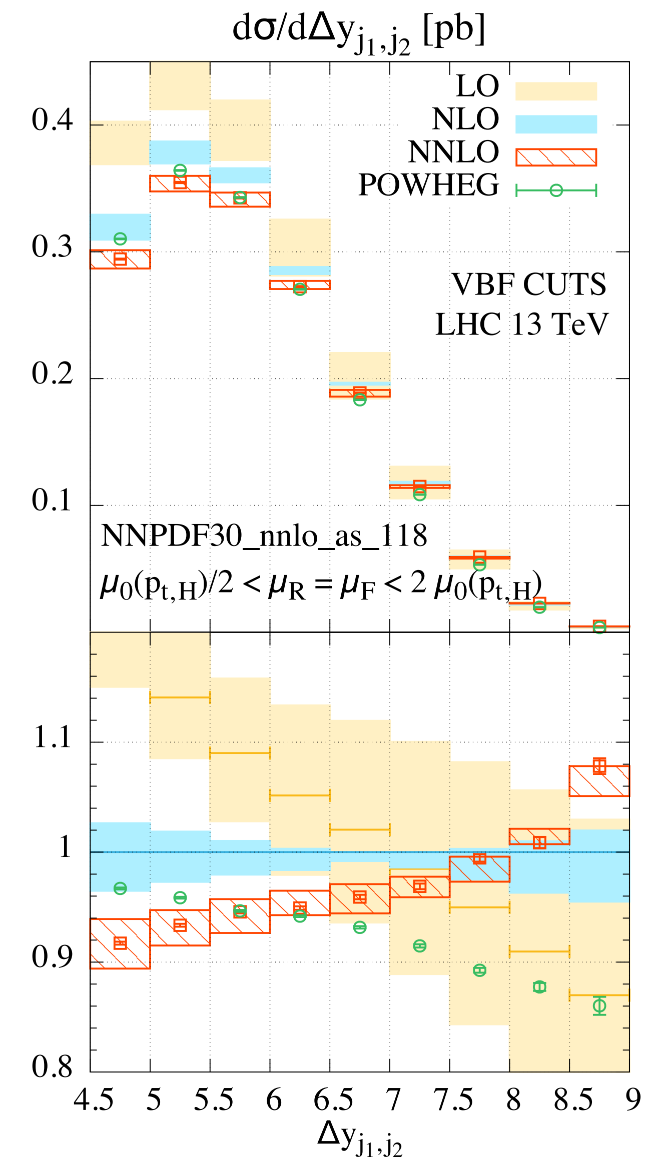
WBF cuts

$$p_\perp^{j_{1,2}} > 25 \text{ GeV}, \quad |y_{j_{1,2}}| < 4.5,$$
$$\Delta y_{j_1,j_2} = 4.5, \quad m_{j_1,j_2} > 600 \text{ GeV},$$
$$y_{j_1} y_{j_2} < 0, \quad \Delta R > 0.4$$

Cross sections with and without WBF cuts

	$\sigma^{\text{nocuts}}[\text{pb}]$	$\sigma^{\text{VBF cuts}}[\text{pb}]$
LO	$4.032^{+0.057}_{-0.069}$	$0.957^{+0.066}_{-0.059}$
NLO	$3.929^{+0.024}_{-0.023}$	$0.876^{+0.008}_{-0.018}$
NNLO	$3.888^{+0.016}_{-0.012}$	$0.826^{+0.013}_{-0.014}$

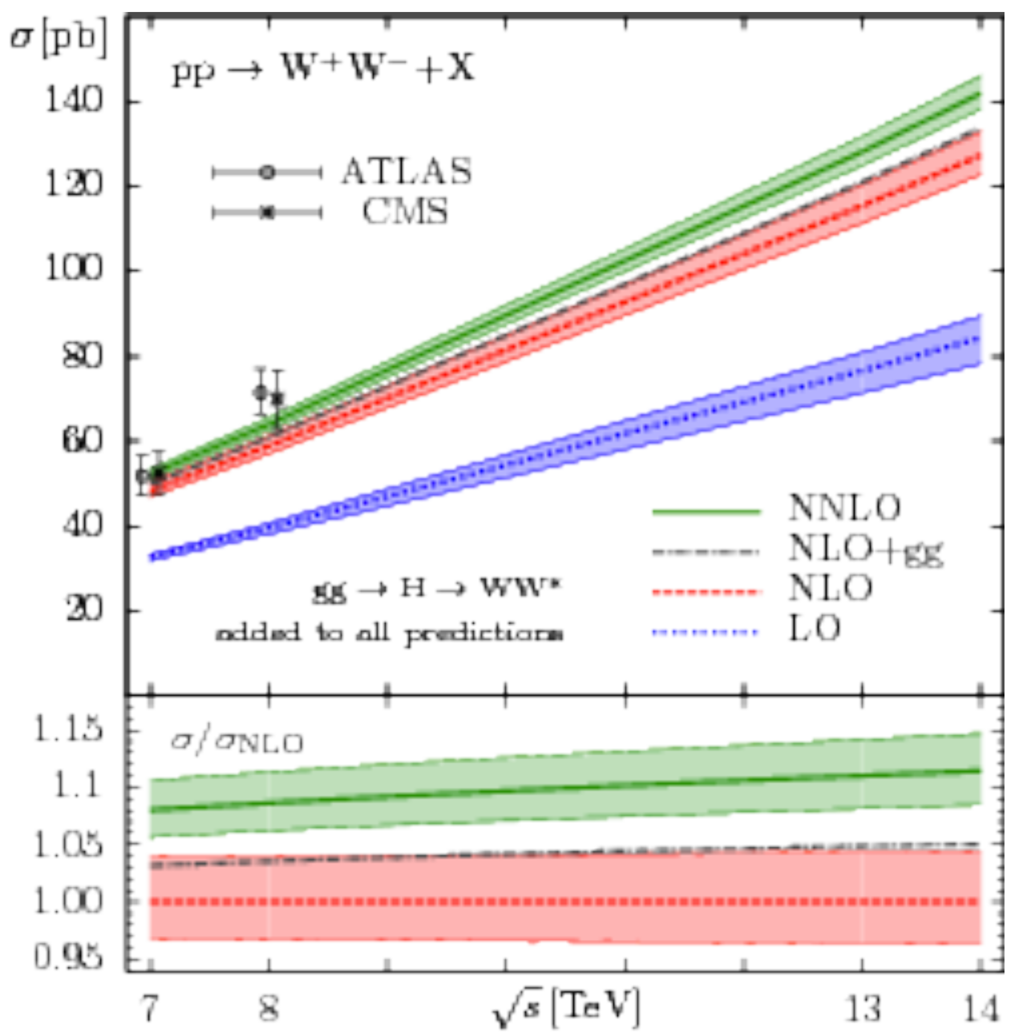




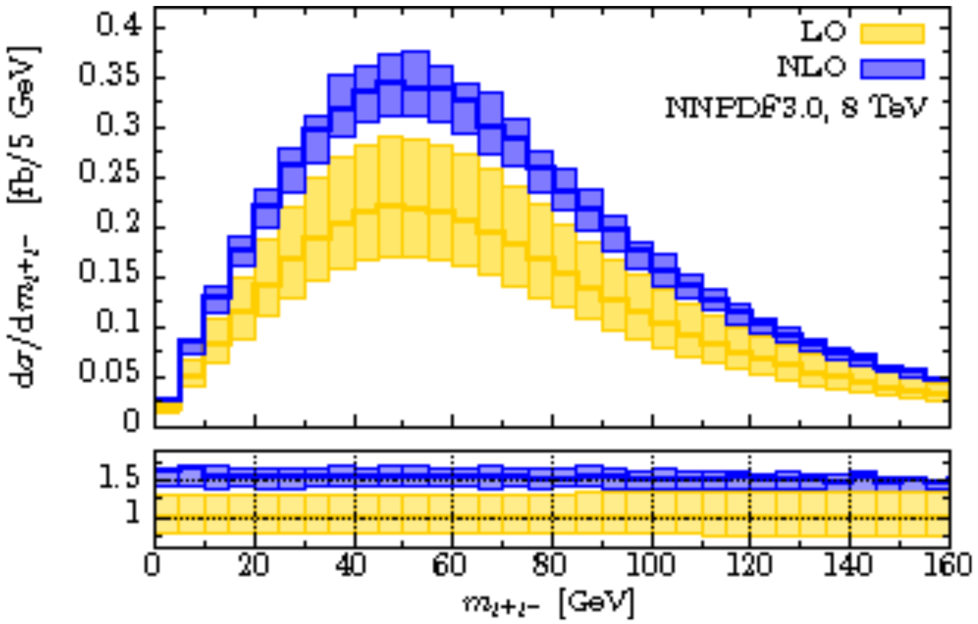
Cacciari, Dreyer, Kalberg, Salam, Zanderighi

W-boson pair production

Interest in this process is related to a two-sigma excess that was observed by both ATLAS and CMS in 7 TeV and 8 TeV data. The NNLO QCD corrections to quark-anti-quark annihilation as well as the NLO QCD corrections to gluon fusion push the theory prediction much closer to experiment.



T. Gehrmann, M. Grazzini, S. Kallweit, P. Maierhoefer, A. von Manteuffel, S. Pozzorini, D. Rathlev, L. Tancredi



F. Caola, K. Melnikov, R. Rontsch, L. Tancredi

$$\sigma^{q\bar{q}+H+gg,\mathrm{NLO}}_{\mu\mu,ee,e\mu+\mu e} = (72.0^{+1.3}_{-2.1},\ 66.3^{+1.2}_{-1.7},\ 337.3^{+6.3}_{-4.5}).$$

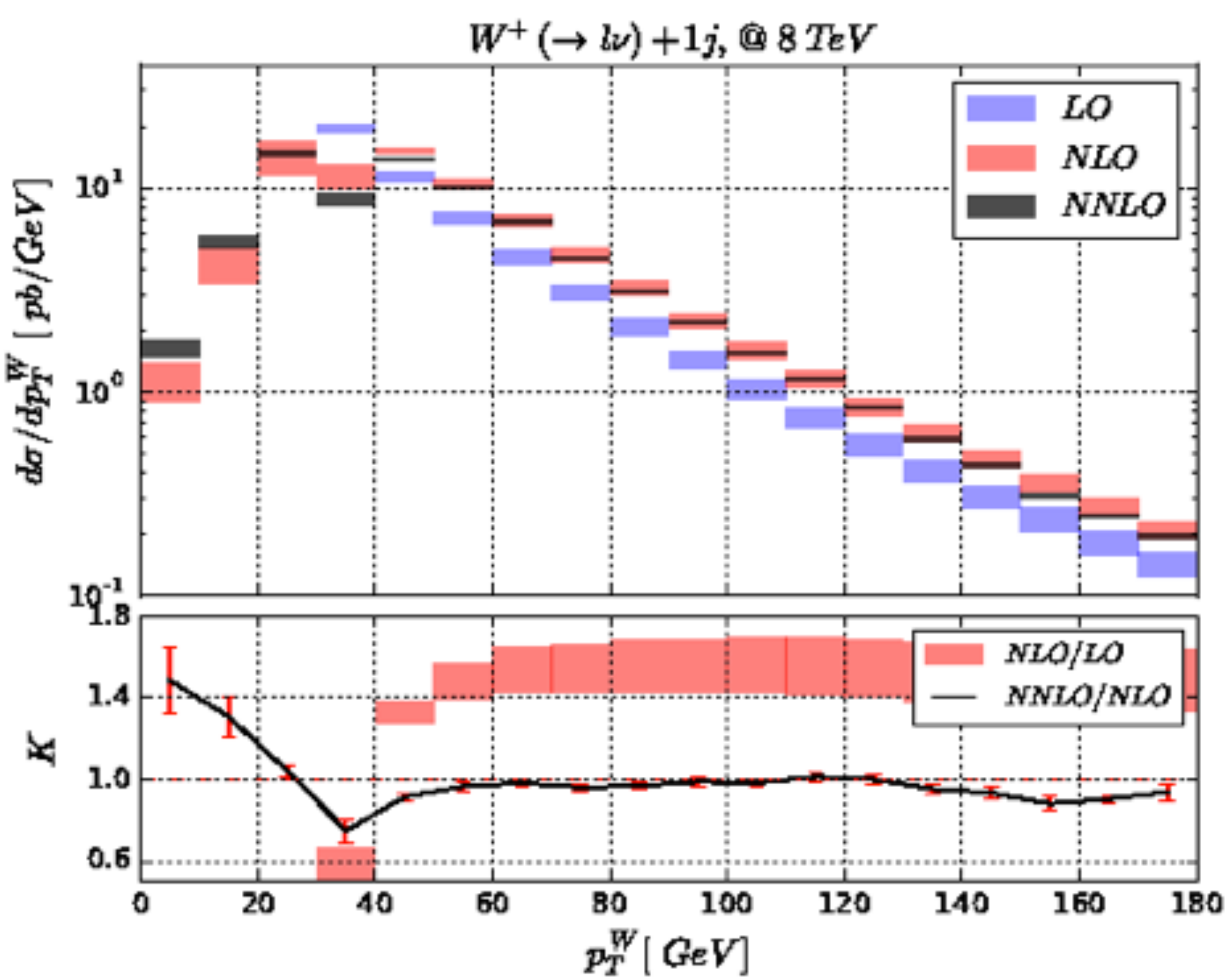
$$\sigma_{\mu\mu,ee,e\mu+\mu e} = (74.4^{+8.1}_{-7.1},\ 68.5^{+9.0}_{-8.0},\ 377.8^{+28.4}_{-25.6})$$

Estimating the NNLO QCD corrections by re-scaling inclusive ones, we find that they can add additional 4-20 fb, for ee and electron-muon channels, respectively. This will make theory and experiment agree to within one sigma.

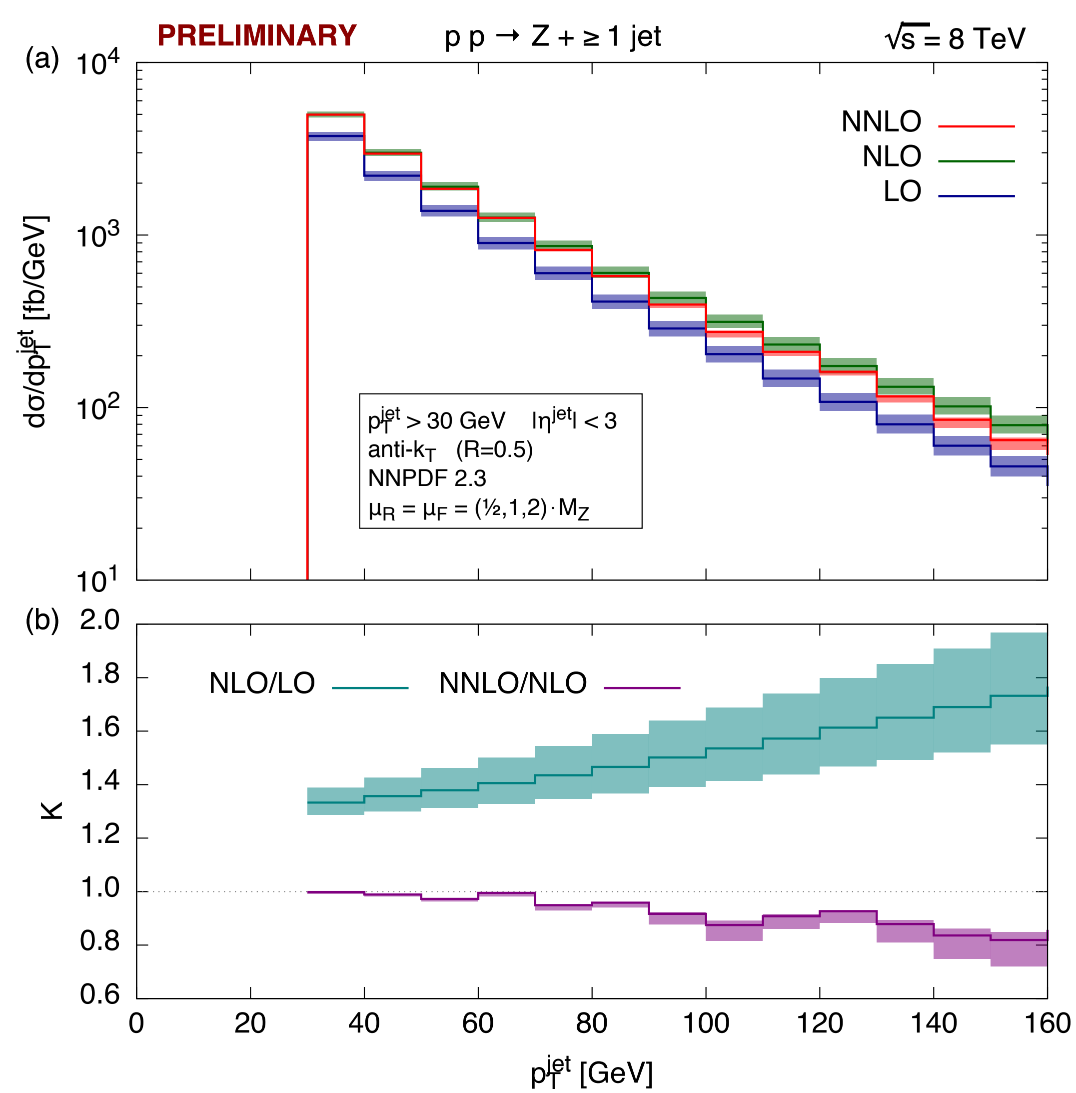
Vector bosons plus jet

NNLO QCD computations for W+j and Z+j are now available. Corrections are found to be quite small.

These results can be used for better background modeling, for improved understanding of the W and Z bosons transverse momentum distribution and for constraining the gluon PDF.



Bougezhal, Focke, Liu, Petriello



Gehrmann-de Ridder, Gehrmann, Glower, Huss, Morgan

Also studied by Bougezhal, Campbell, Ellis, Focke, Giele, Liu, Petriello

Conclusion

Our ability to perform NNLO QCD computations increased dramatically during the past year. Development of robust theoretical methods finally paid off and allowed us to compute large number of 2 -> 2 processes through NNLO QCD in a fully exclusive manner.

NNLO QCD is the ``last perturbative order" that is possible to study without understanding non-perturbative effects at colliders (exceptions are processes with very large NLO QCD corrections).

NNLO is a high enough perturbative order to provide both correct physics and high precision.Use of NNLO should naturally reduce the reliance on resummations and parton showers outside of their applicability region.

NNLO QCD predictions show that after a certain level of precision, it is not possible to rely on the approximate ways of computing radiative corrections; full fixed order calculations are needed. This is especially true for hard fiducial cross sections that, in fixed order calculations, can be computed for the same sets of cuts that are used in the measurement.

Phenomenological reach of these computations is very broad and impacts studies of top quark properties, understanding the Higgs boson couplings, extraction of parton distribution functions, measurements of the strong coupling constant and refined modeling of backgrounds.

Further developments of theoretical methods for these computations will involve massive loops, higher multiplicity final states, unitarity and improvements in the efficiency of subtraction methods.