# Status and Results of the NNLO Penguin Amplitudes

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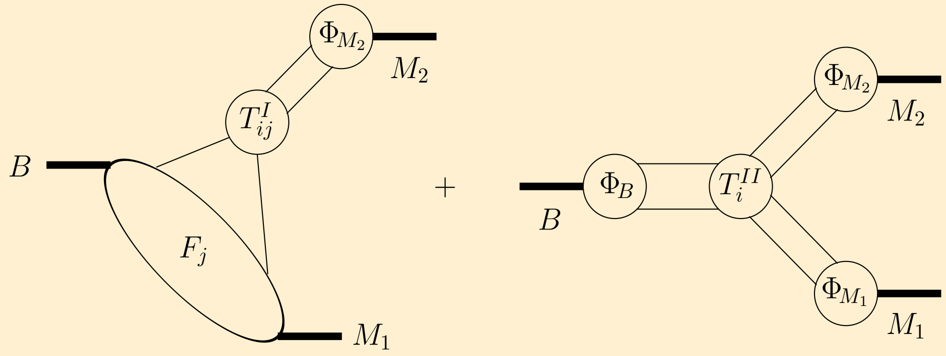
Based on

- Bell, TH, [arXiv:1410.2804], JHEP
- Bell, Beneke, Li, TH, [arXiv:1507.03700], PLB

6$^{\mathrm{th}}$ QFET Workshop, Siegen, January 18$^{\mathrm{th}}$, 2016

# QCD factorisation




- Amplitude in the limit $m_b \gg \Lambda_{\text{QCD}}$ *[Beneke,Buchalla,Neubert,Sachrajda'99-'04]*

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \simeq m_B^2\ F_+^{B\to M_1}(0)\ f_{M_2} \int_0^1 du\ T_i^I(u)\ \phi_{M_2}(u)$$

$$+ f_B\, f_{M_1}\, f_{M_2} \int_0^1 d\omega dv du\ T_i^{II}(\omega, v, u)\ \phi_B(\omega)\ \phi_{M_1}(v)\ \phi_{M_2}(u)$$

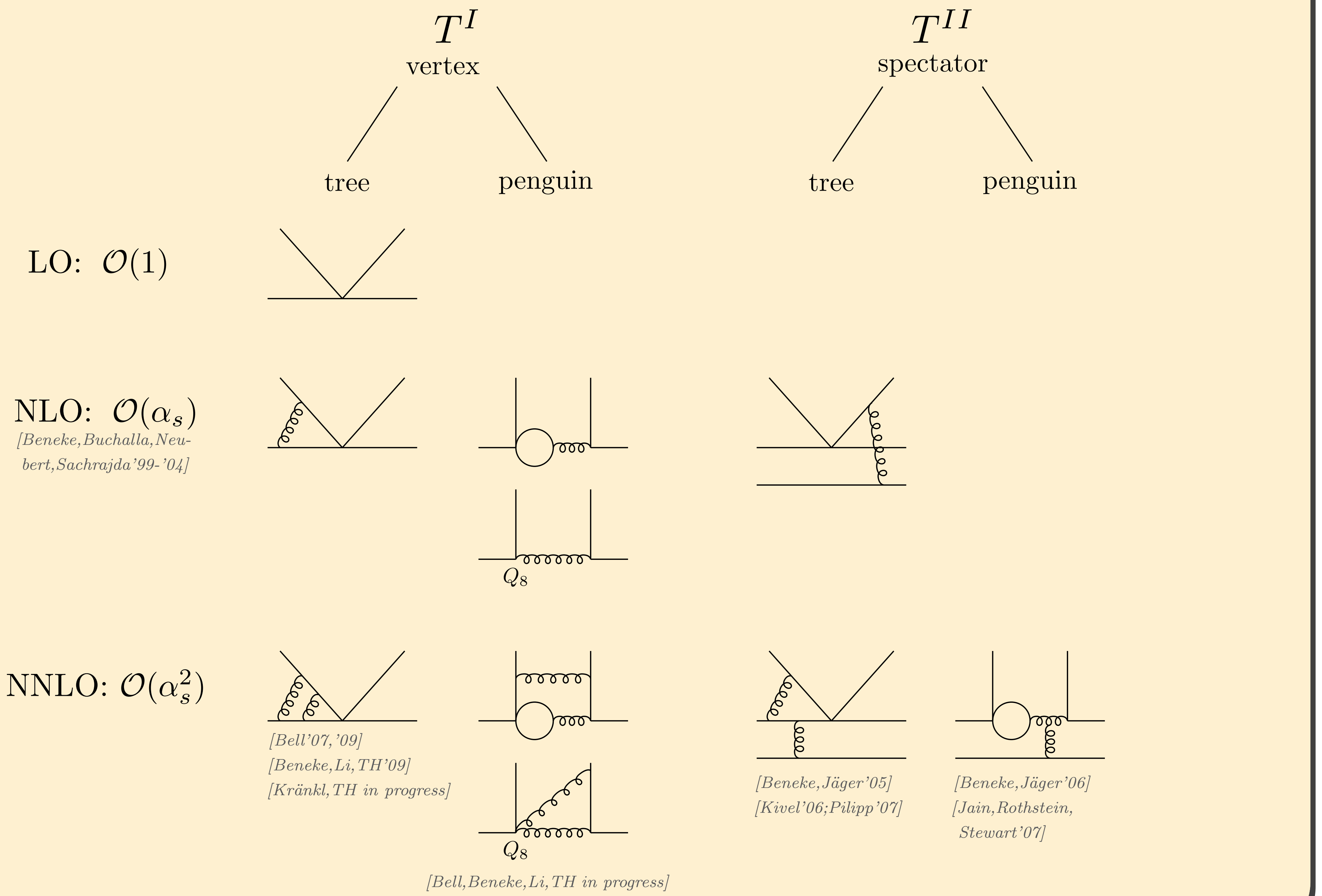
- $T^{I,II}$ : Hard scattering kernels, perturbatively calculable

- $F_+$ : $B \to M$ form factor
  $f_i$ : decay constants
  $\phi_i$ : light-cone distribution amplitudes

  } Universal. From Sum Rules, Lattice

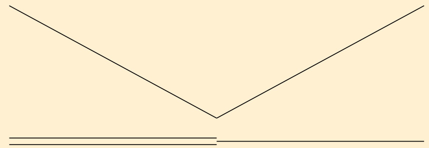
- Strong phases are $\mathcal{O}(\alpha_s)$ and/or $\mathcal{O}(\Lambda_{\text{QCD}}/m_b)$

# Anatomy of QCD factorisation

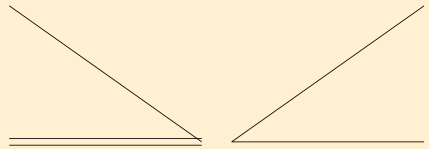


# Classification of amplitudes

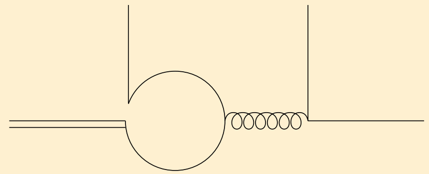
- $\alpha_1$ : colour-allowed tree amplitude



- $\alpha_2$ : colour-suppressed tree amplitude



- $\alpha_4^{u,c}$ : QCD penguin amplitudes



$$\begin{aligned}
\sqrt{2}\,\langle\pi^-\pi^0|\,\mathcal{H}_{eff}\,|B^-\rangle &= A_{\pi\pi}\,\lambda_u\big[\alpha_1(\pi\pi)+\alpha_2(\pi\pi)\big] \\
\langle\pi^+\pi^-|\,\mathcal{H}_{eff}\,|\bar{B}^0\rangle &= A_{\pi\pi}\,\big\{\lambda_u\big[\alpha_1(\pi\pi)+\alpha_4^u(\pi\pi)\big]+\lambda_c\,\alpha_4^c(\pi\pi)\big\} \\
-\,\langle\pi^0\pi^0|\,\mathcal{H}_{eff}\,|\bar{B}^0\rangle &= A_{\pi\pi}\,\big\{\lambda_u\big[\alpha_2(\pi\pi)-\alpha_4^u(\pi\pi)\big]-\lambda_c\,\alpha_4^c(\pi\pi)\big\}
\end{aligned}$$

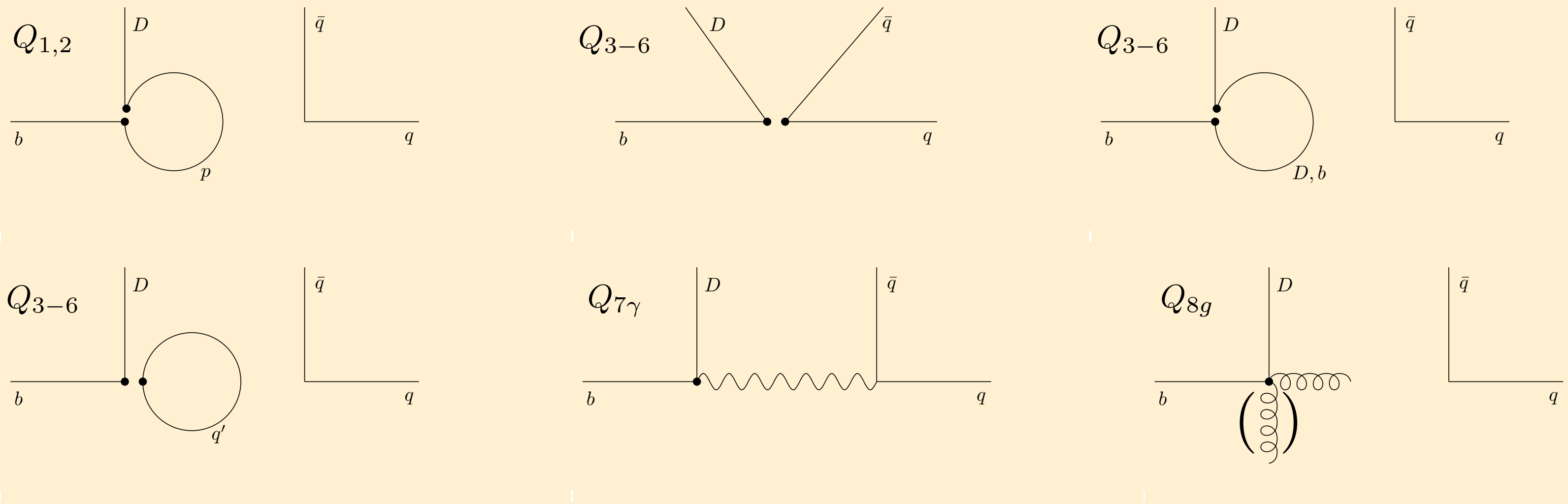
$$\begin{aligned}
\langle\pi^-\bar{K}^0|\,\mathcal{H}_{eff}\,|B^-\rangle &= A_{\pi\bar{K}}\,\Big[\lambda_u^{(s)}\,\alpha_4^u+\lambda_c^{(s)}\,\alpha_4^c\Big] \\
\langle\pi^+K^-|\,\mathcal{H}_{eff}\,|\bar{B}^0\rangle &= A_{\pi\bar{K}}\Big[\lambda_u^{(s)}\,(\alpha_1+\alpha_4^u)+\lambda_c^{(s)}\,\alpha_4^c\Big]
\end{aligned}$$

*[Beneke,Neubert'03]*

- Tree amplitudes $\alpha_1$ and $\alpha_2$ known analytically to NNLO *[Bell'07'09; Beneke,Li,TH'09]*

# Penguin amplitudes $a_4^u$ and $a_4^c$

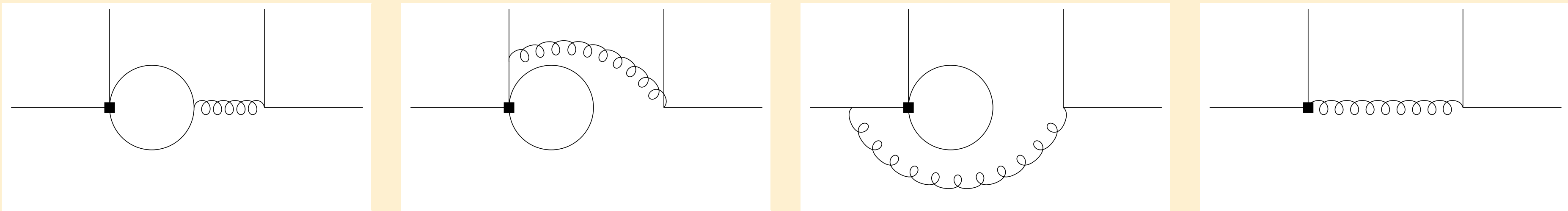
- Various insertions




- Main motivation for NNLO: Direct CP asymmetries
  - They start at $\mathcal{O}(\alpha_s)$
  - Large (scale) uncertainties at NLO
  - NNLO is only first perturbative correction
  - NNLO is NLO for direct CP asymmetries!

# Penguin amplitudes $a_4^u$ and $a_4^c$ at two loops

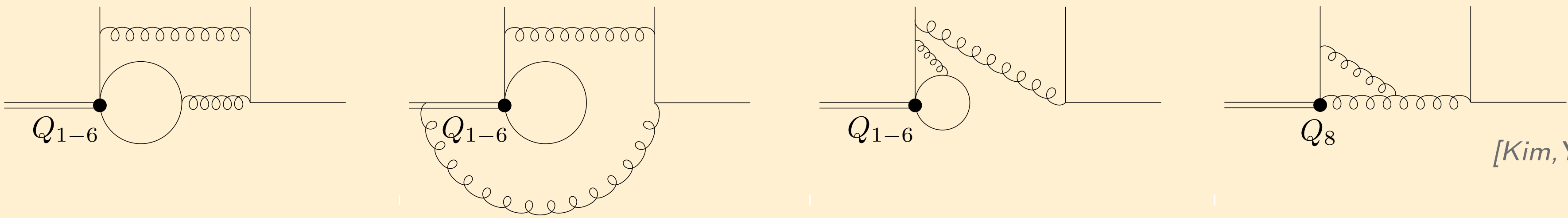
- NLO:



$$a_4^u(\pi\bar{K})_{|\text{NLO}} = (-0.0087 - 0.0172i)_{|Q_{1,2}} + (0.0042 + 0.0041i)_{|Q_{3-6}} + 0.0083_{|Q_{8g}},$$
$$a_4^c(\pi\bar{K})_{|\text{NLO}} = (-0.0131 - 0.0102i)_{|Q_{1,2}} + (0.0042 + 0.0041i)_{|Q_{3-6}} + 0.0083_{|Q_{8g}},$$

- !!! Focus on $Q_1^{u,c}$ and $Q_2^{u,c}$ insertions !!!

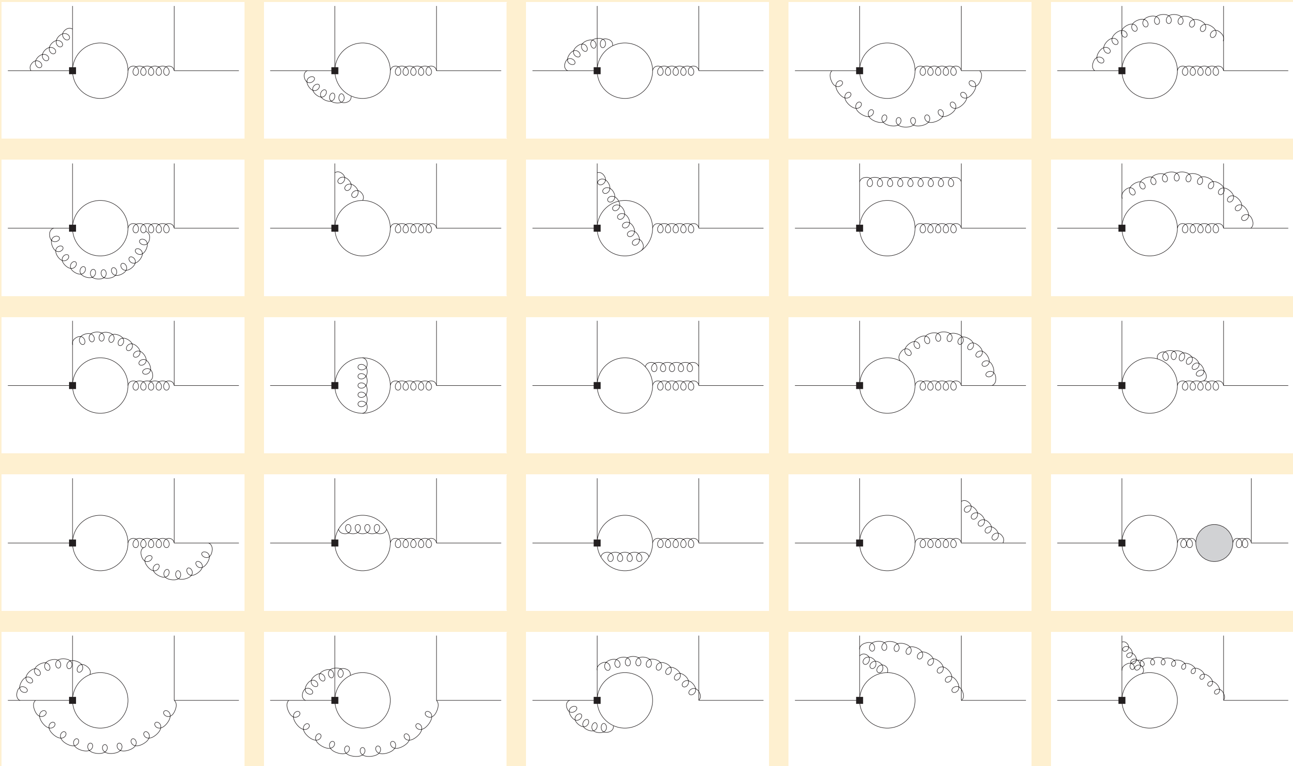
- $\mathcal{O}(70)$ diagrams at NNLO.




[Kim,Yoon'11]

# Penguin amplitudes $a_4^u$ and $a_4^c$ at two loops

- Only two dozens of diagrams contribute to current-current insertion



- Regularize UV and IR divergences dimensionally. Poles up to $1/\epsilon^3$
- SCET operator basis

$$O_1 = \bar{\chi}\,\frac{\not{n}_-}{2}(1-\gamma_5)\chi\;\bar{\xi}\,\not{n}_+(1-\gamma_5)h_v\,,$$

$$\tilde{O}_n = \bar{\xi}\,\gamma_\perp^\alpha\gamma_\perp^{\mu_1}\gamma_\perp^{\mu_2}\ldots\gamma_\perp^{\mu_{2n-2}}\chi\;\bar{\chi}(1+\gamma_5)\gamma_{\perp\alpha}\gamma_{\perp\mu_{2n-2}}\gamma_{\perp\mu_{2n-3}}\ldots\gamma_{\perp\mu_1}h_v$$

# Calculation

- Reduction: Integration-by-parts relations, Laporta algorithm

  *[Tkachov'81; Chetyrkin,Tkachov'81] [Laporta'01; Anastasiou,Lazopoulos'04; Smirnov'08; Studerus,von Manteuffel'10,'12]*

  – Obtain a set of 29 master integrals

- Use differential equations in canonical form *[Henn'13]*
  – First example of canonical basis in case of 2 different internal masses
  – Found analytical solution in terms of iterated integrals ✓ *[Bell,TH'14]*

- Benefits of canonical basis
  – QCD amplitude much simpler, especially pre-factors of masters
  – Catalyses for convolution with LCDA

- UV renormalisation + IR subtraction (matching onto SCET)

$$\widetilde{T}_i^{(2)} = \widetilde{A}_{i1}^{(2)\mathrm{nf}} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(1)} + Z_{ij}^{(2)}\,\widetilde{A}_{j1}^{(0)} + Z_{\alpha}^{(1)}\,\widetilde{A}_{i1}^{(1)\mathrm{nf}} + (-i)\,\delta m^{(1)}\,\widetilde{A}_{i1}^{\prime(1)\mathrm{nf}}$$
$$+ Z_{ext}^{(1)}\left[\widetilde{A}_{i1}^{(1)\mathrm{nf}} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(0)}\right] - \widetilde{T}_i^{(1)}\left[C_{FF}^{(1)} + \widetilde{Y}_{11}^{(1)}\right] + \ldots$$

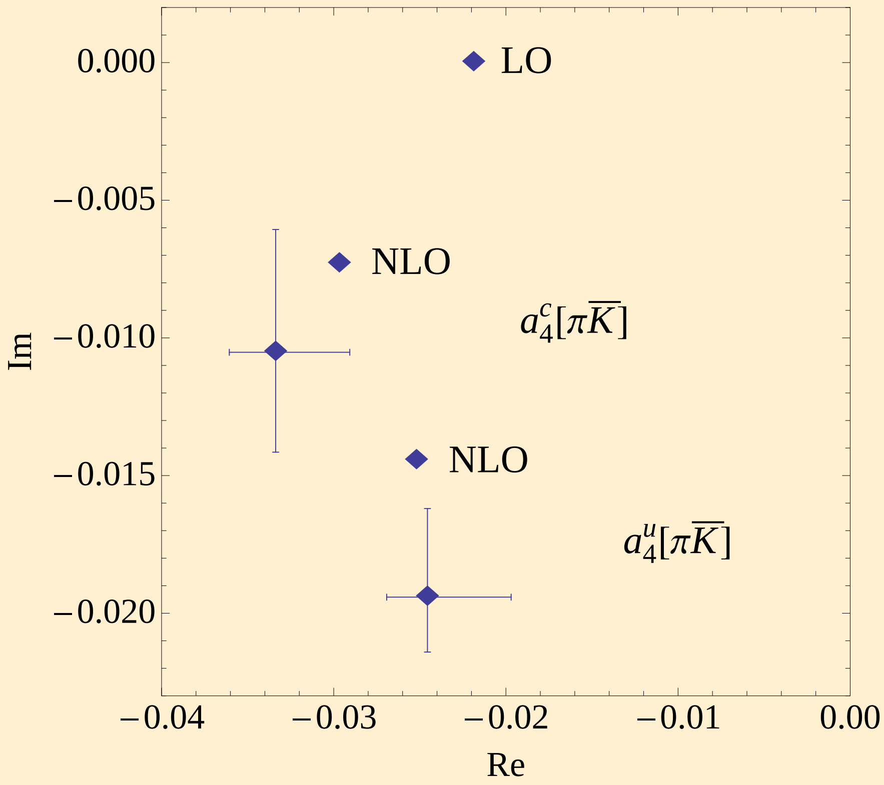
# Results: Penguin Amplitudes

- Only $Q_{1,2}$ contribution. Inputs from *[Beneke,Li,TH'09]*

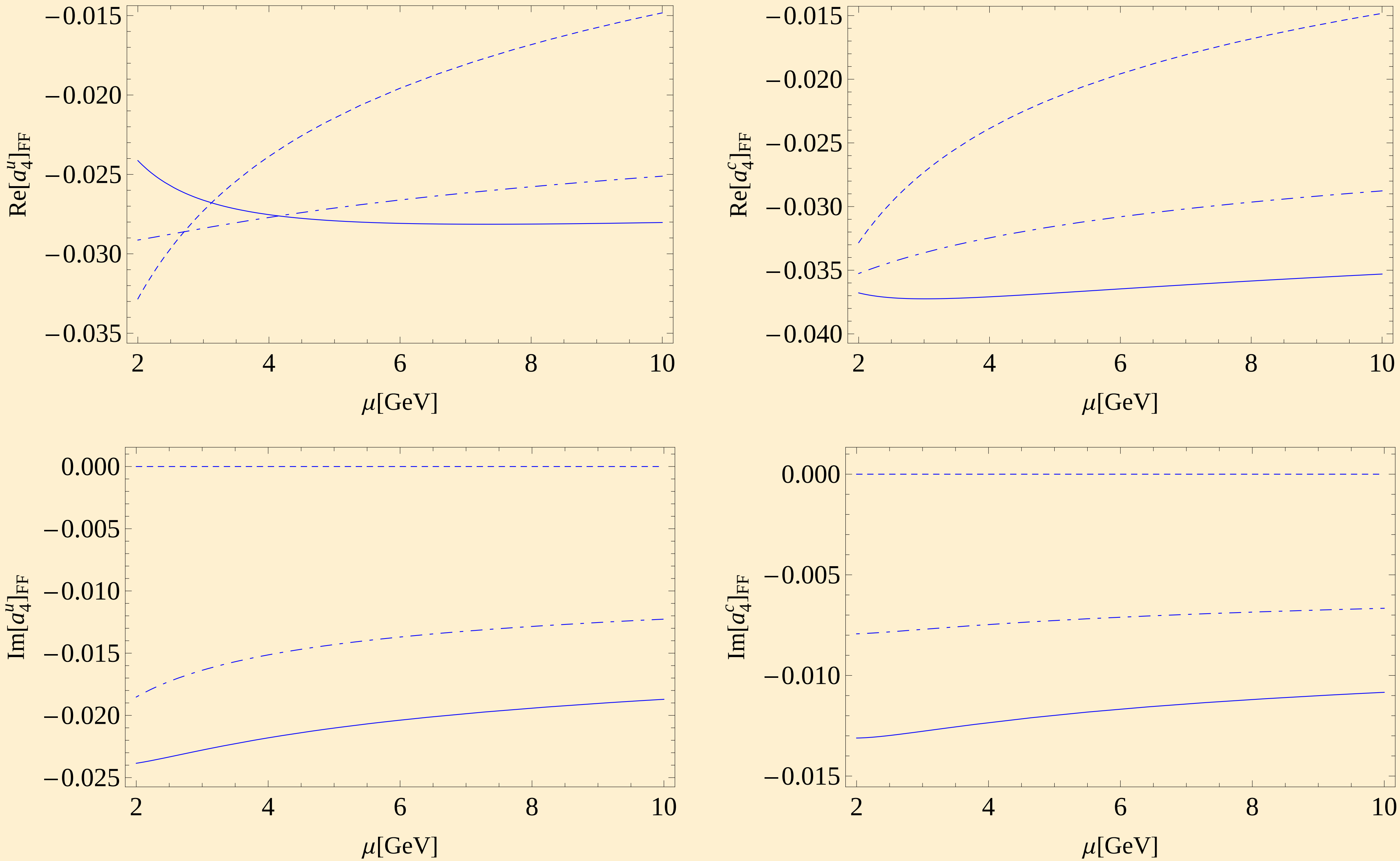
$$
\begin{aligned}
a_4^u(\pi\bar{K})/10^{-2} &= -2.87 - [0.09 + 0.09i]_{\mathrm{V}_1} + [0.49 - 1.32i]_{\mathrm{P}_1} - [0.32 + 0.71i]_{\mathrm{P}_2} \\
&\quad + \left[\frac{r_{\mathrm{sp}}}{0.434}\right] \left\{[0.13]_{\mathrm{LO}} + [0.14 + 0.12i]_{\mathrm{HV}} - [0.01 - 0.05i]_{\mathrm{HP}} + [0.07]_{\mathrm{tw3}}\right\} \\
&= (-2.46^{+0.49}_{-0.24}) + (-1.94^{+0.32}_{-0.20})i\,, \\
a_4^c(\pi\bar{K})/10^{-2} &= -2.87 - [0.09 + 0.09i]_{\mathrm{V}_1} + [0.05 - 0.62i]_{\mathrm{P}_1} - [0.77 + 0.50i]_{\mathrm{P}_2} \\
&\quad + \left[\frac{r_{\mathrm{sp}}}{0.434}\right] \left\{[0.13]_{\mathrm{LO}} + [0.14 + 0.12i]_{\mathrm{HV}} + [0.01 + 0.03i]_{\mathrm{HP}} + [0.07]_{\mathrm{tw3}}\right\} \\
&= (-3.34^{+0.43}_{-0.27}) + (-1.05^{+0.45}_{-0.36})i\,.
\end{aligned}
$$

- NNLO correction sizable, but no breakdown of perturbative expansion

# Results: Penguin Amplitudes



# Results: Scale dependence



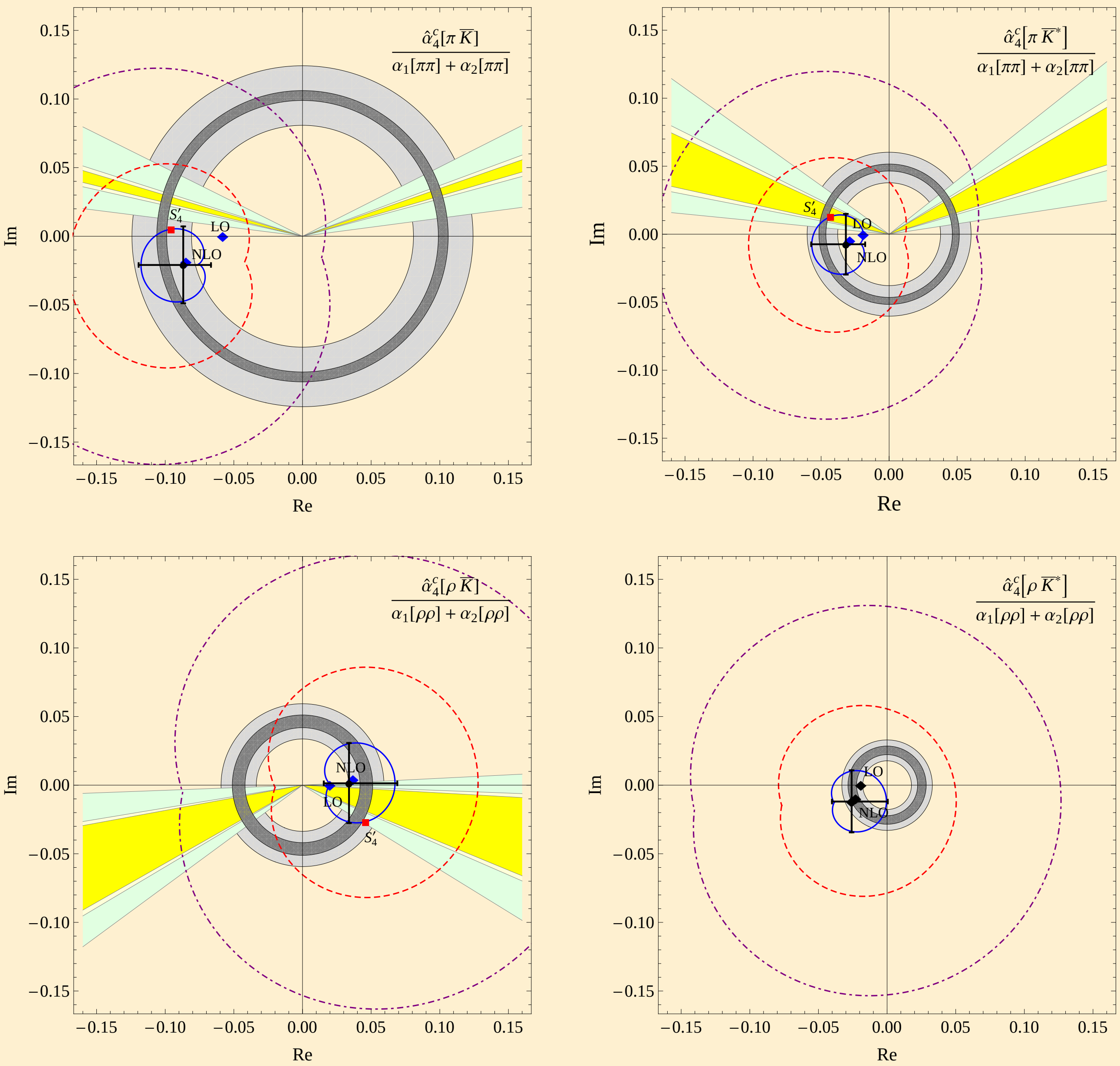

- Only form factor term, no spectator scattering

# Results: Amplitude ratios

| Ratio | NLO | NNLO |
|---|---|---|
| $\frac{P_{\pi\pi}}{T_{\pi\pi}}$ | $-0.121 - 0.021i$ | $-0.124^{+0.031}_{-0.060} + (-0.026^{+0.045}_{-0.046})i$ |
| $\frac{P_{\rho\rho}}{T_{\rho\rho}}$ | $-0.035 - 0.009i$ | $-0.041^{+0.020}_{-0.016} + (-0.014^{+0.019}_{-0.018})i$ |
| $\frac{P_{\pi\rho}}{T_{\pi\rho}}$ | $-0.038 - 0.005i$ | $-0.040^{+0.016}_{-0.030} + (-0.009^{+0.026}_{-0.026})i$ |
| $\frac{P_{\rho\pi}}{T_{\rho\pi}}$ | $0.040 + 0.002i$ | $0.036^{+0.042}_{-0.023} + (-0.001^{+0.033}_{-0.033})i$ |
| $\frac{C_{\pi\pi}}{T_{\pi\pi}}$ | $0.317 - 0.040i$ | $0.320^{+0.255}_{-0.142} + (-0.030^{+0.150}_{-0.091})i$ |
| $\frac{C_{\rho\rho}}{T_{\rho\rho}}$ | $0.165 - 0.064i$ | $0.176^{+0.187}_{-0.133} + (-0.054^{+0.142}_{-0.104})i$ |
| $\frac{C_{\pi\rho}}{T_{\pi\rho}}$ | $0.219 - 0.064i$ | $0.212^{+0.197}_{-0.112} + (-0.062^{+0.114}_{-0.079})i$ |
| $\frac{C_{\rho\pi}}{T_{\rho\pi}}$ | $0.092 - 0.080i$ | $0.112^{+0.189}_{-0.144} + (-0.065^{+0.152}_{-0.115})i$ |
| $\frac{T_{\rho\pi}}{T_{\pi\rho}}$ | $0.821 + 0.016i$ | $0.810^{+0.262}_{-0.200} + (\ 0.010^{+0.062}_{-0.062})i$ |
| $\frac{\alpha_4^c(\pi K)}{\alpha_1(\pi\pi) + \alpha_2(\pi\pi)}$ | $-0.085 - 0.019i$ | $-0.087^{+0.022}_{-0.036} + (-0.021^{+0.029}_{-0.029})i$ |
| $\frac{\alpha_4^c(\pi K^*)}{\alpha_1(\pi\pi) + \alpha_2(\pi\pi)}$ | $-0.029 - 0.005i$ | $-0.030^{+0.015}_{-0.026} + (-0.007^{+0.023}_{-0.023})i$ |
| $\frac{\alpha_4^c(\rho K)}{\alpha_1(\rho\rho) + \alpha_2(\rho\rho)}$ | $0.037 + 0.004i$ | $0.034^{+0.039}_{-0.021} + (\ 0.001^{+0.030}_{-0.030})i$ |
| $\frac{\alpha_4^c(\rho K^*)}{\alpha_1(\rho\rho) + \alpha_2(\rho\rho)}$ | $-0.023 - 0.010i$ | $-0.027^{+0.027}_{-0.016} + (-0.012^{+0.024}_{-0.023})i$ |

- Unpublished numbers.
  Only $Q_{1,2}$ contribution.
  Inputs from *[Beneke,Li,TH'09]*.

# Results: Amplitude ratios



# Results: Direct CP asymmetries I

- Direct CP asymmetries in percent. Errors are CKM and hadronic, respectively.

| $f$ | NLO | NNLO | NNLO + LD | Exp |
|---|---|---|---|---|
| $\pi^-\bar{K}^0$ | $0.71\,^{+0.13}_{-0.14}\,^{+0.21}_{-0.19}$ | $0.77\,^{+0.14}_{-0.15}\,^{+0.23}_{-0.22}$ | $0.10\,^{+0.02}_{-0.02}\,^{+1.24}_{-0.27}$ | $-1.7 \pm 1.6$ |
| $\pi^0 K^-$ | $9.42\,^{+1.77}_{-1.76}\,^{+1.87}_{-1.88}$ | $10.18\,^{+1.91}_{-1.90}\,^{+2.03}_{-2.62}$ | $-1.17\,^{+0.22}_{-0.22}\,^{+20.00}_{-6.62}$ | $4.0 \pm 2.1$ |
| $\pi^+ K^-$ | $7.25\,^{+1.36}_{-1.36}\,^{+2.13}_{-2.58}$ | $8.08\,^{+1.52}_{-1.51}\,^{+2.52}_{-2.65}$ | $-3.23\,^{+0.61}_{-0.61}\,^{+19.17}_{-3.36}$ | $-8.2 \pm 0.6$ |
| $\pi^0\bar{K}^0$ | $-4.27\,^{+0.83}_{-0.77}\,^{+1.48}_{-2.23}$ | $-4.33\,^{+0.84}_{-0.78}\,^{+3.29}_{-2.32}$ | $-1.41\,^{+0.27}_{-0.25}\,^{+5.54}_{-6.10}$ | $1 \pm 10$ |
| $\delta(\pi\bar{K})$ | $2.17\,^{+0.40}_{-0.40}\,^{+1.39}_{-0.74}$ | $2.10\,^{+0.39}_{-0.39}\,^{+1.40}_{-2.86}$ | $2.07\,^{+0.39}_{-0.39}\,^{+2.76}_{-4.55}$ | $12.2 \pm 2.2$ |
| $\Delta(\pi\bar{K})$ | $-1.15\,^{+0.21}_{-0.22}\,^{+0.55}_{-0.84}$ | $-0.88\,^{+0.16}_{-0.17}\,^{+1.31}_{-0.91}$ | $-0.48\,^{+0.09}_{-0.09}\,^{+1.09}_{-1.15}$ | $-14 \pm 11$ |

$$\delta(\pi\bar{K}) = A_{\rm CP}(\pi^0 K^-) - A_{\rm CP}(\pi^+ K^-)$$

$$\Delta(\pi\bar{K}) = A_{\rm CP}(\pi^+ K^-) + \frac{\Gamma_{\pi^-\bar{K}^0}}{\Gamma_{\pi^+K^-}} A_{\rm CP}(\pi^-\bar{K}^0) - \frac{2\Gamma_{\pi^0K^-}}{\Gamma_{\pi^+K^-}} A_{\rm CP}(\pi^0 K^-) - \frac{2\Gamma_{\pi^0\bar{K}^0}}{\Gamma_{\pi^+K^-}} A_{\rm CP}(\pi^0\bar{K}^0)$$

# Results: Direct CP asymmetries II

- Direct CP asymmetries in percent

| $f$ | NLO | NNLO | NNLO + LD | Exp |
|---|---|---|---|---|
| $\pi^- \bar{K}^{*0}$ | $1.36\,^{+0.25}_{-0.26}\,^{+0.60}_{-0.47}$ | $1.49\,^{+0.27}_{-0.29}\,^{+0.69}_{-0.56}$ | $0.27\,^{+0.05}_{-0.05}\,^{+3.18}_{-0.67}$ | $-3.8 \pm 4.2$ |
| $\pi^0 K^{*-}$ | $13.85\,^{+2.40}_{-2.70}\,^{+5.84}_{-5.86}$ | $18.16\,^{+3.11}_{-3.52}\,^{+7.79}_{-10.57}$ | $-15.81\,^{+3.01}_{-2.83}\,^{+69.35}_{-15.39}$ | $-6 \pm 24$ |
| $\pi^+ K^{*-}$ | $11.18\,^{+2.00}_{-2.15}\,^{+9.75}_{-10.62}$ | $19.70\,^{+3.37}_{-3.80}\,^{+10.54}_{-11.42}$ | $-23.07\,^{+4.35}_{-4.05}\,^{+86.20}_{-20.64}$ | $-23 \pm 6$ |
| $\pi^0 \bar{K}^{*0}$ | $-17.23\,^{+3.33}_{-3.00}\,^{+7.59}_{-12.57}$ | $-15.11\,^{+2.93}_{-2.65}\,^{+12.34}_{-10.64}$ | $2.16\,^{+0.39}_{-0.42}\,^{+17.53}_{-36.80}$ | $-15 \pm 13$ |
| $\delta(\pi \bar{K}^*)$ | $2.68\,^{+0.72}_{-0.67}\,^{+5.44}_{-4.30}$ | $-1.54\,^{+0.45}_{-0.58}\,^{+4.60}_{-9.19}$ | $7.26\,^{+1.21}_{-1.34}\,^{+12.78}_{-20.65}$ | $17 \pm 25$ |
| $\Delta(\pi \bar{K}^*)$ | $-7.18\,^{+1.38}_{-1.28}\,^{+3.38}_{-5.35}$ | $-3.45\,^{+0.67}_{-0.59}\,^{+9.48}_{-4.95}$ | $-1.02\,^{+0.19}_{-0.18}\,^{+4.32}_{-7.86}$ | $-5 \pm 45$ |

$$\hat{\alpha}_4^p(M_1 M_2) = a_4^p(M_1 M_2) \pm r_\chi^{M_2} a_6^p(M_1 M_2) + \beta_3^p(M_1 M_2)$$

# Results: Direct CP asymmetries III

- Direct CP asymmetries in percent

| $f$ | NLO | NNLO | NNLO + LD | Exp |
|---|---|---|---|---|
| $\rho^-\bar{K}^0$ | $0.38\,{}^{+0.07}_{-0.07}\,{}^{+0.16}_{-0.27}$ | $0.22\,{}^{+0.04}_{-0.04}\,{}^{+0.19}_{-0.17}$ | $0.30\,{}^{+0.06}_{-0.06}\,{}^{+2.28}_{-2.39}$ | $-12 \pm 17$ |
| $\rho^0 K^-$ | $-19.31\,{}^{+3.42}_{-3.61}\,{}^{+13.95}_{-8.96}$ | $-4.17\,{}^{+0.75}_{-0.80}\,{}^{+19.26}_{-19.52}$ | $43.73\,{}^{+7.07}_{-7.62}\,{}^{+44.00}_{-137.77}$ | $37 \pm 11$ |
| $\rho^+ K^-$ | $-5.13\,{}^{+0.95}_{-0.97}\,{}^{+6.38}_{-4.02}$ | $1.50\,{}^{+0.29}_{-0.27}\,{}^{+8.69}_{-10.36}$ | $25.93\,{}^{+4.43}_{-4.90}\,{}^{+25.40}_{-75.63}$ | $20 \pm 11$ |
| $\rho^0\bar{K}^0$ | $8.63\,{}^{+1.59}_{-1.65}\,{}^{+2.31}_{-1.69}$ | $8.99\,{}^{+1.66}_{-1.71}\,{}^{+3.60}_{-7.44}$ | $-0.42\,{}^{+0.08}_{-0.08}\,{}^{+19.49}_{-8.78}$ | $6 \pm 20$ |
| $\delta(\rho\bar{K})$ | $-14.17\,{}^{+2.80}_{-2.96}\,{}^{+7.98}_{-5.39}$ | $-5.67\,{}^{+0.96}_{-1.01}\,{}^{+10.86}_{-9.79}$ | $17.80\,{}^{+3.15}_{-3.01}\,{}^{+19.51}_{-62.44}$ | $17 \pm 16$ |
| $\Delta(\rho\bar{K})$ | $-8.75\,{}^{+1.62}_{-1.66}\,{}^{+4.78}_{-6.48}$ | $-10.84\,{}^{+1.98}_{-2.09}\,{}^{+11.67}_{-9.09}$ | $-2.43\,{}^{+0.46}_{-0.42}\,{}^{+4.60}_{-19.43}$ | $-37 \pm 37$ |

# Results: Branching ratios

- Unpublished numbers. Only $Q_{1,2}$ contribution. Inputs from [Beneke,Li,TH'09].
- Branching ratios in $10^{-6}$.

| | NNLO | NLO | Experiment |
|---|---|---|---|
| $B^- \to \pi^-\pi^0$ | $5.43^{+2.66}_{-2.14}{}^{+2.05}_{-1.73}{}^{+1.27}_{-0.57}{}^{+0.52}_{-0.50}$ | $5.33$ | $5.48^{+0.35}_{-0.34}$ |
| $\bar{B}^0_d \to \pi^+\pi^-$ | $7.47^{+3.15}_{-2.61}{}^{+3.36}_{-2.76}{}^{+0.30}_{-0.60}{}^{+1.18}_{-0.66}$ | $7.30$ | $5.10^{+0.19}_{-0.19}$ |
| $\bar{B}^0_d \to \pi^0\pi^0$ | $0.35^{+0.14}_{-0.11}{}^{+0.19}_{-0.11}{}^{+0.33}_{-0.09}{}^{+0.20}_{-0.10}$ | $0.33$ | $1.33^{+0.46}_{-0.46}$ |
| $B^- \to \pi^-\bar{K}^0$ | $16.03^{+0.79}_{-0.77}{}^{+9.66}_{-6.68}{}^{+0.87}_{-1.28}{}^{+13.51}_{-5.61}$ | $14.94$ | $23.79^{+0.75}_{-0.75}$ |
| $B^- \to \pi^0K^-$ | $9.57^{+0.79}_{-0.74}{}^{+5.00}_{-3.50}{}^{+0.18}_{-0.39}{}^{+7.15}_{-3.01}$ | $8.97$ | $12.94^{+0.52}_{-0.51}$ |
| $\bar{B}^0_d \to \pi^+K^-$ | $14.01^{+1.09}_{-1.03}{}^{+8.43}_{-5.76}{}^{+0.12}_{-0.26}{}^{+11.92}_{-4.92}$ | $12.88$ | $19.57^{+0.53}_{-0.52}$ |
| $\bar{B}^0_d \to \pi^0\bar{K}^0$ | $5.82^{+0.31}_{-0.31}{}^{+4.05}_{-2.72}{}^{+0.07}_{-0.16}{}^{+5.58}_{-2.26}$ | $5.31$ | $9.93^{+0.49}_{-0.49}$ |

# Conclusion and Outlook

- $Q_{1,2}$-contribution to penguin amplitudes $\alpha_4^u$ and $\alpha_4^c$ at NNLO ready
- NNLO shift in amplitudes is rather sizable
- Shift in amplitude ratios, CP asymmetries, BRs is moderate
- Future plans
  - Include also penguin operators
  - Phenomenology based on NNLO results
  - Connect QCDF with flavour symmetries
  - Power suppressed amplitude $a_6$ at NNLO
  - QED corrections