NioBe : **N**on-leptoni**c** Three-B**o**dy B d**e**cays

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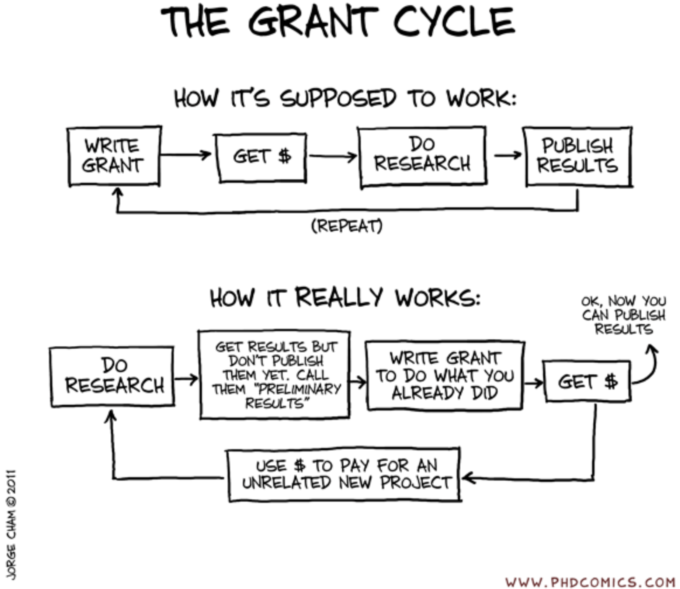
6th QFET WORKSHOP
Siegen – January 18, 2016

[Susanne Kränkl, Thomas Mannel, JV, **Nucl.Phys. B899 (2015) 247-264**]

+ W.I.P with Thomas, Tobias, Alex and Shan

Theoretishe Physik 1

FOR 1873


THE GRANT CYCLE
HOW IT'S SUPPOSED TO WORK:
WRITE GRANT
GET $
DO RESEARCH
PUBLISH RESULTS
(REPEAT)
HOW IT REALLY WORKS:
DO RESEARCH
GET RESULTS BUT DON'T PUBLISH THEM YET. CALL THEM "PRELIMINARY RESULTS"
WRITE GRANT TO DO WHAT YOU ALREADY DID
GET $
OK, NOW YOU CAN PUBLISH RESULTS
USE $ TO PAY FOR AN UNRELATED NEW PROJECT
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Outline

1. $B \to D(\pi\pi)$ at small $m_{\pi\pi}$
2. $B \to \pi(\pi\pi)$ at small $m_{\pi\pi}$
3. $B \to \pi\pi\pi$ at large $m_{\pi\pi}$
4. $B \to \pi X$ for a soft pion
5. $B \to \pi\pi$ form factors at small $m_{\pi\pi}$

B-decay Amplitudes

- Effective Hamiltonian at the hadronic scale $\mu \sim m_B$

$$\mathcal{H}_{\text{eff}} = -\mathcal{L}_{QED+QCD} + \sum_i C_i\, \mathcal{O}_i$$

- C_i – Wilson coefficients (UV physics) $\rightarrow$ perturbation theory

- $\mathcal{O}_i$ – Effective operators (IR physics) [e.g. $\mathcal{O} = (\bar{b}\gamma^\mu u)(\bar{u}\gamma_\mu d)$]

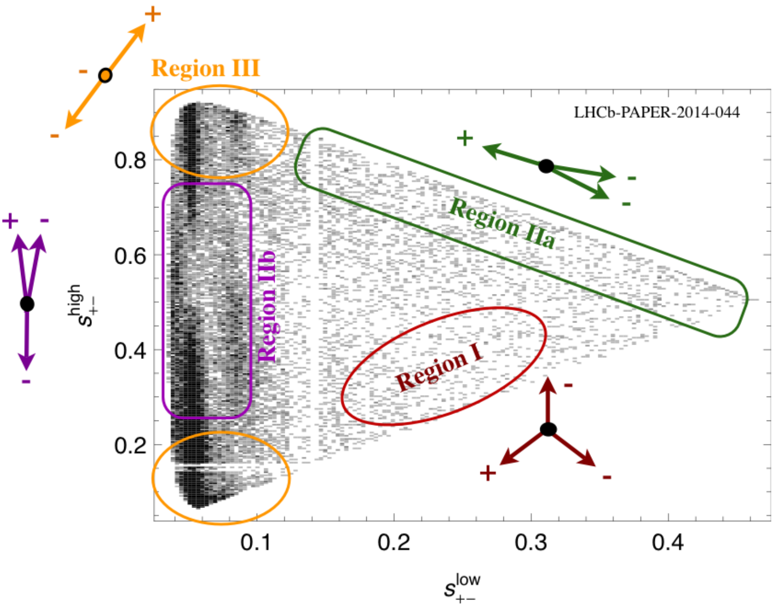
- Amplitudes:

$$\mathcal{A}(B \to f) = \sum_i C_i\, \langle f|\mathcal{O}_i|B\rangle$$

The problem is to compute the **operator matrix elements**:

$\longrightarrow$ non-perturbative, process dependent (non-universal)

Regions



Region III
LHCb-PAPER-2014-044
Region IIa
Region IIb
Region I
0.8
0.6
0.4
0.2
0.1
0.2
0.3
0.4
s_{+-}^{high}
s_{+-}^{low}

$B \to D\pi\pi$

- Effective hamiltonian:

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}}\, V_{ud}^* V_{cb}\, (C_1 Q_1 + C_2 Q_2) + h.c.$$

- Operators:

$$Q_1 = (\bar{d}\gamma_\mu P_L T^a u)\,(\bar{c}\gamma^\mu P_L T^a b)\ , \quad Q_2 = (\bar{d}\gamma_\mu P_L u)\,(\bar{c}\gamma^\mu P_L b)\ .$$

- We need to calculate the matrix elements

$$\langle Q_i \rangle \equiv \langle D^+\pi^-\pi^0 | Q_i | \bar{B}^0 \rangle\ .$$

- We consider the region where $m_{\pi\pi}^2 \ll m_B^2$ (e.g. the "ρ" region)

$B \to D\pi\pi$

Due to the kinematics of the process we can match the operators Q_i onto SCET operators $\widetilde{\mathcal{O}}_k$, for which the matrix elements factorize into the matrix elements of two SCET currents. These SCET currents can in turn be matched to QCD currents, so that one can define a set of operators $\mathcal{O}_k$ in QCD such that:

$$Q_i = \sum_k \int_0^1 dt\, C_{ik}(t)\mathcal{O}_k(t)$$

$$\mathcal{O}_k(t) = [\bar{d}(tn_-)[tn_-, 0]\, \not{\bar{n}}\Gamma_k\, u(0)] \otimes [\bar{c}\, \not{n}_+\, \Gamma'_k\, b]$$

with Γ_k, Γ'_k some dirac structures, and the notation $\mathcal{J}_1 \otimes \mathcal{J}_2$ means that the matrix elements factorize.
Then,

$$\langle \mathcal{O}_k(t) \rangle = \langle \pi^- \pi^0 | \bar{d}(tn_-)[tn_-, 0]\, \not{n}_-\Gamma_k\, u(0)|0\rangle \langle D^+ | \bar{c}\, \not{n}_+\, \Gamma'_k\, b|\bar{B}^0\rangle$$

$B \to D\pi\pi$

- Form factor:

$$\langle D^+|\bar{c}\,\not{n}_+\,\Gamma'_k\,b|\bar{B}^0\rangle = F^{B\to D}(s)$$

- GDA:

$$S^{\bar{d}u}_{\alpha\beta}(z,k_1,k_2) \equiv \frac{k^+_{12}}{4\pi}\int dx^-\,e^{-iz(k^+_{12}x^-)/2}\langle\pi^-\pi^0|\bar{d}_\beta(x)[x,0]u_\alpha(0)|0\rangle_{x^+=x_\perp=0}$$

$$S^{\bar{d}u}_{\alpha\beta} = \frac{1}{4}\Phi^{\bar{d}u}_{\parallel}(z,\zeta,k^2_{12})\,\not{k}_{12} + \Phi^{\bar{d}u}_{\perp}(z,\zeta,k^2_{12})\,\sigma_{\mu\nu}k^\mu_1 k^\nu_2$$

$$\langle\pi^-\pi^0|\bar{d}(tn_-)[tn_-,0]\,\not{n}_-\Gamma_k\,u(0)|0\rangle = \frac{k^+_{12}}{2}\int dz\,e^{iztk^+_{12}}\,\Phi^{\bar{d}u}_{\parallel}(z,\zeta,s)$$

$B \to D\pi\pi$

- Finally, the matrix elements are given by:

$$\langle Q_i \rangle = \frac{1}{2} k_{12}^{+}\, F^{B\to D}(s) \int dz\, T_i(z, k_{12}^{+})\, \Phi_{\parallel}^{\bar{d}u}(z, \zeta, s)$$

- with the usual hard-scattering kernels

$$T_i(z, k_{12}^{+}) = \sum_k \int_0^1 dt\, C_{ik}(t)\, e^{izk_{12}^{+} t}$$

- T_i known at NNLO from 2-body B-decay studies.

GDAs from data

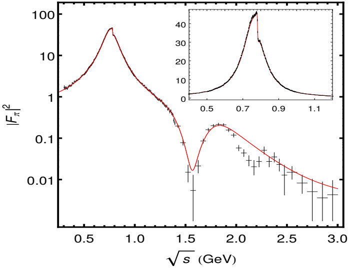
- Definition: $[s = (k_1 + k_2)^2,\ k_1 = \zeta k_{12},\ k_2 = \bar{\zeta} k_{12}]$

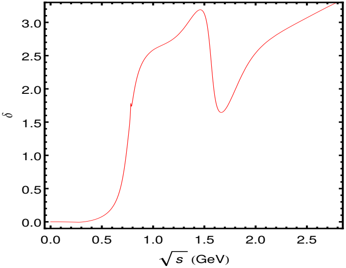
$$\phi^q_{\pi\pi}(z,\zeta,s) = \int \frac{dx^-}{2\pi} e^{iz(k^+_{12}x^-)} \langle \pi^+(k_1)\pi^-(k_2)|\bar{q}(x^- n_-)\not{n}_+ q(0)|0\rangle$$

- Normalization (local correlator):

$$\int dz\, \phi_{\pi\pi}(z,\zeta,s) = (2\zeta - 1)F_\pi(s) \quad \text{(pion time-like FF)}$$

- $F_\pi(s)$: Data (BaBar) + Theory (χPT, $R\chi PT$, Asymptotics...)





$B \to D\pi\pi$

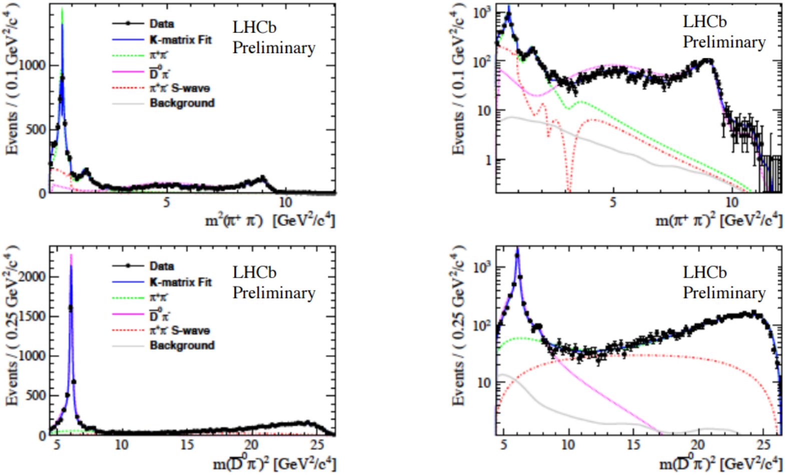
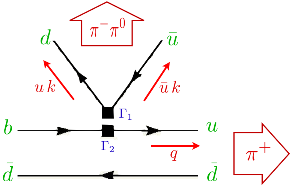
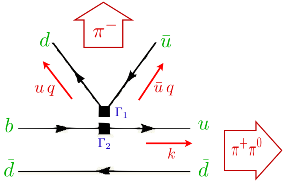


Figure 7: Projections of the data and K-matrix fit onto (a) $m^2(\pi^+\pi^-)$ and (c) $m^2(\overline{D}^0\pi^-)$ with the same projections shown in (b) and (d) with a logarithmic y-axis scale. Components are described in the legend. The lines denoted $\overline{D}^0\pi^-$ and $\pi^+\pi^-$ include the coherent sums of all $\overline{D}^0\pi^-$ resonances, $\pi^+\pi^-$ resonances, and $\pi^+\pi^-$ S-wave resonances.

$B \to \pi\pi\pi$

- Two contributions: 2π GDA *and* $B \to \pi\pi$ FF:





$$
\begin{aligned}
\langle \pi^-_{\bar{n}} \pi^+_{\bar{n}} \pi^-_n | \mathcal{O} | B \rangle &= \langle \pi^-_n | \bar{h}_v \Gamma \xi_n | B \rangle \times \int dz\, T_1(z) \langle \pi^-_{\bar{n}} \pi^+_{\bar{n}} | \bar{\chi}_{\bar{n}}(z\bar{n}) \Gamma' \chi_{\bar{n}}(0) | 0 \rangle \\
&+ \langle \pi^-_{\bar{n}} \pi^+_{\bar{n}} | \bar{h}_v \Gamma \xi_{\bar{n}} | B \rangle \times \int dz\, T_2(z) \langle \pi^-_n | \bar{\chi}_{\bar{n}}(zn) \Gamma' \chi_n(0) | 0 \rangle \\
&\sim F^{B\to\pi}\, T_1 \otimes \phi_{\pi\pi} + F^{B\to\pi\pi}\, T_2 \otimes \phi_\pi
\end{aligned}
$$

- New Non-perturbative input:
  - Generalized Distribution Amplitudes (GDAs) [Diehl, Polyakov, Gousset, Pire...]
  - Generalized Form Factors (GFFs) [Faller, Feldmann, Khodjamirian, Mannel, van Dyk...]

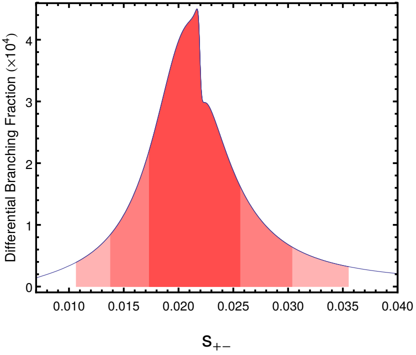
$B \to (\pi\pi)_\rho \, \pi$

⋆ Leading order amplitude:

$$\mathcal{A}|_{s_{+-} \ll 1} = \frac{G_F}{\sqrt{2}} \Big[4 m_B^2 f_0(s_{+-})(2\zeta - 1) F_\pi(s_{+-})(a_2 + a_4) + f_\pi m_\pi (a_1 - a_4) F_t(\zeta, s_{+-}) \Big]$$

⋆ Integrating around the ρ:

$$BR(B^- \to \rho\,\pi^-) \simeq \int_0^1 ds_{++} \int_{s_\rho^-}^{s_\rho^+} ds_{+-} \; \frac{\tau_B \, m_B |\mathcal{A}|^2}{32(2\pi)^3}$$



with $s_\rho^\pm = (m_\rho \pm n\Gamma_\rho)^2 / m_B^2$

$BR(B^+ \to \rho\pi^+) \simeq 9.4 \cdot 10^{-6}$ $(n = 0.5)$

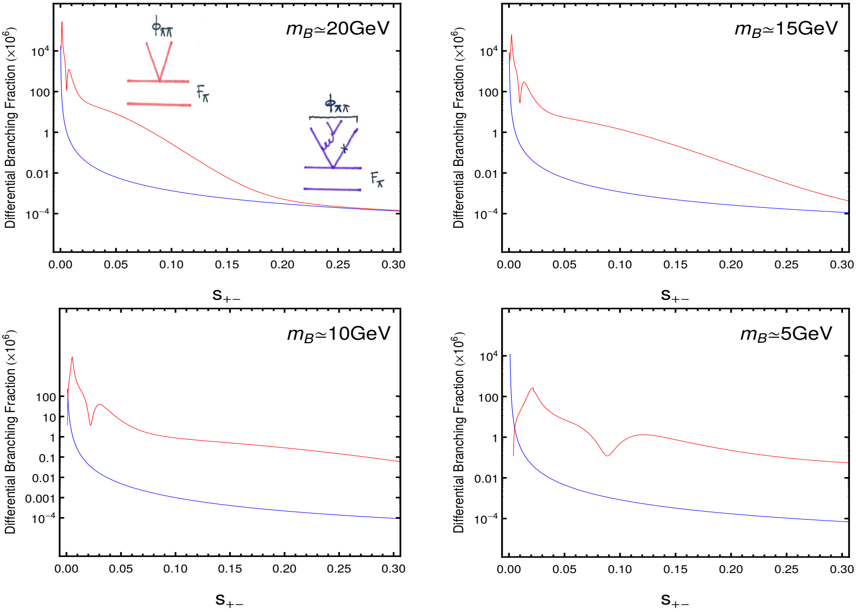
$BR(B^+ \to \rho\pi^+) \simeq 12.8 \cdot 10^{-6}$ $(n = 1)$

$BR(B^+ \to \rho\pi^+) \simeq 14.1 \cdot 10^{-6}$ $(n = 1.5)$

$BR(B^+ \to \rho\pi^+)_{\rm EXP} = (8.3 \pm 1.2) \cdot 10^{-6}$

$BR(B^+ \to \rho\pi^+)_{\rm QCDF} = (11.9^{+7.8}_{-6.1}) \cdot 10^{-6}$

$B \to \pi\pi\pi$ at large $m_{\pi\pi}$ ($\phi_{\pi\pi}$ term)



Soft pions

- Soft pion theorem:

$$\lim_{q^\mu \to 0} \langle \pi^+ \beta | \mathcal{O} | \alpha \rangle = -\frac{i}{\sqrt{2} f_\pi} \langle \beta | [Q_5^+, \mathcal{O}] | \alpha \rangle + \lim_{q^\mu \to 0} i q^\mu R_\mu^k$$

with

$$Q_5^+ = \int d^3x \; \bar{d}(x) \gamma_\mu \gamma_5 u(x) \; .$$

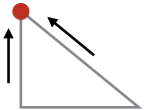
- $B \to \pi\pi\pi$

$$\lim_{q^\mu \to 0} \langle \pi^+(q) [\pi^+ \pi^-] | \mathcal{O} | B^+ \rangle = \frac{i}{\sqrt{2} f_\pi} \langle \pi^+ \pi^- | (\bar{b} \Gamma u)(\bar{d} \gamma_5 \Gamma' d) | B^+ \rangle$$

can be computed in QCDF analogous to $B \to \pi\pi$.

- Should recover the same result from soft-pion limit of edge:

$$\Phi_{\parallel}(z, \zeta = 1, s = 0) = -\Phi_{\parallel}(z, \zeta = 0, s = 0) = \varphi_\pi(z)$$



$B \to \pi\pi$ form factors at low s

Idea: Use LCSR with B-meson DA's (complementary to Hambrock, Khodjamirian)

- **Correlation function**

$$\begin{aligned} F_{\mu\nu}(k,q) &= i\int d^4x e^{ik\cdot x}\langle 0|T\{\bar{d}(x)\gamma_\mu u(x), \bar{u}(0)\gamma_{\nu L} b(0)\}|\bar{B}(k+q)\rangle \\ &= i\varepsilon_{\mu\nu\rho\sigma} q^\rho k^\sigma F^{(\varepsilon)}(k^2,q^2) + \widetilde{F}_{\mu\nu}(k,q) \end{aligned}$$

- **Unitarity relation**

$$2\,\mathrm{Im}F_{\mu\nu} = \int d\tau_{2\pi}\langle 0|\bar{d}\gamma_\mu u|\pi^+\pi^0\rangle\langle\pi^+\pi^0|\bar{u}\gamma_{\nu L} b|\bar{B}(k+q)\rangle + \text{higher states}$$

$$\Rightarrow \mathrm{Im}F^{(\varepsilon)}(k^2,q^2) = \frac{\sqrt{k^2}\beta_\pi^3 F_\pi(k^2)}{24\pi\sqrt{\lambda}} F_\perp^{(1)}(k^2,q^2) + \text{higher states}$$

- **Dispersion relation**

$$F^{(\varepsilon)}(k^2,q^2) = \frac{1}{\pi}\int_{4m_\pi^2}^\infty ds\,\frac{\mathrm{Im}F^{(\varepsilon)}(s,q^2)}{s-k^2} = \int_{4m_\pi^2}^\infty ds\,\frac{\sqrt{s}\beta_\pi^3(s)F_\pi(s)F_\perp^{(1)}(s,q^2)}{24\pi^2\sqrt{\lambda(s)}\,(s-k^2)} + \cdots$$

$B \to \pi\pi$ form factors at low s

- **OPE**

$$F^{(\varepsilon)}_{\rm OPE}(k^2,q^2) = \frac{-if_B m_B\, q_+ k_-}{4[q^2k^2-(q\cdot k)^2]}\int_0^\infty d\omega\,\frac{\phi_+^B(\omega)}{\omega-2k_+} \simeq \frac{if_B}{2}\int_0^\infty d\omega\,\frac{\phi_+^B(\omega)}{m_B\omega-k^2}$$

- **Borel transformation and duality**

$$\frac{if_B}{2m_B}\int_0^\infty ds\, e^{-s/M^2}\,\phi_+^B(s/m_B) = \int_{4m_\pi^2}^\infty ds\,\frac{\sqrt{s}\,\beta_\pi^3(s)F_\pi(s)F_\perp^{(1)}(s,q^2)}{24\pi^2\sqrt{\lambda(s)}}\,e^{-s/M^2}$$

etc...

- Understand the ρ contribution
- see what constraints are derived from this SR, etc...

Conclusions