$B \to \pi\pi$ form factors at small dipion mass from QCD Light-Cone Sum Rules

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(Ch. Hambrock, AK, 1511.02509 [hep-ph] , subm. to Nucl.Phys. B)

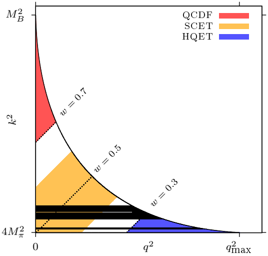
6th QFET Workshop, January 18, 2016, Siegen

Motivation

- why are the $B \to 2\pi\ell\nu_\ell$ form factors important:
  - alternative to $B \to \pi\ell\nu_\ell$ for exclusive $|V_{ub}|$ determination
  - rich set of observables,

    [S. Faller, T. Feldmann, A. Khodjamirian, T. Mannel and D. van Dyk, (2013)]
  - future accurate data expected from Belle-2:

    $B \to \rho$ dominance?, $B \to f_0$ background?, higher partial waves of 2π states?
- Dalitz plot, k^2 - dipion mass vs q^2- momentum transfer



- the region of small dipion mass, $k^2 \lesssim 1$ GeV^2, $0 \leq q^2 \leq$ 12-14 GeV^2.
- the method: similar to the LCSRs for $B \to \pi$ form factors, new nonperturbative input: dipion distribution amplitudes (DAs)
- first exploratory study: only $\bar{B}^0 \to \pi^+\pi^0\ell^-\nu_\ell$, isospin 1, $L = 1, 3, , ..$

The method of LCSRs

- The correlation function: $k = k_1 + k_2$, $\bar{k} = k_1 - k_2$

$$\Pi_\mu(q,k_1,k_2) =$$
$$= i \int d^4x\, e^{iqx} \langle \pi^+(k_1)\pi^0(k_2)| T\{\bar{u}(x)\gamma_\mu(1-\gamma_5) b(x), i m_b \bar{b}(0)\gamma_5 d(0)\}|0\rangle \,.$$
$$= i\epsilon_{\mu\alpha\beta\rho} q^\alpha k_1^\beta k_2^\rho\, \Pi^{(V)} + q_\mu \Pi^{(A,q)} + k_\mu \Pi^{(A,k)} + \bar{k}_\mu \Pi^{(A,\bar{k})} \,,$$

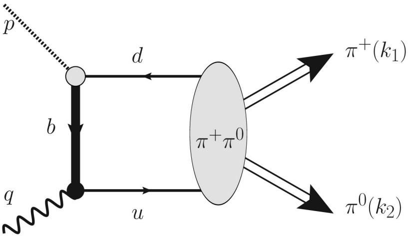
- the invariant amplitudes $\Pi^{(V),(A,q),..)}(p^2, q^2, k^2, q\cdot\bar{k})$, $p = (k+q)$

- OPE valid at $q^2 \ll m_b^2$ (b-quark virtual)
  $k^2 \ll m_b^2$ (2-pion system produced near the LC)

- LO diagram:
  $\langle b(x)\bar{b}(0)\rangle \to S_b(x,0)$

- vacuum $\to$ on-shell dipion hadronic matrix elements of nonlocal $\bar{u}(x)d(0)$ operators

- with ρ-meson "embedded"



Dipion light-cone DAs

- introduced and developed for $\gamma^*\gamma \to 2\pi$ processes
  [M. Diehl, T. Gousset, B. Pire and O. Teryaev, (1998)
  D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes and J. Horejsi, (1994)
  M. V. Polyakov, (1999)]

- twist-2 DAs:

$$\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}(x)\gamma_\mu[x,0]d(0)|0\rangle = -\sqrt{2}k_\mu \int_0^1 du\, e^{iu(k\cdot x)}\Phi_\parallel^{I=1}(u,\zeta,k^2)\,,$$

$$\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}(x)\sigma_{\mu\nu}[x,0]d(0)|0\rangle = 2\sqrt{2}i\frac{k_{1\mu}k_{2\nu}-k_{2\mu}k_{1\nu}}{2\zeta-1}\int_0^1 du\, e^{iu(k\cdot x)}\Phi_\perp^{I=1}(u,\zeta,k^2)$$

- the "angular" variable: $\zeta = k_1^+/k^+,\;\; 1-\zeta = k_2^+/k^+\,,\;\; \zeta(1-\zeta) \geq \frac{m_\pi^2}{k^2}\,.$

$q\cdot\bar{k} = \frac{1}{2}(2\zeta-1)\lambda^{1/2}(p^2,q^2,k^2)\,,$ in dipion c.m. $(2\zeta-1) = (1-4m_\pi^2/k^2)^{1/2}cos\theta_\pi\,,$

- normalization conditions $\to$ pion timelike form factors ,

$$\int_0^1 du \left\{ \begin{array}{c} \Phi_\parallel^{I=1}(u,\zeta,k^2) \\ \Phi_\perp^{I=1}(u,\zeta,k^2) \end{array} \right. = (2\zeta-1)\left\{ \begin{array}{ll} F_\pi^{em}(k^2) & \text{pion e.m. form factor} \\ F_\pi^t(k^2) & \text{pion “tensor” form factor} \end{array} \right. ,$$

- $\Phi_{\perp,\parallel}(u,\zeta,k^2)$ at $k^2 > 4m_\pi^2$ contain Im part
- $F_\pi^{em}(0) = 1$, • "tensor" charge of the pion $F_\pi^t(0) = 1/f_{2\pi}^\perp$

Result for the correlation function in twist-2 approx.

- at LO, twist-2 accuracy:

$$\Pi_\mu(q,k_1,k_2) = i\sqrt{2}m_b \int_0^1 \frac{du}{(q+uk)^2-m_b^2}\Big\{\Big[(q\cdot\bar{k})k_\mu - \Big((q\cdot k)+uk^2\Big)\bar{k}_\mu$$

$$+i\epsilon_{\mu\alpha\beta\rho}q^\alpha k_1^\beta k_2^\rho\Big]\frac{\Phi_\perp(u,\zeta,k^2)}{2\zeta-1} - m_b k_\mu \Phi_\parallel(u,\zeta,k^2)\Big\}.$$

- read off invariant amplitudes: $\Pi^{(V)}$, $\Pi^{(A,k)}$, $\Pi^{(A,\bar{k})}$, $\Pi^{(A,q)} = 0$
- transform to a form of dispersion integral in the variable p^2: $s(u) = \frac{m_b^2 - q^2\bar{u} + k^2 u\bar{u}}{u}$

$$\Pi^{(r)}(p^2,q^2,k^2,\zeta) = \sum_{i=\parallel,\perp} f_i^{(r)}(p^2,q^2,k^2,\xi)\int_{m_b^2}^{\infty}\frac{ds}{s-p^2}\left(\frac{du}{ds}\right)\Phi_i(u(s),\zeta,k^2)\,.$$

- a problem: nonanalyticity of the Källen function:

$$q\cdot\bar{k} = (1/2)(2\zeta-1)\lambda^{1/2}(p^2,q^2,k^2) \quad \rightarrow \text{kinematical singularity}$$

$$\lambda^{1/2}(p^2,q^2,k^2) = (p^2-(\sqrt{q^2}-\sqrt{k^2})^2)^{1/2}(p^2-(\sqrt{q^2}+\sqrt{k^2})^2)^{1/2}.$$

☹ no consistent sum rule for $\Pi^{(A,k)}$

Hadronic dispersion relation

- the ground B-meson state contribution:

$$\Pi_\mu(q,k_1,k_2)=\frac{\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma_\mu(1-\gamma_5)b|\bar{B}^0(p)\rangle f_B m_B^2}{m_B^2-p^2}+\dots ,$$

- expansion of $B\to\pi\pi$ matrix element in form factors:

$$\begin{aligned} i\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma^\mu(1-\gamma_5)b|\bar{B}^0(p)\rangle &= -F_\perp(q^2,k^2,\zeta)\,\frac{4}{\sqrt{k^2\lambda_B}}\,i\epsilon^{\mu\alpha\beta\gamma}\,q_\alpha\,k_{1\beta}\,k_{2\gamma} \\ &+F_t(q^2,k^2,\zeta)\,\frac{q^\mu}{\sqrt{q^2}}+F_0(q^2,k^2,\zeta)\,\frac{2\sqrt{q^2}}{\sqrt{\lambda_B}}\left(k^\mu-\frac{k\cdot q}{q^2}q^\mu\right) \\ &+F_\parallel(q^2,k^2,\zeta)\,\frac{1}{\sqrt{k^2}}\left(\bar{k}^\mu-\frac{4(q\cdot k)(q\cdot\bar{k})}{\lambda_B}\,k^\mu+\frac{4k^2(q\cdot\bar{k})}{\lambda_B}\,q^\mu\right), \end{aligned}$$

- quark-hadron duality in the B-meson channel, Borel transform

LCSRs for the form factors

- in both sum rules only the chiral-odd twist-2 DA contributes:

$$\frac{F_\perp(q^2,k^2,\zeta)}{\sqrt{k^2}\sqrt{\lambda_B}} = \frac{m_b}{\sqrt{2} f_B m_B^2(1-2\zeta)} \int\limits_{u_0}^{1} \frac{du}{u} \Phi_\perp(u,\zeta,k^2)\, e^{\frac{m_B^2}{M^2} - \frac{m_b^2 - q^2\bar{u} + k^2 u\bar{u}}{uM^2}} \,.$$

$$\frac{F_\parallel(q^2,k^2,\zeta)}{\sqrt{k^2}} = \frac{m_b}{\sqrt{2} f_B m_B^2(1-2\zeta)} \int\limits_{u_0}^{1} \frac{du}{u^2} \left(m_b^2 - q^2 + k^2 u^2\right) \Phi_\perp(u,\zeta,k^2)\, e^{\frac{m_B^2}{M^2} - \frac{m_b^2 - q^2\bar{u} + k^2 u\bar{u}}{uM^2}}$$

- an additional relation between the axial-current form factors:

$$F_t(q^2,k^2,\zeta) = \frac{1}{\sqrt{\lambda_B}} \Big[(m_B^2 - q^2 - k^2) F_0(q^2,k^2,\zeta) - 2\sqrt{k^2}\sqrt{q^2}(2\zeta - 1) F_\parallel(q^2,k^2,\zeta) \Big] \,.$$

What do we know about LCDAs

[M. V. Polyakov, Nucl. Phys. B 555 (1999) 231.]

- double expansion in Legendre and Gegenbauer polynomials:

$$\Phi_\perp(u,\zeta,k^2) = \frac{6u(1-u)}{f_{2\pi}^\perp} \sum_{n=0,2,..}^{\infty} \sum_{\ell=1,3,..}^{n+1} B_{n\ell}^\perp(k^2) C_n^{3/2}(2u-1)\beta_\pi P_\ell^{(0)}\left(\frac{2\zeta-1}{\beta_\pi}\right),$$

- $B_{n\ell}^\perp(k^2)$ are analogs of Gegenbauer moments,

  complex functions at $k^2 > 4m_\pi^2$

- instanton vacuum model for the coefficients,
  $n = 0, 2, 4$, valid at small $k^2 \sim 4m_\pi^2$ *[M. V. Polyakov and C. Weiss, (1999)]*

$$B_{01}^\perp(k^2) = 1 + \frac{k^2}{12M_0^2}, B_{21}^\perp(k^2) = \frac{7}{36}\left(1 - \frac{k^2}{30M_0^2}\right), \quad B_{23}^\perp(k^2) = \frac{7}{36}\left(1 + \frac{k^2}{30M_0^2}\right),$$

$$B_{41}^\perp(k^2) = \frac{11}{225}\left(1 - \frac{5k^2}{168M_0^2}\right), B_{43}^\perp(k^2) = \frac{77}{675}\left(1 - \frac{k^2}{630M_0^2}\right), B_{45}^\perp(k^2) = \frac{11}{135}\left(1 + \frac{k^2}{56M_0^2}\right).$$

$f_{2\pi}^\perp = 4\pi^2 f_\pi^2/3M_0 \simeq 650$ MeV, where $f_\pi = 132$ MeV is the pion decay constant.

- Omnes representation for the k^2-dependence (to be updated)
- we confined ourselves by $k^2 \sim k_{min}^2 \simeq 4m_\pi^2$ for an exploratory numerical analysis

Sum rules for partial waves

- The form factors expanded in partial waves:

$$F_{\perp,\parallel}(q^2,k^2,\zeta) = \sum_{\ell=1}^{\infty}\sqrt{2\ell+1}F^{(\ell)}_{\perp,\parallel}(q^2,k^2)\frac{P_\ell^{(1)}(\cos\theta_\pi)}{\sin\theta_\pi}\,,$$

$\zeta \sim \cos\theta$, $P_l^{(m)}$ -the (associated) Legendre polynomials

- sum rules for separate partial waves

$$F^{(\ell)}_{\perp}(q^2,k^2) = \frac{\sqrt{k^2}}{\sqrt{2}f^{\perp}_{2\pi}}\frac{\sqrt{\lambda_B}m_b}{m_B^2 f_B}e^{m_B^2/M^2}\sum_{n=0,2,..}\sum_{\ell'=1,3,..}^{n+1} I_{\ell\ell'}\,B^{\perp}_{n\ell'}(k^2)J^{\perp}_n(q^2,k^2,M^2,s_0^B)\,,$$

$$F^{(\ell)}_{\parallel}(q^2,k^2) = \frac{\sqrt{k^2}}{\sqrt{2}f^{\perp}_{2\pi}}\frac{m_b^3}{m_B^2 f_B}e^{m_B^2/M^2}\sum_{n=0,2,4,..}\sum_{\ell'=1,3,..}^{n+1} I_{\ell\ell'}\,B^{\perp}_{n\ell'}(k^2)J^{\parallel}_n(q^2,k^2,M^2,s_0^B)\,,$$

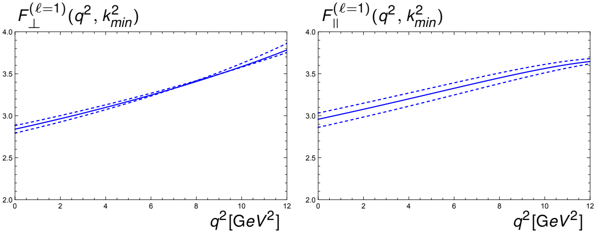
- $I_{\ell\ell'}$ - integrals over Legendre polynomials,

  e.g., $I_{1,1}=1/\sqrt{3}$, $I_{1,3}=-1/\sqrt{3}$, $I_{1,5}=4/(5\sqrt{3})$,
- $J_n^{\perp,\parallel}$ - the Borel-weighted integrals over $C_n^{3/2}(2u-1)$
- $I_{ll'}=0$ at $\ell>\ell'$,

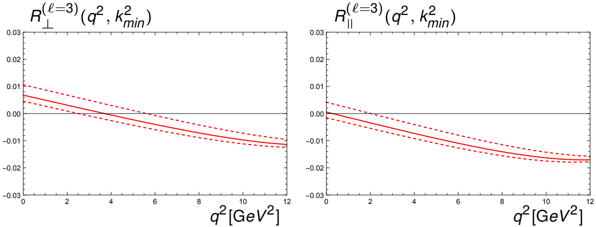
$\Rightarrow$ in the limit of the asymptotic DA, ($B_{01}\neq 0$), only P-wave form factors are $\neq 0$

Numerical results

- *P*-wave form factors: (only twist-2)



- *P*-wave dominance: ratios of *F*- and *P*-wave form factors



- - - - uncertainties from the variation of M^2.

How much $B \to \rho$ contributes to the $B \to 2\pi$?

- dispersion relation for the P-wave ($\ell = 1$) of $B \to \pi\pi$ FFs:

$$\frac{\sqrt{3}F_{\perp}^{(\ell=1)}(q^2,k^2)}{\sqrt{k^2}\sqrt{\lambda_B}} = \frac{g_{\rho\pi\pi}}{m_\rho^2 - k^2 - i m_\rho \Gamma_\rho(k^2)} \frac{V^{B\to\rho}(q^2)}{m_B + m_\rho} + \dots$$

and

$$\frac{\sqrt{3}F_{\parallel}^{(\ell=1)}(q^2,k^2)}{\sqrt{k^2}} = \frac{g_{\rho\pi\pi}}{m_\rho^2 - k^2 - i m_\rho \Gamma_\rho(k^2)} (m_B + m_\rho) A_1^{B\to\rho}(q^2) + \dots$$

$$\Gamma_\rho(k^2) = \frac{m_\rho^2}{k^2}\left(\frac{k^2 - 4m_\pi^2}{m_\rho^2 - 4m_\pi^2}\right)^{3/2} \theta(k^2 - 4m_\pi^2)\Gamma_\rho^{tot}\,,$$

- using the definition of $B \to \rho$ FFs:

$$\langle \rho^+(k)|\bar{u}\gamma_\mu(1-\gamma_5)b|\bar{B}^0(p)\rangle = \epsilon_{\mu\alpha\beta\gamma}\epsilon_\alpha^{*(\rho)} p^\beta k^\gamma \frac{2V^{B\to\rho}(q^2)}{m_B + m_\rho}$$
$$- i\epsilon_\mu^{*(\rho)}(m_B + m_\rho)A_1^{B\to\rho}(q^2) + \dots$$

LCSRs for $B \to \rho$ FFs

e.g., [P. Ball and V. M. Braun, Phys. Rev. D **55** *(1997) 5561]*

- LCSRs for $B \to \rho$ form factors in terms of the ρ-meson DAs in the twist-2 approximation:

$$V^{B\to\rho}(q^2) = \frac{(m_B + m_\rho)m_b}{2m_B^2 f_B} f_\rho^\perp e^{\frac{m_B^2}{M^2}} \int\limits_{u_0}^{1} \frac{du}{u}\, \phi_\perp^{(\rho)}(u)\, e^{-\frac{m_b^2 - q^2\bar{u} + m_\rho^2 u\bar{u}}{uM^2}} ,$$

$$A_1^{B\to\rho}(q^2) = \frac{m_b^3}{2(m_B + m_\rho)m_B^2 f_B} f_\rho^\perp e^{\frac{m_B^2}{M^2}} \int\limits_{u_0}^{1} \frac{du}{u^2}\, \phi_\perp^{(\rho)}(u) \left(1 - \frac{q^2 - m_\rho^2 u^2}{m_b^2}\right) e^{-\frac{m_b^2 - q^2\bar{u} + m_\rho^2 u\bar{u}}{uM^2}} .$$

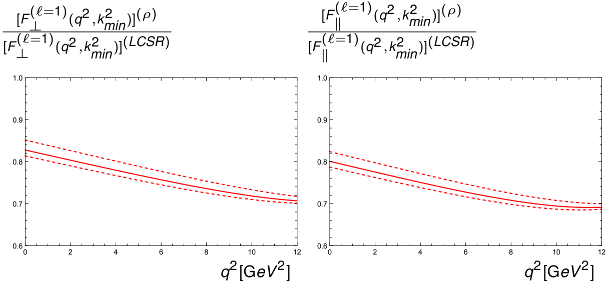
- both sum rules determined by the chiral-odd ρ-meson DA:

$$\langle \rho^+(k)|\bar{u}(x)\sigma_{\mu\nu}[x,0]d(0)|0\rangle = -if_\rho^\perp \left(\epsilon_\mu^{*(\rho)} k_\nu - k_\mu \epsilon_\nu^{*(\rho)}\right) \int\limits_0^1 du e^{iuk\cdot x} \phi_\perp^{(\rho)}(u) ,$$

- the Gegenbauer polynomial expansion:

$$\phi_\perp^{(\rho)}(u) = 6u(1-u)\left(1 + \sum_{n=2,4,,..} a_n^{(\rho)\perp} C_n^{3/2}(2u-1)\right) ,$$

Numerical estimates



The relative contribution of ρ-meson to the P-wave $B \to \pi^+\pi^0$ form factors $F_\perp^{(\ell=1)}(q^2, k^2_{min})$ (left panel) and $F_\parallel^{(\ell=1)}(q^2, k^2_{min})$ (right panel) calculated from LCSRs at central values of the input. Dashed lines indicate the uncertainty due to the variation of the Borel parameter.

Future perspectives

- k^2-ansatz for Gegenbauer functions $B_{nl}(k^2)$
  from LCSRs for pion FFs at $k^2 < 0 \oplus$ dispersion representations at $k^2 > 4m_\pi^2$
- twist-3,4 and $\bar{q}qG$ components of OPE,
  to identify corresponding DAs and their expansions
- NLO gluon radiative corrections
- $B \to \pi^+\pi^-$ channel, including S-wave
- $B \to K\pi(K^*)$ form factors
- alternative method: LCRs with B-meson DA and $\bar{u}\gamma_\mu d$ current

  S.Cheng, AK, J.Virto, work in progress