# Status of the $B \to X_c \tau \nu$ Calculation

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- BaBar reported $3.4\sigma$ deviation from SM in the combination of the ratios $R(D)$ and $R(D^*)$:

$$R(D^{(*)}) = \frac{\Gamma(B \to D^{(*)}\tau\bar{\nu})}{\Gamma(B \to D^{(*)}\ell\bar{\nu})}$$

| | BaBar | Belle | SM |
|---|---|---|---|
| $R(D)$ | $0.440 \pm 0.058 \pm 0.042$ | $0.430 \pm 0.091$ | $0.297 \pm 0.017$ |
| $R(D^*)$ | $0.332 \pm 0.024 \pm 0.018$ | $0.405 \pm 0.047$ | $0.252 \pm 0.003$ |
| correlation | neglected | $-0.27$ | |

- Learn from inclusive $= \sum$ exclusive
  $\mathcal{B}(B^- \to X_c\ell\bar{\nu}) = (10.92 \pm 0.16)\%$ and $R(X_c) = 0.222 \pm 0.003$ [hep-ph/9401226, hep-ph/9811239]
  $\Rightarrow \mathcal{B}(B^- \to X_c\tau\bar{\nu}) = (2.42 \pm 0.05)\%$, LEP : $\mathcal{B}(b \to X\tau^+\bar{\nu}) = (2.41 \pm 0.23)\%$
- The $R(D^{(*)})$ data imply:

$$Data\ : \mathcal{B}(\bar{B} \to D^*\tau\bar{\nu}) + \mathcal{B}(\bar{B} \to D\tau\bar{\nu}) = (2.78 \pm 0.25)\%$$

- SM prediction for $\Gamma(B \to D^{**}\tau\bar{\nu}) \gtrsim 0.2\%$

Inclusive decay rate

$$d\Gamma_{sl} = \frac{G_F^2 |V_{cb}|^2}{2} W_{\mu\nu} dL^{\mu\nu}$$

- Leptonic tensor

$$L^{\mu\nu} = \frac{1}{2} Tr[(\not{p}_\ell + m_\ell)\gamma^\mu (\not{p}_\nu)\gamma^\nu (1 - \gamma_L)]$$

- Hadronic tensor

$$W^{\mu\nu} = (2\pi)^3 \sum_X \delta^4(p_b - q - p_X) \langle B(p)|J^{\mu\dagger}|X\rangle \langle X|J^\nu|B(p)\rangle$$

$$q = p_\ell + p_\nu$$
$$J^\mu = \bar{c}\gamma^\mu(1 - \gamma_5)b$$

can be decomposed as:

$$W^{\mu\nu} = -W_1 g^{\mu\nu} + W_2 v^\mu v^\nu + iW_3 \epsilon^{\mu\nu\rho\sigma} v_\rho q_\sigma + W_4 q^\mu q^\nu + T_5(q^\mu v^\nu + q^\nu v^\mu)$$

Optical theorem :$W_{\mu\nu}$ be expressed as the discontinuity across the cut in the time order product of weak currents in a B meson

$$W_{\mu\nu} = -\frac{1}{\pi} Im T_{\mu\nu}$$

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- Starting from the correlator of two hadronic currents

$$T_{\mu\nu} = \frac{1}{2m_B} \int d^4x\, e^{-iq\cdot x} \langle B(p)|T\{J^\dagger_\mu(x) J_\nu(0)\}|B(p)\rangle$$

- $T^{\mu\nu}$ can be decomposed into

$$T^{\mu\nu} = -T_1 g^{\mu\nu} + T_2 v^\mu v^\nu + iT_3 \epsilon^{\mu\nu\rho\sigma} v_\rho q_\sigma + T_4 q^\mu q^\nu + T_5(q^\mu v^\nu + q^\nu v^\mu)$$

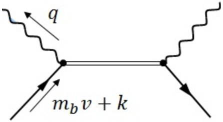
$W_i(q^0, q^2)$ is the imaginary part of the $T_i(q^0, q^2)$, using the identity:

$$-\frac{1}{\pi}\mathrm{Im}\left(\frac{1}{z+i\epsilon}\right) = \frac{(-1)^n}{n!}\delta^{(n)}(z)$$

## Triple differential decay rate

$$\frac{d\Gamma}{dq^2 dE_\ell dE_\nu} = \frac{G_F^2|V_{cb}|^2}{(2\pi)^3}\theta(z_+)\theta(z_-)\Big\{W_1 q^2 + W_2\left(2E_\ell E_\nu - \frac{q^2}{2}\right) + W_3(q^2(E_\ell - E_\nu))$$
$$+ \frac{1}{2}m_\ell^2\Big(-2W_1 + W_2 - 2W_3\,(E_\nu + E_\ell) + q^2 W_4 + 4E_\nu W_5\Big)\Big\}$$

# Calculational scheme for $1/m_b^n$ at tree level [Dassinger, Mannel,Turczyk (2006)]




Background-Field c-quark propagator:

$$iS_{BGF} = \frac{1}{\not{Q} + \not{k} - m_c}, \quad Q = m_b v + k,\ k = iD$$

- multiplying $iS_{BGF}$ to the appropriate Dirac matrices for the left handed current $\Gamma_\mu = 1/2\gamma_\mu(1-\gamma^5)$.
- expanding of the $iS_{BGF}$ to $n^{th}$ order.
- evaluating the forward matrix elements of operators

$$\langle B(p)|b_{v,\alpha}(iD_{\mu_1})\ldots(iD_{\mu_{n-1}})b_{v,\beta}|B(p)\rangle$$

in terms of the *basic parameters* for a certain order in $1/m_b$.

# Calculational scheme for $1/m_b^n$ at tree level [Dassinger, Mannel,Turczyk (2006)]

Basic parameters at $1/m_b^n$

- ...at $1/m_b^2$

$$2M_B\mu_\pi^2 = -\langle B(p)|\bar{b}_v(iD)^2 b_v|B(p)\rangle$$
$$2M_B\mu_G^2 = \langle B(p)|\bar{b}_v(iD_\mu)(iD_\nu)(-i\sigma^{\mu\nu})b_v|B(p)\rangle$$

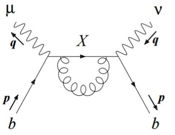
- ...at $1/m_b^3$

$$2M_B\rho_D^3 = \langle B(p)|\bar{b}_v(iD_\mu)(ivD)(iD^\mu)b_v|B(p)\rangle$$
$$2M_B\rho_{LS}^3 = \langle B(p)|\bar{b}_v(iD_\mu)(ivD)(iD_\nu)(-i\sigma^{\mu\nu})b_v|B(p)\rangle$$

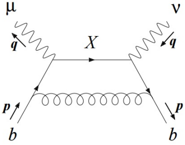
$\rightarrow$ $W_i$'s can be expanded in series of $\Lambda_{QCD}/m_b$

$$W_i = w_i^{(0)} + \frac{\mu_\pi^2}{2m_b^2}w_i^{(\pi,0)} + \frac{\mu_G^2}{2m_b^2}w_i^{(G,0)} + \frac{2\rho_D^3}{3m_b^3}w_i^{(D,0)} + \frac{2\rho_{LS}^3}{3m_b^3}w_i^{(LS,0)}\ldots$$

- Diagrams contributing to $\mathcal{O}(\alpha_s)$ from one-gluon emission



Final quark self-energy diagram.



Box diagram.

Vertex correction diagrams.

# Matching at $\mathcal{O}(\alpha_s)$ [V.Aquila, P. Gambino, G. Ridolfi and N.Uraltsev (2005)]

- The hadronic tensor recieves contributions of order $\alpha_s$ from one-gluon emission at the tree level

$$b(p) \to c(p') + \ell(p_\ell) + \bar{\nu}(p_\nu) + g(k)$$

- Integrated over the charm-gluon phase space $(r = p' + k)$ and from one-loop virtual corrections to the $b(p) \to c(p') + \ell(p_\ell) + \bar{\nu}(p_\nu)$:

$$\to W^{\mu\nu}(p,q) = W^{\mu\nu}_{Real}(p,q) + W^{\mu\nu}_{virtual}(p,q)$$

- $W_i$'s are known. [V.Aquila, P. Gambino, G. Ridolfi and N.Uraltsev (2005)]

$W_i$'s can be expanded in series of $\alpha_s$ and $\Lambda_{QCD}/m_b$

$$W_i = w_i^{(0)} + \frac{C_F\alpha_s}{\pi} w_i^{(1)} + \frac{\mu^2_{\pi,G}}{2m_b^2} w_i^{(\pi,G,0)} + \frac{\rho^3_{D,LS}}{3m_b^3} w_i^{(D,LS,0)} + \mathcal{O}(\alpha_s^2, 1/m_b^4)$$

# Phase space limits

- neutrino energy $E_\nu$ in $q^2 - E_\tau$ plan

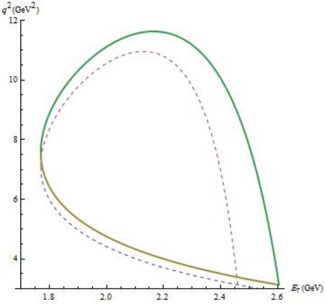
$$\frac{q^2-\eta}{2(y+\sqrt{y^2-\eta})} \leq x \leq \frac{q^2-\eta}{2(y-\sqrt{y^2-\eta})}$$

- Momentum transfer $q^2$

$$y_-(1-\frac{\rho}{1-y_-}) \leq q^2 \leq y_+(1-\frac{\rho}{1-y_+})$$

- $\tau$ lepton energy

$$2\sqrt{\eta} \leq y \leq (1+\eta-\rho)$$

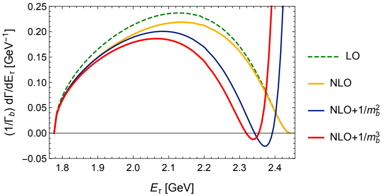



dimensionless variables:

$$y=\frac{2E_\tau}{m_b},\ x=\frac{2E_\nu}{m_b},\ \eta=\frac{m_\tau^2}{m_b^2},\ \rho=\frac{m_c^2}{m_b^2},\ q^2=\frac{q^2}{m_b^2}$$

$$y_\pm=\frac{1}{2}\left(y\pm\sqrt{y^2-4\eta}\right),\ q_\pm=v\cdot q\pm\sqrt{(v\cdot q)^2-q^2}$$

- The dotted (green) curve shows the free-quark decay result, the orange curve include $\mathcal{O}(\alpha_s)$ correction and the red (blue) curve includes both $\alpha_s$ and $\Lambda^3_{QCD}/m_b^3$ ($\Lambda^2_{QCD}/m_b^2$) correction.




- Numerical results

| $m_b$ | $m_c$ | $\mu_\pi^2$ | $\mu_G^2$ | $\rho_D^3$ | $\rho_{LS}^3$ | $V_{cb}\times 10^{-3}$ |
|---|---|---|---|---|---|---|
| 4.548 | 1.092 | 0.428 | 0.344 | 0.158 | -0.146 | 42.24 |

In kinetic scheme ( $\tau_B = 1.582(7)$ps)[P. Gambino, C. Schwanda (2014)]

| BR % | $\alpha_s$ | $\alpha_s + 1/m_b^2$ | $\alpha_s + 1/m_b^3$ | [1]$SM$ | Data |
|---|---|---|---|---|---|
| $BR_{ce\nu}$% | 11.40 | 10.87 | 10.48 | 10.66 | $10.92 \pm 0.16$ [arxiv:1202.1834] |
| $BR_{c\tau\nu}$% | 2.63 | 2.42 | 2.19!? | - | $2.41 \pm 0.23$ [J. Bringer et al.[PDG]] |

[1]:[P. Gambino, C.Schwanda (2014)]

# Outlook

- calculation of the moments
- Calculation of the QCD Radiative corrections to the next to leading terms in the $\Lambda_{QCD}/m_b$ expansion
- Refinement of the methods to compute the form factors of the exclusive decays
- Discussion of possible non-standard effects