Effective Field Theories and Higgs Physics

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QFT

A local gauge-invariant Lagrangian

$$L = \partial_\mu \phi^\dagger \partial_\mu \phi - m^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 + L_{\text{ct}}$$

- Incorporates causality, unitarity, crossing symmetry, etc.
- finite number of couplings
- In weak coupling, an expansion in λ.
- renormalizable (i.e. infinities are of the same form as term in L)

Renormalizability

Add counterterms of the same form as terms in L so that S-matrix elements are finite

Can relate cross sections to each other in a finite way.
QED — 2 measurements give you e and m_e, and then other processes such as $e\gamma \to e\gamma$ are determined. [So what if L is infinite.]

EFT

A local gauge-invariant Lagrangian

$$L = \partial_\mu \phi^\dagger \partial_\mu \phi - m^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 - \frac{c_6}{\Lambda^2} (\phi^\dagger \phi)^3 - \frac{c_8}{\Lambda^4} (\phi^\dagger \phi)^4 + \ldots + L_{\rm ct}$$

- Incorporates causality, unitarity, crossing symmetry, etc.
- power counting scheme (count powers of $1/\Lambda$)
  - finite number of couplings to a given order in $1/\Lambda$
  - expansion in λ and in p/Λ.
  - renormalizable (i.e. infinities are of the same form as term in L)

Power Counting

Graph with one c_6 is order $1/\Lambda^2$, two c_6 vertices or one c_8 vertex is order $1/\Lambda^4$, etc.

Can relate cross sections to each other in a finite way.
2 measurements give you λ and m and you can compute to $1/\Lambda^0$
3 measurements give you λ, m and c_6 and you can compute to $1/\Lambda^2$

An EFT is a theory

An EFT is a field theory, just like QED or QCD with

- A Lagrangian
- A renormalization scheme (usually $\overline{\text{MS}}$).

Given these, you can calculate without external inputs. Even if the EFT is the low-energy limit of a full theory, you can compute using L_{EFT} without reference to the full theory.

No need for the historical distinction between renormalizable and non-renormalizable theories.

There is a small expansion parameter analogous to α in QED.

The new feature is truncation to some order in $1/\Lambda$.

This does not mean the theory is perturbative, e.g. in HQET we systematically include corrections of order $(\Lambda_{\text{QCD}}/m_b)^n$

3 Reasons for using EFT

- Deal with only one scale at a time:
  B meson decay rate depends on M_W, m_b and $\Lambda_{\rm QCD}$
- Makes symmetries manifest:
  HQET has a spin-flavor symmetry not obvious from the QCD Lagrangian

$$b\uparrow,\ b\downarrow, c\uparrow, c\downarrow$$

- Sum logs (including IR logs):
  Fixed order perturbation theory breaks down, and need RG improved perturbation theory:

$$1 + c_1\, \alpha_s \log \frac{M_W}{m_b} + c_2\, \alpha_s^2 \log^2 \frac{M_W}{m_b} + \ldots$$

EFT is the only way in which these logs have been handled in the weak interactions.

EFT Expansion

Parametrize deviations from the SM:

Effective Lagrangian:

$$L_{\rm EFT} = L_{D\le 4} + \frac{O_5}{\Lambda} + \frac{O_6}{\Lambda^2} + \ldots$$

An infinite number of terms (and parameters), but only a finite number to any given order in the expansion.

Usually many operators of a given dimension.

$$L_{d=6} = \sum_i c_i\, O_i$$

and treat c_i as formally of order $1/\Lambda^2$.

Experimental Summary

- Standard Model provides a good description of all observations
- A particle has been seen with a mass $M_h \sim 126$ GeV consistent with the Higgs boson of the standard model
- No evidence for any BSM physics up to energies of ~ 1 TeV

If there are no new particles at the ~ 500 GeV scale, then can use the SM and parametrize new physics by higher dimension operators. New physics effects $\propto p^2/\Lambda^2$.

Standard Model (Cartesian Coordinates)

$$H = \begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix} = \begin{bmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + \mathrm{h} + i\eta) \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} i\phi_1 + \phi_2 \\ \phi_4 - i\phi_3 \end{bmatrix}, \quad \boldsymbol{\phi} = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{bmatrix}$$

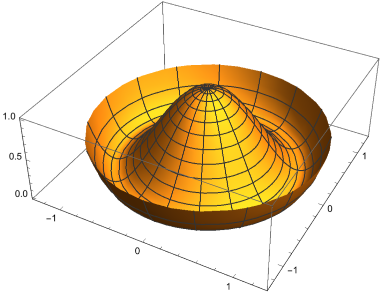
$$\begin{aligned} L &= D_\mu H^\dagger D^\mu H - \lambda \left(H^\dagger H - \frac{v^2}{2} \right)^2 \\ &= \frac{1}{2} D_\mu \boldsymbol{\phi} \cdot D^\mu \boldsymbol{\phi} - \frac{\lambda}{4} \left(\boldsymbol{\phi} \cdot \boldsymbol{\phi} - v^2 \right)^2 \end{aligned}$$

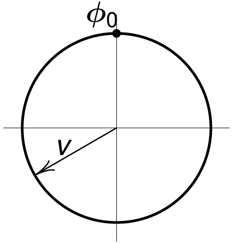
Assume custodial symmetry: $O(4) \sim SU(2) \times SU(2) \supset SU(2) \times U(1)$.

We want to test whether this is the EW symmetry breaking mechanism

Scalar Potential

$$V = -\frac{\lambda}{4}\left(\phi\cdot\phi - v^2\right)^2 \qquad \phi_0 = (0,0,0,v)^T$$





The potential is a minimum on the black sphere S^3.

Any point on the sphere is a suitable vacuum configuration, so pick ϕ_0 to be the North pole of S^3.

SM in Polar Coordinates

Spherical polar coordinates

$$\phi = (v + h)\,\boldsymbol{n} \qquad \boldsymbol{n}\cdot\boldsymbol{n} = 1 \qquad \boldsymbol{n} = (n_1, n_2, n_3, n_4)$$

Components of $\boldsymbol{n}$ are not independent.

$$\boldsymbol{n} = \begin{bmatrix} \boldsymbol{\pi} \\ \sqrt{1 - \boldsymbol{\pi}\cdot\boldsymbol{\pi}} \end{bmatrix} \qquad \boldsymbol{n} = e^{i\boldsymbol{T}\cdot\boldsymbol{\varphi}}\,\boldsymbol{n}_0 = e^{i\boldsymbol{T}\cdot\boldsymbol{\varphi}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

In one lower dimension:

$$\boldsymbol{n} = \begin{bmatrix} x \\ y \\ \sqrt{1 - x^2 - y^2} \end{bmatrix} \qquad \boldsymbol{n} = \begin{bmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{bmatrix},$$

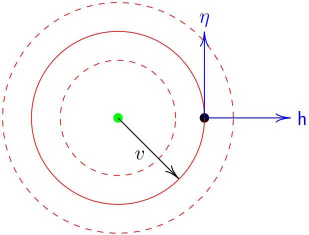
Scalar Manifold $\mathcal{M}$

Would like to explore the Higgs (scalar) landscape

SM the manifold is flat, with fields $(\phi_1, \phi_2, \phi_3, \phi_4) \in \mathbb{R}^4$.

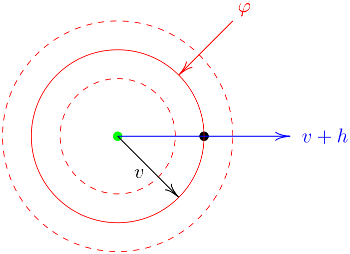
Cartesian coordinates

h, η, ϕ^+



Polar coordinates

h, φ^a



All that is need for EW symmetry breaking is the sphere S^3 of radius v.

Extensions of the SM

Some dynamics at a scale $f \gg v$.

Rather than worry about details about the high energy theory, try and characterize what aspects of the theory can be measured at low (i.e. LHC) energies.

SMEFT (Standard Model Effective Field Theory)

HEFT (Higgs Effective Field Theory)

SMEFT: written in terms of $(v+h)\boldsymbol{n} \to \phi \to H$

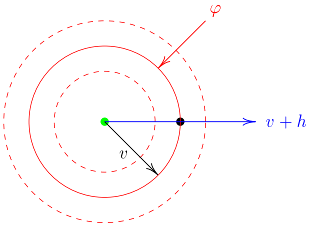
HEFT: written in terms of h and $\boldsymbol{n}$ — no connection between radial and angular directions

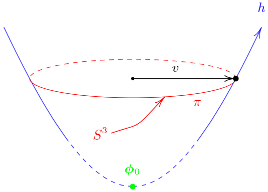
HEFT $\supset$ SMEFT as a special case

$$L = \frac{1}{2}\partial_\mu \phi \cdot \partial^\mu \phi$$

with $\phi = (v + h)\boldsymbol{n}$ becomes

$$L = \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}F(h)^2 v^2 (\partial_\mu \boldsymbol{n})^2 \qquad F(h) = 1 + \frac{h}{v}$$





Depending on $F(h)$, the scalar manifold could be flat or curved.

Generalize SM to SMEFT

Fields are three generations of fermions

$$\mathrm{L}: q_i,\ l_i, \qquad \mathrm{R}: u_i,\ d_i,\ e_i \qquad i = 1, \ldots, n_g = 3$$

the scalar doublet H, and $SU(3) \times SU(2) \times U(1)$ gauge fields.

$$L = L_{SM} + \sum \frac{1}{\Lambda^n} L^{(4+n)} = L_{SM} + \frac{1}{\Lambda^2} L^{(6)} + \ldots \quad \textit{note the dots}$$

Λ is the scale of new physics, and assume $\Lambda > v$

HEFT every operator can be multiplied by $F_O(h)$.

Baryon and Lepton Number Violation

Dimension five operator:

$$\frac{1}{\Lambda_5}(H\,\ell)(H\,\ell)$$

$\Delta L = 2$ operator which gives neutrino masses, e.g. by seesaw mechanism

Not relevant for 1 TeV LHC processes, i.e. can have $\Lambda_5 \gg \Lambda$ since Λ_5 violates lepton number, but Λ does not.

Similarly, baryon number violating operators can be dropped.

SMEFT Operators

- Leading higher dimension operators are $d = 6$.
- Assuming B and L conservation, operators divided into eight operator classes based on field content.

$$1 : X^3 \qquad 2 : H^6 \qquad 3 : H^4 D^2 \qquad 4 : X^2 H^2$$

$$5 : \psi^2 H^3 \qquad 6 : \psi^2 X H \qquad 7 : \psi^2 H^2 D \qquad 8 : \psi^4$$

$$X = G^A_{\mu\nu}, W^I_{\mu\nu}, B_{\mu\nu} \qquad \psi = q, l, u, d, e$$

Buchmuller & Wyler 1986

Grzadkowski, Iskrzynski, Misiak and Rosiek 2010

Dimension Six Operators

1 : X^3

Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$
$Q_{\widetilde{W}}$	$\epsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$

2 : H^6

Q_H	$(H^\dagger H)^3$

3 : $H^4 D^2$

$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
Q_{HD}	$\left(H^\dagger D_\mu H\right)^* \left(H^\dagger D_\mu H\right)$

5 : $\psi^2 H^3 + \text{h.c.}$

Q_{eH}	$(H^\dagger H)(\bar{l}_p e_r H)$
Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \widetilde{H})$
Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$

4 : $X^2 H^2$

Q_{HG}	$H^\dagger H\, G_{\mu\nu}^A G^{A\mu\nu}$
$Q_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$
Q_{HW}	$H^\dagger H\, W_{\mu\nu}^I W^{I\mu\nu}$
$Q_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$
Q_{HB}	$H^\dagger H\, B_{\mu\nu} B^{\mu\nu}$
$Q_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu} B^{\mu\nu}$
Q_{HWB}	$H^\dagger \tau^I H\, W_{\mu\nu}^I B^{\mu\nu}$
$Q_{H\widetilde{W}B}$	$H^\dagger \tau^I H\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$

6 : $\psi^2 X H + \text{h.c.}$

Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I H W_{\mu\nu}^I$
Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$
Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r)\widetilde{H}\, G_{\mu\nu}^A$
Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\tau^I \widetilde{H}\, W_{\mu\nu}^I$
Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\widetilde{H}\, B_{\mu\nu}$
Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H\, G_{\mu\nu}^A$
Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r)\tau^I H\, W_{\mu\nu}^I$
Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) H\, B_{\mu\nu}$

7 : $\psi^2 H^2 D$

$Q_{Hl}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{l}_p \gamma^\mu l_r)$
$Q_{Hl}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar{l}_p \tau^I \gamma^\mu l_r)$
Q_{He}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$
$Q_{Hq}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)$
$Q_{Hq}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar{q}_p \tau^I \gamma^\mu q_r)$
Q_{Hu}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu u_r)$
Q_{Hd}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{d}_p \gamma^\mu d_r)$
Q_{Hud} + h.c.	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_p \gamma^\mu d_r)$

S, T parameter operators are Q_{WB} and Q_{HD}. U parameter operator $\left(H^\dagger W_{\mu\nu} H\right)^2$ is dimension eight.

Dimension Six Operators

$8:(\bar{L}L)(\bar{L}L)$		$8:(\bar{R}R)(\bar{R}R)$		$8:(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$8:(\bar{L}R)(\bar{R}L)+\text{h.c.}$		$8:(\bar{L}R)(\bar{L}R)+\text{h.c.}$	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_{tj})$	$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\epsilon_{jk}(\bar{q}_s^k d_t)$
		$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\epsilon_{jk}(\bar{q}_s^k T^A d_t)$
		$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\epsilon_{jk}(\bar{q}_s^k u_t)$
		$Q_{lequ}^{(3)}$	$(\bar{l}_p^j\sigma_{\mu\nu} e_r)\epsilon_{jk}(\bar{q}_s^k\sigma^{\mu\nu} u_t)$

$$\psi^4 \to JJ,\ (\bar{L}R)(\bar{L}R),\ (\bar{L}R)(\bar{R}L)$$

How many operators at dimension six?

(B conserving)

- 59 entries in the table
- 76 hermitian operators (i.e. real parameters) for $n_g = 1$
  53 CP-even and 23 CP-odd
- 2499 hermitian operators (i.e. real parameters) for $n_g = 3$
  1350 CP-even and 1149 CP-odd
- 156 different irreducible flavor representations

Alonso, Jenkins, AM, Trott, arXiv:1312.2014 counting for any n_g

2499 verified by Henning, Lu, Melia, Murayama, arXiv:1512.03433 for $n_g = 3$

Field Redefinitions

Gaillard and Lee; Gilman and Wise   H.D. Politzer: NPB172 (1980) 349

Operator conversions done by making field redefinitions, since

$$L(\phi + \epsilon\, f(\phi)) = L(\phi) + \epsilon\, \frac{\delta L}{\delta \phi}\, f(\phi) + \ldots$$

- Change of variables in a (functional) integral
- *S*-matrix unchanged
- Green's functions can change

(related to equations of motion)

$$L = L_{\text{SM}} + L_{d=6} + \ldots$$

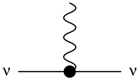
Have to make a field redefinition in the *entire Lagrangian including the SM part*. $h \to h + ah^2$ induces $h^2 \bar{t} t$ interactions in L_{SM}.

No excuse for not doing it right

From Passarino, arXiv:1610.09618

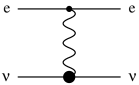
Besides it is an error to believe that rigour is the enemy of simplicity. On the contrary we find it confirmed by numerous examples that the rigorous method is at the same time the simpler and the more easily comprehended. The very effort for rigour forces us to find out simpler methods of proof.
David Hilbert, *Mathematical Problems*, Bulletin of the American Mathematical Society (July 1902), 8, 44.

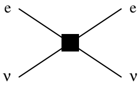
Need to compute an observable (S-matrix) element — typically many operators can contribute.
An old example is the neutrino charge radius:



$$F(q^2) = \frac{1}{6} r_\nu^2 q^2$$

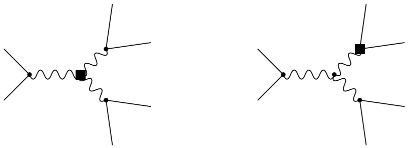
But to measure it, we need νe scattering





both terms contribute

Triple gauge vertex:



Should not drop operators unless you have a symmetry reason.

$$c_O \to 0 + \frac{*}{\epsilon}$$

depends on the choice of scheme, operator basis, etc.
Nothing special about 0 in an interacting QFT

[move a contribution to an operator which is then dropped.]

correct $\gg$ simple

Remember that at present there are *no* deviations from the SM.

No need to overinterpret the results.

1. Can do a SM fit, and see if there are any deviations
2. Can do an EFT fit, and put constraints on EFT coefficients
3. Cannot just randomly rescale vertices in an interacting theory

e.g. just rescaling a vertex by κ gives a change

$$\text{Tree Level:} \qquad (\kappa - 1)$$

$$\text{One Loop:} \qquad (\kappa - 1)\frac{\alpha_W}{4\pi}$$

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EFT allows you to introduce deviations in a sensible way:

$$\frac{1}{\Lambda^2}(H^\dagger H)(\overline{Q}_{tL} H t_R)$$

has tth, tth^2, tth^3 vertices from expanding $(v+h)^3$, and ensures divergences are $1/\Lambda^4$.

Power Counting for the SMEFT

Amplitudes and anomalous dimensions obey power counting:

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C^{(6)} \propto C^{(6)}$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C^{(8)} \propto C^{(8)} + \left[C^{(6)}\right]^2$$

In the SM, because of the dimension two operator $m_H^2 H^\dagger H$, have

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C^{(4)} \propto C^{(4)} + m_H^2 C^{(6)} + \ldots$$

SM parameter RG evolution modified by feedback from dim 6 terms at order m_H^2/Λ^2.
Just as important as dim 6 operators.

Jenkins, AM, Trott, arXiv:1308.2627

RGE for SM parameters from Dim 6

add to usual SM running:

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\lambda = \frac{m_H^2}{16\pi^2}\Big[12C_H + \left(-32\lambda + \frac{10}{3}g_2^2\right)C_{H\Box} + \left(12\lambda - \frac{3}{2}g_2^2 + 6g_1^2\mathsf{y}_H^2\right)C_{HD} + 2\eta_1 + 2\eta_2$$
$$+ 12g_2^2 c_{F,2}C_{HW} + 12g_1^2\mathsf{y}_H^2 C_{HB} + 6g_1 g_2 \mathsf{y}_H C_{HWB} + \frac{4}{3}g_2^2 C^{(3)}_{\substack{Hl\\tt}} + \frac{4}{3}g_2^2 N_c C^{(3)}_{\substack{Hq\\tt}}\Big],$$

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}m_H^2 = \frac{m_H^4}{16\pi^2}\left[-4C_{H\Box} + 2C_{HD}\right],$$

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}[Y_d]_{rs} = \frac{m_H^2}{16\pi^2}\Big[3C^*_{\substack{dH\\sr}} - C_{H\Box}[Y_d]_{rs} + \frac{1}{2}C_{HD}[Y_d]_{rs} + [Y_d]_{rt}\left(C^{(1)}_{\substack{Hq\\ts}} + 3C^{(3)}_{\substack{Hq\\ts}}\right) - C_{\substack{Hd\\rt}}[Y_d]_{ts}$$
$$- [Y_u]_{ts}C^*_{\substack{Hud\\tr}} - 2\left(C^{(1)*}_{\substack{qd\\sptr}} + c_{F,3}C^{(8)*}_{\substack{qd\\sptr}}\right)[Y_d]_{tp} + C_{\substack{ledq\\ptrs}}[Y_e]^*_{pt} + N_c C^{(1)*}_{\substack{quqd\\ptsr}}[Y_u]^*_{tp}$$
$$+ \frac{1}{2}\left(C^{(1)*}_{\substack{quqd\\sptr}} + c_{F,3}C^{(8)*}_{\substack{quqd\\sptr}}\right)[Y_u]^*_{tp}\Big]$$

$$\mu\frac{\mathrm{d}g_3}{\mathrm{d}\mu} = -4\frac{m_H^2}{16\pi^2}g_3 C_{HG}, \qquad \mu\frac{\mathrm{d}g_2}{\mathrm{d}\mu} = -4\frac{m_H^2}{16\pi^2}g_2 C_{HW}, \qquad \mu\frac{\mathrm{d}g_1}{\mathrm{d}\mu} = -4\frac{m_H^2}{16\pi^2}g_1 C_{HB},$$
$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\theta_3 = -\frac{4m_H^2}{g_3^2}C_{H\widetilde{G}}, \qquad \mu\frac{\mathrm{d}}{\mathrm{d}\mu}\theta_2 = -\frac{4m_H^2}{g_2^2}C_{H\widetilde{W}}, \qquad \mu\frac{\mathrm{d}}{\mathrm{d}\mu}\theta_1 = -\frac{4m_H^2}{g_1^2}C_{H\widetilde{B}},$$

Dimension Six Anomalous Dimension Matrix

Groejan, Alonso, Jenkins, AM, Trott: 1301.2588, 1308.2627, 1310.4838, 1312.2014

Computed the running of the SM dimension-four terms and the full dimension-six anomalous dimension at one loop, for general n_g.

γ: $156 \times 156 = 24336$. (or 2499×2499)

$\approx$ 572 cm in JHEP

J. Elias Miro, J.R. Espinosa, E. Masso, A. Pomarol, 1302.5661, 1308.1879

Use $n_g = 1$ and only keep y_t.
Compute γ for 5 classes of operators

$\approx$ 23 cm in JHEP

Features of RG evolution

- There are some big numbers:

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C_H = \frac{1}{16\pi^2} \left[108\,\lambda\, C_H - 160\,\lambda^2\, C_{H\Box} + 48\,\lambda^2\, C_{HD} \right] + \dots$$

- Test minimal flavor violation (MFV)

Chivukula, Georgi   D'Ambrosio, Giudice, Isidori, Strumia

$$Q_{eW} = (\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I H W^I_{\mu\nu} \qquad \text{MFV} \Rightarrow \; C_{\substack{eW \\ pr}} \propto [Y_e]_{pr}$$

- Limits from $h \to \gamma\gamma$, $\mu \to e\gamma$, d_e give constraints in the multi-TeV range. e.g. $d_e < 8.7 \times 10^{-29}\, e\,\text{cm}$ (ACME collab.) gives (in TeV^{-2})

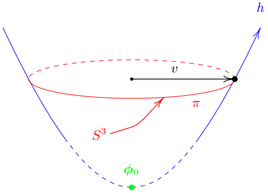
$$\left| C_{H\widetilde{W}B} \right| \lesssim 2 \times 10^{-4} \qquad \qquad \left| \widetilde{\mathscr{C}}_{\gamma\gamma} \right| \lesssim 2 \times 10^{-2}$$

$h \to \gamma\gamma$ gives a similar limit for $|\mathscr{C}_{\gamma\gamma}|$. Trott et al. $h \to \gamma\gamma$ incl. dim 6

SMEFT vs HEFT

Recall:

$$L_{\text{SMEFT}}\left[\phi=(v+h)\boldsymbol{n}\right] \qquad\qquad L_{\text{HEFT}}\left[h,\boldsymbol{n}\right]$$



h, $\boldsymbol{n}(\pi)$ coordinates on scalar manifold $\mathcal{M}$.

$O(4)$ symmetry acts on the angular variables.

L_{SMEFT} and L_{HEFT} have an infinite number of terms. Are they equivalent?

The SMEFT has an $O(4)$ fixed point at $\phi=0$, i.e. at $h=-v$.

HEFT equivalent to SMEFT iff $\mathcal{M}$ has an $O(4)$ invariant fixed point.

Alonso, Jenkins, AM, arXiv:1511.00724, 1605.03602

“Does the Higgs transform linearly or non-linearly?”

$$H = \begin{bmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}\left(v + \mathrm{h} + i\eta\right) \end{bmatrix} \qquad \text{or} \qquad \boldsymbol{n} = U = e^{i\boldsymbol{T}\cdot\boldsymbol{\varphi}},\ h$$

This is not a sensible question

SM

In Cartesian coordinates, H transforms linearly
In Polar coordinates $(h, \boldsymbol{n})$, $\boldsymbol{n}$ transforms non-linearly, h is invariant
Answer is coordinate dependent and not measurable.

Experimental Observable

What is measurable is whether the scalar field space is flat or curved.

Observables and Geometry

In a QFT, the S matrix is unchanged under field redefinitions.

The geometrical properties of a manifold are independent of the coordinate parameterization.

Experimentally measured quantities are given by the curvature of $\mathcal{M}$.

Observables $\longleftrightarrow$ Geometry

$$Q_{H\Box} = (H^\dagger H)\Box(H^\dagger H) \qquad\qquad Q_{HD} = \left(H^\dagger D_\mu H\right)^* \left(H^\dagger D_\mu H\right)$$

change the curvature of $\mathcal{M}$, and so cannot be eliminated.

Trott, Burgess, Passarino

For the SM:

$$F(h) = 1 + \frac{h}{v}.$$

even though S^3 is curved, $\mathcal{M}$ is flat, and all the curvatures vanish.

In HEFT, one considers a general radial function

$$F(h) = 1 + c_1 \left(\frac{h}{v}\right) + \frac{1}{2} c_2 \left(\frac{h}{v}\right)^2 + \ldots,$$

In this case,

$$K_{\pi\pi} = 1 - c_1^2, \qquad K_{\pi h} = -\frac{1}{2} c_2$$

Sectional curvature:

$$K(X, Y) = R_{abcd} X^a Y^b X^c Y^d, \quad X \perp Y, \quad X \cdot X = 1, \quad Y \cdot Y = 1$$

If f is the scale of new physics:

$$R_{abcd} \sim K \sim \frac{1}{f^2}$$

Experimental Consequences

Scalar observables depend on $K_{\pi\pi}, K_{\pi h}$, analogous to S, T, U for the Higgs sector.

Alonso, Jenkins, AM: arXiv:1511.00724 using a calculation from Barbieri, Bellazzini, Rychkov, Varagnolo

The scattering amplitude at high energies of longitudinal W-bosons W_L depends on the curvature:

$$\mathcal{A}(W_L W_L \to W_L W_L) = \underbrace{-4\lambda}_{SM} + \frac{s+t}{v^2} K_{\pi\pi}$$

$$\mathcal{A}(W_L W_L \to hh) = \underbrace{2\lambda}_{SM} - \frac{2s}{v^2} K_{\pi h} .$$

The scale of new physics governing the mass of these resonances is $\Lambda \sim 4\pi v/\sqrt{K}$

Note that this result is in accordance with the scenario of the Higgs boson as a Goldstone boson (Georgi, Kaplan) where resonances are expected at $\Lambda \sim 4\pi f$.

In composite Higgs models, all sectional curvatures are non-negative, and the sign is fixed; $K_{\pi\pi}, K_{\pi h} \geq 0$

Holomorphy

R. Alonso, E. Jenkins, AM: 1409.0868

Definition

The holomorphic part of the Lagrangian, $\mathcal{L}_{\mathfrak{h}}$, is the Lagrangian constructed from the fields X^+, R, $\overline{L}$, but none of their hermitian conjugates. These transform as $(0, \frac{1}{2})$ or $(0, 1)$ under the Lorentz group, i.e. only under the $SU(2)_R$ part of $SU(2)_L \times SU(2)_R$.

$$\mathcal{L}^{d=6} = \mathcal{L}_{\mathfrak{h}} + \mathcal{L}_{\overline{\mathfrak{h}}} + \mathcal{L}_{\mathfrak{n}} = C_{\mathfrak{h}} Q_{\mathfrak{h}} + C_{\overline{\mathfrak{h}}} Q_{\overline{\mathfrak{h}}} + C_{\mathfrak{n}} Q_{\mathfrak{n}}$$

$$Q_{\mathfrak{h}} \subset \left\{ {X^+}^3,\ {X^+}^2 H^2,\ \left(\overline{L}\sigma^{\mu\nu} R\right) X^+ H,\ (\overline{L}R)(\overline{L}R) \right\}$$

$$Q_{\overline{\mathfrak{h}}} \subset \left\{ {X^-}^3,\ {X^-}^2 H^2,\ \left(\overline{R}\sigma^{\mu\nu} L\right) X^- H,\ (\overline{R}L)(\overline{R}L) \right\} = Q_{\mathfrak{h}}^\dagger$$

$$Q_{\mathfrak{n}} \subset \left\{ H^6,\ H^4 D^2,\ \psi^2 H^3,\ \psi^2 H^2 D,\ (\overline{L}R)(\overline{R}L),\ JJ \right\}$$

Holomorphy

	$(X^+)^3$	$(X^+)^2H^2$	ψ^2X^+H	$(\bar{L}R)(\bar{L}R)$	$(\bar{L}R)(\bar{R}L)$	JJ	ψ^2H^3	H^6	H^4D^2	ψ^2H^2D
$(X^+)^3$	$\to\mathfrak{h}$	$\to 0$	0	0	0	0	0	0	0	0
$(X^+)^2H^2$	$\to\mathfrak{h}$	$\to\mathfrak{h}$	$\to\mathfrak{h}$	0	0	$\nexists$	0	0	$\to 0$	$\to 0$
ψ^2X^+H	$\to\mathfrak{h}$	$\to\mathfrak{h}$	$\to\mathfrak{h}$	$\mathfrak{h}_F$	$\to 0$	$\to 0$	$\to 0$	0	$\nexists$	$\to 0$
$(\bar{L}R)(\bar{L}R)$	$\to 0$	$\nexists$	$\mathfrak{h}_F$	$\mathfrak{h}_F$	$Y_u^\dagger Y_{e,d}^\dagger$	$Y_u^\dagger Y_{e,d}^\dagger$	$\nexists$	$\nexists$	$\nexists$	$\to 0$
$(\bar{L}R)(\bar{R}L)$	$\to 0$	$\nexists$	$\to 0$	Y_uY_d, $Y_u^\dagger Y_e^\dagger$	$\mathfrak{h}_F$	$*$	$\nexists$	$\nexists$	$\nexists$	$\to 0$
JJ	$\to 0$	$\nexists$	$\to 0$	$Y_uY_{e,d}$	$*$	$*$	$\nexists$	$\nexists$	$\nexists$	$*$
ψ^2H^3	$\to 0$	$\to\mathfrak{h}$	$\to\mathfrak{h}$	$\to\mathfrak{h}$	$*$	$*$	$*$	$\nexists$	$*$	$*$
H^6	$\to 0$	$\star$	$\nexists$	$\nexists$	$\nexists$	$\nexists$	$*$	$*$	$*$	$*$
H^4D^2	$\to 0$	$\to 0$	$\to 0$	$\nexists$	$\nexists$	$\nexists$	$\to 0$	$\nexists$	$*$	$*$
ψ^2H^2D	$\to 0$	$\to 0$	$\to 0$	$\to 0$	$\to 0$	$*$	$\to 0$	$\nexists$	$*$	$*$

0: Vanishes by NDA, i.e. NDA gives a negative loop order

$\nexists$: There is no one-loop diagram (including from EOM)

$\mathfrak{h}_F$: Holomorphic. Nonholomorphic terms forbidden by NDA and flavor symmetry

$\to 0$: Vanishes by explicit computation, after adding all contributions. Individual graphs need not vanish.

$\to\mathfrak{h}$: Holomorphic, by explicit computation

$*$: Non-zero

RGE of SM parameters

Recall that

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C^{(4)} \propto m_H^2 C^{(6)}$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \tau = \mu \frac{\mathrm{d}}{\mathrm{d}\mu} \left(\frac{4\pi}{g_X^2} - i \frac{\theta_X}{2\pi} \right) = \frac{2 m_H^2}{\pi g_X^2} C_{HX,+}$$

τ is the SUSY holmorphic gauge coupling

Clifford Cheung and Chia-Hsien Shen arXiv:1505.01844
can explain zeros in all but 3 entries:

⋆ Entry: Some Numerology

$$\dot{C}_H = -3g_2^2\left(g_1^2 + 3g_2^2 - 12\lambda\right)\text{Re}(C_{HW,+})$$
$$- 3g_1^2\left(g_1^2 + g_2^2 - 4\lambda\right)\text{Re}(C_{HB,+})$$
$$- 3g_1 g_2\left(g_1^2 + g_2^2 - 4\lambda\right)\text{Re}(C_{HWB,+}) + \ldots$$

The $C_{HB,+}$ and $C_{HWB,+}$ terms vanish if $g_1^2 + g_2^2 = 4\lambda$:

$$m_H^2 = 2m_Z^2 = (129\,\text{GeV})^2\,,$$

and the $C_{HW,+}$ term vanishes if $g_1^2 + 3g_2^2 = 12\lambda$:

$$m_H^2 = \frac{2}{3}m_Z^2 + \frac{4}{3}m_W^2 = (119\,\text{GeV})^2\,,$$

$$\frac{2}{3}(129\,\text{GeV})^2 + \frac{1}{3}(119\,\text{GeV})^2 = (125.7\,\text{GeV})^2$$

Summary

- EFT — an efficient way to parametrize deviations from the SM.
- Complete RGE of dimension-six operators of SM EFT has been computed, including contribution of dimension-six operators to running of SM parameters. (Many checks).
- Flavor mixing allows for a test of MFV hypothesis.
- Holomorphy of 1-loop anomalous dimension matrix.
- The S matrix depends on the geometric properties of $\mathcal{M}$, which can be measured experimentally.
  - Is $\mathcal{M}$ flat or curved? (not whether theory is linear or non-linear)
  - Is there an $O(4)$ invariant fixed point?
- Geometrical method provides an efficient way to compute radiative corrections.