Direct and Indirect Top and W mass measurements, and constraints on the Higgs from the Tevatron

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On behalf of CDF and D0
Higgs couplings 11/11/16
Thanks to all CDF & D0 colleagues
Tevatron Unique Dataset

- Different collision energy, $\sqrt{s_{\text{eff}}}$
  - Cross sections
  - Different (QCD) backgrounds
- $p\bar{p}$ collisions instead of $pp$
  - asymmetries; e.g., top, electroweak
- Top quark forward-backward asymmetries:
- is an initial CP invariant state ($B$ physics)
- Complementary! Production processes different mix of $q\bar{q}$ vs. $gg$ collisions
  - $tt$ spin correlations
- Well understood detector
  (plus lower level of pileup, only getting worse at LHC)
  - $W$ boson mass
  - top quark mass
Most recent Direct top Mass measurements from Tevatron

\[ m_t = 174.98 \pm 0.58 \text{(stat)} \pm 0.49 \text{(syst)} \text{GeV} \]

\[ \Delta m_t / m_t = 0.43\% \]

PRL 113, 032002 (2014),
PRD 91, 112003 (2015)

\[ m_t = 173.93 \pm 1.61 \text{(stat)} \pm 0.88 \text{(syst)} \text{GeV} \]

\[ \Delta m_t / m_t = 1.05\% \]

Phys. Rev. D 94,
032004 (2016)


\[ m_t = 173.32 \pm 1.36 \text{(stat)} \pm 0.85 \text{(syst)} \text{GeV} \]

\[ \Delta m_t / m_t = 0.93\% \]

D0 note 6484

\[ m_t = 173.50 \pm 1.31 \text{(stat)} \pm 0.84 \text{(syst)} \text{GeV} \]
D0 Top mass Combination (July 2016)

**D0 combination (preliminary)**

\[ m_t = 174.95 \pm 0.40 \text{ (stat)} \pm 0.64 \text{ (syst) GeV} \]

\[ \Delta m_t / m_t = 0.43\% \]

\[ \chi^2 / \text{ndof} = 2.5/3, \text{ prob} = 47\% \]
Tevatron Top mass Combination (July 2016)

\[ m_t = 174.30 \pm 0.35 \text{ (stat)} \pm 0.54 \text{ (syst)} \text{ GeV} \]
\[ \Delta m_t / m_t = 0.37\% \]
\[ \chi^2/\text{dof} = 10.8/11, \text{ prob} = 46\% \]
But what is the mass difference between the directly measured mass and the top pole mass?

Let’s look now at the $M_{\text{top}}$, $M_W$, $M_H$ constraints.
• Experimentally interesting situation

• In the SM: \( \sin^2 \theta_W = 1 - \frac{m^2_W}{m^2_Z} \)
  \( \sin^2 \theta_W \) indirectly measures \( m_W \)

• Derive \( \sin^2 \theta_{\text{eff}}^{\text{lept}} \) from angular distribution of leptons in Drell-Yan (Z/\( \gamma^* \rightarrow l^+l^- \)) events.

\[
\frac{dN}{d\theta} \approx 1 + \cos^2 \theta + A_4 \cos \theta
\]

• Forward-backward asymmetry

\[
A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3}{8} A_4
\]

• Measurement \( A_{FB} \rightarrow \sin^2 \theta_{\text{eff}}^{\text{lept}} \) (effective Z/lepton coupling)

\[
\sin^2 \theta_{\text{eff}}^{\text{lept}} = 1.037 \cdot \sin^2 \theta_W \quad \text{[ ZFITTER } \kappa_e (\sin^2 \theta_W M_Z) \text{ form factor]}
\]

\( \rightarrow \sin^2 \theta_W \rightarrow m_W \)
DØ $e^+e^- 9.7 \text{ fb}^{-1}$ $\sin^2\theta_{\text{eff}}^{\text{lept}}$ analysis

- two electrons with $p_T > 25$ GeV
- CC ($|\eta|<1.1$) and EC ($1.5<|\eta|<3.2$)
- Use $75<M_{ee}<115$ GeV $\rightarrow 560k$ events
- New Lepton energy calibration
  - Apply scale factor as a function of $L_{\text{inst}}$ first then as a function of $\eta$
  - $M_{ee}$ peak scaled to Z-LEP value in each bin
- Separate calibrations for data and MC
Corrections are applied to MC to account for:

- Smearing of electron energy
- Efficiency corrections in $p_T(e)$, $\eta(e)$
- $L_{\text{inst}}$ and $z_{\text{PV}}$ reweighting to match data
- Produce 2D templates of $M_{ee}$ and $\cos\theta^*$ by reweighing default MC ($\sin^2\theta_{\text{eff}}=0.232$) as a function of $\sin^2\theta_{\text{eff}}$

Extract $\sin^2\theta_{\text{eff}}$ by fitting raw AFB to templates with different $\sin^2\theta_{\text{eff}}$ values

- No unfolding: MC is carefully corrected to describe the data

$\sin^2\theta_{\text{eff}} = 0.23138 \pm 0.00043\text{(stat)} \pm 0.00008\text{(syst)} \pm 0.00017\text{(NNPDF2.3 PDFs)}$

(no EW radiative corrections)
DØ e⁺e⁻ 9.7 fb⁻¹ sin²θₑᶠᶠ lep analysis

An approximate way to correct for the flavor dependence of sin²θₑᶠᶠ from EW radiative corrections is used by the D0 collaboration. This is done by making the following corrections (proposed by Baur and collaborators [8]):

\[
\sin^2 \theta_{\text{eff}}^{u-quark} = \sin^2 \theta_{\text{eff}}^{\text{lept}} - 0.0001 \\
\sin^2 \theta_{\text{eff}}^{d-quark} = \sin^2 \theta_{\text{eff}}^{\text{lept}} - 0.0002
\]

Change is +0.00008

Final results : DØ ee

\[
\sin^2 \theta_{\text{eff}}^{\text{leptonic (Mz)}} = 0.23147 ± 0.00047 \text{ (total)}
\]

Situation in 2015
Main improvements are in the following areas:
(some details in the next slides and in the backup)

Precise lepton momentum/energy scale for muons and electrons using a new method- (will also reduce scale error for Mw measurement)

Use event weighting method for AFB analyses (systematic errors in acceptance and efficiencies cancel). Extract $A_{FB}$, then, unfold and correct.

Since $\sin^2 \theta_W$ is constant while $\sin^2 \theta_{eff}^{lept} (M_{ee}, \text{flavor})$ is not

$$\Rightarrow$$ Implement full ZFITTER ElectroWeak radiative corrections,

Use Drell-Yan forward-backward asymmetry to constrain parton distribution functions - (will also reduce PDF errors for Mw measurement)
Unfolding/corrections in CDF $e^+e^-$ analysis

$e^+e^- : A_{FB}$ Background subtracted
Raw no corrections

$e^+e^- : A_{FB}$ unfolded, fully corrected
CDF $\mu^+\mu^- \text{ and } e^+e^- 9.7$ fb$^{-1}$ $A_{FB}$ measurements

PRD 89, 072005 (2014)

$$|y_{\mu\mu}| < 1.0$$

First/Last Bin: Underflows/Overflows

PRD 93, 112016 (2016)

$$|y_{ee}| < 1.7$$

First/last bin: underflow overflow

Measurements corrected, templates are pure calculations and for the specified kinematic range.

RESBOS and POWHEG-BOX best-fit templates include EWK radiative corrections

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CDF $\mu^+\mu^- \text{ & } e^+e^-$ \(9.7\text{ fb}^{-1}\) $\sin^2\theta_{\text{eff}}^{\text{lept}}$ extraction

- Asymmetry measurements corrected for direct fits to calculations
  - Measurement: angular-weighted event sums method [EPJ C 76, 321 (2010)]
  - Simulation: matrix unfolding of detector and QED FSR smearing; residual bias correction of a few percent

- Simulation
  - PYTHIA 6.2(CTEQ5L) $\oplus$ PHOTOS 2.0(QED FSR) $\oplus$ CDF detector simulation
  - Higher order QCD effect corrections applied to generated events

- Templates
  - POWHEG-BOX(NLO) $\oplus$ NNPDF v3.0(NNLO) PDFs $\oplus$ PYTHIA 6.4 parton showers
  - ZFITTER 6.43 electroweak radiative corrections incorporated
    - fermion-type dependent effective mixing angles $\sin^2\theta_{\text{eff}}$

$\sin^2\theta_{\text{eff}}^{\text{lept}}$ values from template fits

- $\mu\mu$ analysis: $0.23141 \pm 0.00086$ (stat) $\leftarrow$ refit - same template framework as ee
- $ee$ analysis: $0.23248 \pm 0.00049$ (stat) $\chi^2$'s simply combined into a joint $\chi^2$

- Best-fit value of joint $\chi^2$: $0.23221 \pm 0.00046$ (total)

\[ \Downarrow \]

| statistics: | 0.00043 |
| PDF:       | 0.00016 |
| other systematics: | 0.00006 |
Common PDF and electroweak correction baselines for consistency
- NNPDF v3.0
  • Includes LHC data
  • Improved implementation for PDFs and ensembles
- ZFITTER SM electroweak radiative corrections
  • Used by LEP-1 and SLD for standard-model analysis at Z pole

Standardization paths for CDF and D0
- CDF: Already at baseline
- D0: Standardization corrections to $\sin^2\theta_{\text{eff}}^{\text{lept}}$ value

D0 standardization corrections

- $\Delta$(PDF): NNPDF v2.3 $\rightarrow$ v3.0 offset $= -0.00024 \pm 0.00004$
  • Difference of v3.0 pseudodata $\sin^2\theta_{\text{eff}}^{\text{lept}}$ and v2.3 template fit value
    $A_{\text{fb}}$ pseudodata: v3.0 default PDF with reference value of $\sin^2\theta_{\text{eff}}^{\text{lept}}$
    Templates: v2.3 default PDF with varying values of $\sin^2\theta_{\text{eff}}^{\text{lept}}$

- $\Delta$(RadCor): ZGrad+ResBos $\rightarrow$ ZFITTER offset $= +0.00014 \pm 0.00004$
  • Difference of $\sin^2\theta_{\text{eff}}^{\text{lept}}$ results \textit{with} and \textit{without} ZFITTER corrections

\textbackslash \textdagger\ analog of PYTHIA templates
Combination of $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ results

Input observable values
- Standardized D0 value: $0.23137 \pm 0.00043$ (stat) $\pm 0.00019$ (syst)
- CDF $e\bar{e}@\mu\bar{\mu}$ value: $0.23221 \pm 0.00043$ (stat) $\pm 0.00018$ (syst)

Input uncertainty categories
- Statistics: CDF: 0.00043, D0: 0.00043
- PDF: CDF: 0.00016, D0: 0.00017 (100% correlated)
- Other systematics: CDF: 0.00007, D0: 0.00008 (uncorrelated)
- Standardization: D0: 0.00005 (only applies to D0)

Results of BLUE method

$\sin^2 \theta_{\text{eff}}^{\text{lept}} = 0.23179 \pm 0.00030$ (stat)
$\pm 0.00017$ (syst)

$\chi^2$ of combination: 1.8 (18% probability)

Uncertainties
- Statistics: 0.00030
- PDF: 0.00017
- Other systematics: 0.00005
- Standardization: 0.00003

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Indirect measurement of W mass

$M_W$ also can be determined indirectly via the relation

$$\sin^2 \theta_W^{\text{on-shell}} = 1 - M_w^2 / M_z^2$$

±0.00040 error in $\sin^2 \theta_W$ is equivalent to ±20 MeV error in $M_w$ (indirect)

Both $\sin^2 \theta_W^{\text{on-shell}}$ and $\sin^2 \theta_{\text{eff leptonic}} (Mz)$ can be extracted from Drell-Yan forward-backward asymmetry (Afb) if we include EW radiative corrections.

$\Rightarrow$ $M_w^{\text{indirect}}$ can be extracted from $\sin^2 \theta_W^{\text{on-shell}}$

If the SM is correct, then both direct and indirect measurements of $M_W$ should agree. Deviations may imply the possibility of new physics.

Similarly different measurements of $\sin^2 \theta_{\text{eff leptonic}} (Mz)$ should also agree and deviations may imply new physics.
Indirect W mass results

Indirect measurements

LEP-1 and SLD ($m_t$) 80.363±0.020

NuTeV 80.135±0.085

CDF $\mu\mu$ 9 fb$^{-1}$ 80.365±0.047

CDF $ee$ 9 fb$^{-1}$ 80.313±0.027

CDF $ee+\mu\mu$ 9 fb$^{-1}$ 80.328±0.024

D0 $ee$ 10 fb$^{-1}$ August 2016: preliminary 80.373±0.024

TeV combined: CDF+D0 August 2016: preliminary 80.351±0.018

Direct measurement

TeV and LEP-2 80.385±0.015

$\sin^2\theta_W$ and $M_W$ equivalent in SM on-shell renormalization scheme (ZFITTER)

- $\sin^2\theta_W = 1 - M_W^2/M_Z^2$ all orders definition
- $M_Z = 91.1875±0.0021$ GeV

Standard model help from ZFITTER is needed

- $\sin^2\theta_{eff}^{lept} = \text{Re}[k(M_Z^2,\sin^2\theta_W)]\sin^2\theta_W \approx 1.037$
- Form factors depend on standard-model input parameters
  - Most sensitive to top-quark mass 173.2±0.9 GeV
  - Form factor uncertainty to $\sin^2\theta_W$ : 0.00008
  - Higgs mass value: 125 GeV

$$
\begin{align*}
\sin^2\theta_W & = 0.22400±0.00041±0.00019 & M_W & = 80328±21±10 \text{ MeV} \\
\text{CDF only} & & & \\
\sin^2\theta_W & = 0.22313±0.00041±0.00020 & M_W & = 80373±21±10 \text{ MeV} \\
\text{D0 only} & & & \\
\sin^2\theta_W & = 0.22356±0.00029±0.00019 & M_W & = 80351±15±10 \text{ MeV} \\
\text{Combination} & & & \\
 & & & (\text{stat}) (\text{syst}) (\text{stat}) (\text{syst})
\end{align*}
$$
Summary of CDF & DØ $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ analysis / indirect $M_W$ results

CDF and D0 have
- extracted $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ from Drell-Yan lepton-pair asymmetries
  CDF: electron and muon pairs
  D0: electron pairs
- combined the resulting values of $\sin^2 \theta_{\text{eff}}^{\text{lept}}$

Using ZFITTER SM calculations, inferred $\sin^2 \theta_w$ or equivalently $M_W$

D0 muon-pair asymmetry analysis is in progress — stay tuned!
W Mass direct Measurement

- Tevatron Combination based on CDF (2.2 fb\(^{-1}\)) and D0 (5.1 fb\(^{-1}\)) results:
  \[ M_w = (80387 \pm 16) \text{ MeV} \]
  
  \[ \Rightarrow 0.02\% \text{ precision!} \]
  dominates the World combination (15 MeV)

- Full-data results from both CDF and D0 in progress \[ \Rightarrow \text{ further reduction of uncertainties} \]
  - Statistics gain \[ \Rightarrow \text{ energy scale uncertainty} \]
  - Extension to forward leptons and PDF improvements will allow significant gain

- Target 10 MeV world average with new CDF and D0 measurements using full statistics.

- Consistency check of the SM (top-W-Higgs)
Top, and W Direct Mass Measurements

With a known Higgs mass, the SM is over-constrained.

2014: tension ~1.5σ between the direct Measurements of $M_\text{W}$ and SM

(http://pdg.lbl.gov)

2015: Tension would be ~2σ with the most recent measurement $M_t$ at CMS
(~1.3σ with Tevatron $M_t$)

$M_\text{W} = 80.385\pm0.015$ GeV (TeV/LEP2)

$M_{\text{TOP-TEVATION}} = 174.34\pm0.64$ GeV

$M_{\text{TOP-CMS 2015}} = 172.44\pm0.48$ GeV

174.30\pm0.65$ GeV / July 2016
Direct and Indirect W mass measurements

**Direct measurements:**

Waiting for update from CDF and D0 on full 10 fb\(^{-1}\) dataset (currently 2 and 5 fb\(^{-1}\) )

**Indirect measurements:**

Tevatron:

18 MeV indirect Mw uncertainty is similar to 15 MeV direct Mw uncertainty

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**Graph:**

- **LEP2:** 80.376±0.033, \(\chi^2/\text{dof} = 49/41\)
- **CDF:** 80.389±0.019
- **D0:** 80.383±0.023
- **Tevatron:** 80.387±0.016, \(\chi^2/\text{dof} = 4.2/6\)
- **Overall average:** 80.385±0.015

**Indirect measurements graph:**

- LEP-1 and SLD \((m_t)\):
  - 80.363±0.020
- NuTeV:
  - 80.135±0.085
- CDF \(\mu\mu\) 9 fb\(^{-1}\):
  - 80.365±0.047
- CDF \(ee\) 9 fb\(^{-1}\):
  - 80.313±0.027
- CDF \(ee+\mu\mu\) 9 fb\(^{-1}\):
  - 80.328±0.024
- D0 \(ee\) 10 fb\(^{-1}\) August 2016: preliminary
  - 80.373±0.024
- TeV combined: CDF+D0 August 2016: preliminary
  - 80.351±0.018
- Direct measurement TeV and LEP-2
  - 80.385±0.015

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Top, and W Direct Mass Measurements

With a known Higgs mass, the SM is over-constrained.

2014: tension $\sim 1.5\sigma$ between the direct Measurements of $M_W$ and SM

$M_W = 80.385 \pm 0.015$ GeV (TeV/LEP2)

$M_{top-2014} = 173.34 \pm 0.76$ GeV

(http://pdg.lbl.gov)

2015: Tension would be $\sim 2\sigma$ with the most recent measurement $M_t$ at CMS,
($\sim 1.3\sigma$ with Tevatron $M_t$)

$M_{W,\text{Tevatron}} = 174.34 \pm 0.64$ GeV

$M_{W,\text{Tevatron}} = 172.44 \pm 0.48$ GeV

$M_W = 80.385 \pm 0.015$ GeV (TeV/LEP2)

$M_{W,\text{Tevatron}} = 174.30 \pm 0.65$ GeV / July 2016

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Conclusions / Tevatron Legacy

• The CDF and DZero collaborations are producing final competitive physics results in particular in the electroweak sector.

• The Top mass measurements are essentially complete, the combination gives:
  
  
  174.30 ± 0.65 GeV  
  
  (172.8 ± 3.3 GeV top pole mass from D0 ttbar cross sections)

• Currently the Tevatron direct (L= 2.2 fb-1) and indirect (L=9.4 fb-1) measurements of $M_W$ have similar errors (15 vs.18 MeV)

• Tevatron Run II Legacy measurements of $\sin^2\theta_w$ and $M_w$ indirect are in agreement with SM predictions from $M_H$ and $M_{\text{top}}$, and with direct measurement of $M_W$

• $A_{FB}(M)$ data can also be used to put additional constraints on PDFs. These constraints will help reduce PDF errors in the ongoing Tevatron Run II Legacy (L=9.4 fb-1) direct measurement of $M_W$. 
Top quark mass from $t\bar{t}$ cross-section

- compare the experimental $t\bar{t}$ cross section measurement with the theory computation (depend differently on the top quark mass)
- cross section vs $m_t$ parametrized with $(\text{third order polynomial})/m_t^4$
- theoretical cross section computed at NNLO with top++
- experimental $ll$ & $l+$jets with 9.7 fb$^{-1}$

$$\sigma_{t\bar{t}} = 7.26 \pm 0.13(\text{stat})^{+0.57}_{-0.50}(\text{syst}) \text{ pb}$$

Advantage/Drawback
- extract the top quark mass in a well defined renormalization scheme (pole mass in theory computation)
- less precise than direct measurements

$$L(m_t) = \int f_{\text{exp}}(\sigma|m_t) \left[ f_{\text{scale}}(\sigma|m_t) \otimes f_{\text{PDF}}(\sigma|m_t) \right] d\sigma.$$  

$\begin{align*}
m_t &= 172.8 \pm 1.1 \text{ (theo.)}^{+3.3}_{-3.1} \text{ (exp.) GeV} \\
m_t &= 172.8^{+3.4}_{-3.2} \text{ (tot.) GeV} \\
\Delta m_t/m_t &= 1.9\% 
\end{align*}$

[Graph showing $\sigma_{t\bar{t}}$ versus top quark pole mass]
New technique used for both $\mu+\mu-$ and $e+e-$ for both data and hit level MC

1: Remove the correlations between the scale for the two leptons by getting an initial calibration using Z events and requiring that the mean $<1/PT>$ of each lepton in bins of $\eta$, $\Phi$ and charge be correct.

2: Use the Z mass for calibration. The Z mass as a function of $\eta, \Phi$, (and charge for $\mu+\mu-$) of each lepton must be correct

- Reference for electrons: expected Z mass (post FSR + clustered FSR photons), smeared by resolution (with acceptance cuts).

- Reference for muons: expected Z mass (post FSR) smeared by resolution (with acceptance cuts).
CDF: Use event weighting Method

Event weighting method for $A_{FB}$ analyses
\[ \frac{dN}{d\cos\theta} = 1 + \cos^2\theta + A_0(M, P_T) \left( 1 - 3\cos^2\theta \right)/2 + A_4(M) \cos\theta \]

Angular event weighting is equivalent to extraction of $A_4(M)$ in bins of $\cos\theta$, and averaging the results. Events at large $\cos\theta$ provide better determination of $A_4$, so they are weighted more than events at small $\cos\theta$.

For each $\cos\theta$, acceptance and efficiencies cancel to first order and the statistical errors are 20% smaller.

Uncertainty in $A_{FB}$ is reduced if we have more acceptance at large $\cos\theta$

Standard $A_{FB}$ method requires precise knowledge of acceptance & efficiencies.

Measure $A_4 \rightarrow A_{FB}$
CDF e^+e^- : sin^2\theta_W extraction using templates

- Comparison \( \chi^2: \sum_m \Delta A_{ib}(M) \times E \times \Delta A_{fb}(M) \)
  - Measurement: Fully corrected \( A_{fb}(M) \)
  - Calculated templates: \( A_{fb}(M,\sin^2\theta_W) \) for 16 values of \( \sin^2\theta_W \)
  - E: Measurement error matrix

- Extraction of \( \sin^2\theta_W \) from the scan points
  - Fit \( \chi^2(\sin^2\theta_W) \) scan points to a parabola: \( \chi^2_{\text{min}} + (\sin^2\theta_W - \sin^2\theta_W^{\text{min}})^2/\sigma_{\text{min}}^2 \)
    - Assign each scan point \( \chi^2(\sin^2\theta_W) \) an error of 0.1 in the parabolic fit
    - \( (\sin^2\theta_W^{\text{min}}, \sigma_{\text{min}}) \) are the fit values of \( \sin^2\theta_W \) and its uncertainty
    - \( \chi^2_{\text{min}} \) = minimum \( \chi^2(\sin^2\theta_W) \) at \( \sin^2\theta_W^{\text{min}} \) for 15 mass bins

This analysis is repeated with
1. POWEG  
2. RESBOS  
3. Tree-Level LO

For the POWHEG analysis, the extraction is repeated for all 100 NNPDF3.0 Replicas to get PDF uncertainty.
CDF: Implement ZFITTER EBA EW radiative corrections

\(\sin^2 \theta_W\) (on-shell) is a constant while \(\sin^2 \theta_{\text{eff \ lept}}\ (M_{ee}, \text{flavor})\) is not.

Full ZFITTER EW radiative corrections \(\Rightarrow\) Enhanced Born Approximation (EBA), include full complex form factors implemented in private versions of RESBOS, POWHEG, and LO \(\Rightarrow\) Phys. Rev. D 88, 072002 (2013) Appendix A’

\[
g_V^f \gamma_\mu + g_A^f \gamma_\mu \gamma_5. \ \text{The Born-level couplings are}
\]

\[
g_V^f = T_3^f - 2 Q_f \sin^2 \theta_W \\
g_A^f = T_3^f
\]

They are modified by ZFITTER 6.43 form factors (which are complex)

\[
g_V^f \rightarrow \sqrt{\rho_{eq}}(T_3^f - 2 Q_f \kappa_f \sin^2 \theta_W), \ \text{and} \\
g_A^f \rightarrow \sqrt{\rho_{eq}} T_3^f
\]

- \(T_3\) and \(\sin^2 \theta_W\) \(\rightarrow\) effective \(T_3\) and \(\sin^2 \theta_W\): 1-4\% multiplicative form factors
- On-mass shell scheme: \(\sin^2 \theta_W \equiv 1 - M_W^2/M_Z^2\) to all orders

Accounts for \(\sin^2 \theta_{\text{eff \ lept}}\) dependence on quark flavor and dilepton mass

\(\Rightarrow\) get \(\sin^2 \theta_{\text{eff \ lept}}(M_Z)\) using \(A_{\text{FB}}\) over a range of dilepton mass
### Systematics on $M_W$

<table>
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<tr>
<th>Source</th>
<th>Uncertainty 2.2 fb$^{-1}$ (MeV)</th>
</tr>
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<tbody>
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<td>Lepton energy scale</td>
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<tr>
<td>Lepton energy resolution</td>
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<tr>
<td>Recoil energy scale</td>
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<tr>
<td><strong>Total</strong></td>
<td><strong>19</strong></td>
</tr>
</tbody>
</table>

Many of the systematic uncertainties scale with the size of the calibration samples ($Z$ decays)

With full data sample (10 fb$^{-1}$) measurement becomes dominated by PDF and theoretical uncertainties

**Reaching 10 MeV precision possible with**

- Improved determination of PDF at Tevatron ($W$ charge asymmetry)
- Extension of measurements to endcap region (reduced sensitivity to production modeling)

**Systematic uncertainties in CDF measurement**
From effective to on-shell $\sin^2\theta_W$

- Drell-Yan lepton angular distributions are used to infer $\sin^2\theta_W$
  - Lepton (e, $\mu^-$) angular distributions ($\theta, \phi$) measured in $\gamma^*/Z$ CM frame: separates boson production and decay kinematics
  - Born level polar angle distribution: $1 + \cos^2\theta + A_4 \cos\theta$
  - $\cos\theta$ forward-backward asymmetry: sensitive to $\sin^2\theta_W$
- Vector with axial-vector interference: contributes to $A_4$ coefficient
  - $A_4$ term due to Z-boson self-interference $\propto (1-4|Q^e|\sin^2\theta_W)(1-4|Q^q|\sin^2\theta_W)$
  - Fermion (f) couplings to $\gamma^*/Z$ bosons: $(g_V^f + g_A^f \gamma^5)\gamma^\mu$
    - $q\bar{q} \to \gamma^* \to e^+e^-$: $g_V^f = Q_f^f$
    - $q\bar{q} \to Z \to e^+e^-$: $g_V^f = T_3^f (1-4|Q_f^e|\sin^2\theta_W)$, $g_A^f = T_3^f$ ($T_3$: weak isospin)
    - $\sin^2\theta_W = 1 - M_W^2/M_Z^2$ ($\sin^2\theta_W$ equiv. to $M_W$)
- Weak radiative corrections complicate this simple Born interpretation
  - Implementation: ZFITTER 6.43 enhanced Born approximation (EBA)
  - $T_3$ and $\sin^2\theta_W \to$ effective $T_3$ and $\sin^2\theta_W$: 1-4% multiplicative form factors
  - On-mass shell scheme: $\sin^2\theta_W \equiv 1 - M_W^2/M_Z^2$ to all orders