Higgs Couplings 2016

11 November 2016

Higgs Boson Pair Production @ NLO in QCD

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Borowka, Greiner, Heinrich, Kerner, Schlenk, Schubert, Zirke

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PoS LL2016 (2016) 069

MAX-PLANCK-GESELLSCHAFT

higgstools

Motivation

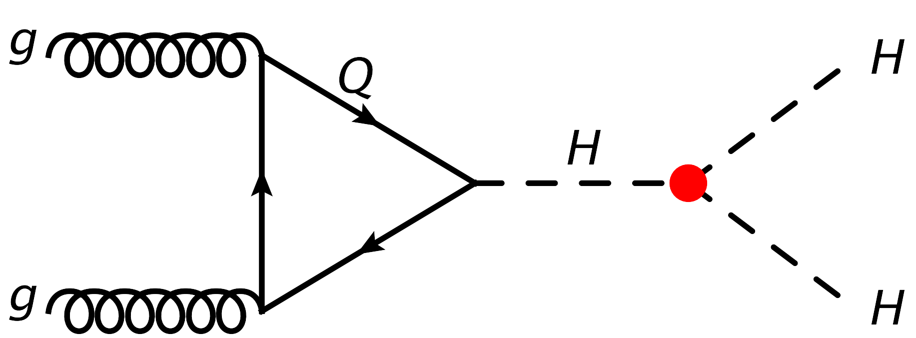
Higgs Lagrangian:

$$\mathcal{L} \supset -V(\Phi^\dagger\Phi), \quad V(\Phi^\dagger\Phi) = -\frac{1}{2}\mu^2\Phi^\dagger\Phi + \frac{1}{4}\lambda(\Phi^\dagger\Phi)^2$$

EW symmetry breaking

$$\frac{m_H^2}{2}H^2 + \boxed{\frac{m_H^2}{2v}H^3} + \frac{m_H^2}{8v^2}H^4$$

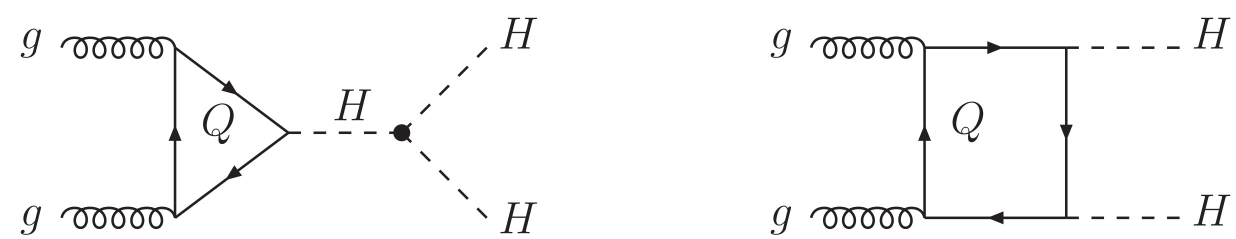
Higgs pair production probes triple-Higgs coupling



Production Channels

$\sigma(pp \to HH + X)$ @ 14 TeV

Gluon Fusion

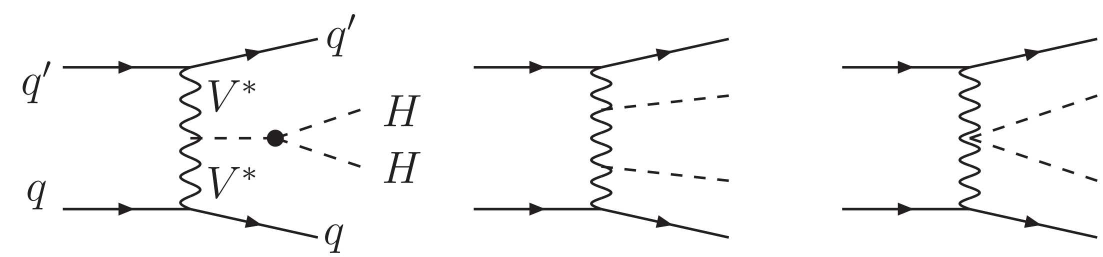


Vector Boson Fusion (VBF)

NLO [1,2] NNLO [3]

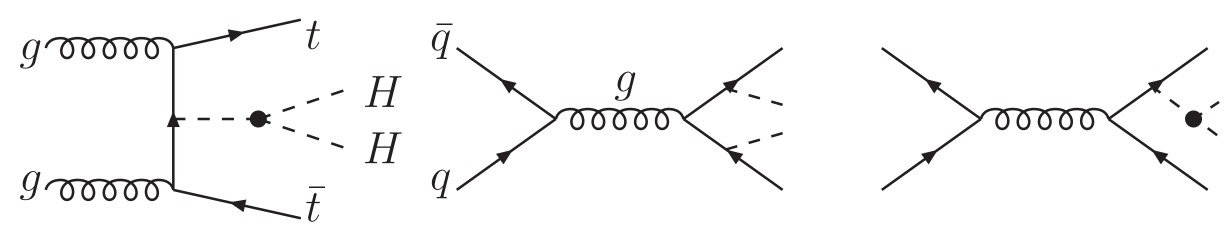
+ non-negligible contribution

from $gg \to HHjj$ LO [5]



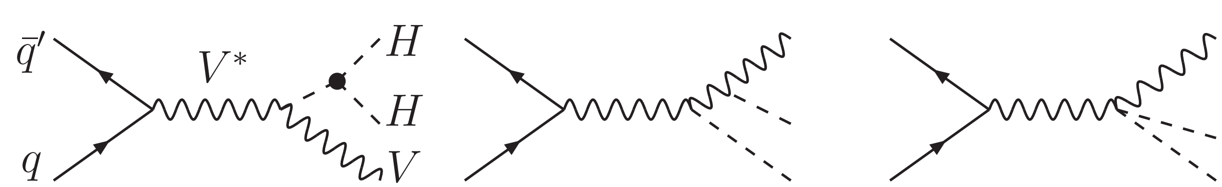
Top-Quark Associated

NLO [2]



Higgs-strahlung

NLO [1,2] NNLO [1,4]



Baglio, Djouadi et al. 12

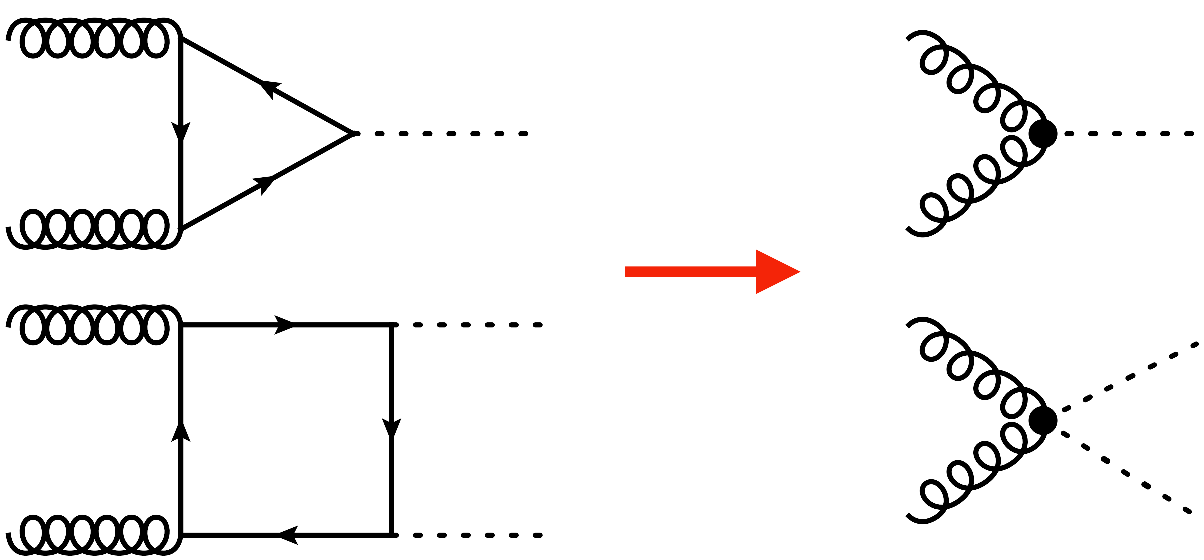
[1] Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, Spira 12;
[2] Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Torrielli, Vryonidou, Zaro 14;
[3] Ling, Zhang, Ma, Guo, Li, Li 14 [4] Li, Wang 16
[5] Dolan, Englert, Greiner, Nordstrom, Spannowsky 15;

Higgs EFT

H(iggs)EFT: $m_T \to \infty$

Effective tree-level couplings between gluons and Higgs

Lowers number of loops by 1



HEFT valid for

$$\sqrt{\hat{s}} \ll 2m_T$$

HH production for

$$2m_H < \sqrt{\hat{s}}$$

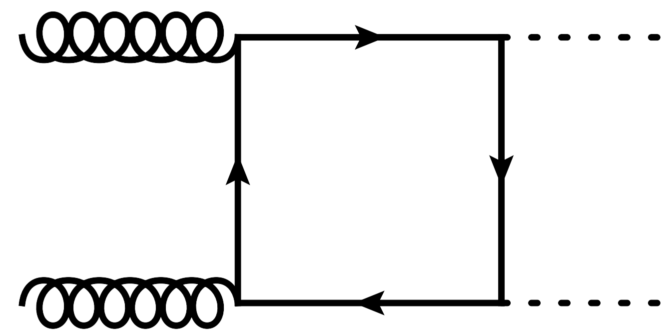
Small energy range in which HEFT is technically justified

Born improved NLO HEFT:

$$\mathrm{d}\sigma_{\mathrm{NLO}}(m_T) \approx \mathrm{d}\bar{\sigma}_{\mathrm{NLO}}(m_T) \equiv \frac{\mathrm{d}\sigma_{\mathrm{NLO}}(m_T \to \infty)}{\mathrm{d}\sigma_{\mathrm{LO}}(m_T \to \infty)} \mathrm{d}\sigma_{\mathrm{LO}}(m_T)$$

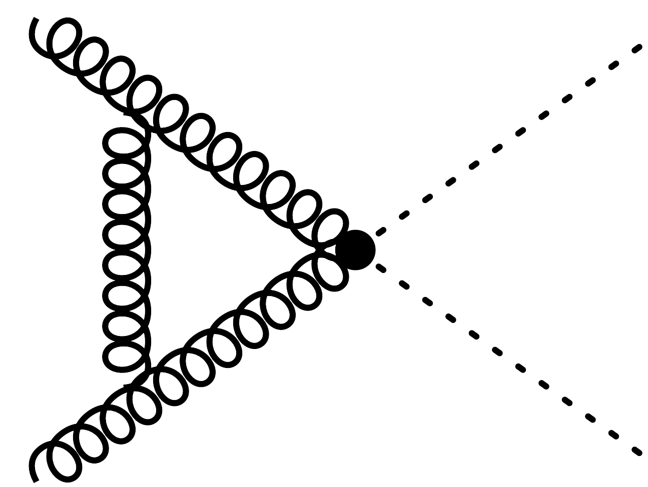
Spira et al. (HPAIR)

Gluon Fusion



1. LO (1-loop), Dominated by top (bottom <1%)
   Glover, van der Bij 88

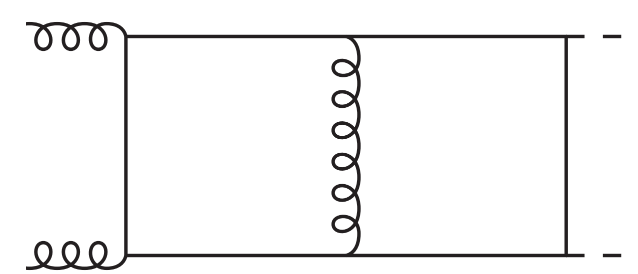
2. Born Improved NLO H(iggs)EFT $m_T \to \infty$ **K≈ 2**
   Dawson, Dittmaier, Spira 98



A. Including m_T in Real radiation **-10%**
   Maltoni, Vryonidou, Zaro 14

B. Including $\mathcal{O}(1/m_T^{12})$ terms in Virtual MEs **±10%**
   Grigo, Hoff, Melnikov, Steinhauser 13; Grigo, Hoff 14; Grigo, Hoff, Steinhauser 15

3. NLO (2-loop) with full top mass ← **this talk**
   Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16; Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16



(Transverse momentum) NLL + NLO
Ferrera, Pires 16

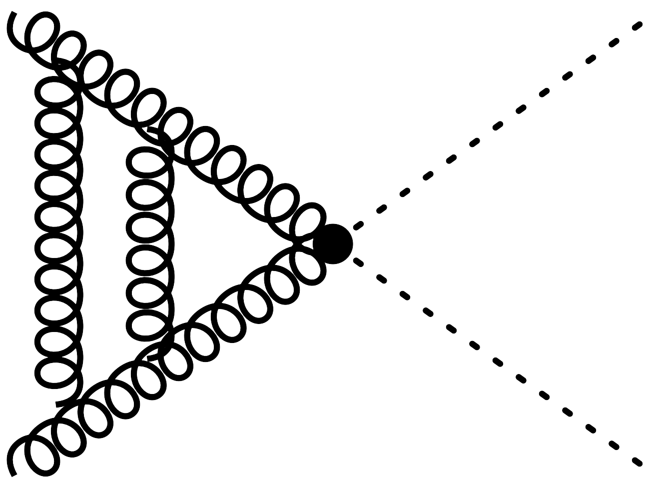
Gluon Fusion (II)

4. Born Improved NNLO HEFT **+20%**
   De Florian, Mazzitelli 13

   Including matching coefficients
   Grigo, Melnikov, Steinhauser 14

   Including terms $\mathcal{O}(1/m_T^4)$ in Virtual MEs
   Grigo, Hoff, Steinhauser 15

   (Threshold) NNLL + NNLO Matching **+9%**
   (SCET) Shao, Li, Li, Wang 13; de Florian, Mazzitelli 15

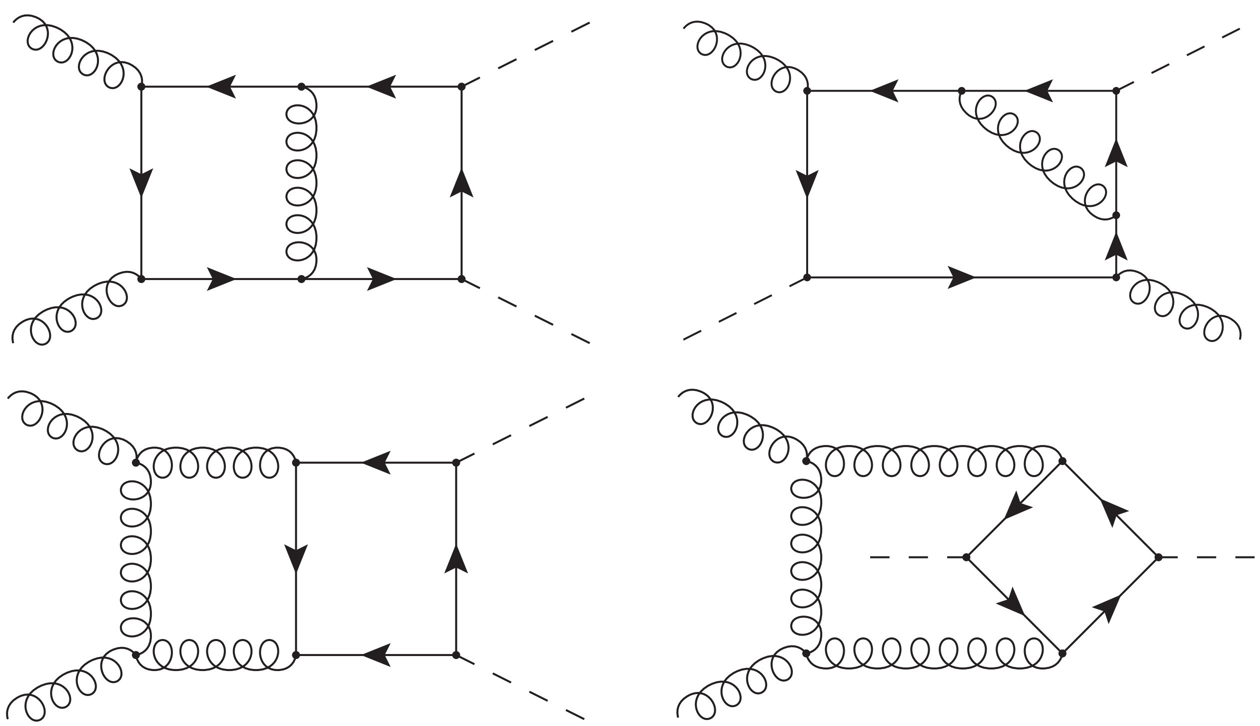


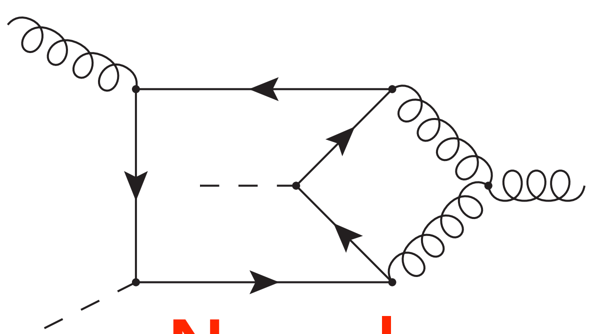
5. NNLO HEFT (Differential)
   de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16

NLO Calculation

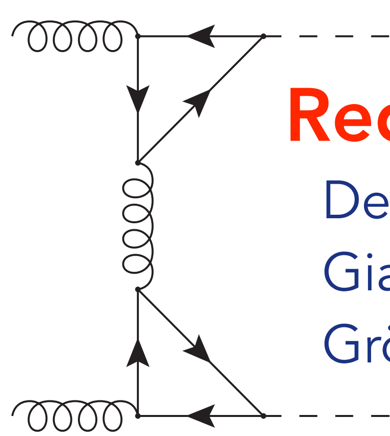
Virtual MEs: $gg \to HH$ $\cancel{q\bar{q} \to HH}$ ← **NNLO**

Yukawa only (≤ 4-point)





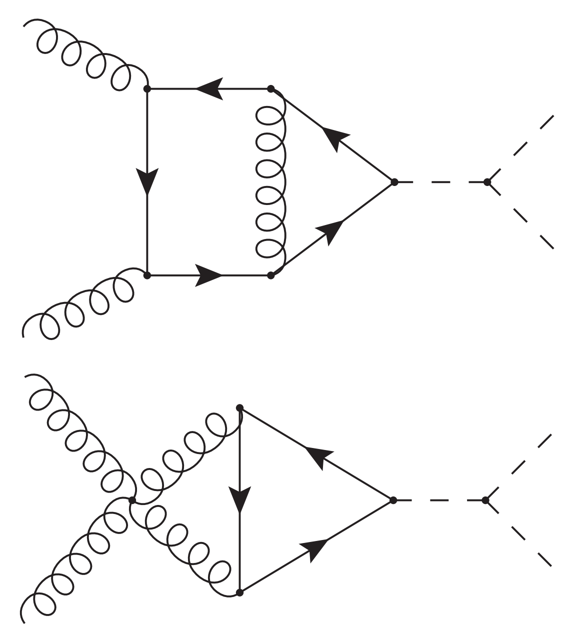
Non-planar



Reducible

Degrassi, Giardino, Gröber 16

Self-coupling (≤3-point)



Integrals Known

$gg \to H$

Spira, Djouadi et al. 93, 95; Bonciani, P. Mastrolia 03,04; Anastasiou, Beerli et al. 06;

Many integrals not known analytically, except:

$H \to Z\gamma$ Bonciani, Del Duca, Frellesvig et al. 15; Gehrmann, Guns, Kara 15;

Form Factor Decomposition

Expose tensor structure: $\mathcal{M} = \epsilon^1_\mu \epsilon^2_\nu \mathcal{M}^{\mu\nu}$

Form Factors (Contain integrals)

$$\mathcal{M}^{\mu\nu} = F_1(\hat{s}, \hat{t}, m_h^2, m_t^2, D) T_1^{\mu\nu} + F_2(\hat{s}, \hat{t}, m_h^2, m_t^2, D) T_2^{\mu\nu}$$

(Tensor) Basis, built from external momenta & metric

Choose:

$$\mathcal{M}^{++} = \mathcal{M}^{--} = -F_1$$
$$\mathcal{M}^{+-} = \mathcal{M}^{-+} = -F_2$$

Glover, van der Bij 88

Construct projectors such that:

$$P_1^{\mu\nu} \mathcal{M}_{\mu\nu} = F_1(\hat{s}, \hat{t}, m_h^2, m_t^2, D)$$
$$P_2^{\mu\nu} \mathcal{M}_{\mu\nu} = F_2(\hat{s}, \hat{t}, m_h^2, m_t^2, D)$$

Integral Reduction

Tensor integrals rewritten as inverse propagators

Scalar products:

$$S = \frac{l(l+1)}{2} + lm$$

$l = 2$ # Loops

$m = 3$ # L.I External momenta

$$\boxed{S = 9}$$

4 scales $\hat{s}, \hat{t}, m_T^2, m_H^2$

Choose 5 planar + 3 non-planar integral families

Integrals	1-loop	2-loop
Direct	63	9865
+ Symmetries	21	1601
+ IBPs	8	~260-270 (currently 327)

Reduction with `REDUZE 2`

von Manteuffel, Studerus 12

Up to 4 inverse propagators

Simplification, fix:

$$m_T = 173 \text{ GeV}, \quad m_H = 125 \text{ GeV}$$

(Mostly) Finite Basis

Panzer 14; von Manteuffel, Panzer, Schabinger 15

Non-planar integrals computed mostly without reduction

Amplitude Evaluation

All master integrals processed with SecDec Borowka, Heinrich, Jahn, SJ, Kerner, Schlenk, Zirke

Sector decompose Feynman integrals Hepp 66; Denner, Roth 96; Binoth, Heinrich 00

Contour deformation Soper 00; Binoth, Guillet, Heinrich et al. 05; Nagy, Soper 06; Borowka et al. 12

Use Quasi-Monte-Carlo (QMC) integration $\mathcal{O}(n^{-1})$ error scaling

Review: Dick, Kuo, Sloan 13; Li, Wang, Yan, Zhao 15

Implemented in OpenCL, evaluated on GPUs

Entire 2-loop amplitude evaluated with a single code

$$F = \sum_i \underbrace{\left(\sum_j C_{i,j} \epsilon^j \right)}_{\text{coeff.}} \underbrace{\left(\sum_k I_{i,k} \epsilon^k \right)}_{\text{integral}} = \epsilon^{-2} \left[C^{(L)}_{1,-2} I^{(L)}_{1,0} + \ldots \right] + \epsilon^{-1} \left[C^{(L)}_{1,-1} I^{(L)}_{1,0} + \ldots \right] + \ldots$$

compute once

Dynamically set target precision for each sector, minimising time:

$$T = \sum_i t_i + \bar{\lambda} \left(\sigma^2 - \sum_i \sigma_i^2 \right), \quad \sigma_i \sim t_i^{-e}$$

$\bar{\lambda}$ – Lagrange multiplier

σ – precision goal

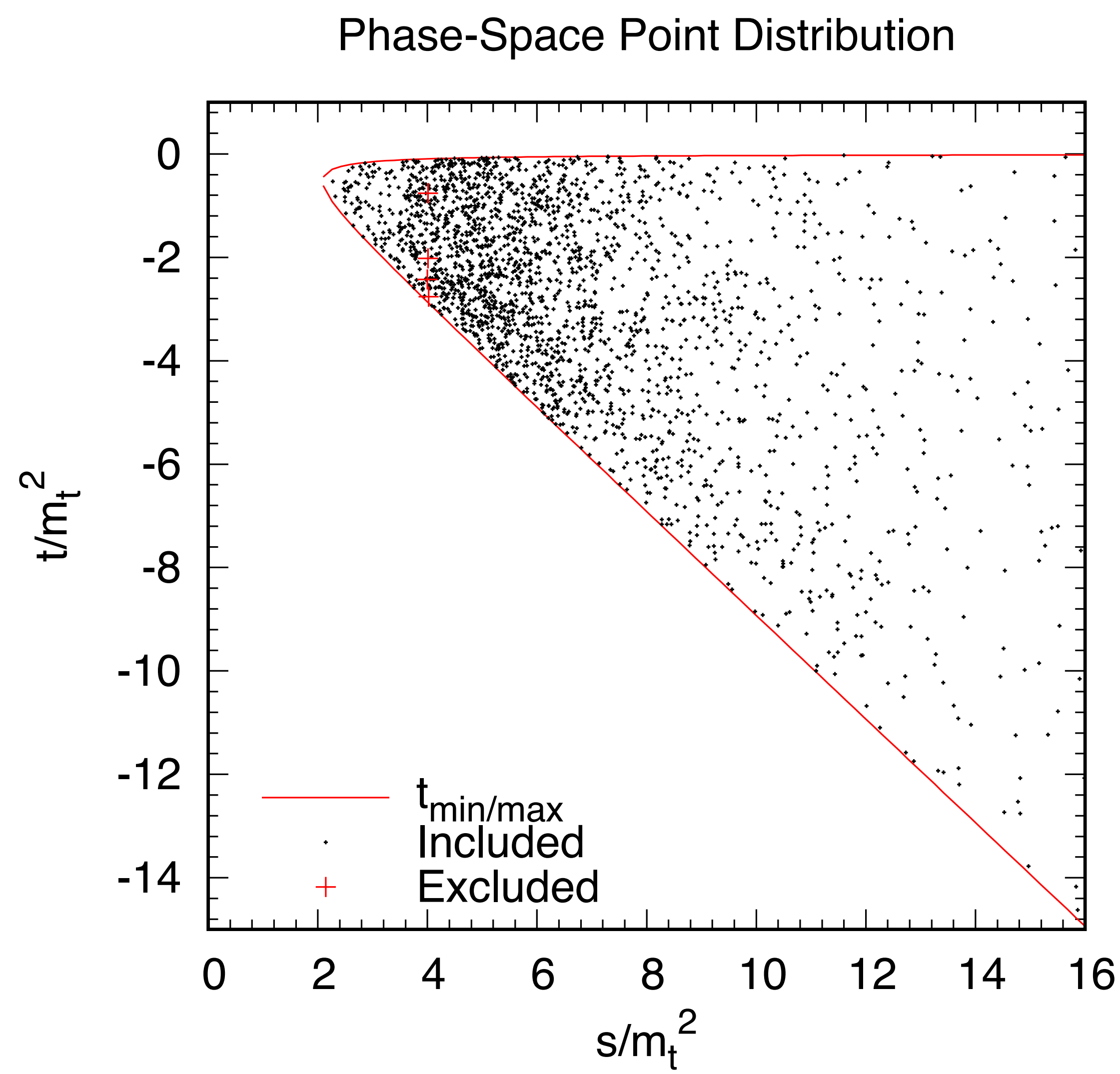
σ_i – error estimate

Phase-space Sampling

1) VEGAS algorithm applied to LO calculation; $\mathcal{O}(100k)$ events computed

2) unweighted LO events using accept/reject method; $\mathcal{O}(30k)$ events remain

3) Randomly select 917+150 events, compute at NLO, exclude 4+1

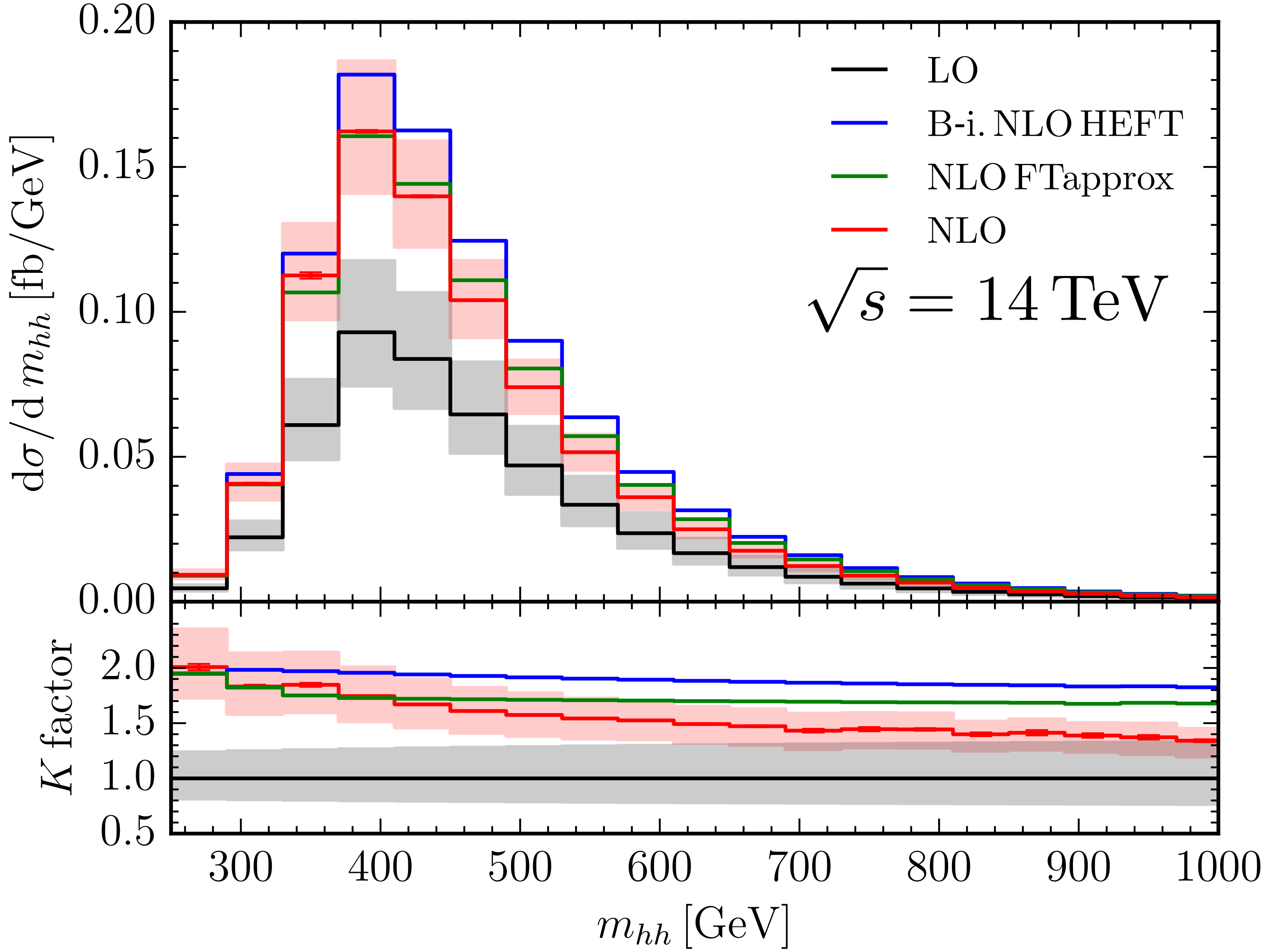


Accuracy goal: 3% for F_1
5-20% for F_2 (depending on F_2/F_1)

GPU Time/PS point: 80 min - 2 d (=wall-clock limit)
median 2 h

Additional 1488 events now on disk (not used yet)

Results: Invariant Mass



PDF4LHC15_nlo_30_pdfas
$m_H = 125$ GeV
$m_T = 173$ GeV
Uncertainty:
$\mu_0 = \frac{m_{HH}}{2}$
$\mu_{R,F} \in \left[\frac{\mu_0}{2}, 2\mu_0\right] \quad (7-\text{point})$

HEFT: Outside scale var.

$m_{hh} > 420$ GeV

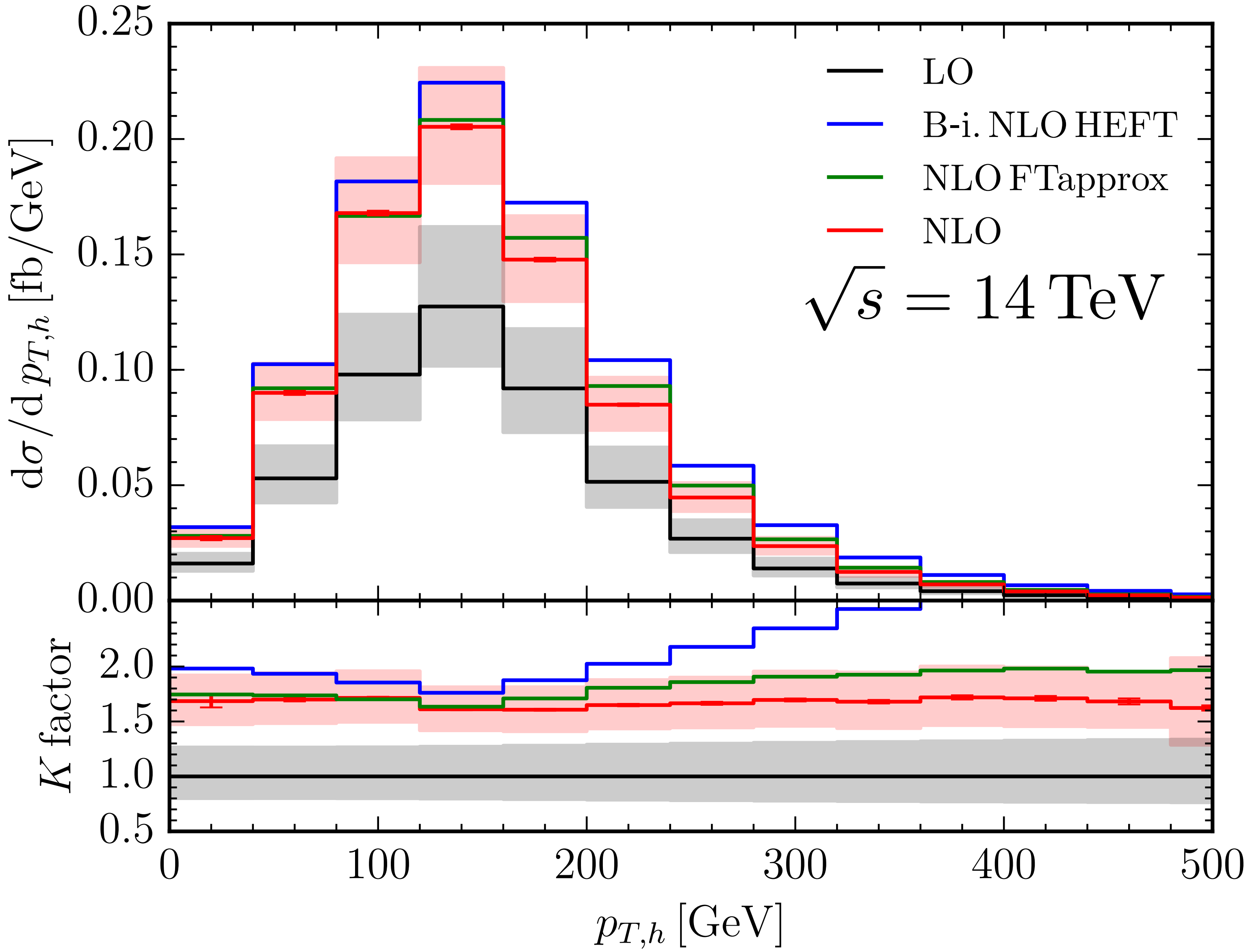
FTapp: Outside scale var.

$m_{hh} > 620$ GeV

	σ_{LO} (fb)	σ_{NLO} (fb)
B.I. HEFT	$19.85^{+27.6\%}_{-20.5\%}$	$38.32^{+18.1\%}_{-14.9\%}$
FTapprox	$19.85^{+27.6\%}_{-20.5\%}$	$34.26^{+14.7\%}_{-13.2\%}$
Full Theory	$19.85^{+27.6\%}_{-20.5\%}$	$\mathbf{32.91^{+13.6\%}_{-12.6\%}} \pm \mathbf{0.3}\%(\text{stat.}) \pm \mathbf{0.1}\%(\text{int.})$

HEFT overestimates by 16%
FTap. overestimates by 4%

Results: pT either Higgs



HEFT: Can poor approx. for larger $p_{T,h}$

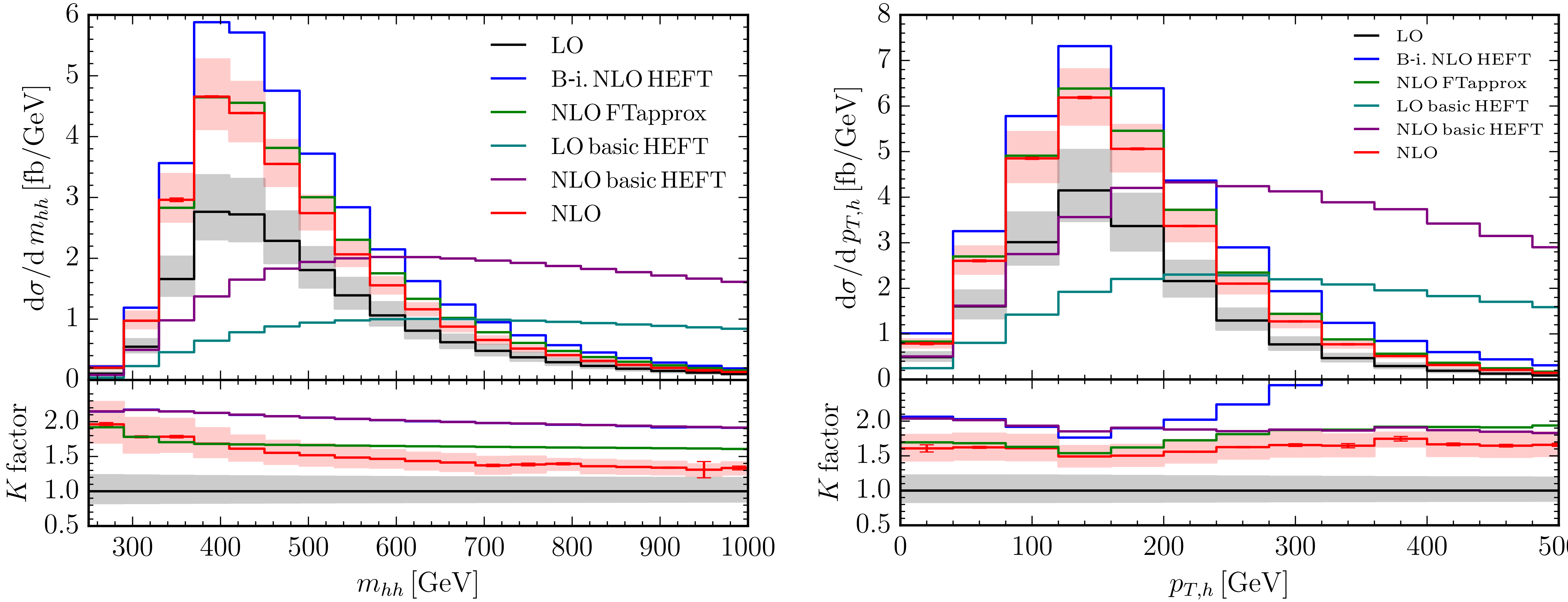
Note: ambiguous how to rescale HEFT reals by full LO born differentially

FTapp: Significantly better but still overestimating

Real radiation plays larger role for large $p_{T,h}$

(As hoped) Including full reals does improve over HEFT in tails

Results: 100TeV



	$\sigma_{\rm LO}$ (fb)	$\sigma_{\rm NLO}$ (fb)
B.I. HEFT	–	$1511^{+16.0\%}_{-13.0\%}$
FTapprox	–	$1220^{+11.9\%}_{-10.7\%}$
Full Theory	$731.3^{+20.9\%}_{-15.9\%}$	$1149^{+10.8\%}_{-10.0\%}$

HEFT overestimates by 32%

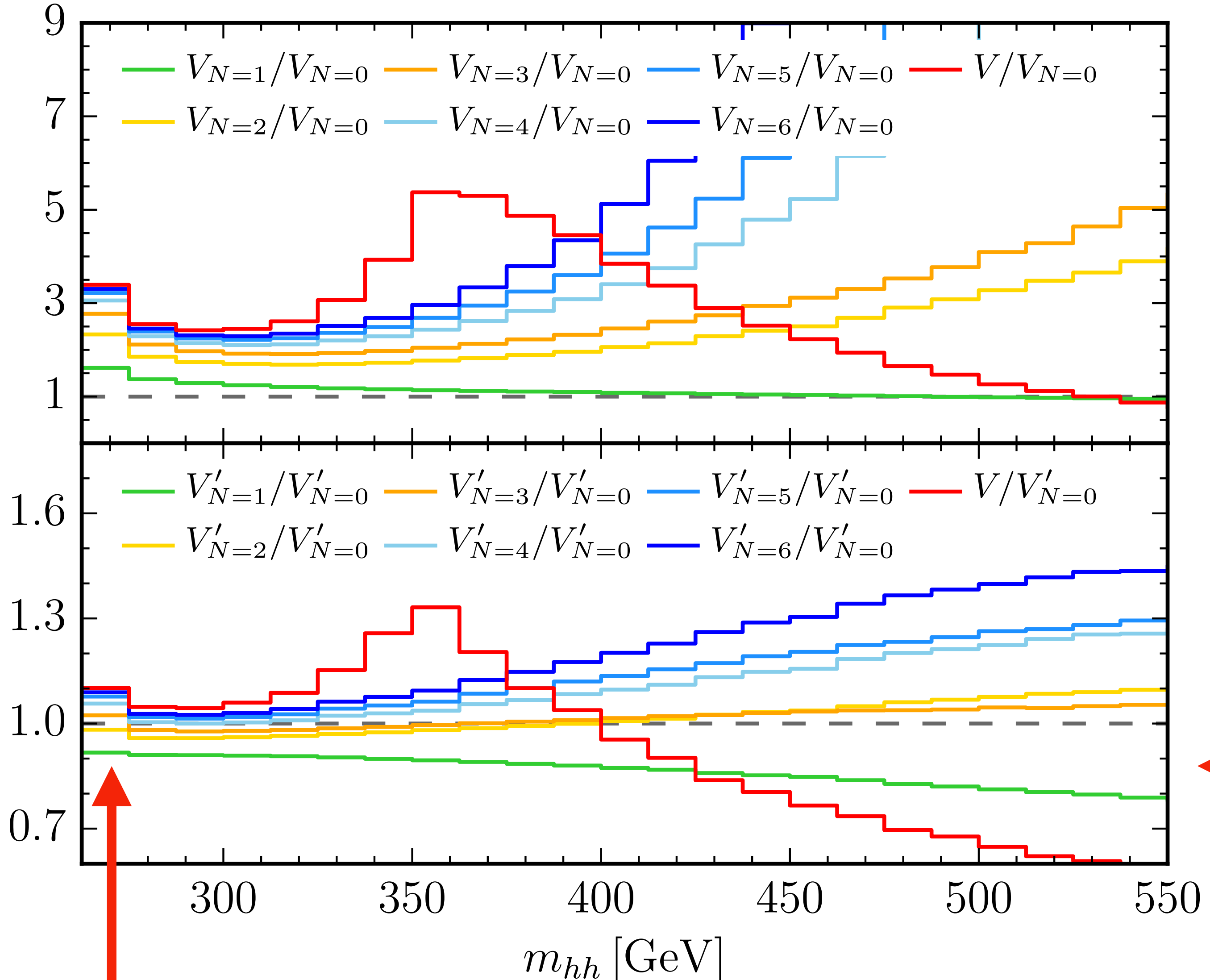
FTap. overestimates by 6%

Difference between full theory and HEFT more pronounced

Comparison to Expansion

Can compare just virtual ME to expansion:

$$d\hat{\sigma}_N = \sum_{\rho=0}^{N} d\hat{\sigma}^{(\rho)} \left(\frac{\Lambda}{m_t}\right)^{2\rho} \qquad \Lambda \in \left\{\sqrt{\hat{s}}, \sqrt{\hat{t}}, \sqrt{\hat{u}}, m_h\right\}$$



$$V_N = \left(d\hat{\sigma}_N^V + d\hat{\sigma}_N^{LO} \otimes \mathbf{I}\right)$$

$$V'_N = V_N \frac{d\hat{\sigma}^{LO}}{d\hat{\sigma}_N^{LO}}$$

Rescaled better but does not describe full above threshold

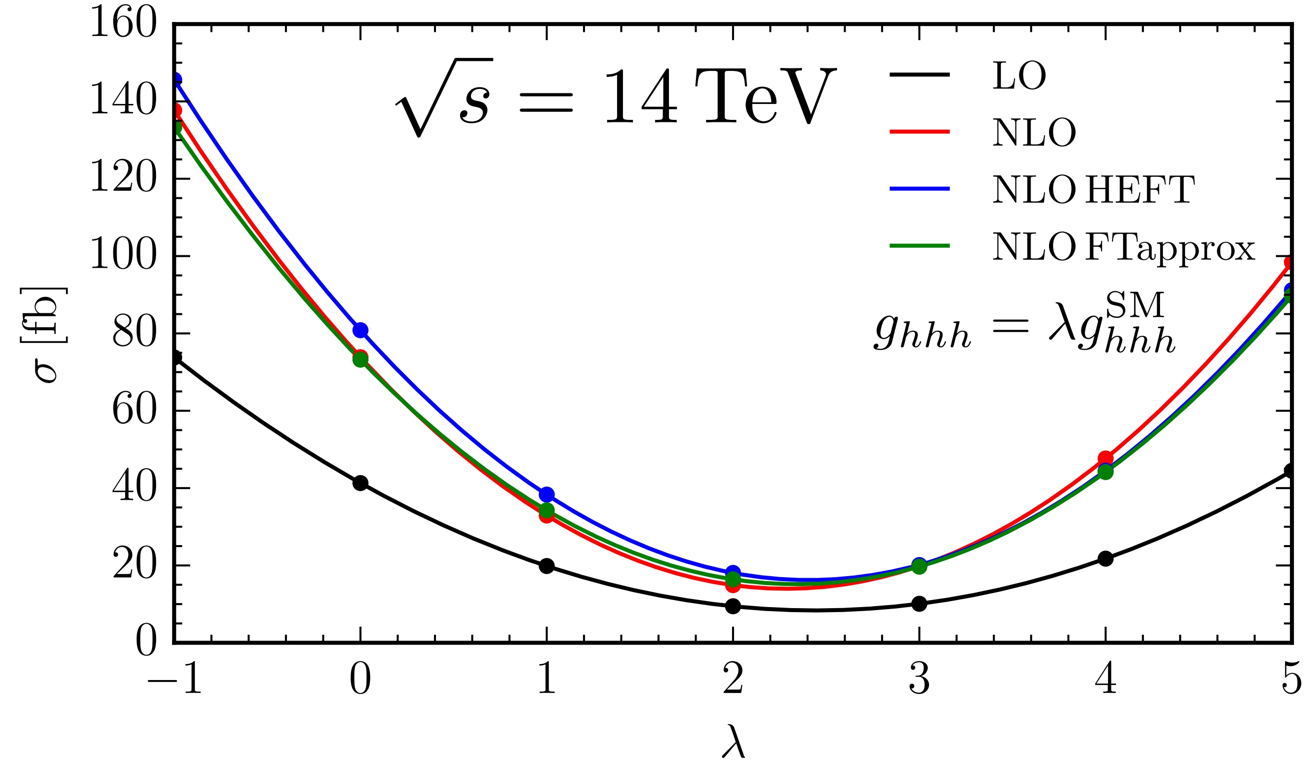
Expansion converges on full $\sqrt{\hat{s}} < 2m_T$

$V_{N\geq 4}$ thanks to J. Hoff

Grigo, Hoff, Steinhauser 15

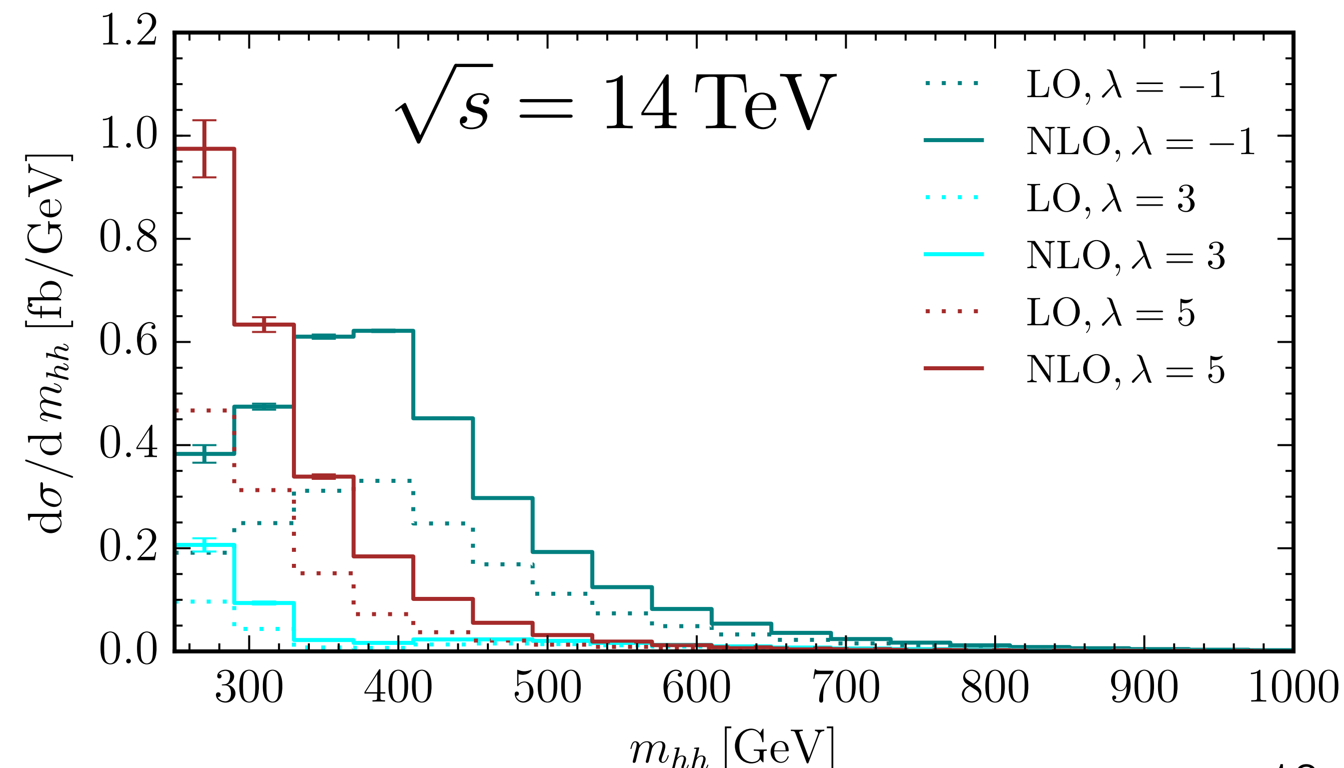
Triple-Higgs Coupling Sensitivity

Note: Just varying λ: one ``direction'' in EFT parameter space



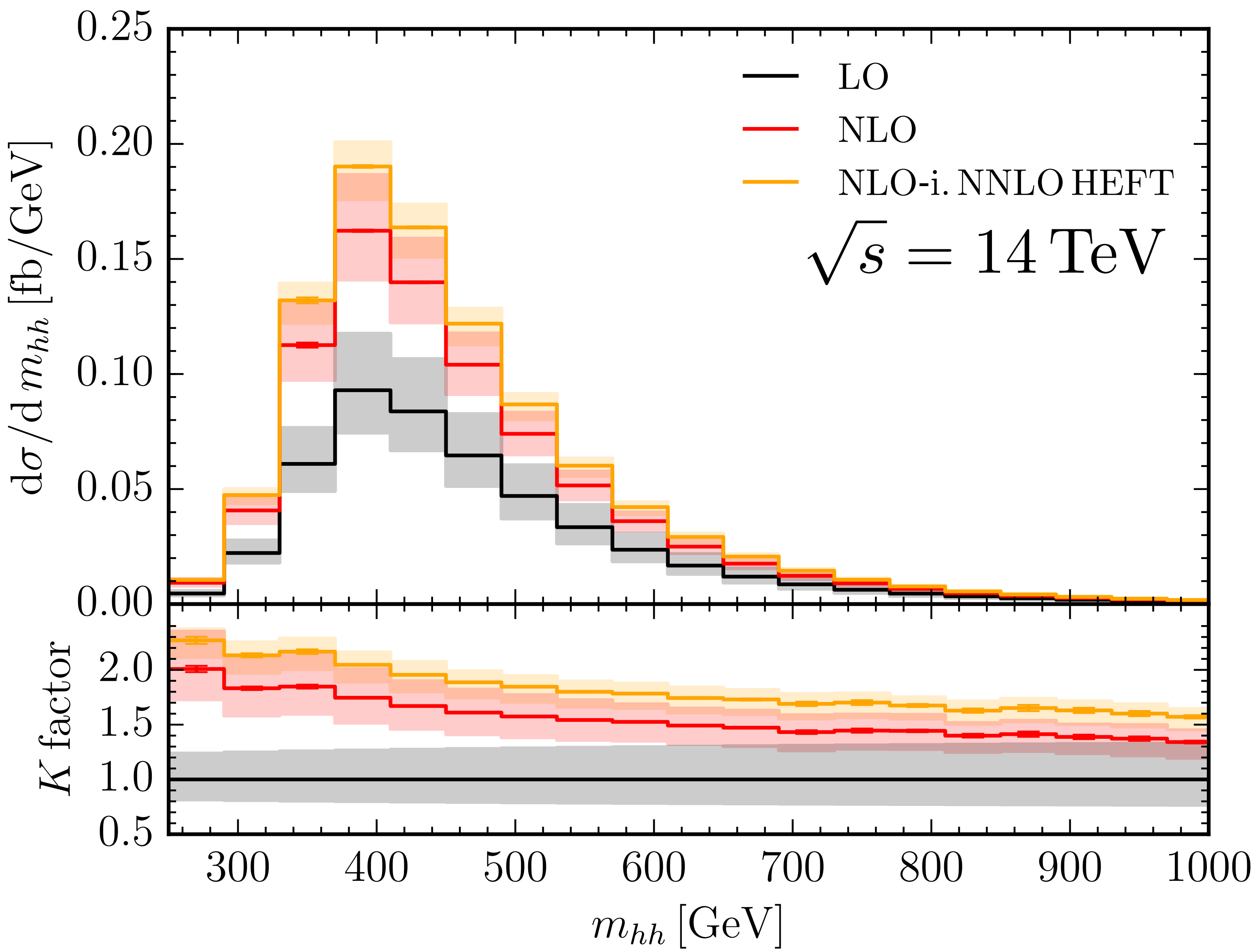
SM: Destructive interference between g_{hhh} and y_T^2 contrib.

Quadratic dependence on λ (at LO in λ)



Distributions: can help to distinguish between λ values that give same total cross-section

NLO Improved NNLO HEFT



First attempt to combine full NLO

Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16

+

NNLO HEFT (Differential)

de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16

$$\frac{\mathrm{d}\sigma^{\mathrm{approx.}}}{\mathrm{d}m_{hh}} \equiv \frac{\mathrm{d}\sigma_{\mathrm{NLO}}}{\mathrm{d}m_{hh}} \times \frac{\mathrm{d}\sigma^{\mathrm{HEFT}}_{\mathrm{NNLO}}/\mathrm{d}m_{hh}}{\mathrm{d}\sigma^{\mathrm{HEFT}}_{\mathrm{NLO}}/\mathrm{d}m_{hh}}$$

Bin-by-bin rescaling of NLO by NNLO HEFT K-factor

$$\sigma^{approx.} = 38.67^{+5.2\%}_{-7.6\%}$$

Conclusion

Gluon Fusion

- Key measurement for probing the self coupling (HL-LHC era)
- NLO deviates from Born Improved HEFT
  -14% @ 14 TeV, -24% @ 100 TeV
- Distributions altered significantly

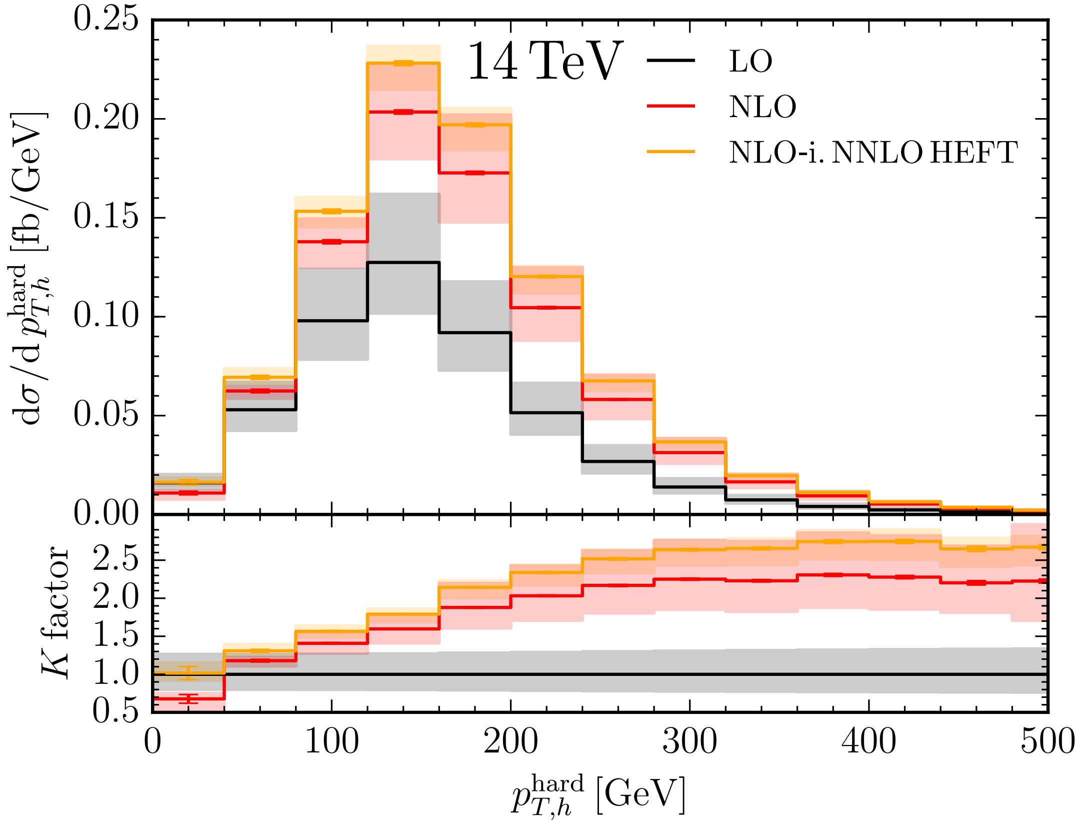
Future

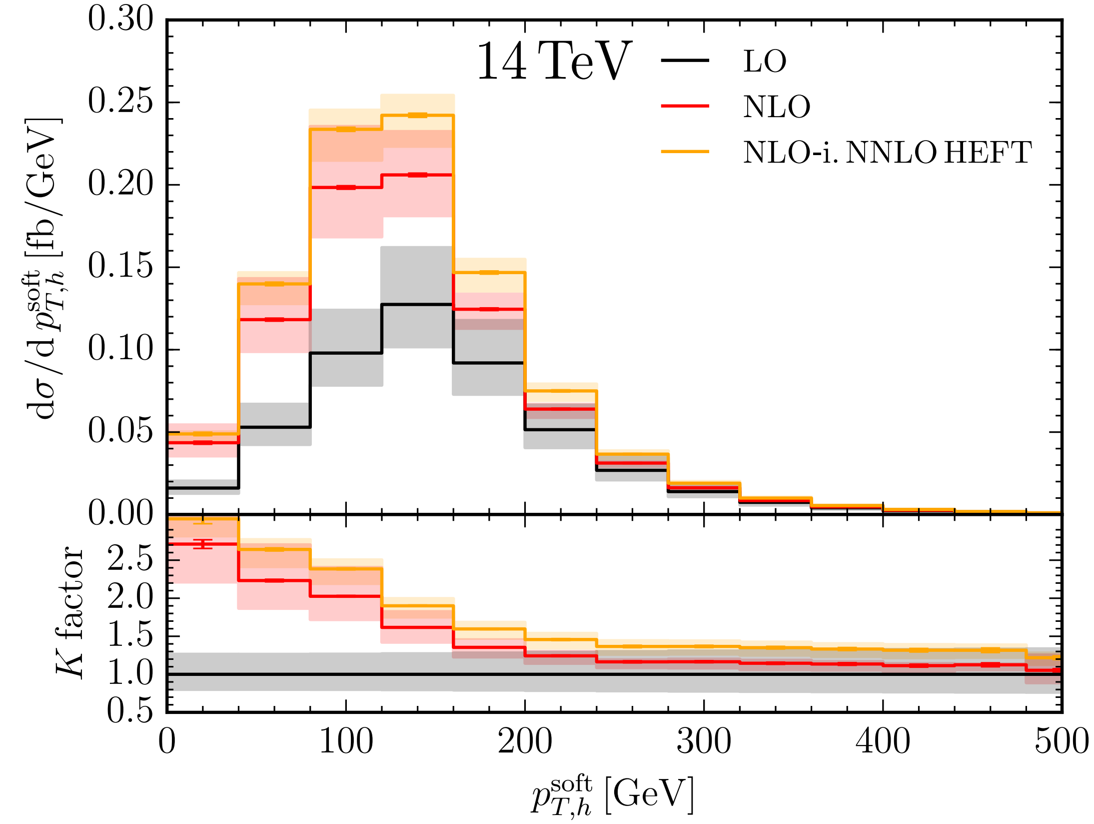
- Fully differential/improved combination with NNLO HEFT
- Grid (faster evaluation of virtuals)
- Parton Shower: `POWHEG`, `MG5_aMC@NLO`, `Herwig`, `Sherpa`
- EFT/2HDM analysis (?)
- Apply methods/framework `GoSam-2L+SecDec` to other processes

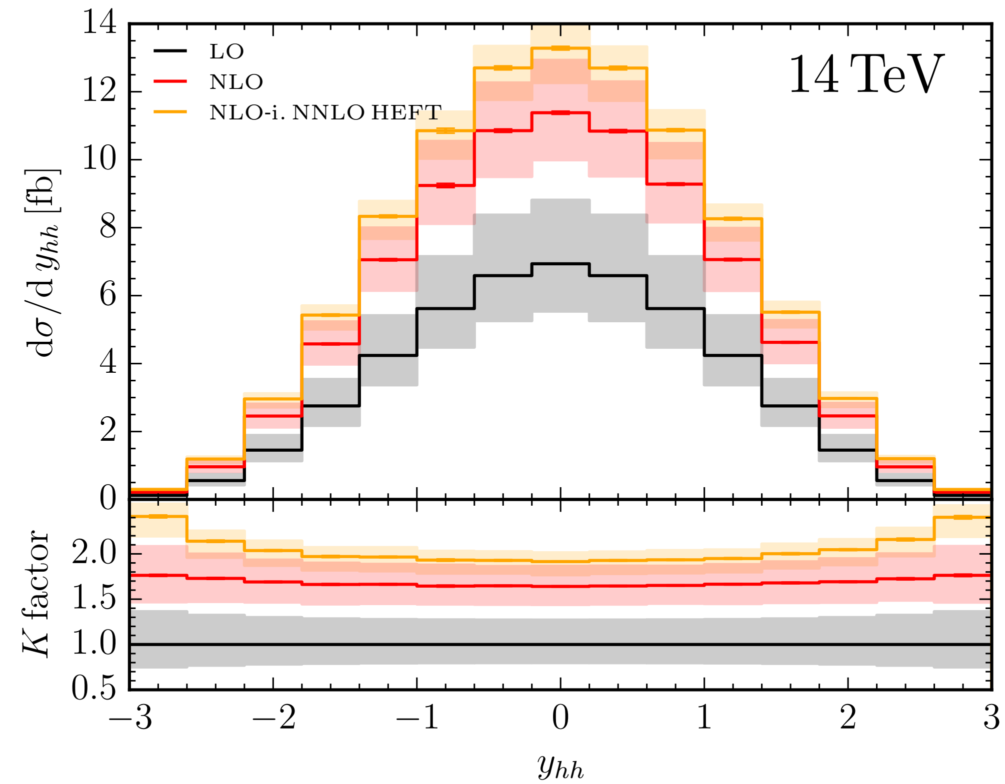
Thank you for listening!

Backup

NLO Improved NNLO HEFT (II)







Total Cross Section @ 14 TeV

	σ_{LO} (fb)	σ_{NLO} (fb)	σ_{NNLO} (fb)
HEFT	$17.07^{+30.9\%}_{-22.2\%}$	$31.93^{+17.6\%}_{-15.2\%}$	$37.52^{+5.2\%}_{-7.6\%}$
B.I. HEFT	$19.85^{+27.6\%}_{-20.5\%}$	$38.32^{+18.1\%}_{-14.9\%}$	$43.63^{+5.2\%}_{-7.6\%}*$
FTapprox	$19.85^{+27.6\%}_{-20.5\%}$	$34.26^{+14.7\%}_{-13.2\%}$	–
Full Theory	$19.85^{+27.6\%}_{-20.5\%}$	$32.91^{+13.6\%}_{-12.6\%}$	–
N.I. HEFT	–	$32.91^{+13.6\%}_{-12.6\%}$	$38.67^{+5.2\%}_{-7.6\%}*$

PDF4LHC15_nlo_30_pdfas

$m_H = 125$ GeV

$m_T = 173$ GeV

Uncertainty:

$$\mu_R = \mu_F = \frac{m_{HH}}{2}$$

$$\mu \in \left[\frac{\mu_0}{2}, 2\mu_0\right] \quad (7-\text{point})$$

* re-weighted on total cross-section level

de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16;
Maltoni, Vryonidou, Zaro 14 (recalculated by us); Borowka, Greiner, Heinrich, Kerner, Schlenk, Schubert, Zirke 16;
Dawson, Dittmaier, Spira 98 (recalculated by us); Glover, van der Bij 88 (recalculated by us)

Comparison to Full Theory

	$\Delta\sigma^{\text{Full}}_{\text{LO}}$	$\Delta\sigma^{\text{Full}}_{\text{NLO}}$
HEFT	-14%	-3.0%
B.I. HEFT	0%	$+16\%$
FTapprox	0%	$+4.1\%$

Can do a similar exercise @ 100 TeV, differences typically larger

YR4 Numbers

YR4 Prescription:

$$\sigma(gg \to hh)^{exact}_{NLO} = \sigma(gg \to hh)^{HEFT}_{NLO}(1 + \delta_t)$$

$$\sigma'_{NNLL} = \sigma_{NNLL} + \delta_t \sigma^{HEFT}_{NLO}$$

$\sqrt{s}$	σ'_{NNLL} (fb)	Scale Unc. (%)	PDF Unc. (%)	α_S Unc. (%)
7 TeV	7.078	$+4.0 - 5.7$	± 3.4	± 2.8
8 TeV	10.16	$+4.1 - 5.7$	± 3.1	± 2.6
13 TeV	33.53	$+4.3 - 6.0$	± 2.1	± 2.3
14 TeV	39.64	$+4.4 - 6.0$	± 2.1	± 2.2

Checks

Real Emission & Catani-Seymour Subtraction Terms Catani, Seymour 96

Independence of dipole cut parameter Nagy 03

Real + HEFT agrees with `MG5_AMC@NLO`

Maltoni, Vryonidou, Zaro 14

Virtual Corrections

2 calculations of unreduced amplitude

2 calculations of mass renormalization (CT vs $\mathrm{d}\mathcal{M}^{\mathrm{LO}}/\mathrm{d}m_T^2$ numerically)

(Some) integrals cross-checked with VEGAS Lepage 80; Hahn (Cuba)

Amplitude invariant under crossing

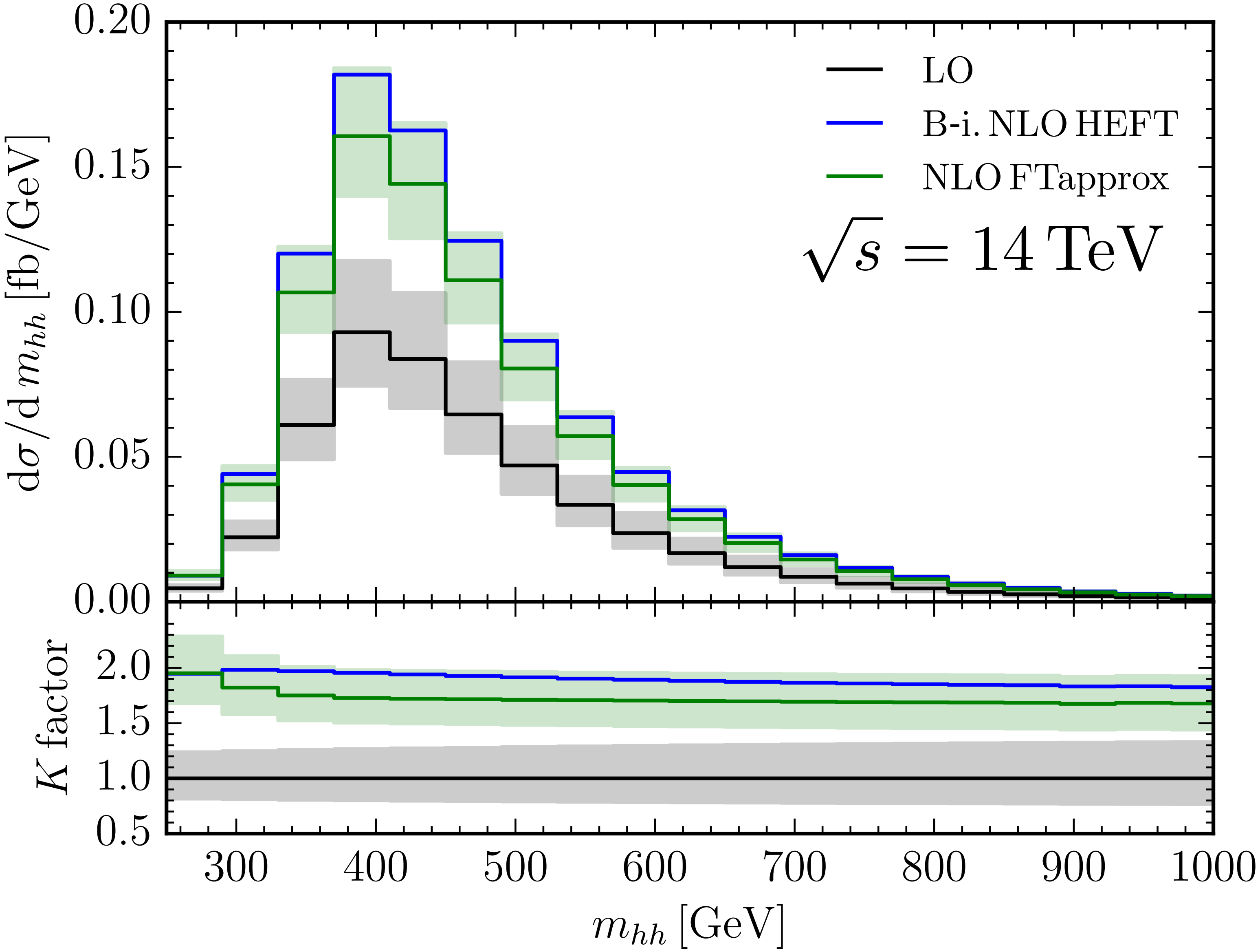
Numerical pole cancellation (5 digits)

Single Higgs production part agrees with `SusHi` Harlander, Liebler, Mantler 13,16;

$1/m_T$ result converges to full result below top threshold

Grigo, Hoff, Steinhauser 15

A. FT approx

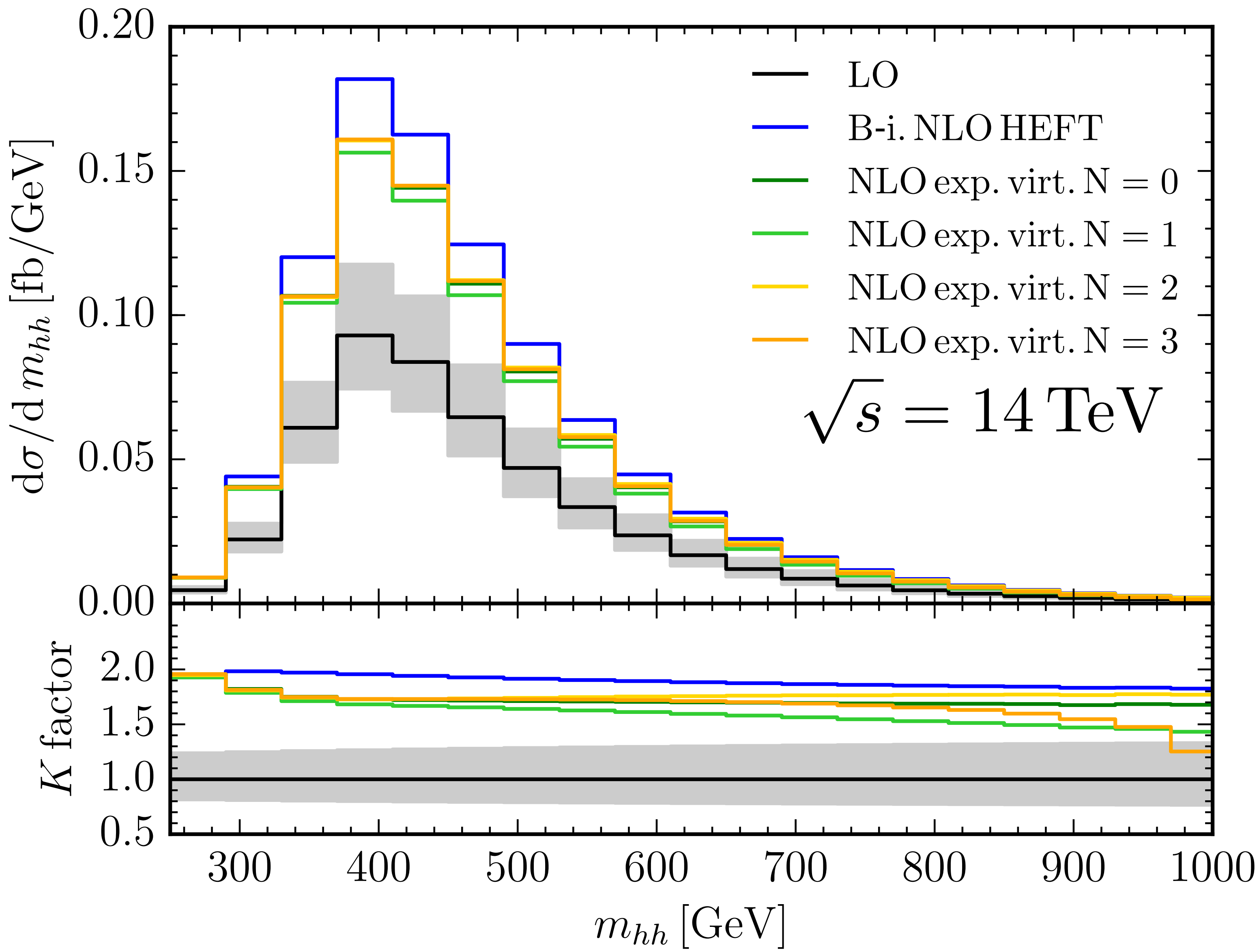


Distribution: Agreement between HEFT approximations in first bin where $\sqrt{\hat{s}} \approx 2m_H$, not much hard real emission

	σ_{LO} (fb)	σ_{NLO} (fb)
B.I. HEFT	$19.85^{+27.6\%}_{-20.5\%}$	$38.32^{+18.1\%}_{-14.9\%}$
FTapprox	$19.85^{+27.6\%}_{-20.5\%}$	$34.26^{+14.7\%}_{-13.2\%}$
Full Theory	$19.85^{+27.6\%}_{-20.5\%}$	...

Total: m_T in only reals suppresses XS by 11% compared to HEFT

B. Expansion in Top Quark Mass



Low m_{hh} **:** Expansion seems ok in first bin

$$\sqrt{\hat{s}} < 2m_T$$

Increasing m_{hh} **:** Fewer reasons to trust expansion

Total: $\mathcal{O}(5\%)$ differences between first few terms of expansion

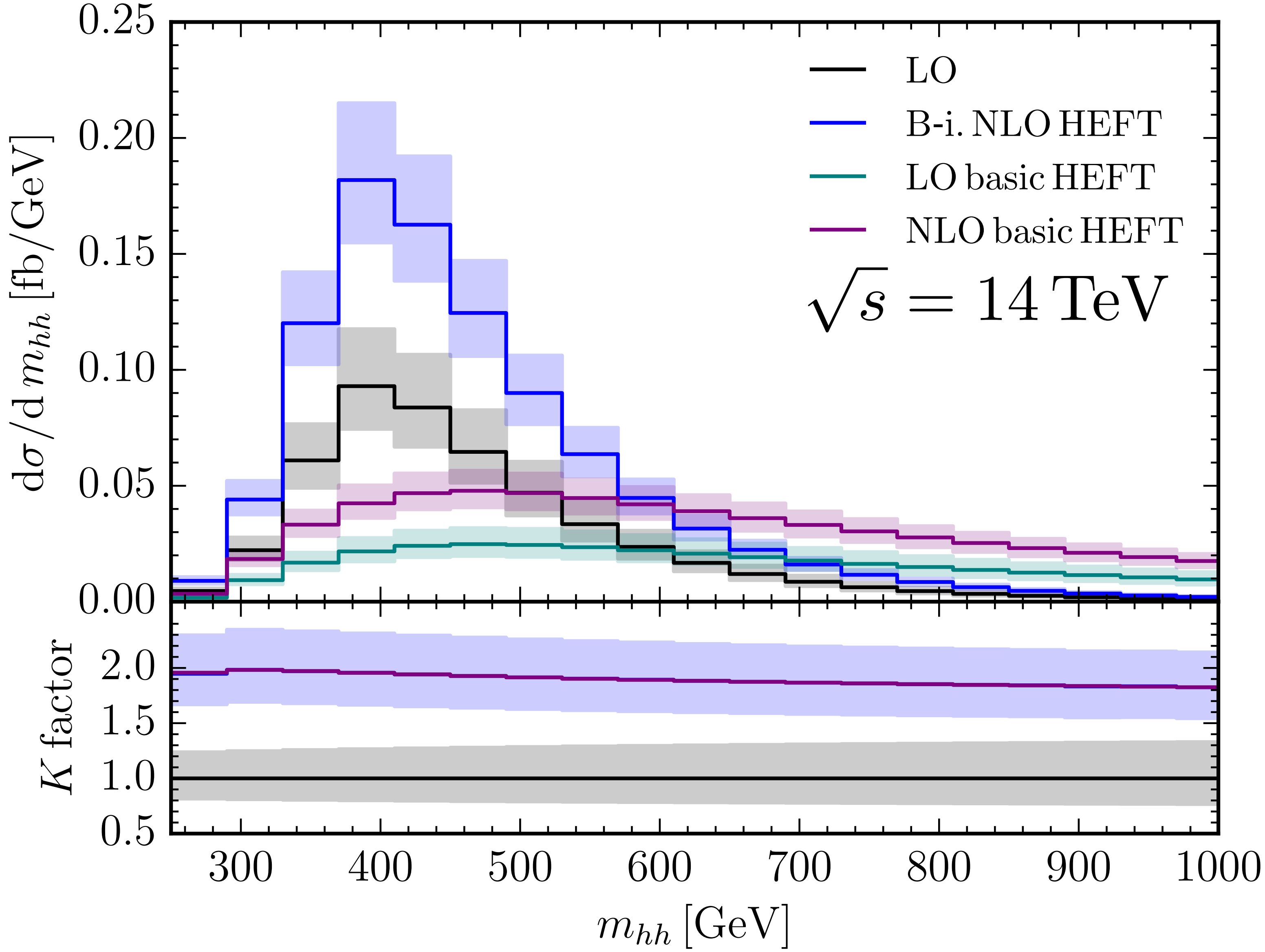
(Tom Zirke) Virtuals: asymptotic expansion in $1/m_T^2$ (`q2e/exp`+ `Reduze` + `matad`)

Harlander, Seidensticker, Steinhauser 97,99; von Manteuffel, Studerus 12; Steinhauser 00

Mass effects give large uncertainty

Required NLO calculation with full mass dependence

LO & Born Improved NLO HEFT



PDF4LHC15_nlo_30_pdfas

$m_H = 125$ GeV

$m_T = 173$ GeV

Uncertainty:

$\mu_R = \mu_F = \frac{m_{HH}}{2}$

$\mu \in \left[\frac{\mu_0}{2}, 2\mu_0\right] \quad (7 - \text{point})$

	σ_{LO} (fb)	σ_{NLO} (fb)
HEFT	$17.07^{+30.9\%}_{-22.2\%}$	$31.93^{+17.6\%}_{-15.2\%}$
B.I. HEFT	$19.85^{+27.6\%}_{-20.5\%}$	$38.32^{+18.1\%}_{-14.9\%}$
Full Theory	$19.85^{+27.6\%}_{-20.5\%}$	$\ldots$

LO: HEFT describes distributions poorly, underestimates XS @ LO by 14%

NLO: HEFT indicates

$$K \approx 2$$

NLO HEFT

Born Improved NLO QCD HEFT

$$\mathrm{d}\sigma_{\mathrm{NLO}}(m_T) \approx \mathrm{d}\bar{\sigma}_{\mathrm{NLO}}(m_T) \equiv \frac{\mathrm{d}\sigma_{\mathrm{NLO}}(m_T \to \infty)}{\mathrm{d}\sigma_{\mathrm{LO}}(m_T \to \infty)} \mathrm{d}\sigma_{\mathrm{LO}}(m_T)$$

$$K \approx 2$$

A. FTapprox

Maltoni et al.14

$$\mathrm{d}\overline{\sigma}^V(m_T)$$

$$\mathrm{d}\sigma^R(m_T)$$

$$-10\%$$

B. Expansion

Grigo, Hoff, Steinhauser 15

$$\mathrm{d}\hat{\sigma}(m_T) \equiv \mathrm{d}\sigma_0 + \mathrm{d}\sigma_1 \frac{m_H^2}{m_T^2} + \ldots + \mathrm{d}\sigma_6 \frac{m_H^{12}}{m_T^{12}}$$

$$\mathrm{d}\bar{\sigma}^{SV}_{\mathrm{NLO}}(m_T) \equiv \mathrm{d}\hat{\sigma}^{SV}_{\mathrm{NLO}}(m_T) \frac{\mathrm{d}\sigma^V_{\mathrm{LO}}(m_T)}{\mathrm{d}\hat{\sigma}^V_{\mathrm{LO}}(m_T)}$$

$$\mathrm{d}\bar{\sigma}^{H}_{\mathrm{NLO}}(m_T) \equiv \mathrm{d}\hat{\sigma}^{H}_{\mathrm{NLO}}(m_T) \frac{\sigma^V_{\mathrm{LO}}(m_T)}{\hat{\sigma}^V_{\mathrm{LO}}(m_T)}$$

$$\pm 10\%$$

Top-quark Width Effects

Total XS @ LO: reduced by 2% by including top-quark width

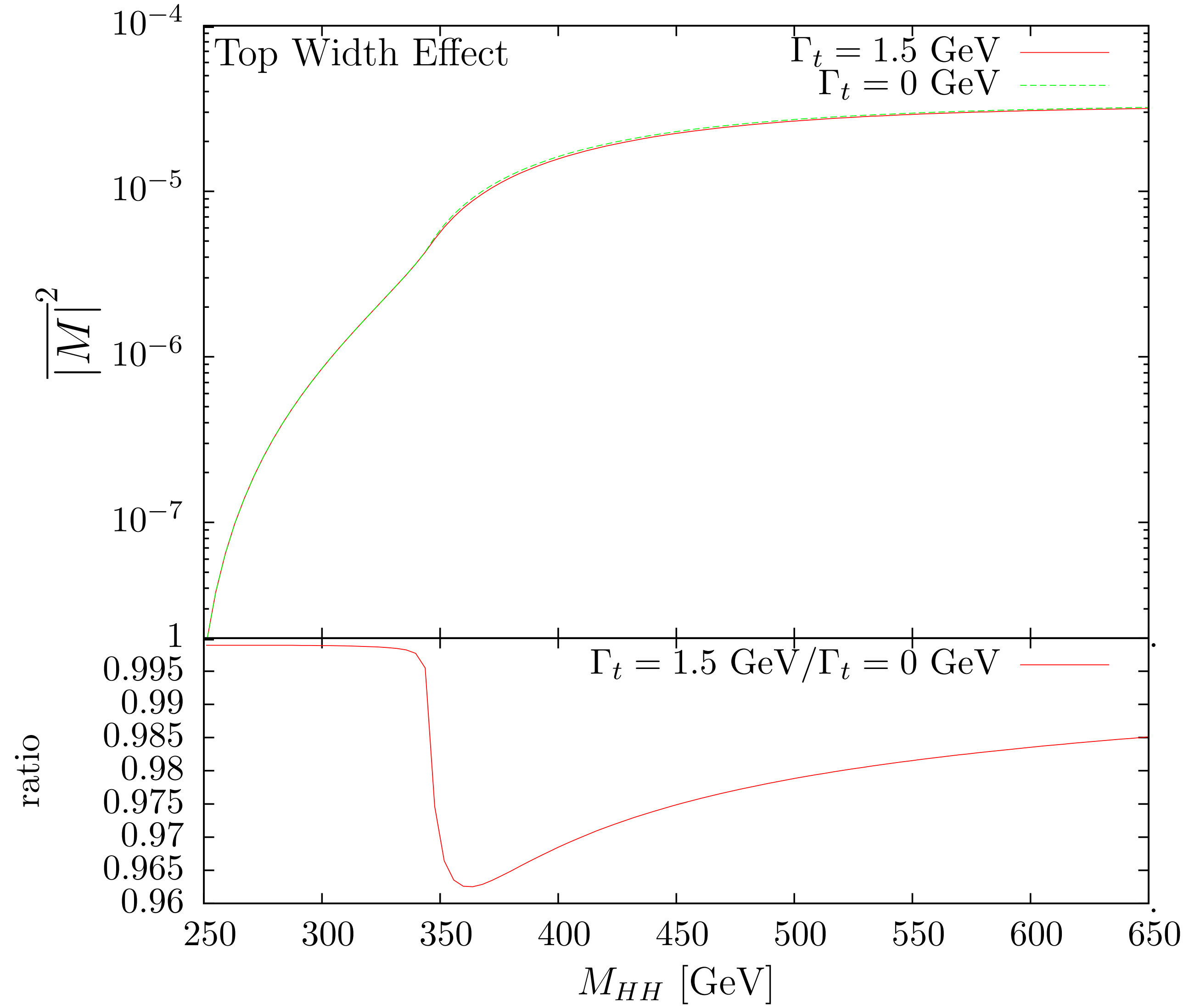
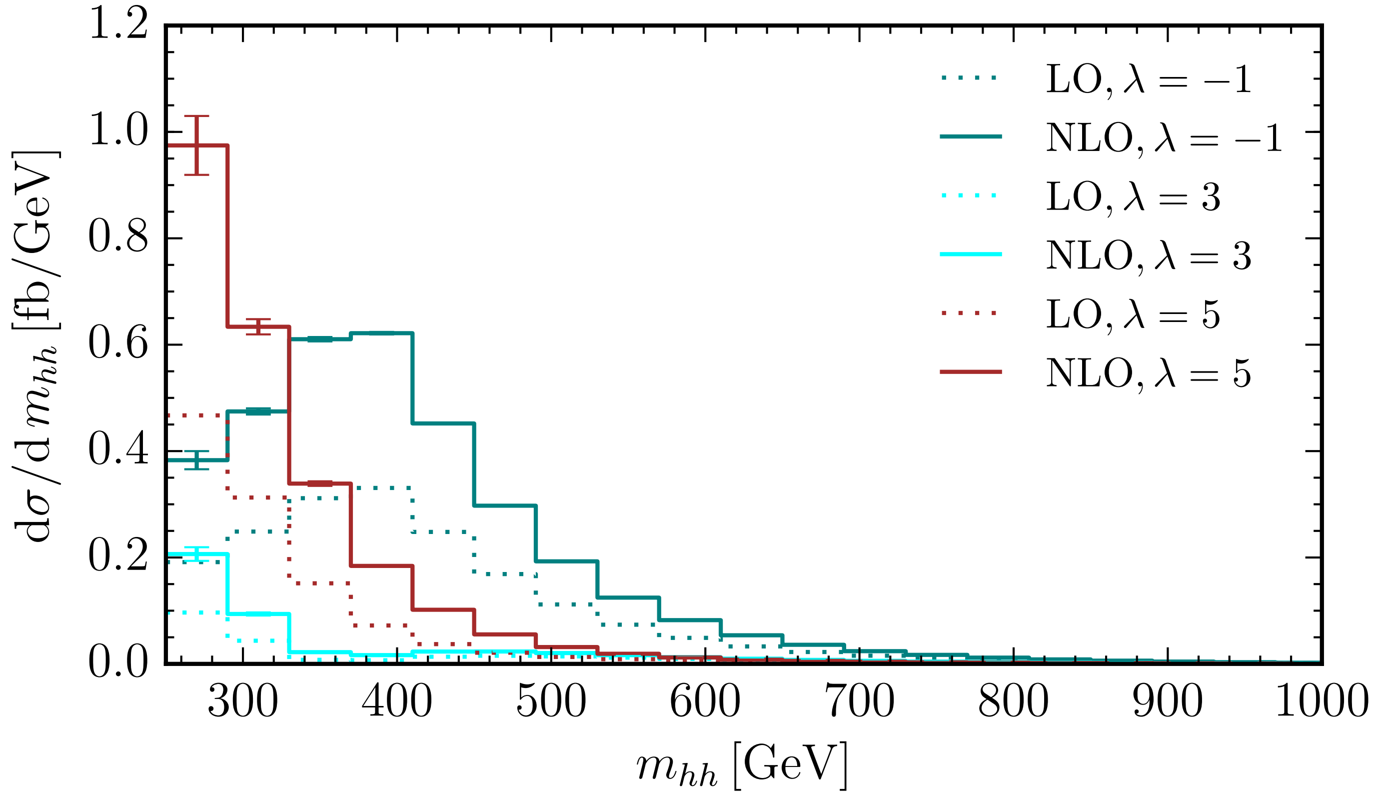


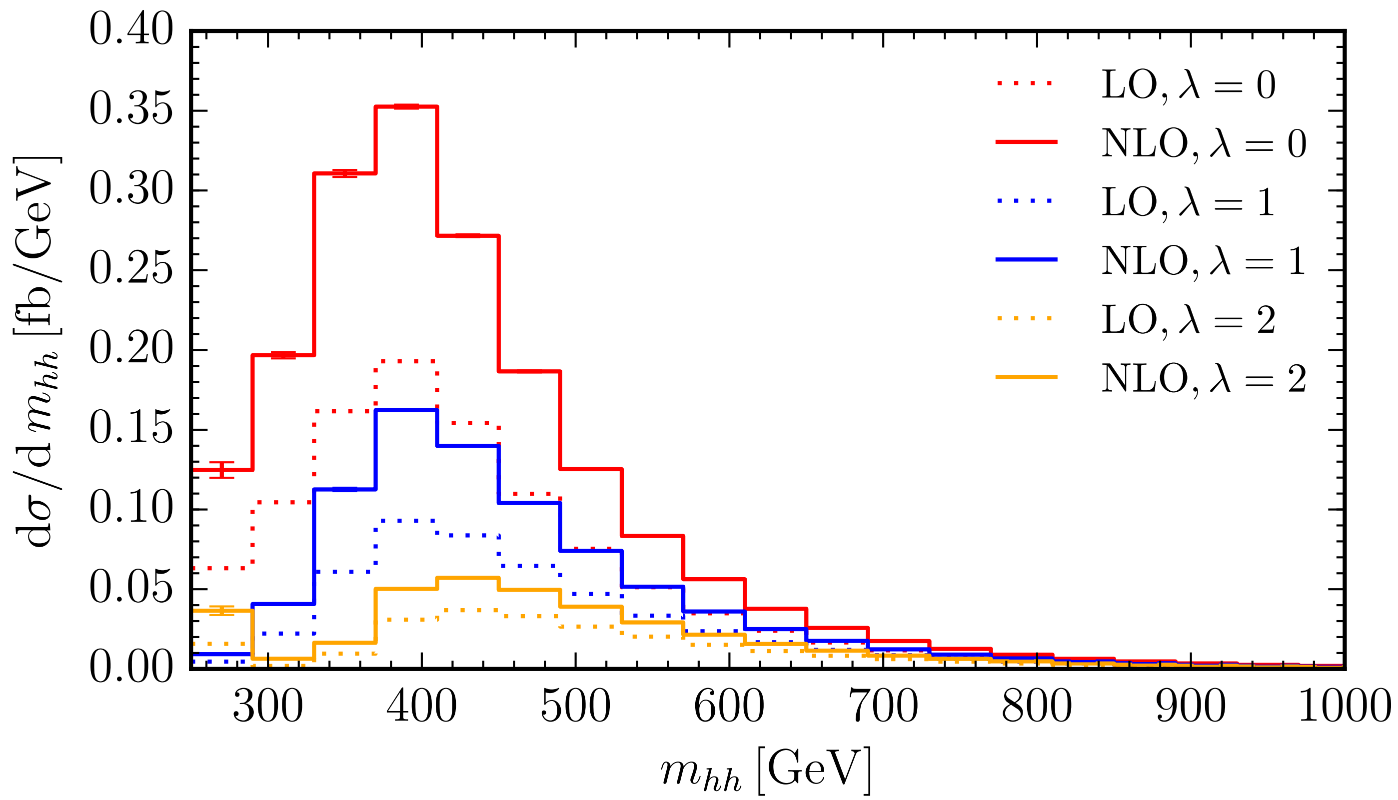
Figure 3: Top width effect on the one-loop (Born) matrix element squared for $gg \to HH$. The results for $\Gamma_t = 0$ and 1.5 GeV are shown along with the corresponding ratio.

Maltoni, Vryonidou, Zaro 14

Lambda Variation

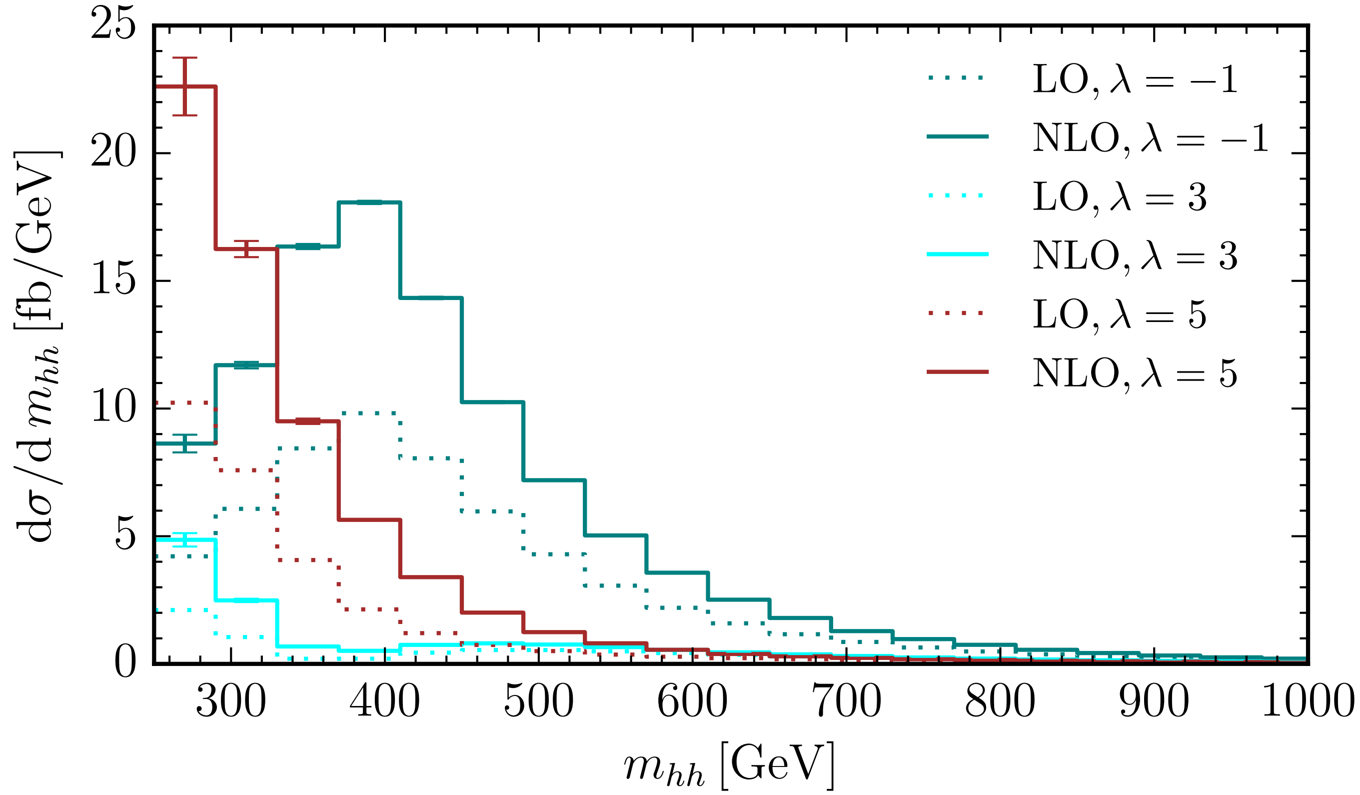


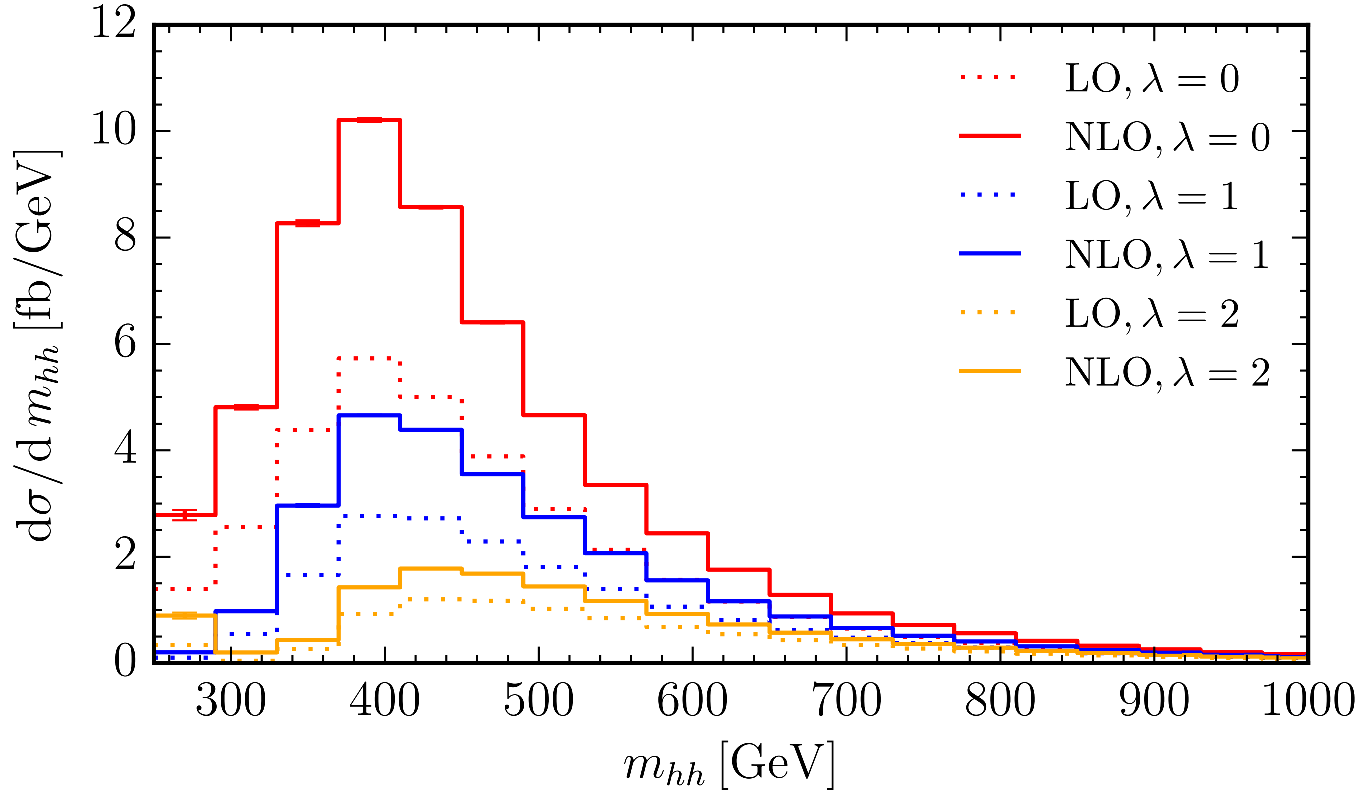
$\sqrt{s} = 14\,\mathrm{TeV}$



Lambda Variation

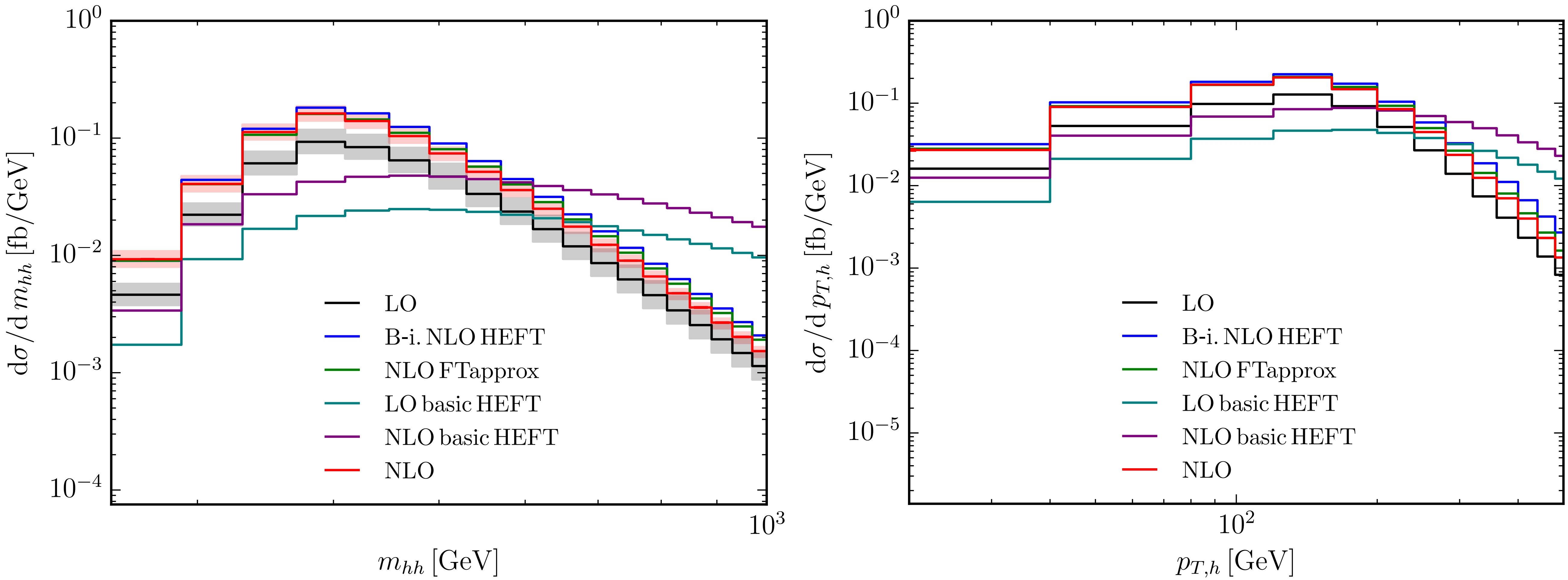
$\sqrt{s} = 100\,\mathrm{TeV}$





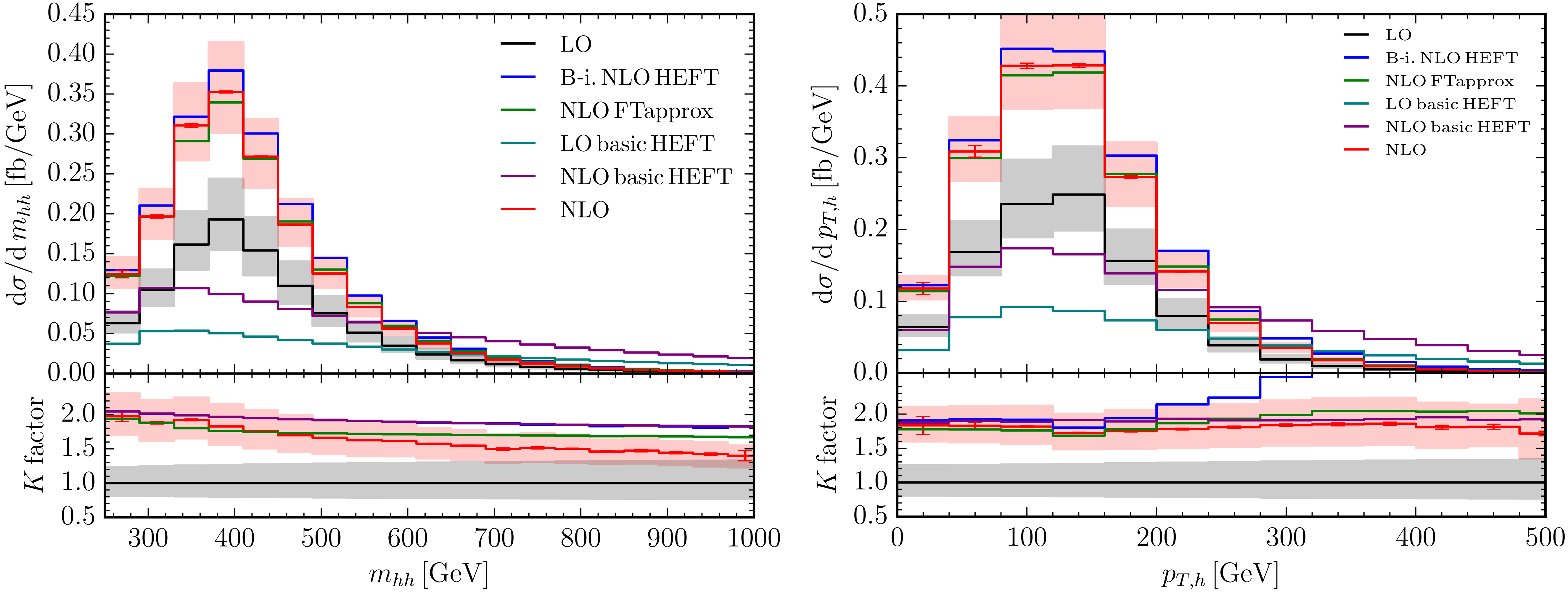
Scaling

$\sqrt{s} = 14\,\text{TeV}$



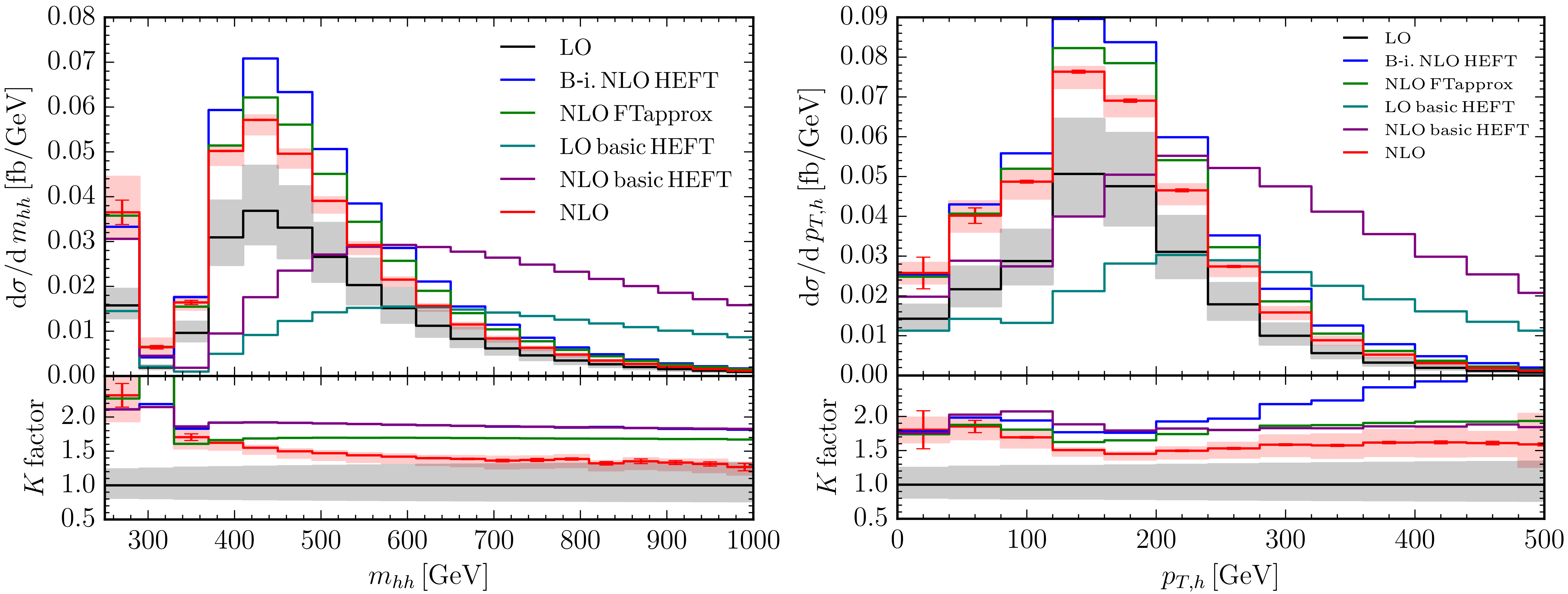
Lambda 0 x SM

$\sqrt{s} = 14\,\mathrm{TeV}$



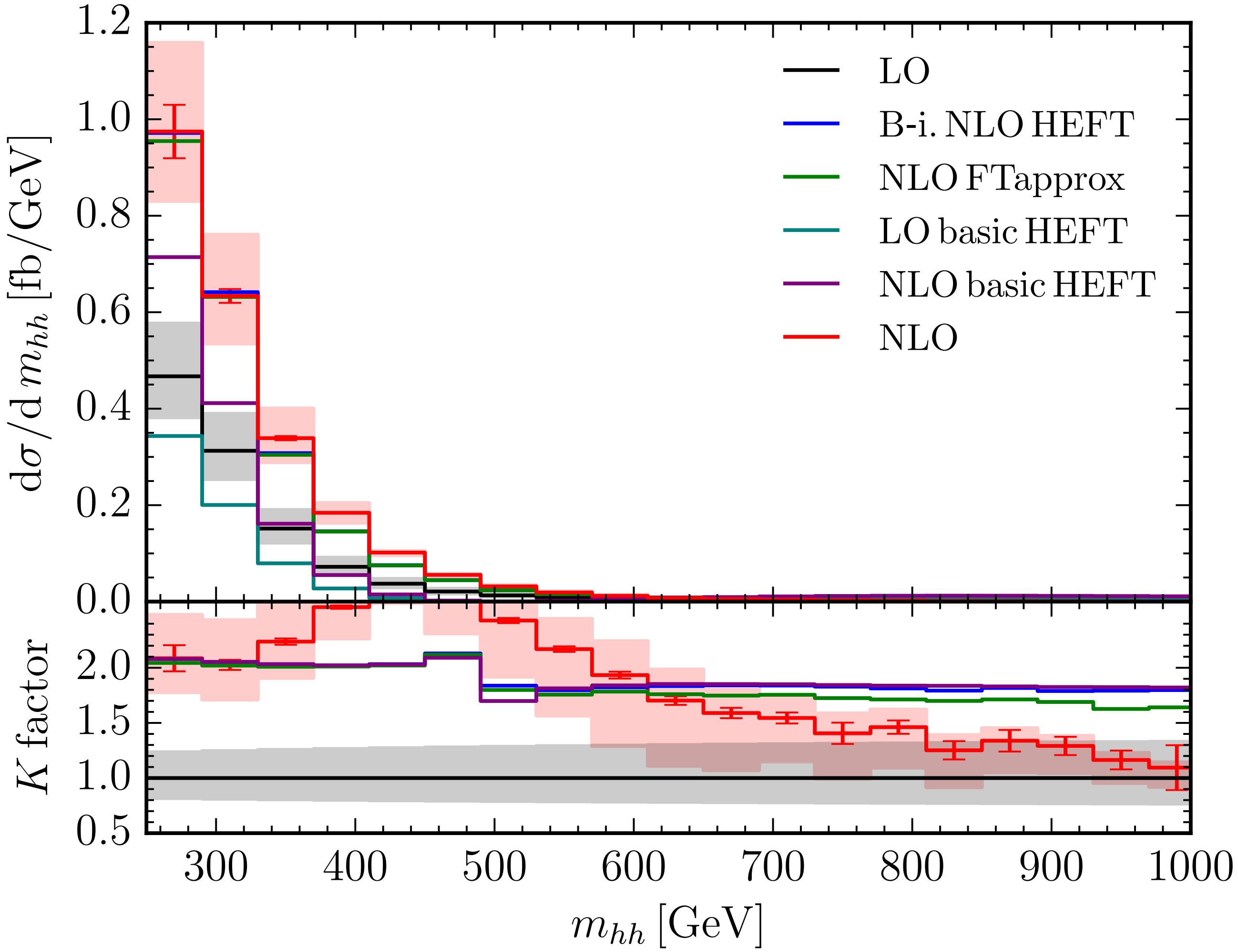
Lambda 2 x SM

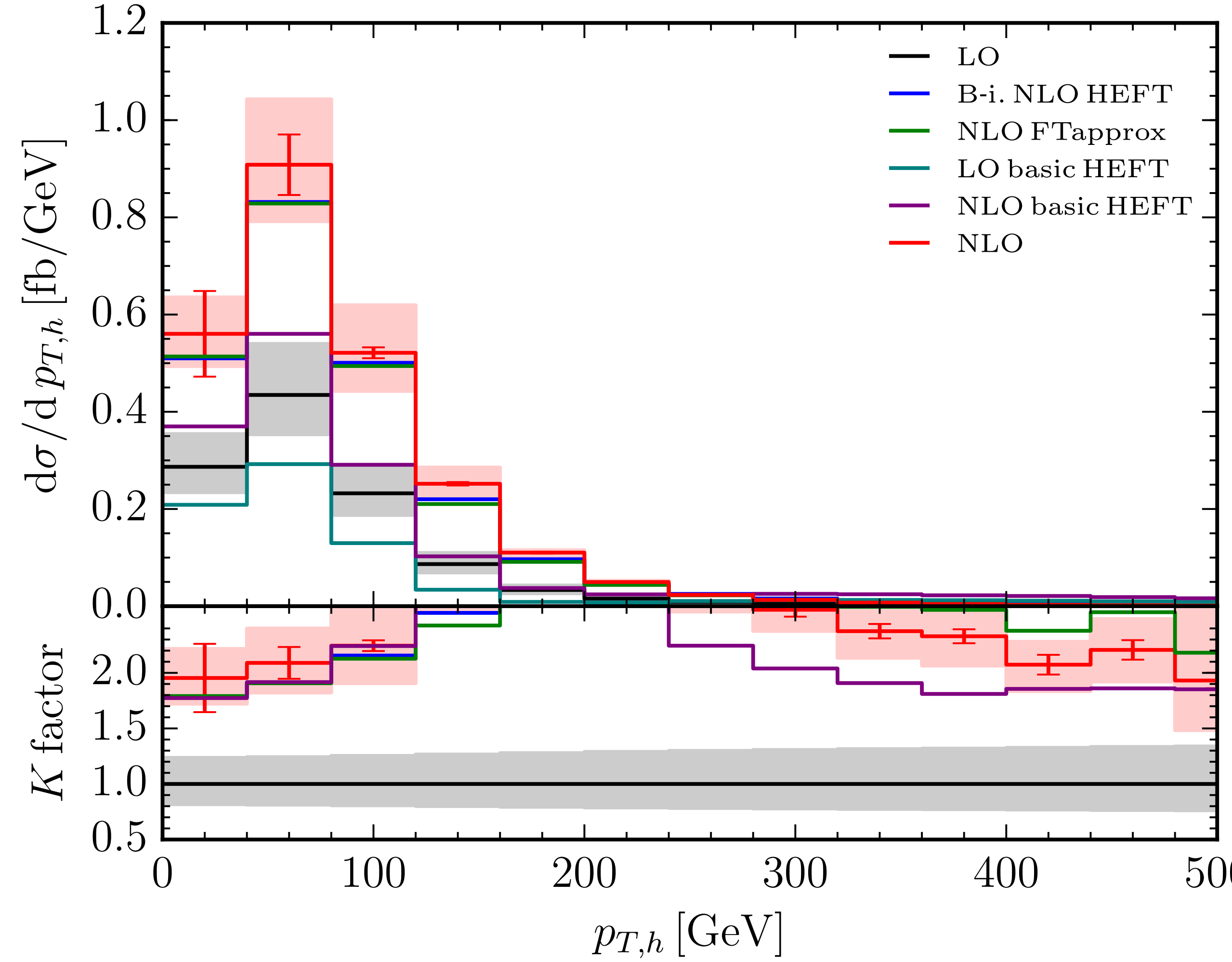
$\sqrt{s} = 14\,\mathrm{TeV}$



Lambda 5 x SM

$\sqrt{s} = 14\,\mathrm{TeV}$





Amplitude Structure

$\overline{\mathrm{MS}}$ scheme strong coupling a and OS top-quark mass:

$$\mathrm{F} = a\mathrm{F}^{(1)} + a^2(\delta Z_A + \delta Z_a)\mathrm{F}^{(1)} + a^2 \delta m_t^2 \mathrm{F}^{ct,(1)} + a^2 \mathrm{F}^{(2)} + O(a^3)$$

$$\mathrm{F}^{(1)} = \left(\frac{\mu_R^2}{M^2}\right)^{\epsilon} \left[b_0^{(1)} + b_1^{(1)}\epsilon + b_2^{(1)}\epsilon^2 + \mathcal{O}(\epsilon^3) \right]$$ ⟵ 1-loop

$$\mathrm{F}^{ct,(1)} = \left(\frac{\mu_R^2}{M^2}\right)^{\epsilon} \left[c_0^{(1)} + c_1^{(1)}\epsilon + \mathcal{O}(\epsilon^2) \right]$$ ⟵ Mass Counter-Terms

$$\mathrm{F}^{(2)} = \left(\frac{\mu_R^2}{M^2}\right)^{2\epsilon} \left[\frac{b_{-2}^{(2)}}{\epsilon^2} + \frac{b_{-1}^{(2)}}{\epsilon} + b_0^{(2)} + \mathcal{O}(\epsilon) \right]$$ ⟵ 2-loop

Red terms contain integrals, computed numerically at each PS point, not re-evaluated for scale variations

Real Radiation (HH + j…):

$gg \to HH + g$ $\quad g\bar{q} \to HH + \bar{q}$

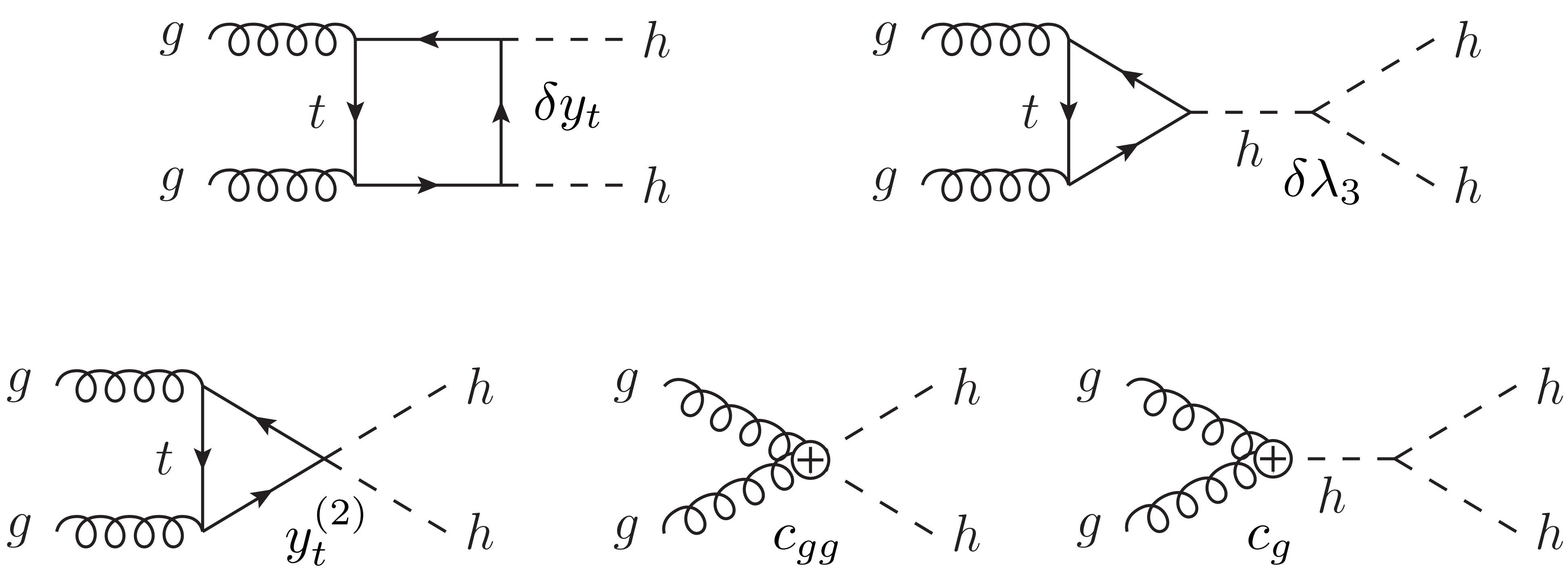
$q\bar{q} \to HH + g$ $\quad gq \to HH + q$

GoSam for MEs Cullen et al. 14

Catani-Seymour Dipole Subtraction Catani, Seymour 96

BSM EFT

Parametrise **non-resonant** new physics with EFT (5 parameters):



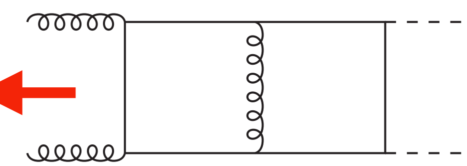
Azatov, Contino, Panico, Son 15;
(Cluster analysis) Dall'Osso, Dorigo, Gottardo, Oliveira, Tosi, Goertz 15;
+ Carvalho, Manzano, Dorigo, Gouzevich 16;
(B.I. HEFT) Gröber, Mühlleitner, Spira, Streicher 15;

12 representative
``clusters''

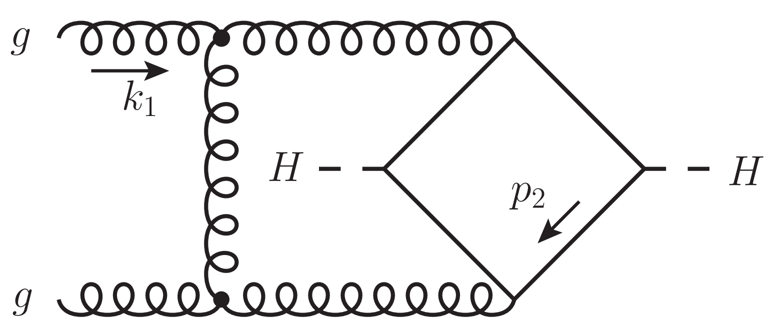
Amplitude Evaluation (II)

Contributing integrals: $\sqrt{s} = 327.25\,\text{GeV},\ \sqrt{-t} = 170.05\,\text{GeV},\ M^2 = s/4$

integral	value	error	time [s]
...			
F1_011111110_ord0	(0.484, 4.96e-05)	(4.40e-05, 4.23e-05)	11.8459
...			
N3_111111100_k1p2k2p2_ord0	(0.0929, -0.224)	(6.32e-05, 5.93e-05)	235.412
N3_111111100_1_ord0	(-0.0282, 0.179)	(8.01e-05, 9.18e-05)	265.896
N3_111111100_k1p2k1p2_ord0	(0.0245, 0.0888)	(5.06e-05, 5.31e-05)	282.794
N3_111111100_k1p2_ord0	(-0.00692, -0.108)	(3.05e-05, 3.05e-05)	433.342



$$I(s,t,m_t^2,m_h^2) = -\left(\frac{\mu^2}{M^2}\right)^{2\varepsilon} \Gamma(3+2\epsilon) M^{-4} \left(\frac{A_{-2}}{\epsilon^2} + \frac{A_{-1}}{\epsilon^1} + A_0 + \mathcal{O}(\epsilon)\right)$$



Sector Decomposition

sector	integral value	error	time [s]	#points
5	(-1.34e-03, 2.00e-07)	(2.38e-07, 2.69e-07)	0.255	1310420
6	(-1.58e-03, -9.23e-05)	(7.44e-07, 5.34e-07)	0.266	1310420
...				
41	(0.179, -0.856)	(1.10e-05, 1.22e-05)	29.484	79952820
42	(0.359, -1.308)	(1.40e-06, 1.58e-06)	80.24	211436900
44	(0.0752, -1.185)	(5.44e-07, 6.76e-07)	99.301	282904860

Slide:
Matthias Kerner
(LL 2016)

Rank 1 Shifted Lattices

$\mathcal{O}(n^{-1})$ algorithm for numerical integration: Review: Dick, Kuo, Sloan 13

$$I_s[f] \equiv \int_{[0,1]^s} \mathrm{d}^s x f(\vec{x})$$

$$f : \mathbb{R}^s \to \mathbb{C}$$

$$I_s[f] \approx \bar{Q}_{s,n,m}[f] \equiv \frac{1}{m} \sum_{k=1}^{m} \frac{1}{n} \sum_{i=0}^{n-1} f\left(\left\{\frac{i\vec{z}}{n} + \vec{\Delta}_k\right\}\right)$$

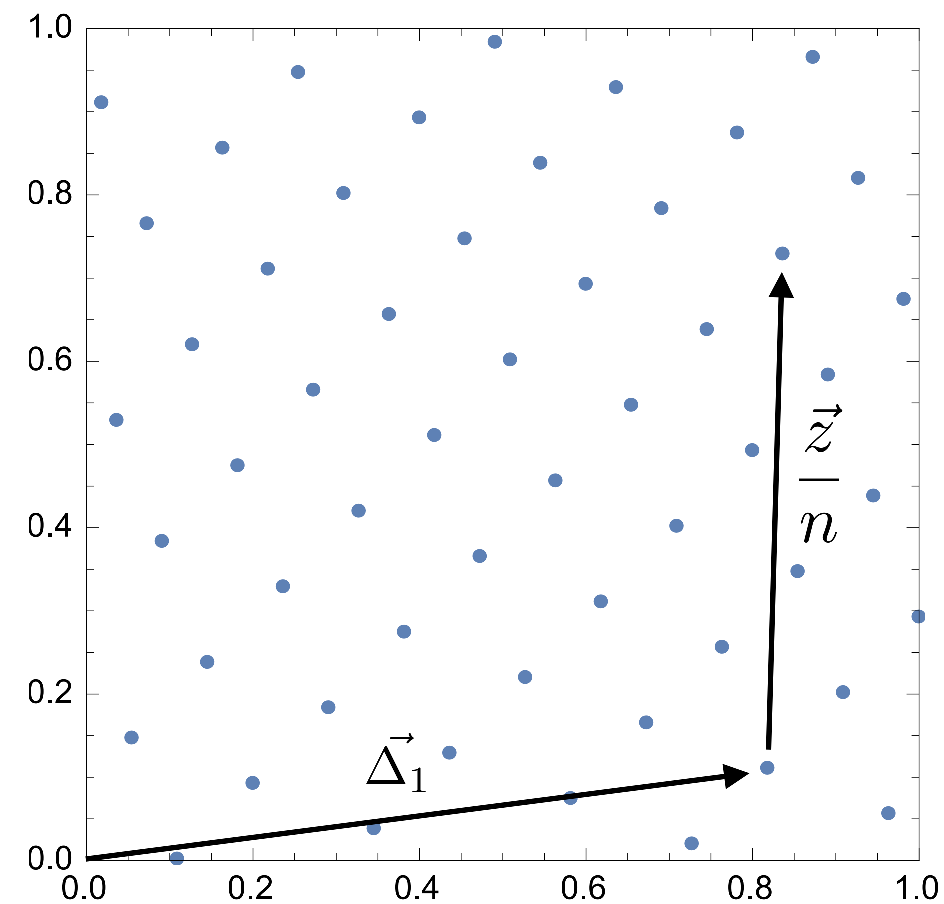
$\vec{z}$ - Generating vec.

$\vec{\Delta}_k$ - Random shift vec.

$\{\}$ - Fractional part

n - # Lattice points

m - # Random shifts

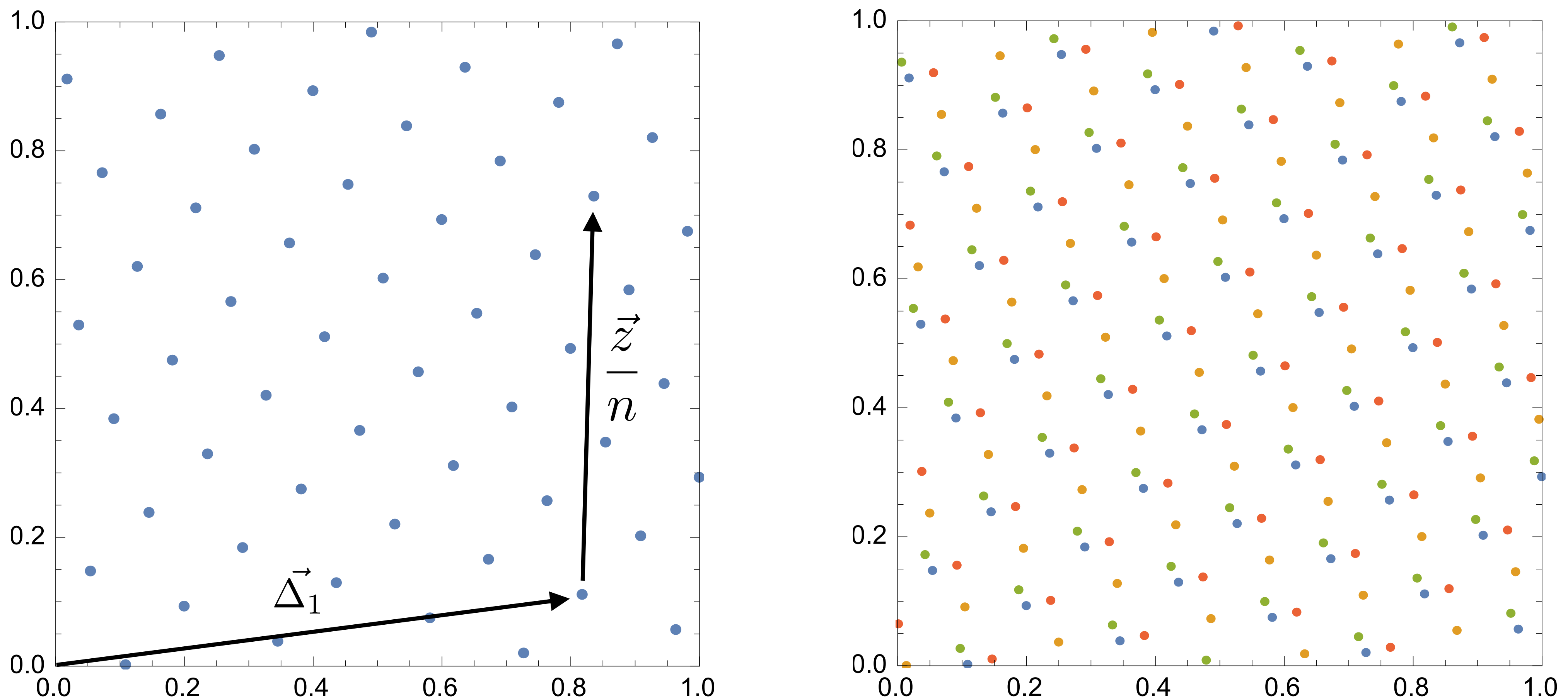


Generating vector $\vec{z}$ precomputed for a **fixed** number of lattice points, chosen to minimise worst-case error Nuyens 07

Rank 1 Shifted Lattices (II)

Unbiased error estimate computed from random shifts:

$$\mathrm{Var}[\bar{Q}_{s,n,m}[f]] \approx \frac{1}{m(m-1)} \sum_{k=1}^{m} (Q_{s,n,k} - \bar{Q}_{s,n,m})^2$$



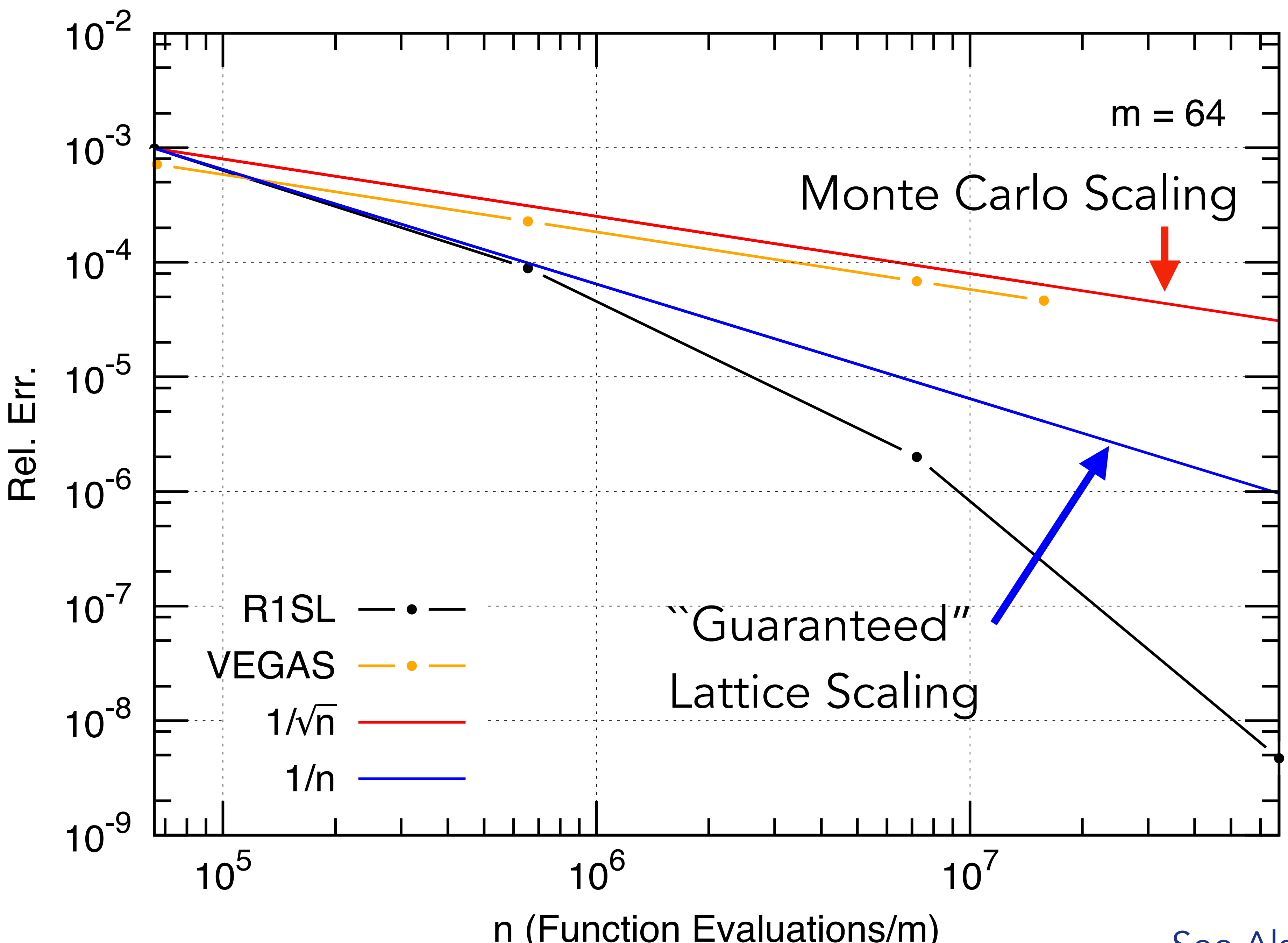
1 Shift

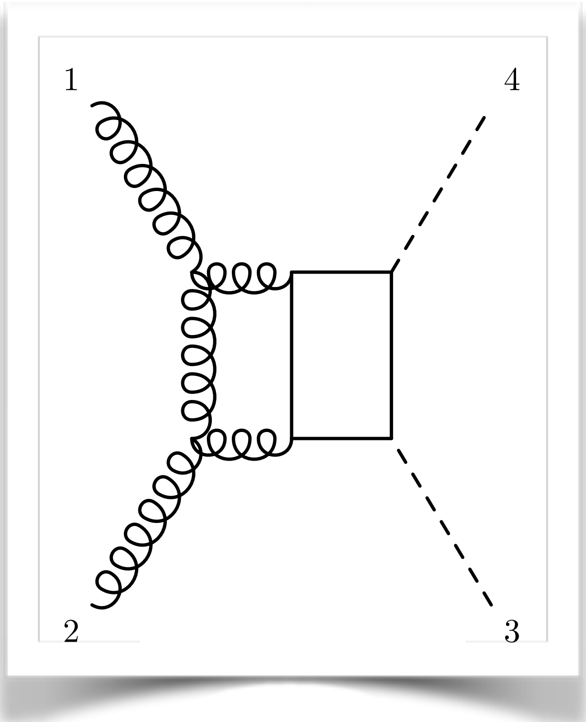
4 Shifts

Typically 10-50 shifts, production run: 20 shifts

R1SL: Algorithm Performance

Example: Rel. Err. of one sector of sector decomposed loop integral





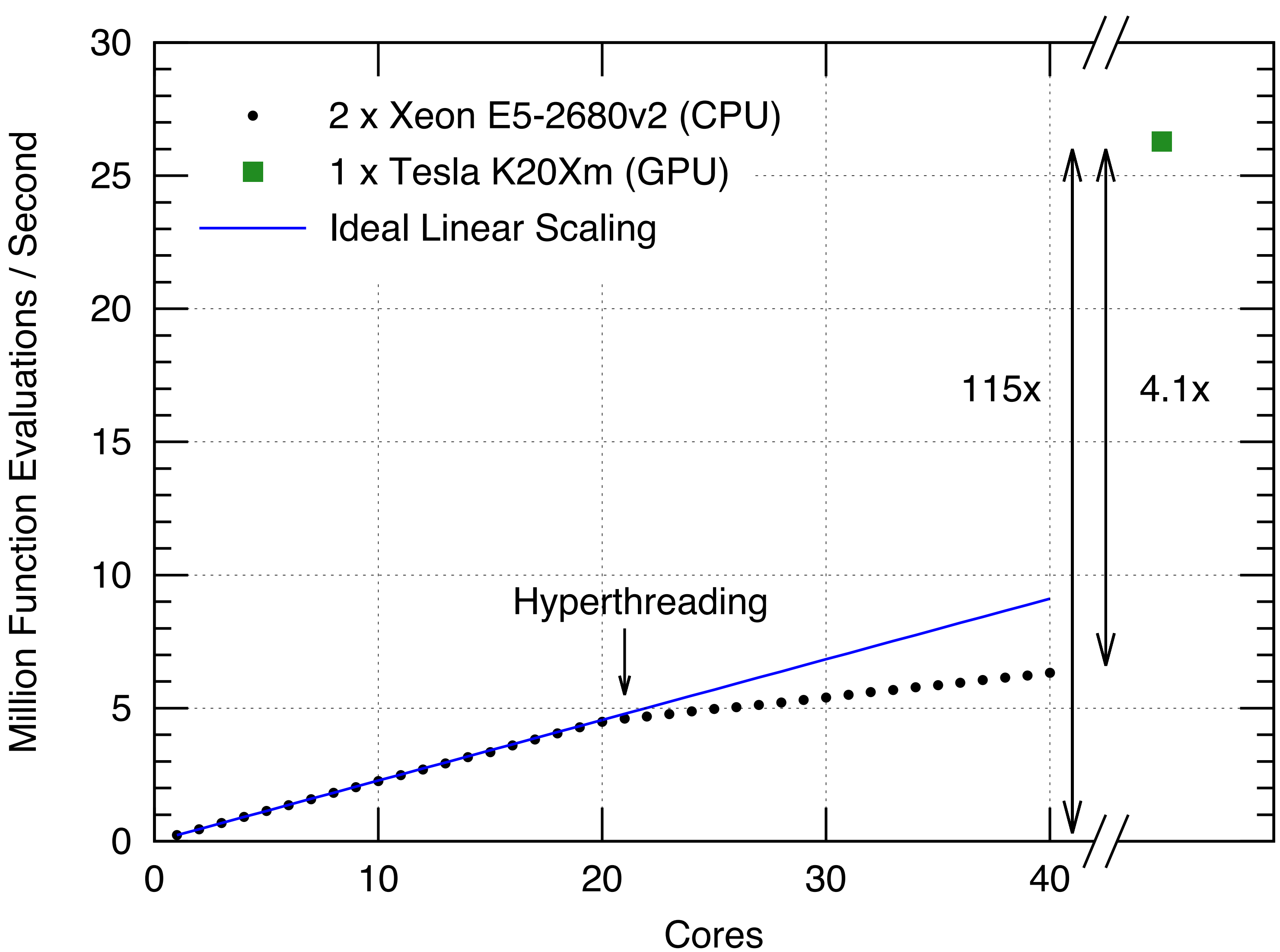
6 dimensional numerical integral

See Also: Li, Wang, Yan, Zhao 15

R1SL: Implementation Performance

Accuracy limited primarily by number of function evaluations

Implemented in OpenCL 1.1 for CPU & GPU, generate points on GPU/CPU core, sum blocks of points (reduce memory usage/transfers)



2 CPUs (20 Cores + HT)

1 GPU

n	CPU (s)	GPU(s)	C/G
655357	6.63	1.60	4.1
7208951	72.3	16.4	4.4
67264993	674.2	152.2	4.4

Current Experimental Limits

Decay Ch.	B.R.	95% Excl.	Analysis $\left(\left[fb^{-1}\right], \sqrt{s}\ [\text{TeV}]\right)$
$b\bar{b}b\bar{b}$	33%	$< 29 \cdot \sigma_{\text{SM}}$	ATLAS-CONF-2016-017 (3.2,13)
			ATLAS-CONF-2016-049 (13.3,13)
$b\bar{b}WW$	25%	–	–
$b\bar{b}\tau\tau$	7.3%	$< 200 \cdot \sigma_{\text{SM}}$	CMS PAS HIG-16-012 (2.7,13)
			CMS PAS HIG-16-028 (12.9,13)
			CMS PAS HIG-15-013 (18.3,8)
$b\bar{b}ZZ$	3.0%	–	–
$WW\tau\tau$	2.71%	–	–
$WWZZ$	1.13%	–	–
$b\bar{b}\gamma\gamma$	0.26%	$< 3.9pb$	ATLAS-CONF-2016-004 (3.2,13)
		$< 74 \cdot \sigma_{\text{SM}}$	CMS-HIG-13-032 (19.7,8)
$\gamma\gamma\gamma\gamma$	0.001%	–	–
$b\bar{b}VV(\to l\nu l\nu)$	1.23%	$400 \cdot \sigma_{\text{SM}}$	CMS PAS HIG-16-024 (2.3,13)
$\gamma\gamma WW^*(\to l\nu jj)$	–	$< 25pb$	ATLAS-CONF-2016-071 (13.3,13)
Comb Ch.	–	$< 70 \cdot \sigma_{\text{SM}}$	ATLAS arXiv:1509.04670v2 (20.3,8)

Future Experimental Prospects

HL-LHC (14 TeV)

ATLAS+CMS bbγγ + bbττ: Expected significance 1.9 sigma

CERN-LHCC-2015-10

ATLAS bbγγ: Signal significance 1.3 sigma ATL-PHYS-PUB-2014-019

ATLAS bbττ: Signal significance 0.6 sigma ATL-PHYS-PUB-2015-046

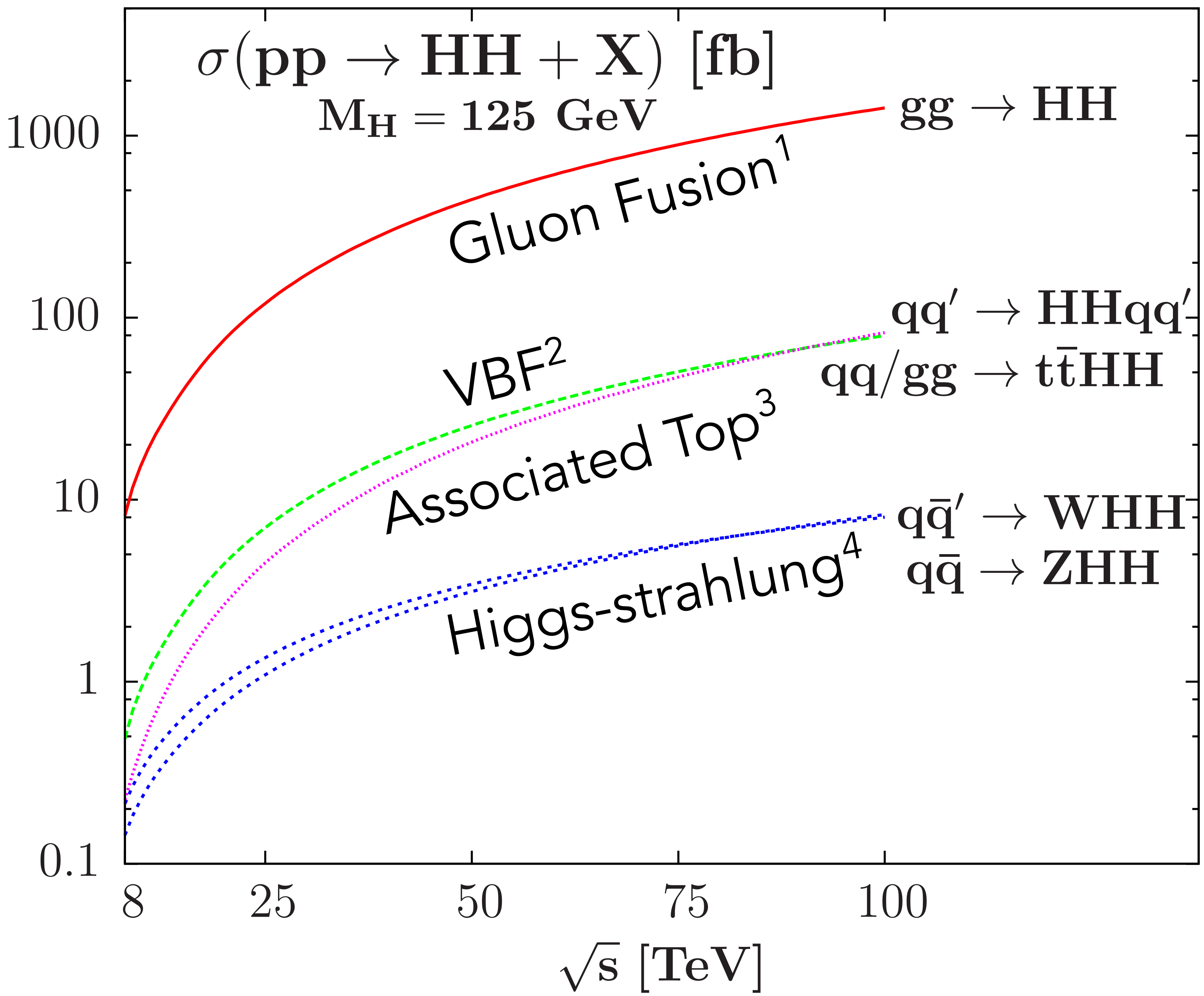
FCC (100 TeV)

This rate is expected to provide a clear signal in the $HH \to (b\bar{b})(\gamma\gamma)$ channel and to allow determination of λ_{3H} with an accuracy of $30-40\%$ with a luminosity of 3 ab^{-1}, and of $5-10\%$ with a luminosity of 30 ab^{-1} [497–499]. A rare decay channel which is potentially interesting is $HH \to (b\bar{b})(ZZ) \to (b\bar{b})(4l)$, with a few expected signal events against $\mathcal{O}(10)$ background events at 3 ab^{-1} [500].

arXiv:1607.01831

Production Channels (II)

$$\sigma(pp \to HH + X) \sim \frac{1}{1000}\sigma(pp \to H + X)$$



[1] NLO QCD HEFT `HPAIR`

[2] NLO QCD `VBFNLO`

[3] LO QCD (NLO, `aMC@NLO`)

[4] NNLO QCD

Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, Spira 12

Resonant Production

YR4 details two benchmark scenarios for initial study

Higgs Singlet Model

$$V = -m^2\Phi^\dagger\Phi - \mu^2 S^2 + \lambda_1(\Phi^\dagger\Phi)^2 + \lambda_2 S^4 + \lambda_3 \Phi^\dagger\Phi S^2$$

$$\Phi^T = (\phi^+, \tilde{\phi}_0 = \frac{\phi_0 + v}{\sqrt{s}})$$

$$S = \frac{s + \langle S \rangle}{\sqrt{2}}$$

Large $\mathcal{O}(20-30\%)$ $H \to hh$

Cross-section can be enhanced by up to 10-20x

2 Higgs Doublet Model (2HDM)

2 neutral scalars → h^0, H^0, A, H^+, H^- ← 2 charged Higgs

↑

Pseudoscalar

Behaviour strongly depends on the scenario

Hespel, López-Val, Vryonidou 14

Integral Families

tensor integrals: scalar products $\rightarrow$ inverse propagators

Slide: Matthias Kerner

l.i. scalar products:

$$S = \frac{l(l+1)}{2} + lm \qquad \begin{array}{ll} l = 2: & \#\,\text{loops} \\ m = 3: & \#\,\text{l.i. external momenta} \end{array} \qquad \Rightarrow \qquad S = 9$$

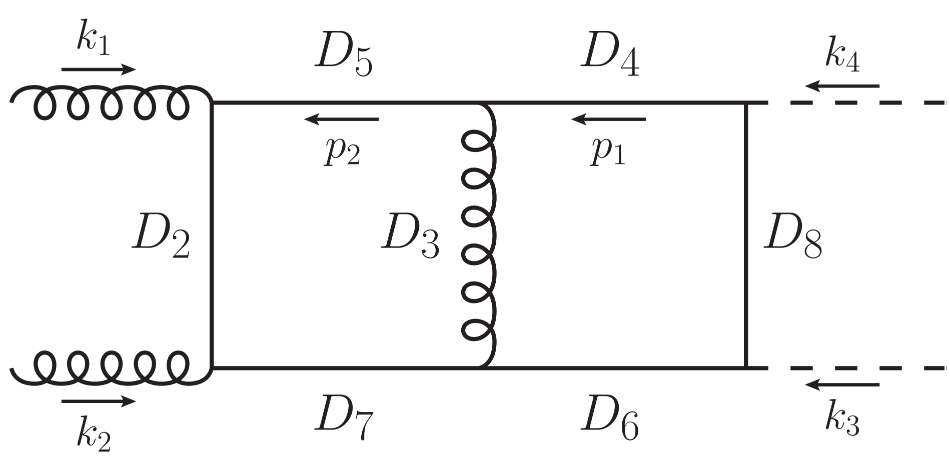
$\rightarrow$ integral families with 9 propagators

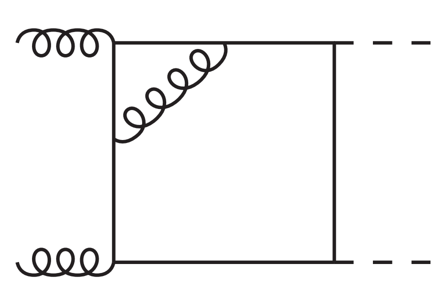
$\rightarrow$ general loop integral:

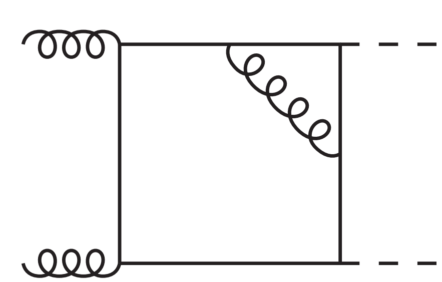
$$I^{\mathrm{fam}_j}_{\nu_1,\ldots,\nu_9} = \int \mathrm{d}^d p_1 \int \mathrm{d}^d p_2 \frac{1}{D_1^{\nu_1} D_2^{\nu_2} \cdots D_9^{\nu_9}} \qquad \nu_i \in \mathbb{Z}$$

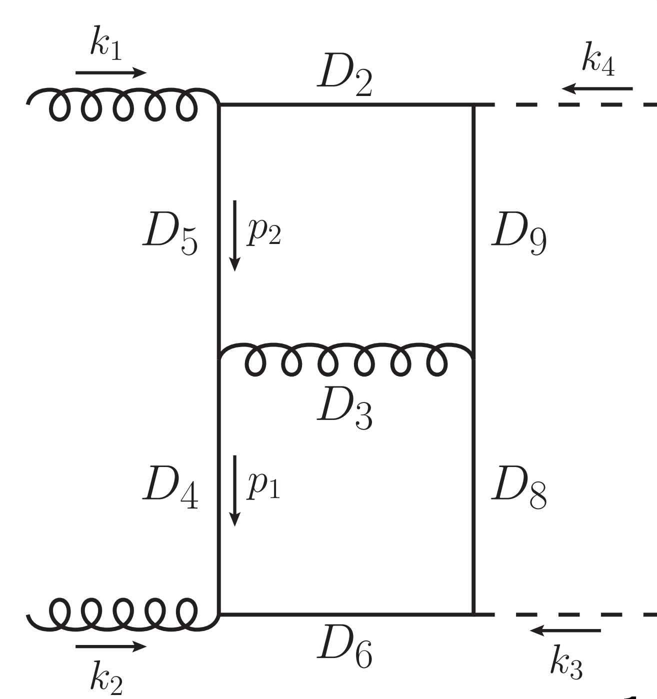
planar family 1:

$$\begin{aligned}
D_1 &= p_1^2 - m_t^2 \\
D_2 &= p_2^2 - m_t^2 \\
D_3 &= (p_1 - p_2)^2 \\
D_4 &= (p_1 + k_1)^2 - m_t^2 \\
D_5 &= (p_2 + k_1)^2 - m_t^2 \\
D_6 &= (p_1 - k_2)^2 - m_t^2 \\
D_7 &= (p_2 - k_2)^2 - m_t^2 \\
D_8 &= (p_1 - k_2 - k_3)^2 - m_t^2 \\
D_9 &= (p_2 - k_2 - k_3)^2 - m_t^2
\end{aligned}$$









Integral Families

tensor integrals: scalar products $\rightarrow$ inverse propagators

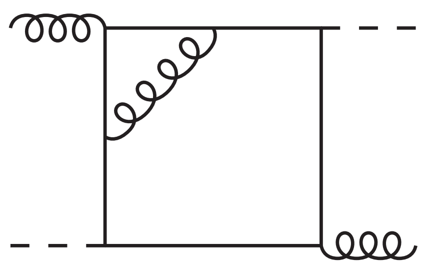
Slide: Matthias Kerner

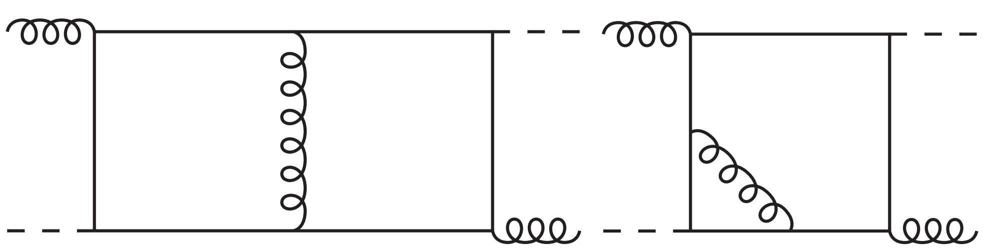
l.i. scalar products:

$$S = \frac{l(l+1)}{2} + lm \qquad \begin{array}{ll} l = 2: & \#\,\text{loops} \\ m = 3: & \#\,\text{l.i. external momenta} \end{array} \quad \Rightarrow \quad S = 9$$

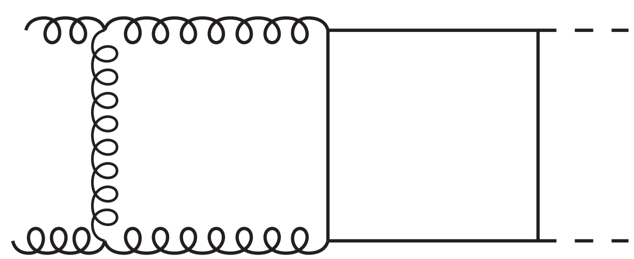
$\rightarrow$ integral families with 9 propagators

planar family 2:

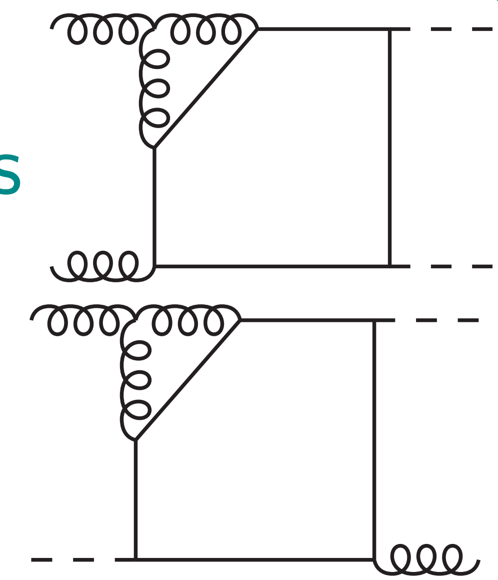




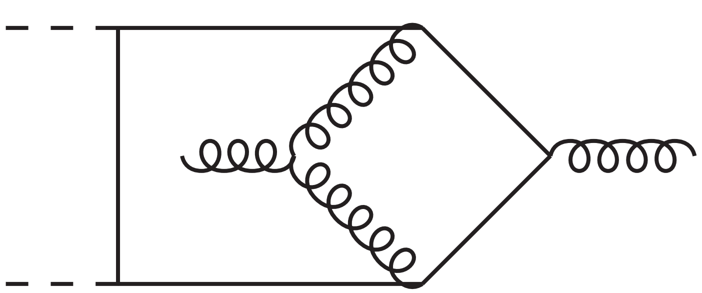
planar family 3:

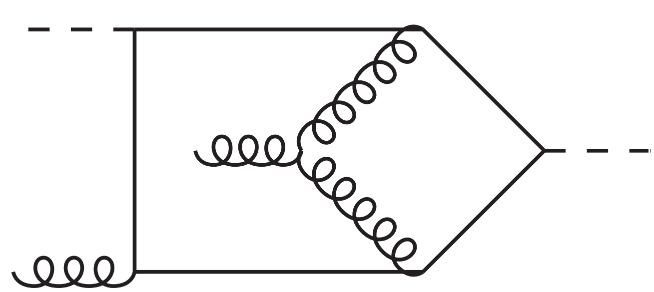


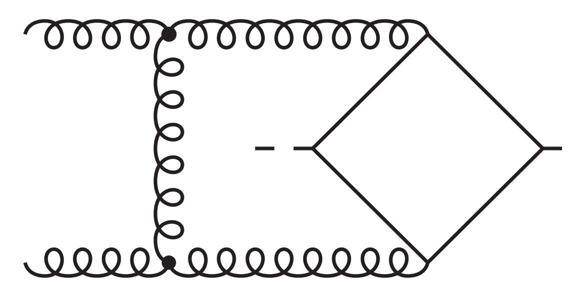
planar families 4/5:



3 non-planar families:







Form Factor Decomposition (II)

$$T_1^{\mu\nu} = g^{\mu\nu} - \frac{p_2^\mu p_1^\nu}{p_1 \cdot p_2} \qquad\qquad p_T^2 = \frac{ut - m_H^4}{s}$$

$$T_2^{\mu\nu} = g^{\mu\nu} + \frac{m_H^2 p_2^\mu p_1^\nu}{p_T^2 p_1 \cdot p_2} - \frac{2 p_1 \cdot p_3 p_2^\mu p_3^\nu}{p_T^2 p_1 \cdot p_2} - \frac{2 p_2 \cdot p_3 p_3^\mu p_1^\nu}{p_T^2 p_1 \cdot p_2} + \frac{2 p_3^\mu p_3^\nu}{p_T^2}$$

Glover, van der Bij 88

Projectors (CDR $D = 4 - 2\epsilon$):

$$P_1^{\mu\nu} = \frac{1}{4}\frac{D-2}{D-3} T_1^{\mu\nu} - \frac{1}{4}\frac{D-4}{D-3} T_2^{\mu\nu}$$

$$P_2^{\mu\nu} = -\frac{1}{4}\frac{D-4}{D-3} T_1^{\mu\nu} + \frac{1}{4}\frac{D-2}{D-3} T_2^{\mu\nu}$$

Same Basis as amplitude

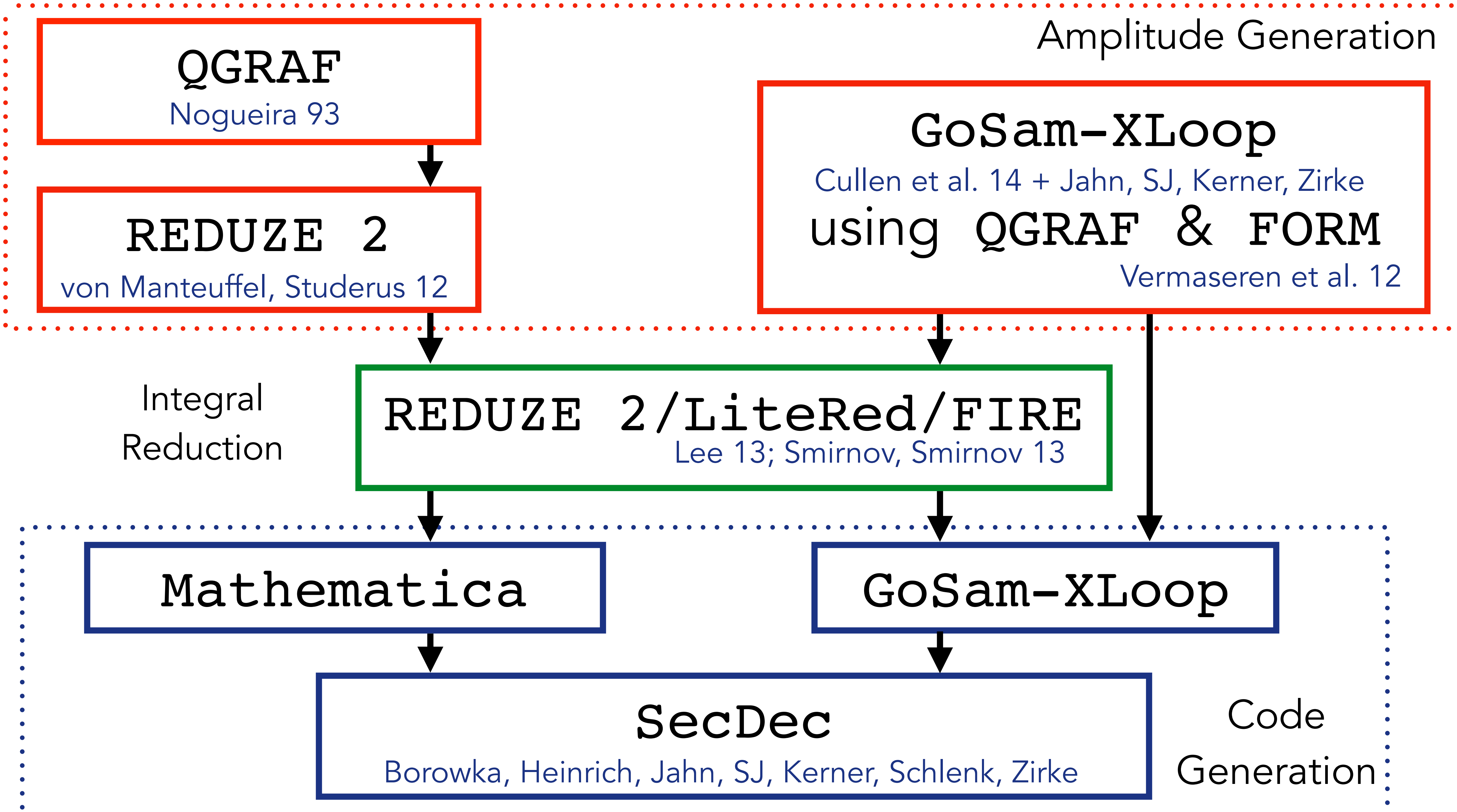
Compute:

$$P_1^{\mu\nu}\mathcal{M}_{\mu\nu} = F_1(\hat{s}, \hat{t}, m_h^2, m_t^2, D)$$

$$P_2^{\mu\nu}\mathcal{M}_{\mu\nu} = F_2(\hat{s}, \hat{t}, m_h^2, m_t^2, D)$$

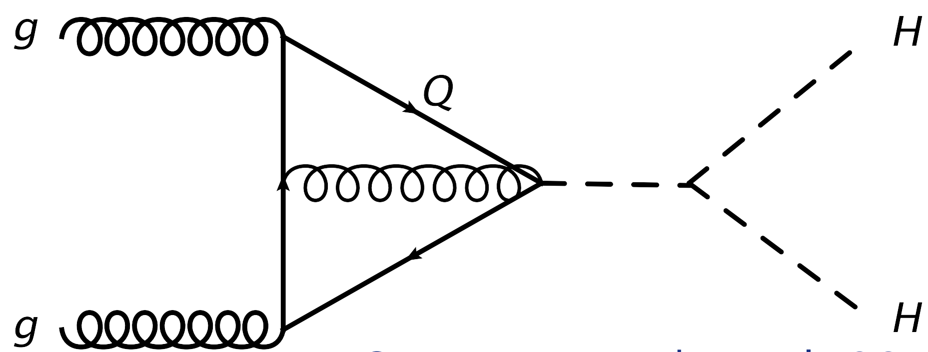
Virtual MEs: Tool Chain

Partial cross-check: 2 Implementations



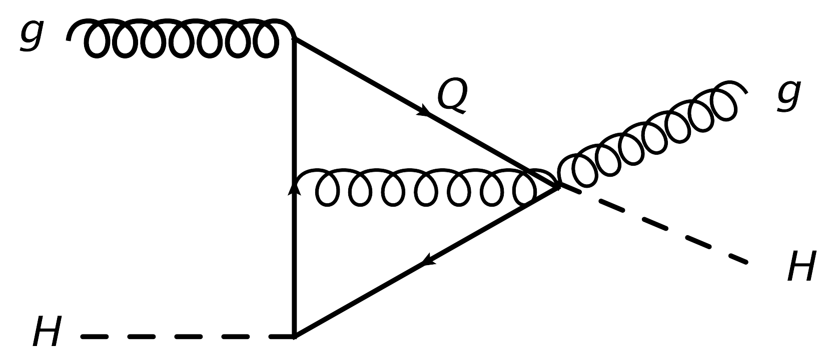
Master Integrals

Known Analytically:



Spira, Djouadi et al. 93, 95;
Bonciani, P. Mastrolia 03,04;
Anastasiou, Beerli et al. 06;

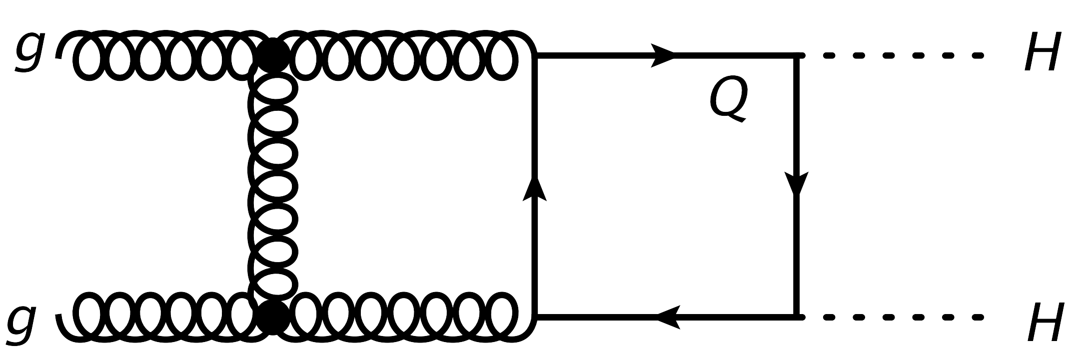
3-point, 1 off-shell leg
HPLs



Gehrmann, Guns, Kara 15

3-point, 2 off-shell legs
Generalized HPLs, 12 Letters

Numeric Evaluation:



Up to 4-point,
4 scales s, t, m_T^2, m_H^2
`SecDec`

Slide: Matthias Kerner

Numerical Master Integrals

To evaluate Master Integrals we use `SecDec` which implements Sector Decomposition   Collaboration: Borowka, Heinrich, Jahn, SJ, Kerner, Schlenk, Zirke

Completely automated procedure

Sector Decomposition

1) Feynman Parametrise integral and compute momentum integrals

$$\boxed{G = (-1)^{N_\nu} \frac{\Gamma(N_\nu - LD/2)}{\prod_{j=1}^{N} \Gamma(\nu_j)} \int_0^\infty \prod_{j=1}^{N} \mathrm{d}x_j \; x_j^{\nu_j - 1} \delta(1 - \sum_{i=1}^{N} x_i) \frac{\mathcal{U}^{N_\nu - (L+1)D/2}(\vec{x})}{\mathcal{F}^{N_\nu - LD/2}(\vec{x}, s_{ij})}}$$

Here $\mathcal{U}, \mathcal{F}$ are 1st, 2nd Symanzik Polynomials

We have exchanged L momentum integrals for N parameter integrals

Sector Decomposition

2) After integrating out δ we are faced with integrals of the form:

$$G_i = \int_0^1 \left(\prod_{j=1}^{N-1} \mathrm{d}x_j x_j^{\nu_j - 1} \right) \frac{\mathcal{U}_i(\vec{x})^{\mathrm{expo}\mathcal{U}(\epsilon)}}{\mathcal{F}_i(\vec{x}, s_{ij})^{\mathrm{expo}\mathcal{F}(\epsilon)}}$$

Powers depending on ϵ

Polynomials in F.P

Which may contain overlapping singularities which appear when several $x_j \to 0$ simultaneously (corresponding to UV/IR singularities)

Sector decomposition maps each integral into integrals of the form:

$$G_{ik} = \int_0^1 \left(\prod_{j=1}^{N-1} \mathrm{d}x_j x_j^{a_j - b_j \epsilon} \right) \frac{\mathcal{U}_{ik}(\vec{x})^{\mathrm{expo}\mathcal{U}(\epsilon)}}{\mathcal{F}_{ik}(\vec{x}, s_{ij})^{\mathrm{expo}\mathcal{F}(\epsilon)}}$$

Singularity structure can be read off

$$\mathcal{U}_{ik}(\vec{x}) = 1 + u(\vec{x})$$

$$\mathcal{F}_{ik}(\vec{x}) = -s_0 + f(\vec{x})$$

$u(\vec{x}), f(\vec{x})$ **have no constant term**

Hepp 66; Denner, Roth 96; Binoth, Heinrich 00

Sector Decomposition (II)

One technique **Iterated Sector Decomposition** repeat:

Binoth, Heinrich 00

$$\int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \frac{1}{(x_1+x_2)^{2+\epsilon}} \longleftarrow \textbf{Overlapping singularity for } x_1, x_2 \to 0$$

$$= \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \frac{1}{(x_1+x_2)^{2+\epsilon}} \left(\theta(x_1 - x_2) + \theta(x_2 - x_1)\right)$$

$$= \int_0^1 \mathrm{d}x_1 \int_0^{x_1} \mathrm{d}x_2 \frac{1}{(x_1+x_2)^{2+\epsilon}} + \int_0^1 \mathrm{d}x_2 \int_0^{x_2} \mathrm{d}x_1 \frac{1}{(x_1+x_2)^{2+\epsilon}}$$

$$= \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}t_2 \frac{x_1}{(x_1+x_1 t_2)^{2+\epsilon}} + \int_0^1 \mathrm{d}x_2 \int_0^1 \mathrm{d}t_1 \frac{x_2}{(x_2 t_1+x_2)^{2+\epsilon}}$$

$$= \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}t_2 \frac{x_1^{-1-\epsilon}}{(1+t_2)^{2+\epsilon}} + \int_0^1 \mathrm{d}x_2 \int_0^1 \mathrm{d}t_1 \frac{x_2^{-1-\epsilon}}{(t_1+1)^{2+\epsilon}} \longleftarrow \textbf{Singularities factorised}$$

If this procedure terminates depends on order of decomposition steps

An alternative strategy **Geometric Sector Decomposition** always terminates; both strategies are implemented in `SecDec`.

Kaneko, Ueda 10; See also: Bogner, Weinzierl 08; Smirnov, Tentyukov 09

Sector Decomposition (III)

3) Expand in ϵ (simple case $a = -1$):

$$\int_0^1 \mathrm{d}x^{-1-b\epsilon} g(x) = \underbrace{\frac{g(0)}{-b\epsilon}}_{\text{Poles}} + \int_0^1 \mathrm{d}x x^{-b\epsilon} \left[\frac{g(x)-g(0)}{x}\right] \leftarrow \textbf{Finite}$$

Note: `subtraction' of $g(0)$

By Definition: $g(0) \neq 0, g(0)$ finite

4) Numerically integrate

SecDec supports: numerators, inverse propagators, ``dots'', physical kinematics, arbitrary loops & legs (within reason)

Soper 00; Nagy, Soper 06; Borowka 14

Key Point: Sector Decomposed integrals can be expanded in ϵ and numerically integrated

SecDec as a Library

Single program to compute **all** coefficients & integrals to obtain **amplitude** to given accuracy

desired precision

list of GPUs & CPUs

```
Amplitude si(epsrel,devinds,crossings);

// coeffs/coeff1.cpp
si.addTerm(
           string("ReduzeF1L2_230000010ord0"),
           ReduzeF1L2_230000010ord0nfunc(),
           crossing,
           &ReduzeF1L2_230000010ord0Integrand,
           &ReduzeF1L2_230000010ord0findoptlam,
           ReduzeF1L2_230000010ord0ndim(),
           params,
           termCoeff1
           );

// coeffs/coeff204.cpp
si.addTerm(
           string("ReduzeF3L2diminc2_131010100ord1"),
           ReduzeF3L2diminc2_131010100ord1nfunc(),
           crossing,
           &ReduzeF3L2diminc2_131010100ord1Integrand,
           &ReduzeF3L2diminc2_131010100ord1findoptlam,
           ReduzeF3L2diminc2_131010100ord1ndim(),
           params,
           termCoeff2
           );

si.optimizeLambda();
si.integrate();
```

name & reference to integrand to integrate

$(\hat{s}, \hat{t}, m_t^2, m_h^2)$

vector of coefficients $C_{1,-2}, C_{1,-1}, \ldots$ for all Form Factors, evaluated at this phase-space point

Find contour deformation (physical region) in parallel for all integrals in amplitude

Computes integrals in parallel on GPUs & CPUs. Dynamically adjusts # points per sector to reduce amplitude error

Slide: Tom Zirke

Approximate top-mass effects at NLO

$$\sigma^{NLO}(p) = \int d\phi_3 \left[\left(d\sigma^R(p)\right)_{\epsilon=0} - \left(\sum_{\text{dipoles}} d\sigma^{LO}(p) \otimes dV_{\text{dipole}} \right)_{\epsilon=0} \right] ✅$$
$$+ \int d\phi_2 \left[d\sigma^V(p) + d\sigma^{LO}(p) \otimes \mathbf{I} \right]_{\epsilon=0}$$
$$+ \int_0^1 dx \int d\phi_2 \left[d\sigma^{LO}(xp) \otimes (\mathbf{P}+\mathbf{K})(x) \right]_{\epsilon=0} ✅$$

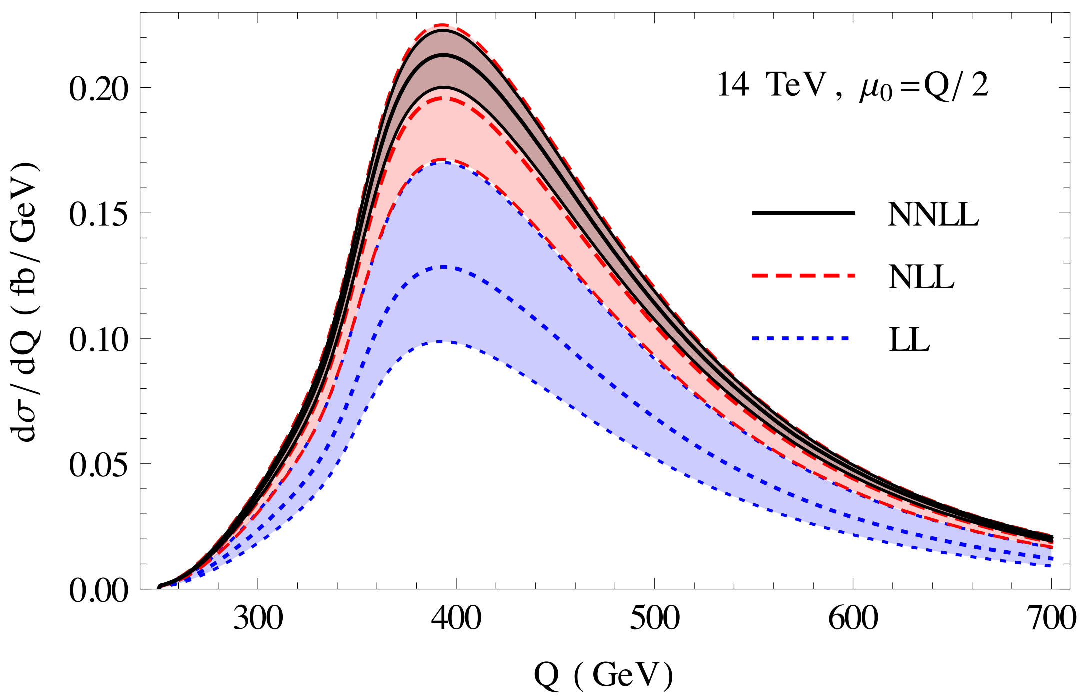
$$d\sigma^V + d\sigma^{LO}(\epsilon) \otimes \mathbf{I} \approx d\sigma^V_{\exp,N} \frac{d\sigma^{LO}(\epsilon)}{d\sigma^{LO}_{\exp,N}(\epsilon)} + d\sigma^{LO}(\epsilon) \otimes \mathbf{I}$$
$$= \left(d\sigma^V_{\exp,N} + d\sigma^{LO}_{\exp,N}(\epsilon) \otimes \mathbf{I} \right) \frac{d\sigma^{LO}(\epsilon)}{d\sigma^{LO}_{\exp,N}(\epsilon)}$$
$$= \left(d\sigma^V_{\exp,N} + d\sigma^{LO}_{\exp,N}(\epsilon) \otimes \mathbf{I} \right) \frac{d\sigma^{LO}(\epsilon=0)}{d\sigma^{LO}_{\exp,N}(\epsilon=0)} + \mathcal{O}(\epsilon)$$

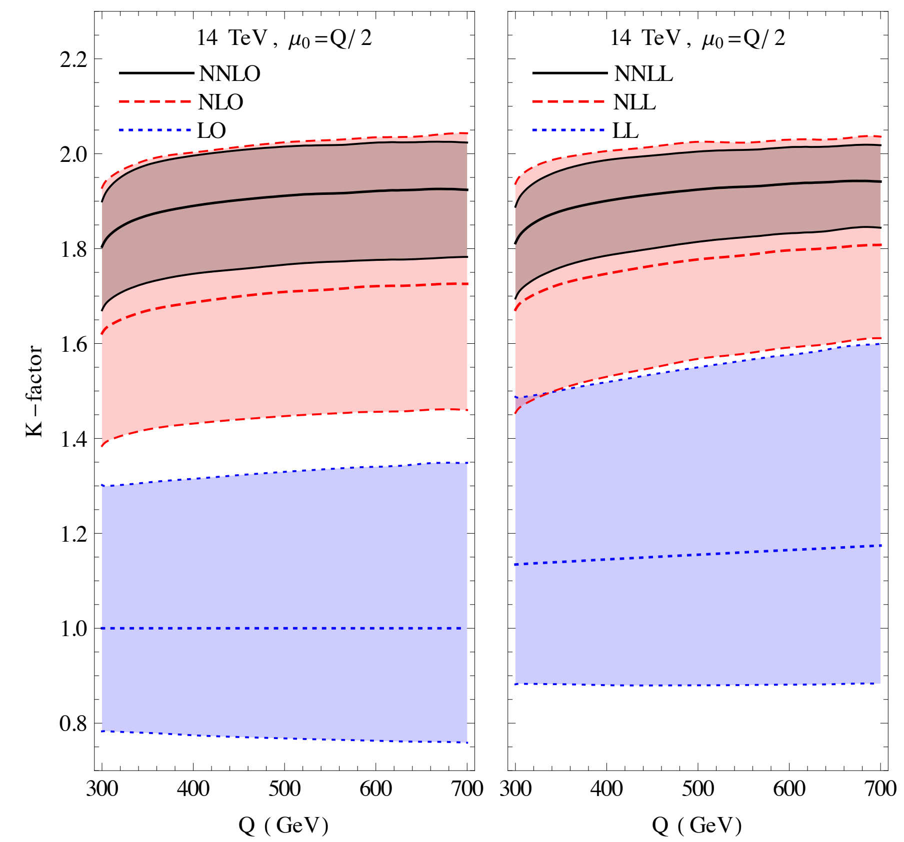
$$d\sigma_{\exp,N} = \sum_{k=0}^{N} d\sigma^{(k)} \left(\frac{\Lambda}{m_t} \right)^{2k}$$

$$\Lambda \in \{ \sqrt{s}, \sqrt{t}, \sqrt{u}, m_h \}$$

- full real-emission matrix elements and dipoles
- virtual corrections as asymptotic expansion in $1/m_t^2$ with `q2e/exp` [Harlander, Seidensticker, Seidensticker] + `Reduze` [von Manteuffel, Studerus] + `matad` [Steinhauser]
- not directly comparable with [Grigo, Hoff, Steinhauser], (real radiation treated differently, expansion parameter $(m_H/m_t)^2$)

HEFT NNLO + NNLL

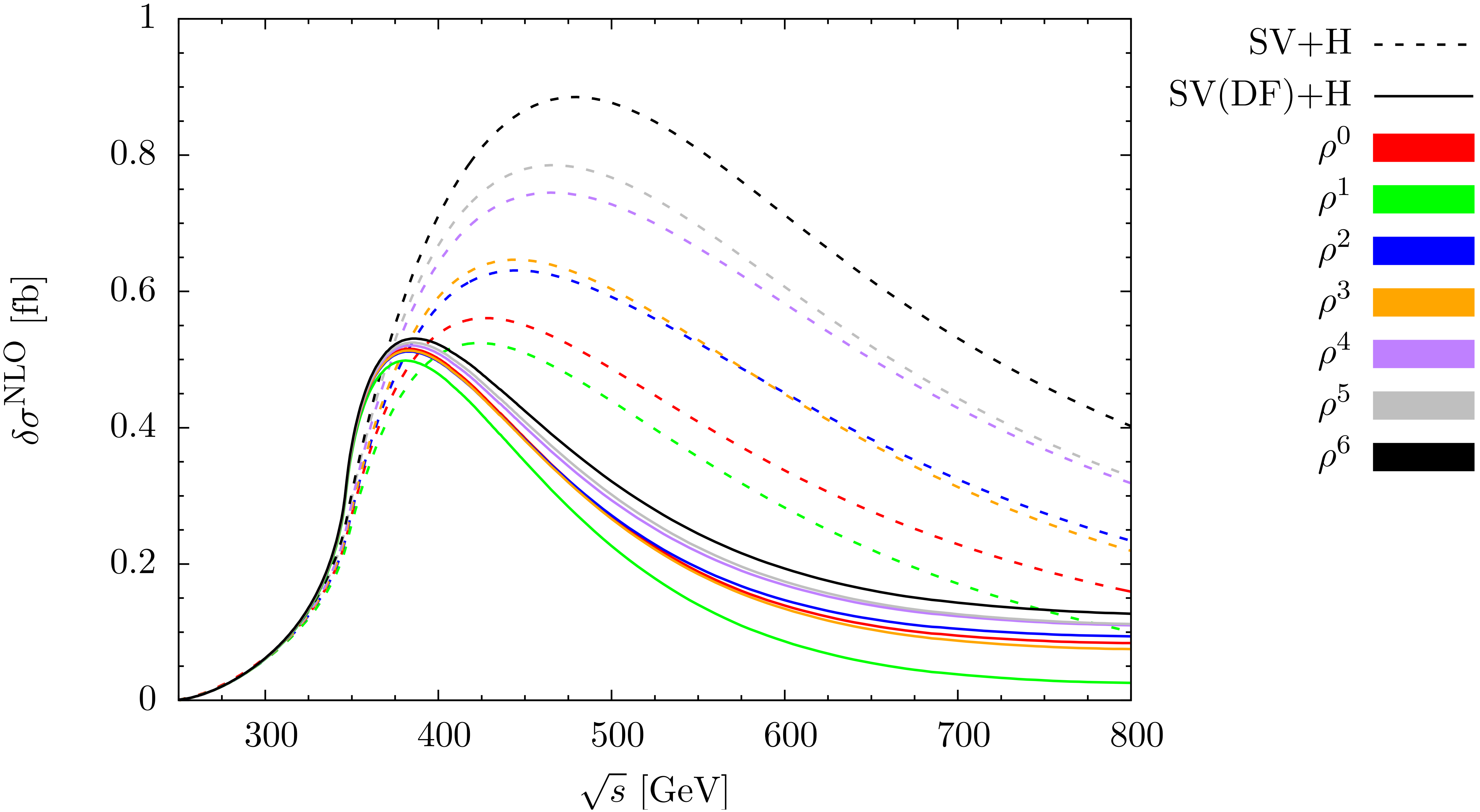




de Florian, Mazzitelli 15

$\mu_0 = Q$	NNLO (fb)	scale unc. (%)	NNLL (fb)	scale unc. (%)	PDF unc. (%)	PDF+α_S unc. (%)
8 TeV	9.92	$+9.3 - 10$	10.8	$+5.4 - 5.9$	$+5.6 - 6.0$	$+9.3 - 9.2$
13 TeV	34.3	$+8.3 - 8.9$	36.8	$+5.1 - 6.0$	$+4.0 - 4.3$	$+7.7 - 7.5$
14 TeV	40.9	$+8.2 - 8.8$	43.7	$+5.1 - 6.0$	$+3.8 - 4.0$	$+7.5 - 7.3$
33 TeV	247	$+7.1 - 7.4$	259	$+5.0 - 6.1$	$+2.2 - 2.8$	$+6.1 - 6.1$
100 TeV	1660	$+6.8 - 7.1$	1723	$+5.2 - 6.1$	$+2.1 - 3.0$	$+5.7 - 5.8$
$\mu_0 = Q/2$	NNLO (fb)	scale unc. (%)	NNLL (fb)	scale unc. (%)	PDF unc. (%)	PDF+α_S unc. (%)
8 TeV	10.8	$+5.7 - 8.5$	11.0	$+4.0 - 5.6$	$+5.8 - 6.1$	$+9.6 - 9.3$
13 TeV	37.2	$+5.5 - 7.6$	37.4	$+4.2 - 5.8$	$+4.1 - 4.3$	$+7.8 - 7.6$
14 TeV	44.2	$+5.5 - 7.6$	44.5	$+4.2 - 5.9$	$+3.9 - 4.1$	$+7.6 - 7.4$
33 TeV	264	$+5.3 - 6.6$	265	$+4.6 - 6.1$	$+2.4 - 2.7$	$+6.3 - 6.1$
100 TeV	1760	$+5.3 - 6.7$	1762	$+4.9 - 6.4$	$+2.2 - 3.1$	$+6.2 - 7.0$

G.H.S Top Mass Expansion



Grigo, Hoff, Steinhauser 15