Shell-Model Approach and Structure of Exotic Nuclei

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Shell-Model Approach and Structure of Exotic Nuclei

I. Physics case: structure of nuclei far from the valley of β-stability

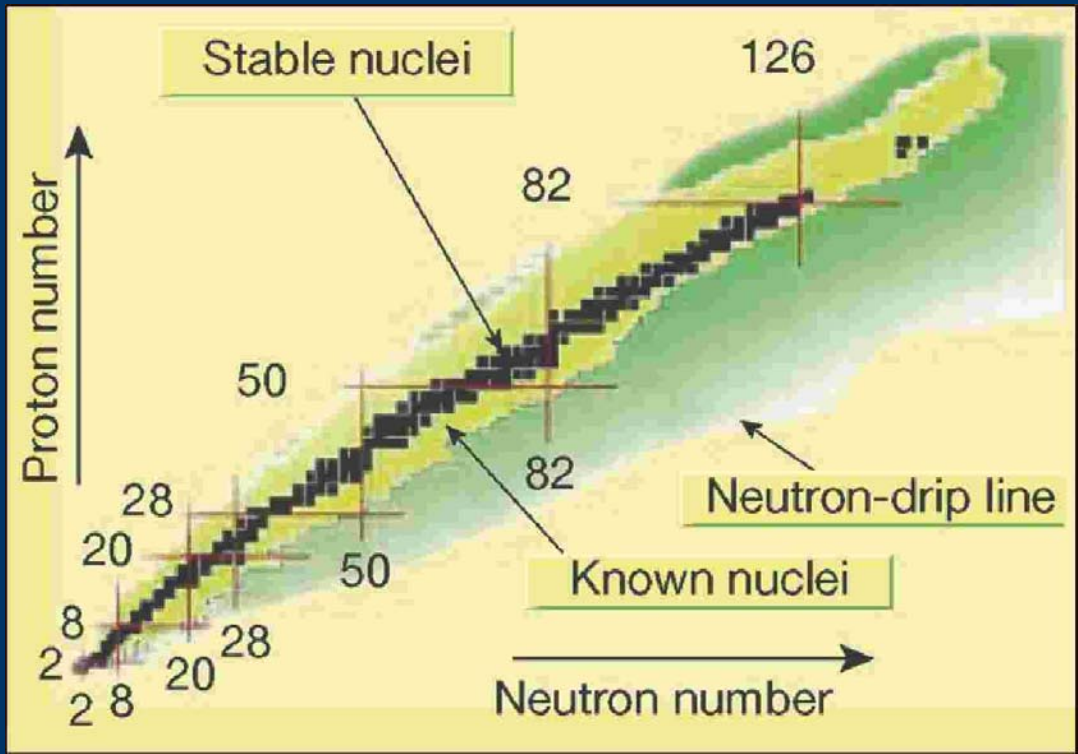
II. The nuclear shell model: formalism, advantages and modern capabilities

III. Some frontier developments and applications:

- Deformation and shape coexistence
- Evolution of the shell structure far from stability
- Drip-line systems and effects of the continuum
- No-core shell model
- Astrophysics applications
- Precision measurements and weak-interaction physics

IV. Perspectives

I. Structure of Nuclei far from Stability



- Where are particle drip-lines ?
- How nuclear structure changes near particle drip-lines ?
- What is the effective nucleon-nucleon interaction in nuclei far from stability ?
- …

Demands from modern nuclear theory : to explain the observed properties and make predictions, e.g. for astrophysics, from a microscopic model and nucleonic interactions

II. Microscopic many-body theories

$$H = \sum_{k=1}^{A} \frac{p_k^2}{2m_k} + \sum_{k<l=1}^{A} W(k,l) + \sum_{k<l<m=1}^{A} W(k,l,m) + \ldots$$

$$k \equiv \{\vec{r}_k, \vec{\sigma}_k, \vec{\tau}_k\}, \quad \vec{p}_k = -i\hbar\vec{\nabla}_k$$

Mean-field theories: search for the 'best' mean-field potential starting from a given two-body interaction + correlations

Shell-model type theories: schematic spherically symmetric average potential + residual interaction

The nuclear shell model : from an independent particle model to extreme correlations

$$H = \underbrace{\sum_{k=1}^{A} \frac{p_k^2}{2m_k} + \sum_{k=1}^{A} U(k)}_{H_{IP}} + \underbrace{\sum_{k<l=1}^{A} W(k,l) - \sum_{k=1}^{A} U(k)}_{V_{res}} = H_{IP} + V_{res}$$

$$U(k) = \frac{m\omega^2 r_k^2}{2} + \alpha\left(\vec{l} \cdot \vec{l}\right) + \beta\left(\vec{l} \cdot \vec{s}\right)$$

M. Goeppert-Mayer, Phys. Rev. 75(1949)
O. Haxel et al, Phys.Rev. 75 (1949)

Diagonalization of the residual interaction in a spherical basis of Slater determinants

$$\Psi_{i_1,\ldots,i_A}(1,\ldots,A) = \det\left\{\phi_{i_1}(1),\ldots,\phi_{i_A}(A)\right\}$$

All the physics – evolution of the spherical mean-field or deformation – is in the <u>residual interaction</u>

Model Space and Effective Operators

Hilbert space → valence space (perturbation theory)

$$V_{NN} \Rightarrow V^{eff}$$

$$\hat{H}\,|\Psi\rangle = E\,|\Psi\rangle \Rightarrow \hat{H}^{eff}\left|\Psi^{eff}\right\rangle = E\,\left|\Psi^{eff}\right\rangle$$

$$\langle\Psi|\hat{O}|\Psi\rangle \Rightarrow \left\langle\Psi^{eff}\middle|\hat{O}^{eff}\middle|\Psi^{eff}\right\rangle$$

A bare NN-potential - CD-Bonn (Machleidt, 2001), Nijmegen II (Stoks et al, 1993), AV18 (Wiringa et al, 1995), chiral N3LO potential (Entem, Machleidt, 2004) - requires regularization

- Brueckner-Bethe-Goldstone theory : *G-matrix*

  M. Hjorth-Jensen et al, Phys.Rep.261 (1995) and refs. therein

- Renormalization group approach : $V_{low\text{-}k}$ (cutoff Λ)

  S. Bogner et al, Phys. Rep. 386 (2003)

Alternative approaches (non-perturbative resummations) :

coupled-cluster theory, …

Effective shell-model interactions

Microscopic effective interactions fail to reproduce nuclear properties when the number of valence particles increases

A.P.Zuker and collaborators: the monopole part of the interaction if deficient (lack of 3-body forces)

$\Rightarrow$ phenomenological adjustment to data

- Monopole part of the interaction adjusted (KB3, KB3G for pf-shell)

A.Poves, A.P.Zuker, Phys. Rep. 70 (1981)
G. Martinez-Pinedo et al, Phys. Rev. C55 (1997)

- Least-square fit of all the m.e. – by a linear-combination method (USD interaction for sd-shell, GXPF1 for pf-shell)

B.A.Brown, B.H Wildenthal, Ann. Rev. Nucl Part. Sci. 38 (1988)
B.A.Brown, W.A.Richter, Phys. Rev. C74 (2006)

M. Honma et al, Phys. Rev. C65 (2002); idem 69 (2004)

Shell-Model Codes

<u>M-scheme codes</u>

- Antoine (Caurier)
- Mshell (Mizusaki)
- Redstick (Ormand, Johnson)
- Vecsse (Sebe)
- Oxbash (Brown et al) ->(JT)
- Oslo code (Engeland et al)
- …

<u>Coupled codes (J-scheme)</u>

- Nathan (Caurier, Nowacki)
- DUPSM (Novoselsky, Vallières)
- Ritsschil (Zwarts)
- …

Exact diagonalization by Lanczos algorithm

Basis dimensions : $\sim 10^{10}$

All nuclei up to A~70,
Heavier nuclei : with a few valence particles above the closed shell core
Example : N=126 isotones have been studied from Pb to Pu

E. Caurier et al, Phys. Rep. C67 (2003)

Alternative techniques : Quantum Monte-Carlo Diagonalization method (Honma et al, 1995), Shell-Model Monte Carlo (Koonin et al, 1991), DMRG (Dukelsky et al, 2001), extrapolation methods (Dean et al)…

Advantages and Capabilities

- Related to the NN potential
- Spherical basis and preserve all symmetries
- Detailed information on nuclear structure (E, J^{π}, μ, Q, B(OL), $\tau_{1/2}$, $S^{1/2}$, ...)
- Very accurate description of many known nuclei at low energies, provided all the physically important degrees of freedom are in the model space and the effective interaction is appropriate

III. Some frontier applications and developments

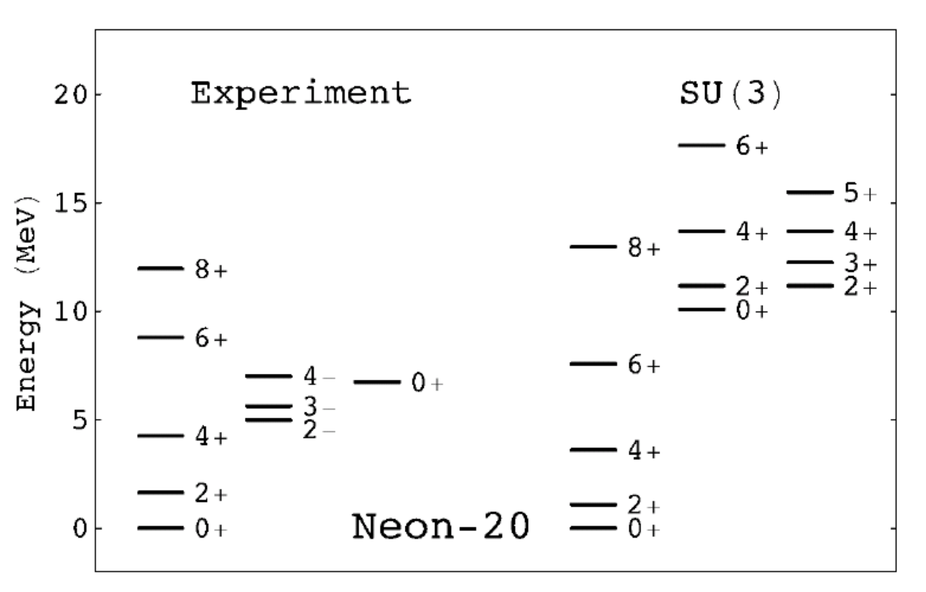
A. Deformation and shape coexistence
B. Evolution of the shell structure far from stability
C. Islands of inversion
D. Drip-line systems and effects of the continuum
E. No-core shell model
F. Astrophysics applications
G. Precision measurements and weak-interaction physics

A. Shell-model description of deformation and shape coexistence

$$H = \sum_{k=1}^{A} \left[\frac{p_k^2}{2m_k} + \frac{1}{2} m \omega^2 r_k^2 \right] + Q \cdot Q$$

J.P.Elliott (1958)

Rotational spectrum



Deformed states (bands) in spherical nuclei based on np-nh excitations between two oscillator shells can come low at energy due to the large quadrupole correlation energy – <u>shape coexistence</u>

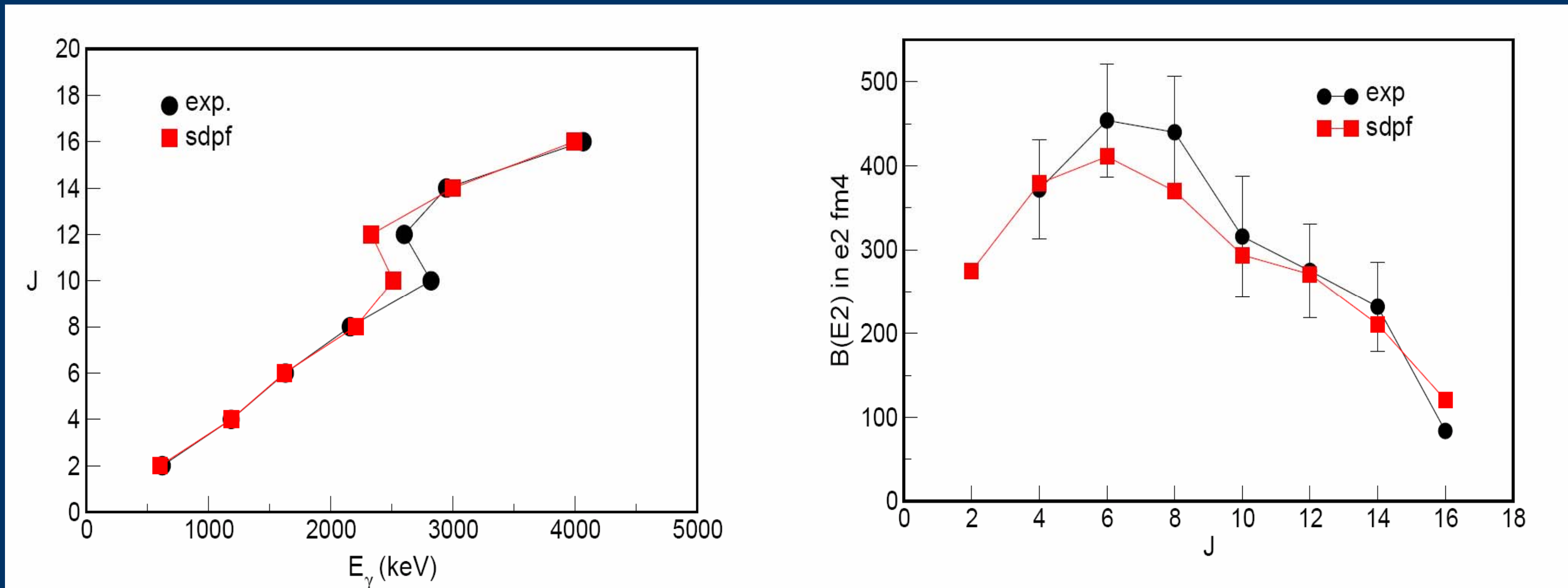
J.L. Wood et al, Phys. Rep. 102 (1992)

Superdeformation in ^{36}Ar

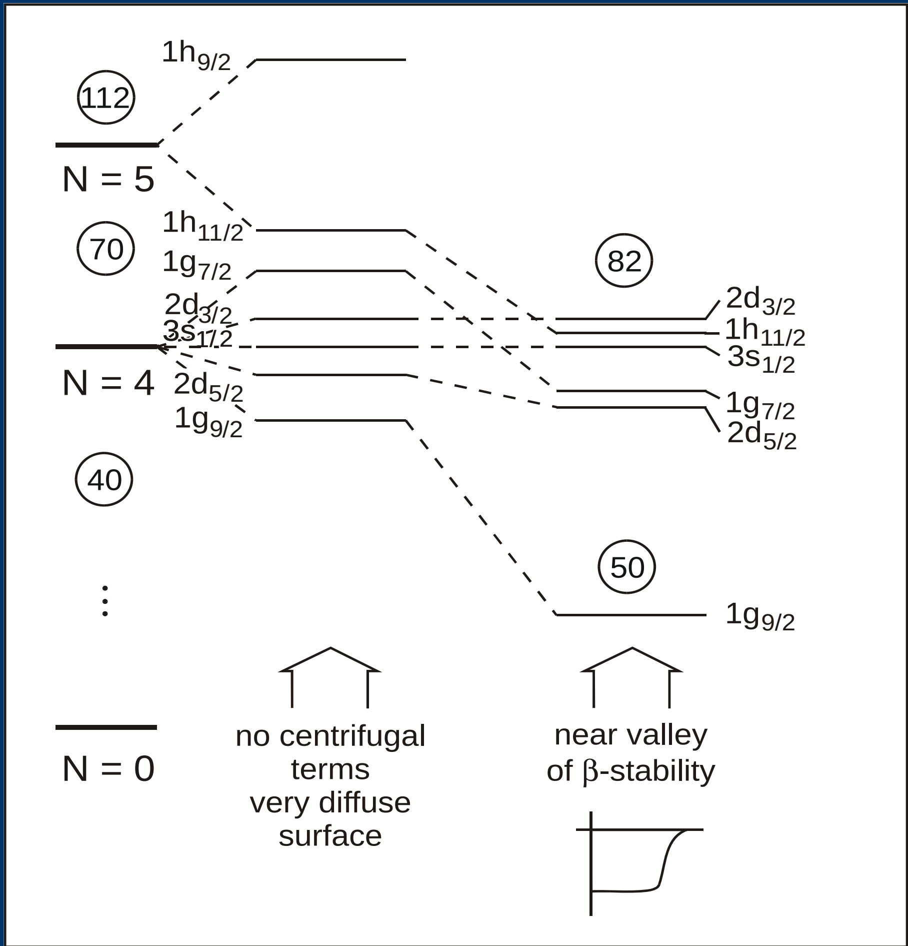
E. Caurier et al, PRL 95, 042502 (2005)

$[sd]^{16}[pf]^{0}$ - 0p0h – spherical configuration

$[sd]^{12}[pf]^{4}$ - 4p4h – deformed configuration



B. Change of single-particle spectrum far from stability



Neutron mean field in very neutron-rich nuclei:

- Diffuse neutron density
- Uniform distribution of levels
- Quenching of the shell gaps

$$H = \sum_{k=1}^{A} \left(\frac{p_k^2}{2m} + \frac{m\omega^2 r_k^2}{2} + \beta\left(\vec{l} \cdot \vec{s}\right) \right)$$

HF(B) theory with a Skyrme-type force
J.Dobaczewski et al, PRL72, 981 (1994); PRC50, 2860 (1994); PRC53, 2809 (1996)

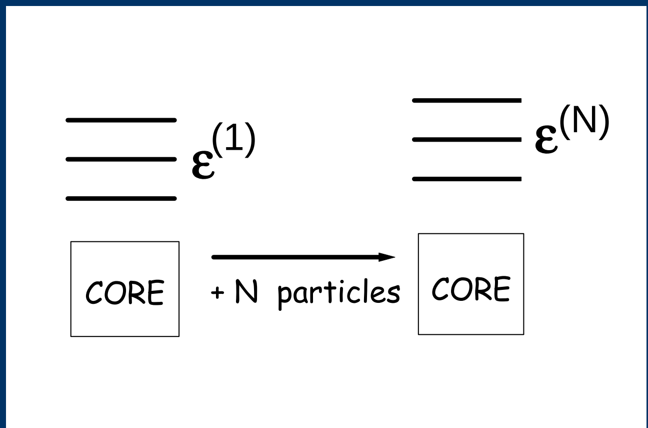
Shell-Model Approach to the Nuclear Mean Field

$$H = \sum_i \varepsilon_{\nu_i} \hat{n}_{\nu_i} + \sum_i \varepsilon_{\pi_i} \hat{n}_{\pi_i} + \sum_{ij} \hat{n}_{\nu_i} \hat{n}_{\pi_i} \overline{V}_{\nu_i \pi_j} + \ldots$$

The spherical mean-field is given by the *monopole part* of the interaction (J=0), all terms involving the particle number operators

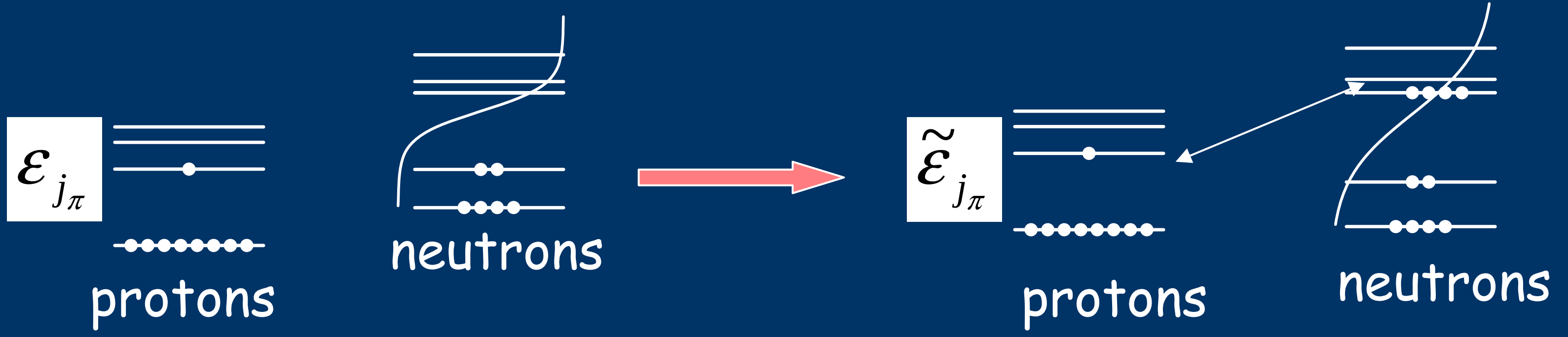
$$\langle H_M \rangle = \sum_i \varepsilon_i n_i + \sum_{i \le j} \frac{n_i (n_j - \delta_{ij})}{1 + \delta_{ij}} \overline{V}_{ij}$$

$$[n_1, n_2, \ldots, n_k]$$

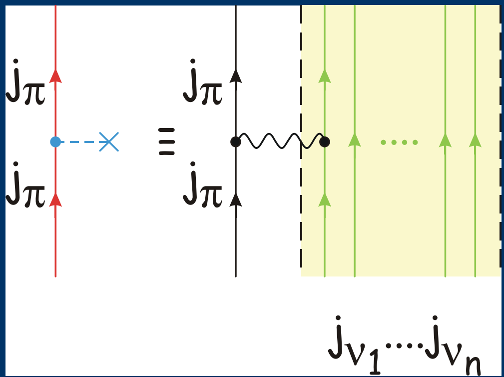


R.K.Bansal, J.B.French, Phys.Lett. 11, 145 (1964)
A.Poves, A.P.Zuker, Phys. Rep. 70, 235 (1981)
M. Dufour, A.P.Zuker, PRC54, 1641 (1996)
E.Caurier et al, Rev. Mod. Phys. 77, 427 (2005)

Proton self-energy correction in 'HF+BCS' approximation



occupation probabilities (from BCS)



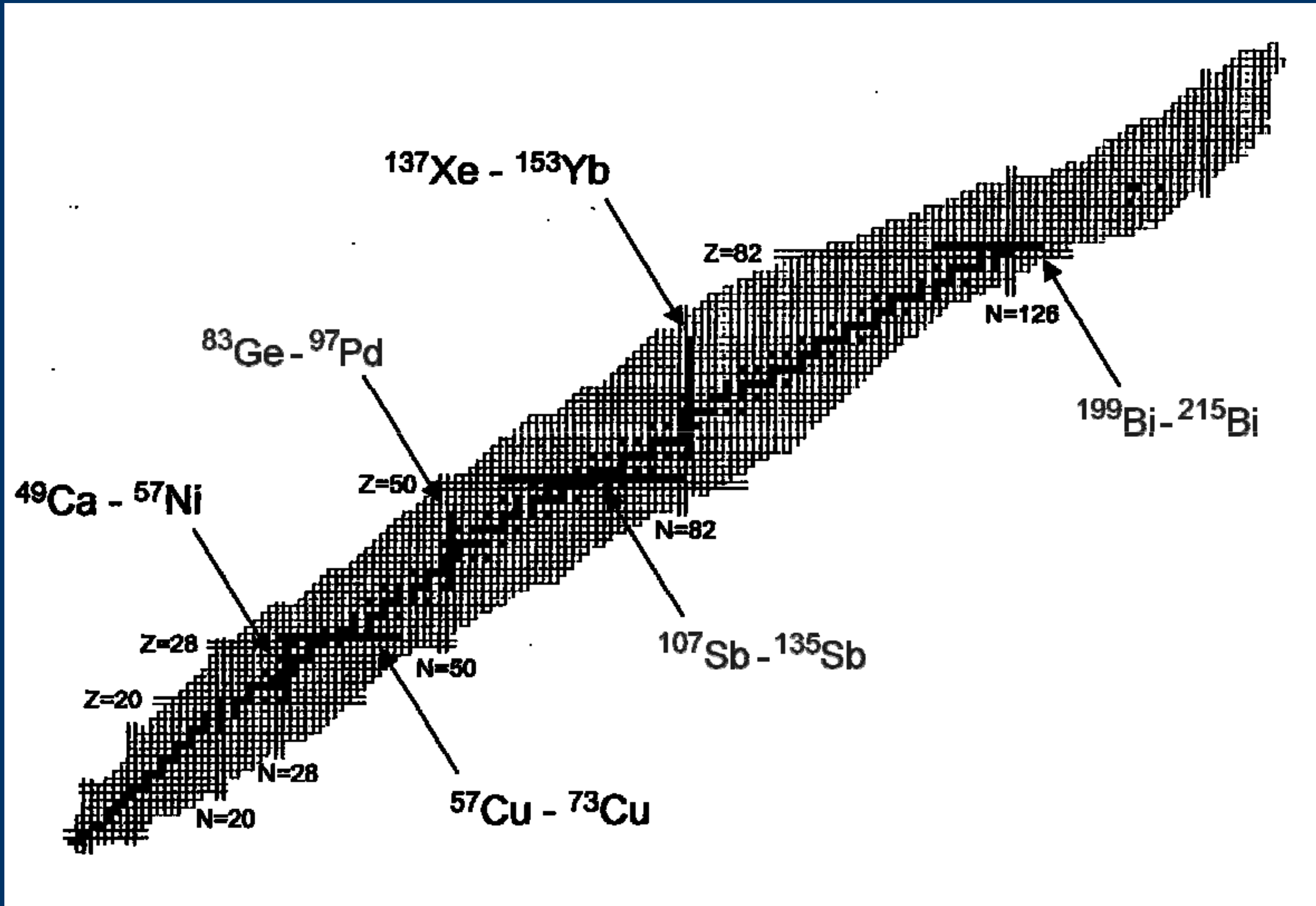
$$\tilde{\varepsilon}_{j_\pi} = \varepsilon_{j_\pi} + \sum_{j_\nu} \overline{V}_{j_\pi j_\nu} (2 j_\nu + 1) v_{j_\nu}^2$$

average interaction matrix element

$$\overline{V}_{j_\pi j_\nu} = \frac{\sum_J \langle j_\pi j_\nu | V | j_\pi j_\nu \rangle_J (2J+1)}{\sum_J (2J+1)}$$

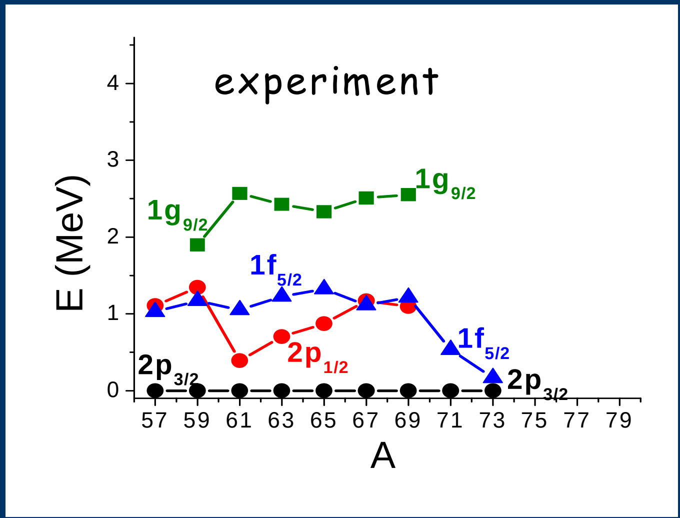
A.L.Goodman, NPA267, 1 (1977)
R.A.Sorensen,NPA420, 221 (1984)
K.Heyde et al,NPA466, 189 (1987)
A.P.Zuker, NPA576, 65 (1994)

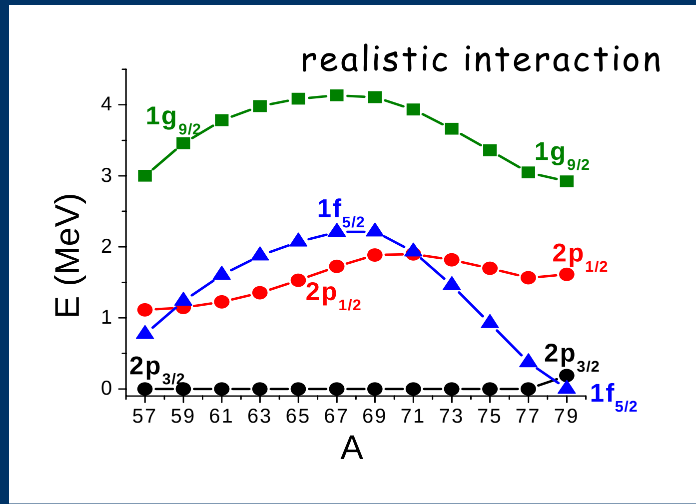
Regions of the Monopole Shifts in Nuclei



S.Franchoo, Ph.D. thesis, K.U.Leuven (1999)

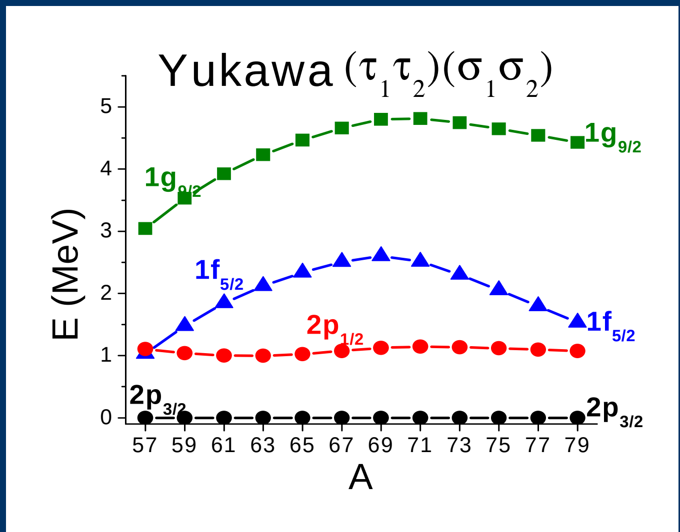
Monopole shift in neutron-rich Cu-isotopes

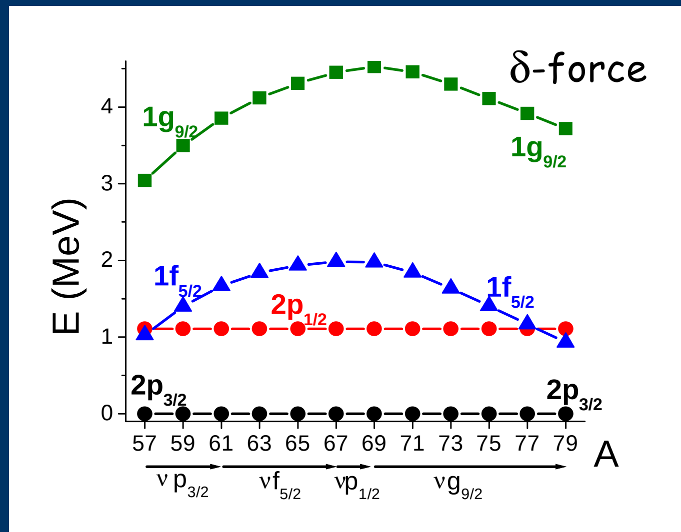




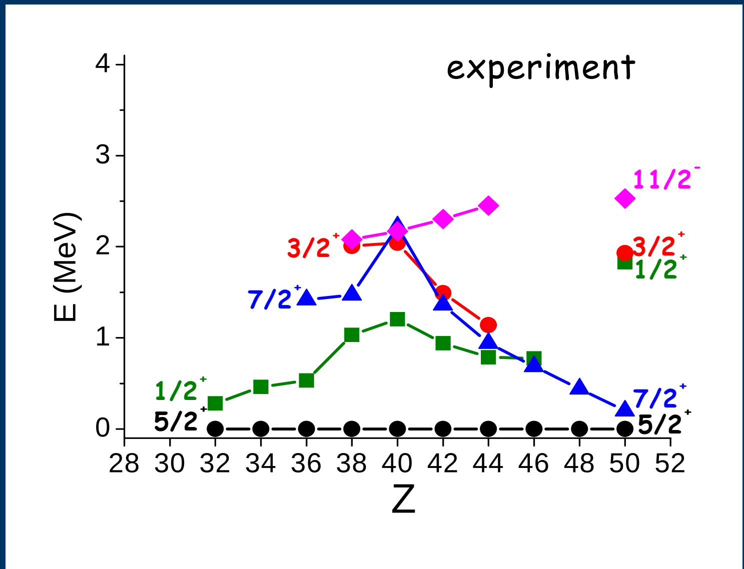
NNDC; S.Franchoo et al, PRC67 (2003)

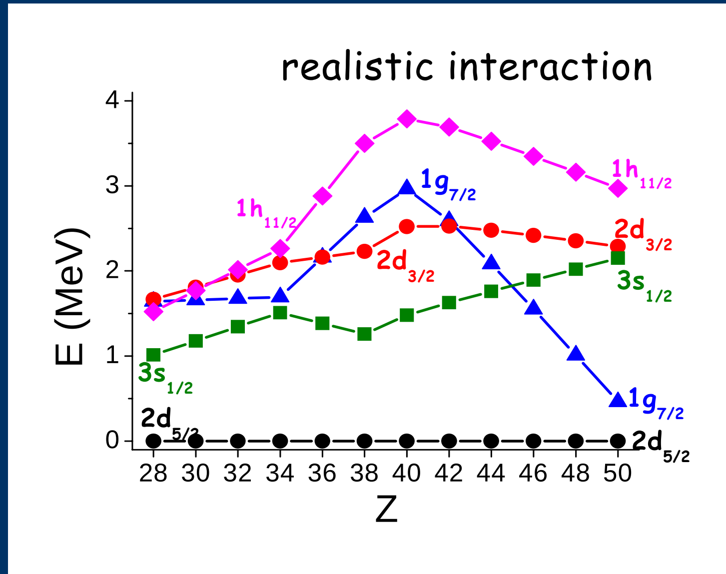
N.Smirnova et al, PRC69 (2004)



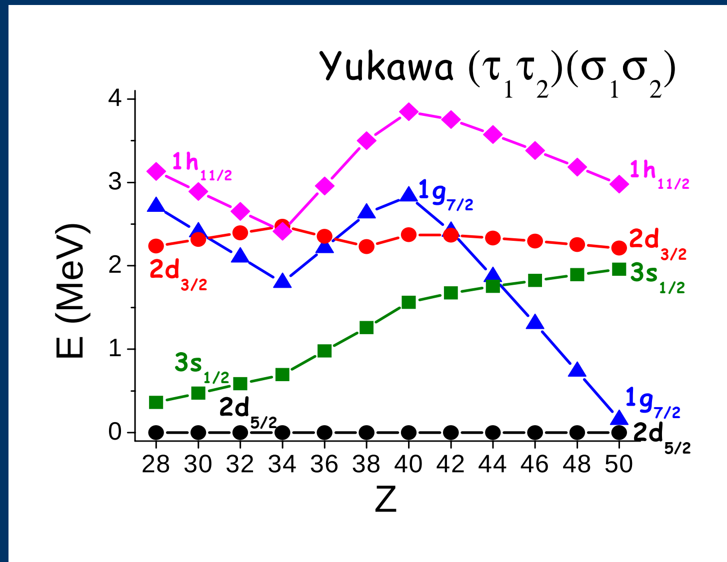


Monopole shift in N=51 isotones (^{79}Ni - ^{101}Sn)





NNDC
J.S.Thomas et al, PRC71, 021302 (2005)

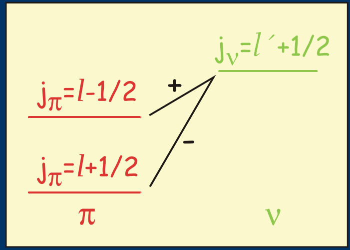


Realistic interaction in the $\pi(2p_{1/2}1g_{9/2})$ $\nu(2d3s1g_{7/2}1h_{11/2})$ shell-model space based on the G-matrix (M. Hjorth-Jensen) + modifications

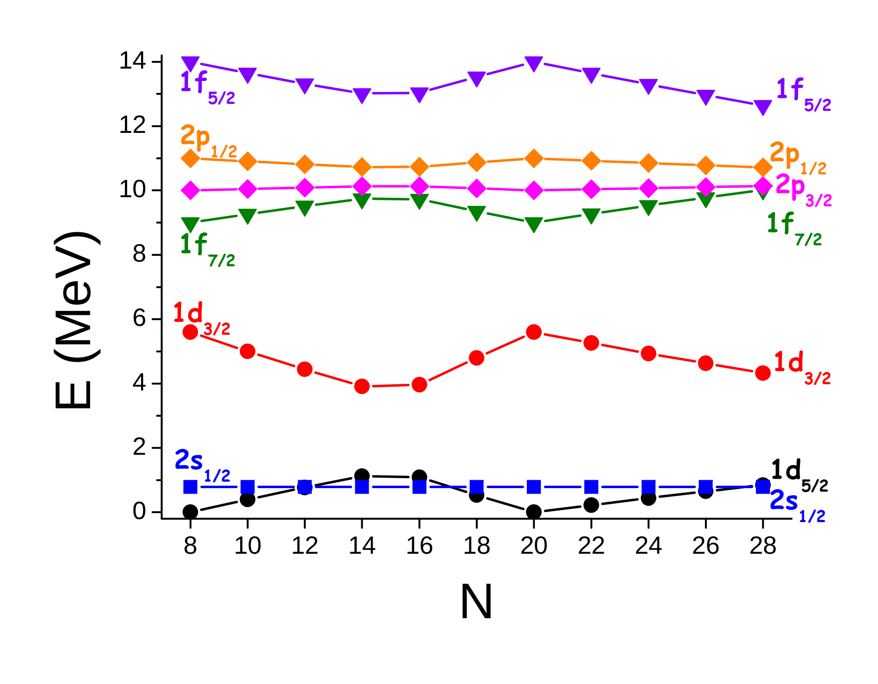
H. Grawe et al, Eur. Phys. J A 25 (2005)

Role of the tensor interaction in the shell evolution

$$V_T(1,2) = V_0(r)\left(\frac{3(\vec{\sigma}_1\cdot\vec{r})(\vec{\sigma}_2\cdot\vec{r})}{r^2} - (\vec{\sigma}_1\cdot\vec{\sigma}_2)\right)(\vec{\tau}_1\cdot\vec{\tau}_2)$$



Generic property



(sdpf)-shell model space

Shell model :

T.Otsuka et al, PRL95 (2005)
N.Smirnova et al, AIP Conf.Proc.(2006)

Need for experimental spectroscopic factors in very proton- or neutron-rich nuclei !

Important : phenomenological functionals (Skyrme, Gogny, …)

T.Otsuka et al, PRL97 (2006) : GT2
J. Dobaczewski, nucl-th/0604043

C. Island(s) of inversion

- N=20 (Na-Mg)

C. Thibault et al, PRC12 (1975)
C. Detraz et al, PRC19 (1979), …

- Early mean-field : X. Campi et al (1975), …
- Shell model : K. Heyde, J.L. Woods (1991); A.Poves, J. Retamosa (1994); Warburton et al (1990); Otsuka, Fukunushi (1996) ; …

<u>General mechanism</u> : lowering of the effective single-particle gaps (monopole field) + gain in correlation energy from promotion of the particle to the next oscillator shell

- N=28 (S - Mg)

E. Caurier et al, NPA742 (2004)

TABLE I: N=28 isotones: quasiparticle neutron gaps, difference in correlation energies between the 2p-2h and the 0p-0h configurations and their relative position

	^{40}Mg	^{42}Si	^{44}S	^{46}Ar	^{48}Ca	^{50}Ti	^{52}Cr	^{54}Fe	^{56}Ni
gap	3.35	3.50	3.23	3.84	4.73	5.33	5.92	6.40	7.12
ΔE_{Corr}	8.45	6.0	6.66	5.98	4.08	7.59	10.34	10.41	6.19
E^*_{2p-2h}	-1.75	1.0	-0.2	1.7	5.38	3.07	1.50	2.39	8.05

D. Drip-line systems and continuum effects

- Continuum shell model

G. Mahaux, H. Weidenmueller (1969)
I. Rotter, Rep. Prog. Phys. 54 (1991) and refs. therein

- SMEC

K. Bennaceur et al, NPA651 (1999)

- Gamow Shell Model
  (Berggren basis in the complex k-plane)

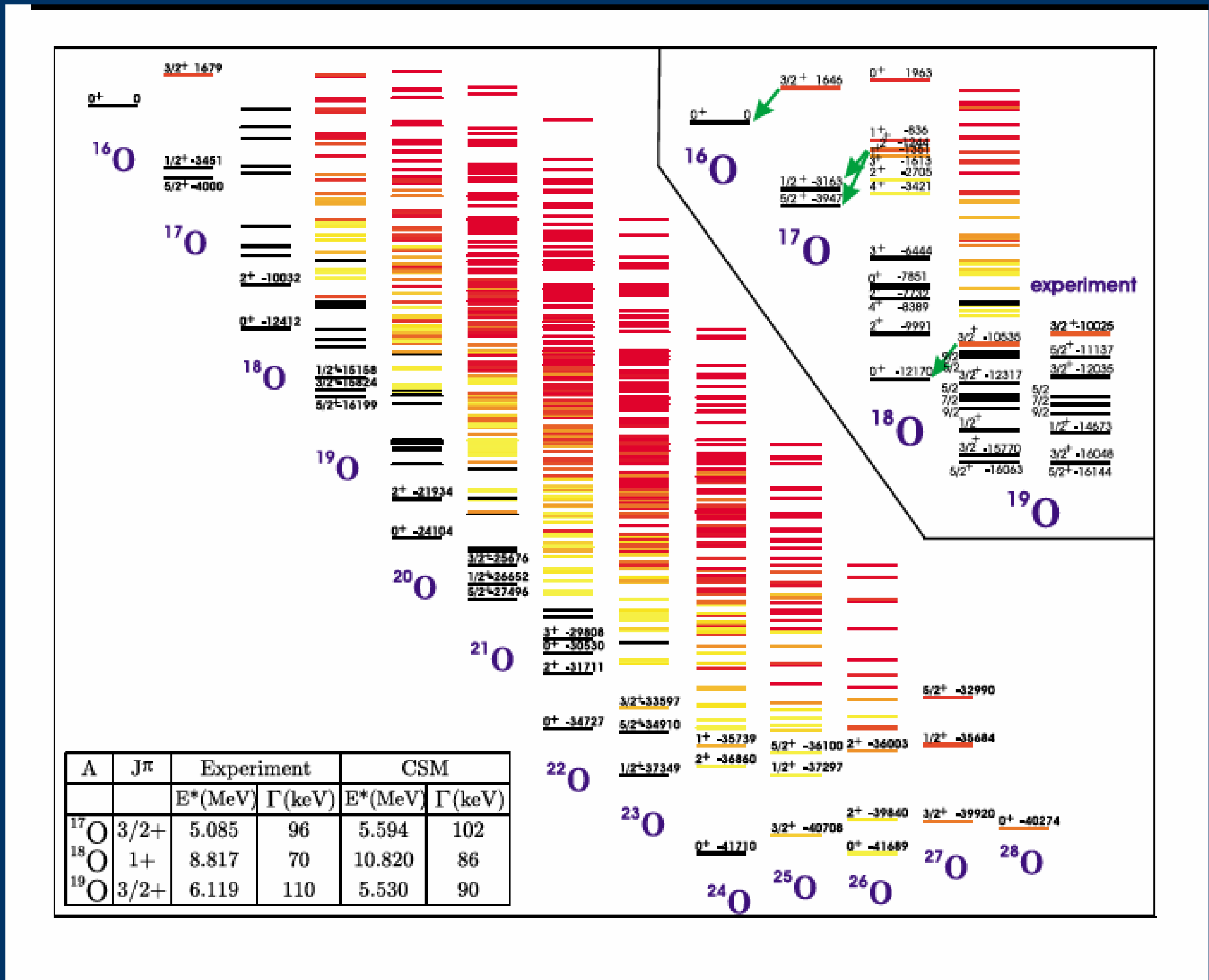
N. Michel et al, PRL89 (2002)
R. Id.Betan et al, PRL 89 (2002)
G. Hagen et al, PRC 71 (2005)

- Continuum Shell Model
  (Feshbach projection method for non-Hermitean H)

A. Volya, V. Zelevinsky, PRL 94 (2005)

O-isotopes in the continuum shell model

A. Volya, V. Zelevinsky, PRL 94 (2005)



A	Jπ	Experiment E*(MeV)	Experiment Γ(keV)	CSM E*(MeV)	CSM Γ(keV)
^{17}O	3/2+	5.085	96	5.594	102
^{18}O	1+	8.817	70	10.820	86
^{19}O	3/2+	6.119	110	5.530	90

E. *Ab-initio* no-core shell model for light nuclei

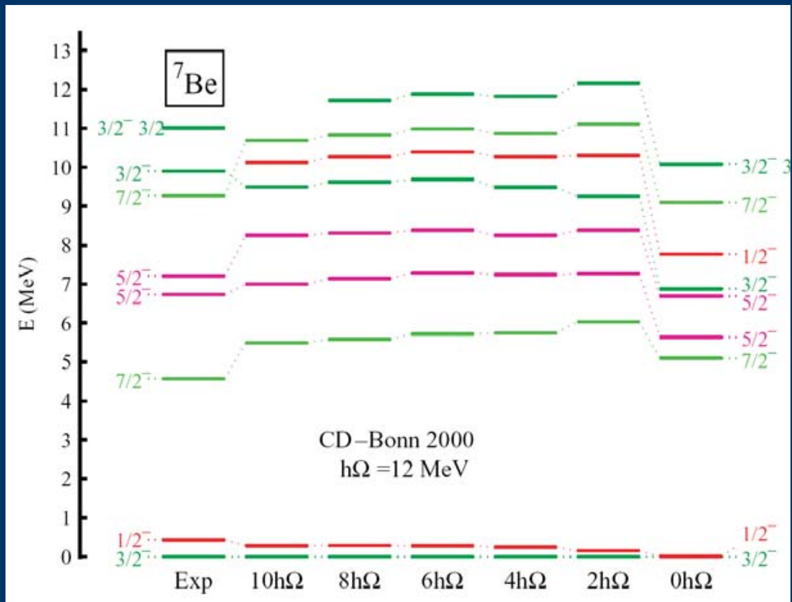
Shell-model calculations for all A-nucleons in $N\hbar\Omega$ space

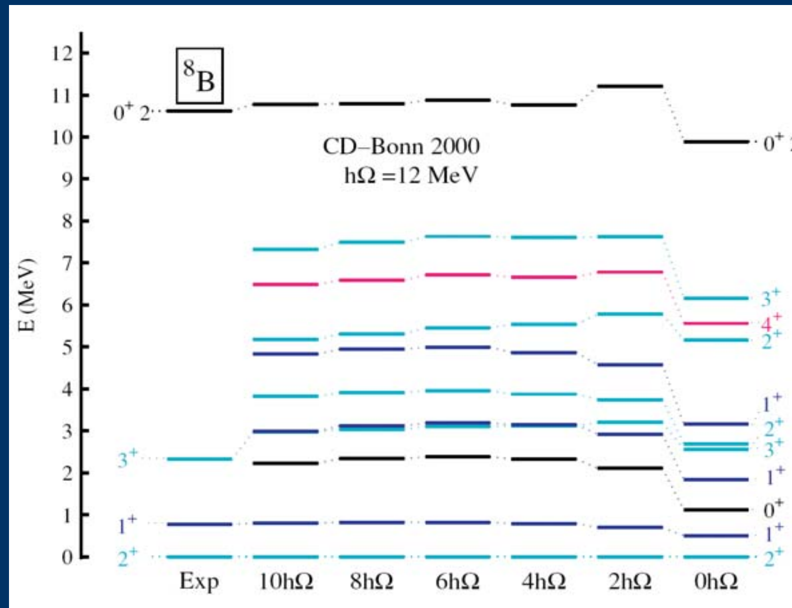
$\Rightarrow$ V_{eff} from the unitary transformation method (exact decoupling of the m.e.)

P. Navrátil, B. Barrett, Phys. Rev. C54 (1996)
P. Navrátil, J.P.Vary, B. Barrett, PRL84 (2000); PRC62 (2000)

$\Rightarrow$ 3-body forces

P. Navrátil, W.E. Ormand, PRL88 (2002); PRC67 (2003)

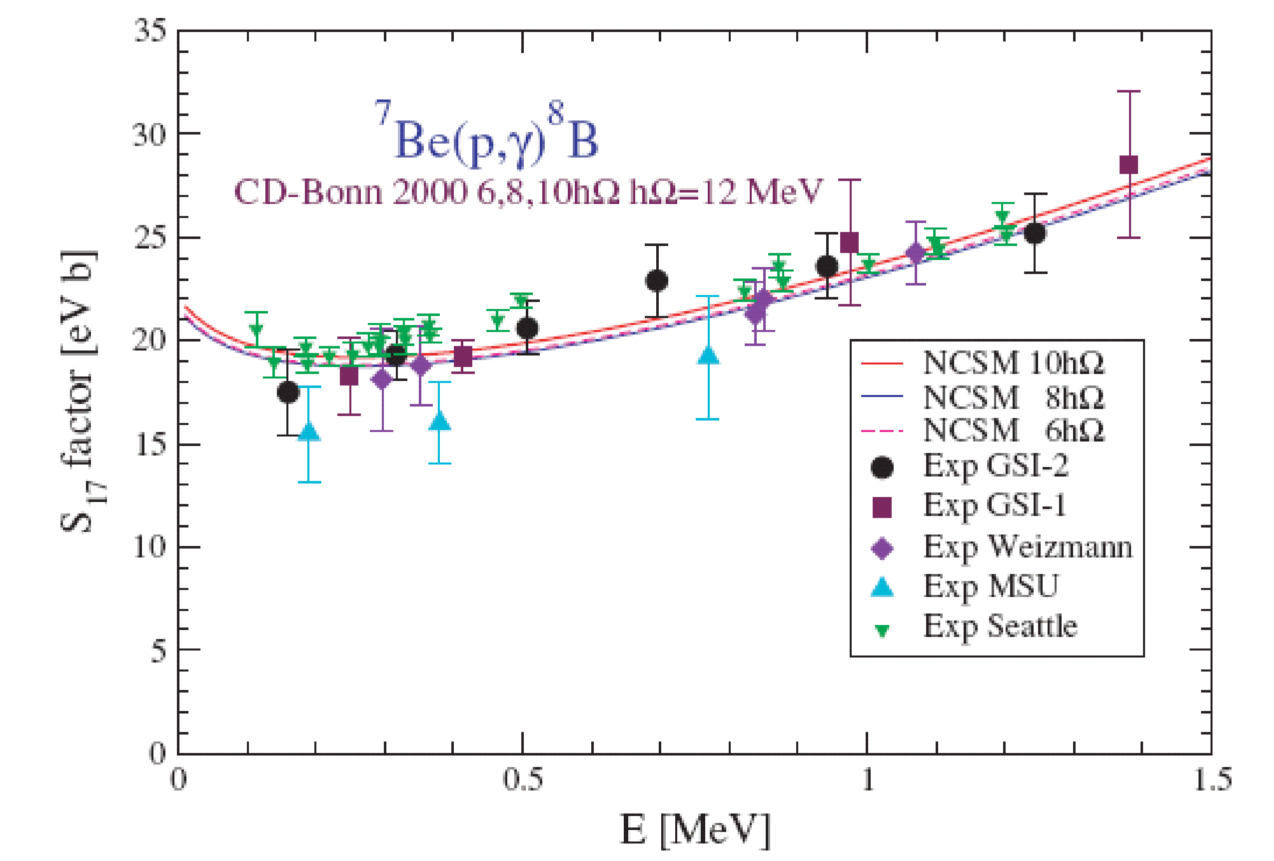




Figures taken from: P. Navrátil, E. Caurier, C. Bertulani, PRC73, 065801 (2006)

$^7Be(p,\gamma)\ ^8B$ S-factor within the no-core shell model

P. Navrátil, E. Caurier, C. Bertulani, PLB 634, 191 (2006); PRC73, 065801 (2006)



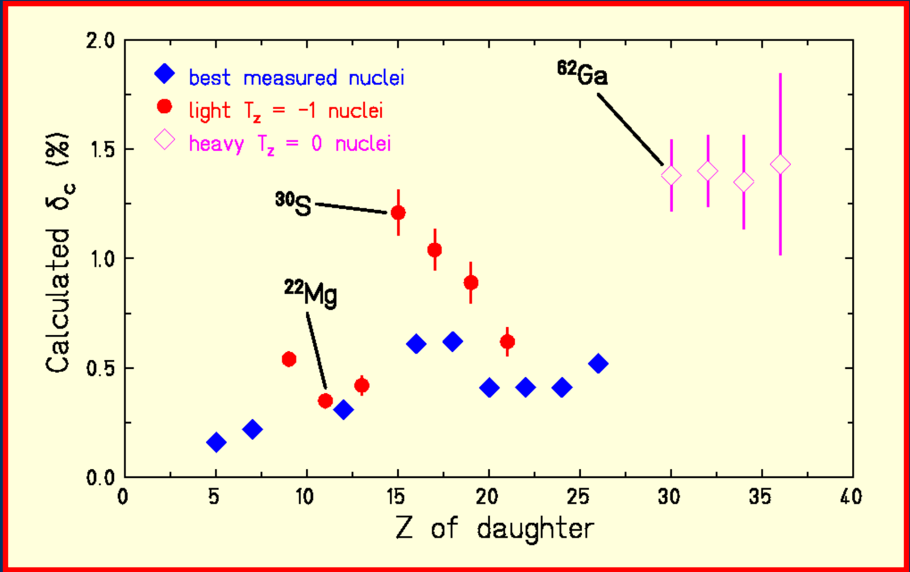
G. Nucleus as a laboratory to study weak interactions

- Superallowed $0^+ \rightarrow 0^+$ transitions and unitarity of CKM matrix
- Search for G-parity violating terms in the axial-vector weak current (mirror transitions, correlation experiments)
- First-forbidden decay and structure of the weak interaction beyond V-A hypothesis
- …

All this requires accurate description of the nuclear states, including isospin-symmetry breaking

- Displacement of energy levels in mirror nuclei
- Isospin-forbidden proton or neutron emission
- Isospin-forbidden Fermi β-decay
- Electromagentic transitions violating isospin symmetry (E1 in N=Z nuclei)
- β-delayed two proton emission,…

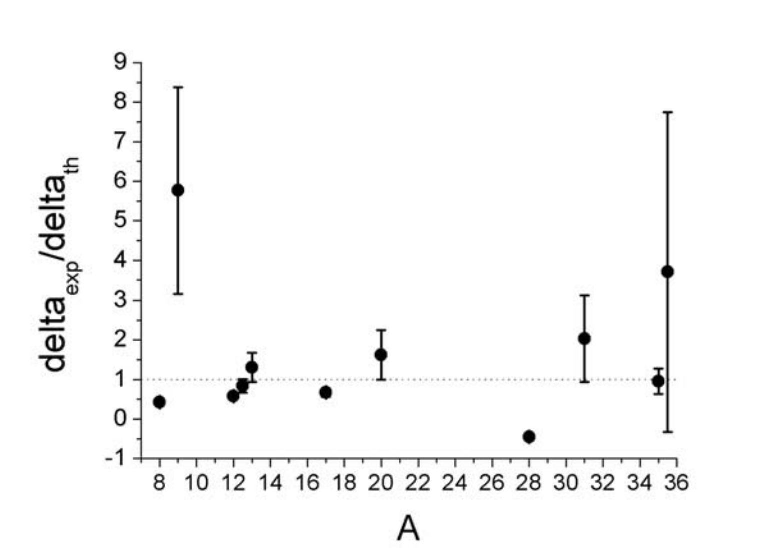
Fermi and GT matrix elements from isospin-symmetry breaking interactions



$$Ft \equiv ft(1+\delta_R)(1-\delta_C) = \frac{K}{2G_F^2 V_{ud}^2\left(1+\Delta_R^V\right)}$$

Towner, Hardy (2002, 2005)
Ormand, Brown (1989, 1995)

$$\delta = \frac{(ft)_+}{(ft)_-} - 1$$



Smirnova, Volpe, NPA714 (2003)

IV. Perspectives

- Investigation of highly unstable nuclei requires accurate description of their properties and predictions

- The shell-model can be regarded as a fundamental microscopic approach to nuclear structure

- Further challenges

  - high-precision effective interactions, including many-body forces
  - treatment of many-nucleon systems in large spaces
  - systematic accounting for continuum effects at drip-lines
  - systematic data for nuclear astrophysics applications

- The experimental data on more and more exotic nuclei (e.g. from future EURISOL facility) will be very helpful for further theoretical developments

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B. Blank (CENBG)
A. Andreyev (TRIUMF, Vancouver)
G. Georgiev (CSNSM, Orsay)
F. De Oliveira, J.-Ch. Thomas (GANIL)
J.L.Wood (GT, Atlanta)