Hierarchical Soft Terms and Flavor Physics

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SUSY flavour breaking sources

- New sources of flavour breaking (squarks only)

$$\tilde{q}_L^\dagger \tilde{m}^2_{q_L} \tilde{q}_L + \tilde{d}_R^\dagger \tilde{m}^2_{d_R} \tilde{d}_R + \tilde{u}_R^\dagger \tilde{m}^2_{u_R} \tilde{u}_R$$
$$+ \left(\tilde{d}_R^\dagger Y_D A_D \tilde{q}_L h_D + \tilde{u}_R^\dagger Y_U A_U \tilde{q}_L h_U + \text{h.c.} \right)$$

- D-squark mass matrix (in the super-CKM basis)

$$\mathcal{M}_D^2 = \begin{pmatrix} LL & LR \\ RL & RR \end{pmatrix} = \mathcal{W}_D \, \mathcal{M}_D^{2\,\text{diag}} \, \mathcal{W}_D^\dagger \qquad (\tilde{d}_L^\dagger, \tilde{d}_R^\dagger) \, \mathcal{M}_D^2 \begin{pmatrix} \tilde{d}_L \\ \tilde{d}_R \end{pmatrix}$$

$$LL = \tilde{m}^2_{q_L} + M_D^\dagger M_D + M_Z^2 z_D c_{2\beta} \mathbf{1}$$
$$RL = -M_D (A_D + \mu \tan\beta)$$

$M_D =$ down quark mass matrix $= M_D^{\text{diag}}$

Degeneracy and hierarchy

- About 100 physical real parameters
- Large FCNC (and CPV) processes in most of the parameter space, (SUSY flavour problem)
- For $\tilde{m}$ < TeV: need small 12/3 mixing + $\tilde{m}_d \approx \tilde{m}_s$ (barring alignment)
- Note: fermion masses also have peculiar structure:
  small 12/3 mixing + m_d, $m_s \ll m_b$; moreover, both correspond to an approximate U(2) symmetry
- U(3) badly broken by $Y_t = O(1)$ → expect $\tilde{m}_b \neq \tilde{m}_{d,s}$ How much?
  Two opposite (complementary) limits:
  - Degeneracy: $\tilde{m}_b \approx \tilde{m}_s \approx \tilde{m}_d$
  - Hierarchy: $\tilde{m}_b \ll \tilde{m}_{s,d}$ (not incompatible with naturalness, see below)

Model-independent analysis

- Expand in small off-diagonal elements of squark mass matrix

$$\mathcal{M}^2 = \mathcal{M}_0^2 + \mathcal{M}_1^2$$

- Degeneracy: $\mathcal{M}_0^2 = \tilde{m}^2 \mathbf{1}$ [MIA, Gabbiani Gabrielli Masiero Silvestrini '96]

$$A(\Delta F = 1)_{ij} = x\, f^{(1)}(x)\, \delta_{ij}$$

$$A(\Delta F = 2)_{ij} = \frac{x^2}{3!} g^{(3)}(x)\, \delta_{ij}^2$$

$$\delta_{ij} \equiv \frac{(\mathcal{M}_1^2)_{ij}}{\tilde{m}^2}, \quad x \equiv \frac{\tilde{m}^2}{M^2}$$

- δ_{ij}: source of flavour violation, process independent
- f, g: loop functions (process dependent, flavour conserving)
- # of derivatives and factorial ↔ # of identical propagators (-1)
- OK in case of short RGE running, in any case useful recipe up to (harmless? see below) O(1) differences between f and f'

Model-independent analysis

- Expand in small off-diagonal elements of squark mass matrix

$$\mathcal{M}^2 = \mathcal{M}_0^2 + \mathcal{M}_1^2$$

- Hierarchy: [Cohen Kaplan Lepeintre Nelson '97]

$$\mathcal{M}_0^2 = \left(\begin{array}{c|c} \text{heavy} \quad \tilde{m}^2 & \\ \hline & \text{heavy} \quad \tilde{m}^2 \end{array}\right)$$

$$A(\Delta F = 1)_{ij} = f(x)\,\hat{\delta}_{ij}$$

$$A(\Delta F = 2)_{ij} = g^{(1)}(x)\,\hat{\delta}_{ij}^2$$

- $\hat{\delta}_{ij} \equiv \mathcal{W}_{i3}\mathcal{W}_{3j}^\dagger \quad x = \dfrac{\tilde{m}^2}{M^2} \qquad \hat{\delta}_{a3} = \dfrac{\mathcal{M}_{a3}^2}{\tilde{m}_a^2} \quad (a = 1, 2)$
- $\boxed{\hat{\delta}_{ab} \approx \hat{\delta}_{a3}\,\hat{\delta}_{3b}}$ (a,b, = 1,2)
- $\boxed{\hat{\delta}_{a3}^{LR} \approx \hat{\delta}_{a3}^{LL}\,\hat{\delta}_{33}^{LR}} \qquad \hat{\delta}_{33}^{LR} = -\dfrac{m_b(A_{33}^D + \mu\tan\beta)}{\tilde{m}^2}$ (under assumptions)

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- $\boxed{\hat{\delta}_{ab} \approx \hat{\delta}_{a3}\,\hat{\delta}_{3b}}$ (a,b, = 1,2)
- $\boxed{\hat{\delta}^{LR}_{a3} \approx \hat{\delta}^{LL}_{a3}\,\hat{\delta}^{LR}_{33}} \qquad \hat{\delta}^{LR}_{33} = -\dfrac{m_b(A^D_{33} + \mu\tan\beta)}{\tilde{m}^2}$ (under assumptions)

4 complex parameters + 3rd family mixing

Degeneracy vs Hierarchy

- Different correlation between ΔF = 1 and ΔF = 2 in the two cases

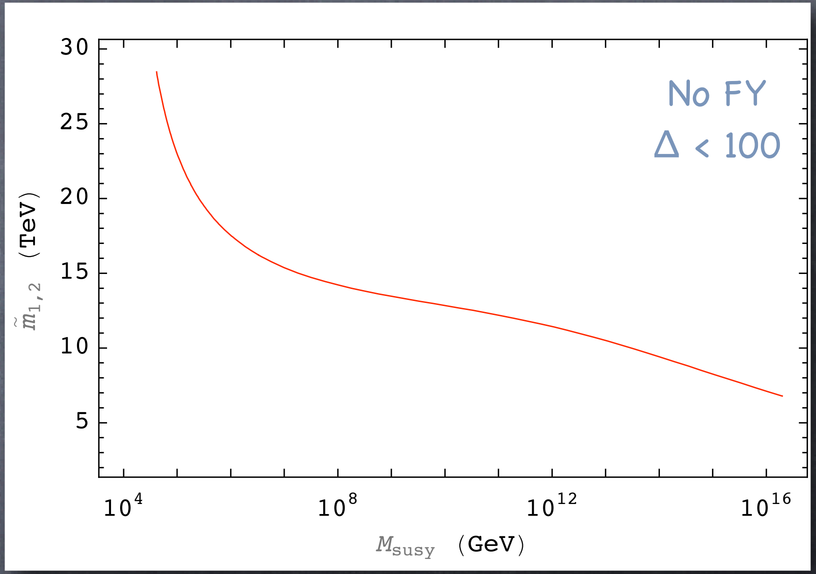
$$\left.\frac{A(\Delta F=2)}{[A(\Delta F=1)]^2}\right|_{\text{degeneracy}} = \frac{1}{6}\frac{g^{(3)}}{g^{(1)}}\left(\frac{f}{f^{(1)}}\right)^2 \left.\frac{A(\Delta F=2)}{[A(\Delta F=1)]^2}\right|_{\text{hierarchy}} .$$

E.g.: $A(\Delta m_{Bs})$ vs $A(b\to s\gamma)$
LL insertion

$$\left.\frac{g^{(3)}}{g^{(1)}}\left(\frac{f}{f^{(1)}}\right)^2\right|_{x=1} \approx \frac{25}{27}$$

Theoretical considerations on the hierarchical case

- The naturalness bound on $\tilde{m}_{1,2}$ is milder than the one on $\tilde{m}_3$ [Dimopoulos Giudice '95]



- Effective supersymmetry [Cohen Kaplan Lepeintre Nelson '97]
- Alleviates the SUSY flavour problem: $\hat{\delta} \lesssim \tilde{m}/\tilde{m}_{1,2}$
- Complementary to degeneracy
- Related to physical quantities: squark masses, CKM angles

Bounds on δ's

Hierarchy | Degeneracy

$D_0 - \bar{D}_0$ mixing

$\left\vert\hat\delta^{LL}_{ut}\hat\delta^{LL*}_{ct}\right\vert < 8.0 \times 10^{-3}\left(\frac{m_{\tilde t}}{350\ \mathrm{GeV}}\right)$	$\left\vert\delta^{LL}_{uc}\right\vert < 3.4 \times 10^{-2}\left(\frac{m_{\tilde q}}{350\ \mathrm{GeV}}\right)$

$B \to X_s\gamma$

$\left\vert\mathrm{Re}\left(\hat\delta^{LL}_{sb}\right)\right\vert < 2.2 \times 10^{-2}\left(\frac{m_{\tilde b}}{350\ \mathrm{GeV}}\right)^2\left(\frac{10}{\tan\beta}\right)$	$\left\vert\mathrm{Re}\left(\delta^{LL}_{sb}\right)\right\vert < 3.8 \times 10^{-2}\left(\frac{m_{\tilde q}}{350\ \mathrm{GeV}}\right)^2\left(\frac{10}{\tan\beta}\right)$
$\left\vert\mathrm{Im}\left(\hat\delta^{LL}_{sb}\right)\right\vert < 6.7 \times 10^{-2}\left(\frac{m_{\tilde b}}{350\ \mathrm{GeV}}\right)^2\left(\frac{10}{\tan\beta}\right)$	$\left\vert\mathrm{Im}\left(\delta^{LL}_{sb}\right)\right\vert < 1.1 \times 10^{-1}\left(\frac{m_{\tilde q}}{350\ \mathrm{GeV}}\right)^2\left(\frac{10}{\tan\beta}\right)$

$|\hat\delta_{sb}| \gtrsim 4\ 10^{-2}$

Δm_{B_s}

$\left\vert\mathrm{Re}\left(\hat\delta^{LL}_{sb}\right)\right\vert < 9.4 \times 10^{-2}\left(\frac{m_{\tilde b}}{350\ \mathrm{GeV}}\right)$	$\left\vert\mathrm{Re}\left(\delta^{LL}_{sb}\right)\right\vert < 4.0 \times 10^{-1}\left(\frac{m_{\tilde q}}{350\ \mathrm{GeV}}\right)$
$\left\vert\mathrm{Im}\left(\hat\delta^{LL}_{sb}\right)\right\vert < 7.2 \times 10^{-2}\left(\frac{m_{\tilde b}}{350\ \mathrm{GeV}}\right)$	$\left\vert\mathrm{Im}\left(\delta^{LL}_{sb}\right)\right\vert < 3.1 \times 10^{-1}\left(\frac{m_{\tilde q}}{350\ \mathrm{GeV}}\right)$

$\tilde m = M_3 = \mu$

$A = 0$

B^0_d–$\bar B^0_d$ mixing

$\left\vert\mathrm{Re}\left(\hat\delta^{LL}_{db}\right)\right\vert < 4.3 \times 10^{-3}\left(\frac{m_{\tilde b}}{350\ \mathrm{GeV}}\right)$	$\left\vert\mathrm{Re}\left(\delta^{LL}_{db}\right)\right\vert < 1.8 \times 10^{-2}\left(\frac{m_{\tilde q}}{350\ \mathrm{GeV}}\right)$
$\left\vert\mathrm{Im}\left(\hat\delta^{LL}_{db}\right)\right\vert < 7.3 \times 10^{-3}\left(\frac{m_{\tilde b}}{350\ \mathrm{GeV}}\right)$	$\left\vert\mathrm{Im}\left(\delta^{LL}_{db}\right)\right\vert < 3.1 \times 10^{-2}\left(\frac{m_{\tilde q}}{350\ \mathrm{GeV}}\right)$

$|\hat\delta_{db}| \gtrsim 0.8\ 10^{-2}$

Δm_K

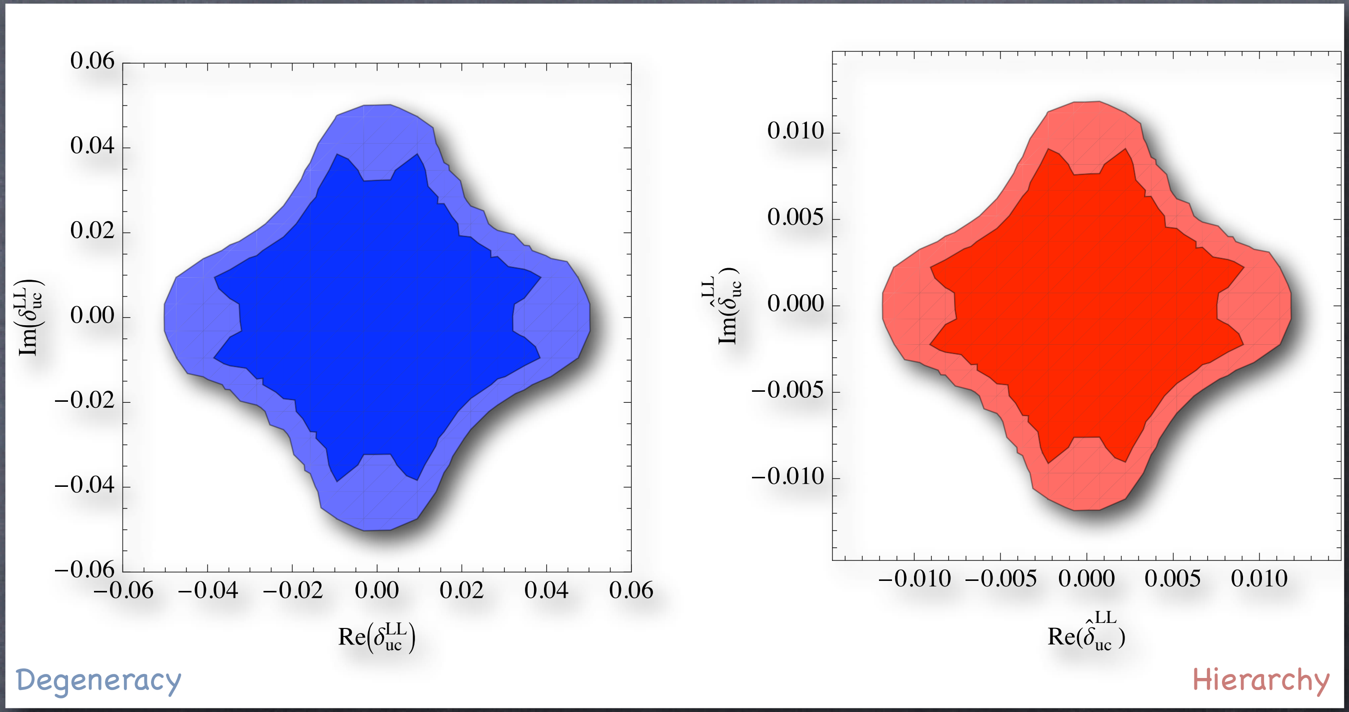
$\sqrt{\left\vert\mathrm{Re}\left(\hat\delta^{LL}_{db}\hat\delta^{LL*}_{sb}\right)^2\right\vert} < 1.0 \times 10^{-2}\left(\frac{m_{\tilde b}}{350\ \mathrm{GeV}}\right)$	$\sqrt{\left\vert\mathrm{Re}\left(\delta^{LL}_{ds}\right)^2\right\vert} < 4.2 \times 10^{-2}\left(\frac{m_{\tilde q}}{350\ \mathrm{GeV}}\right)$

$|\hat\delta_{ds}| \gtrsim 3\ 10^{-4}$

ϵ_K

$\sqrt{\left\vert\mathrm{Im}\left(\hat\delta^{LL}_{db}\hat\delta^{LL*}_{sb}\right)^2\right\vert} < 4.4 \times 10^{-4}\left(\frac{m_{\tilde b}}{350\ \mathrm{GeV}}\right)$	$\sqrt{\left\vert\mathrm{Im}\left(\delta^{LL}_{ds}\right)^2\right\vert} < 1.8 \times 10^{-3}\left(\frac{m_{\tilde q}}{350\ \mathrm{GeV}}\right)$

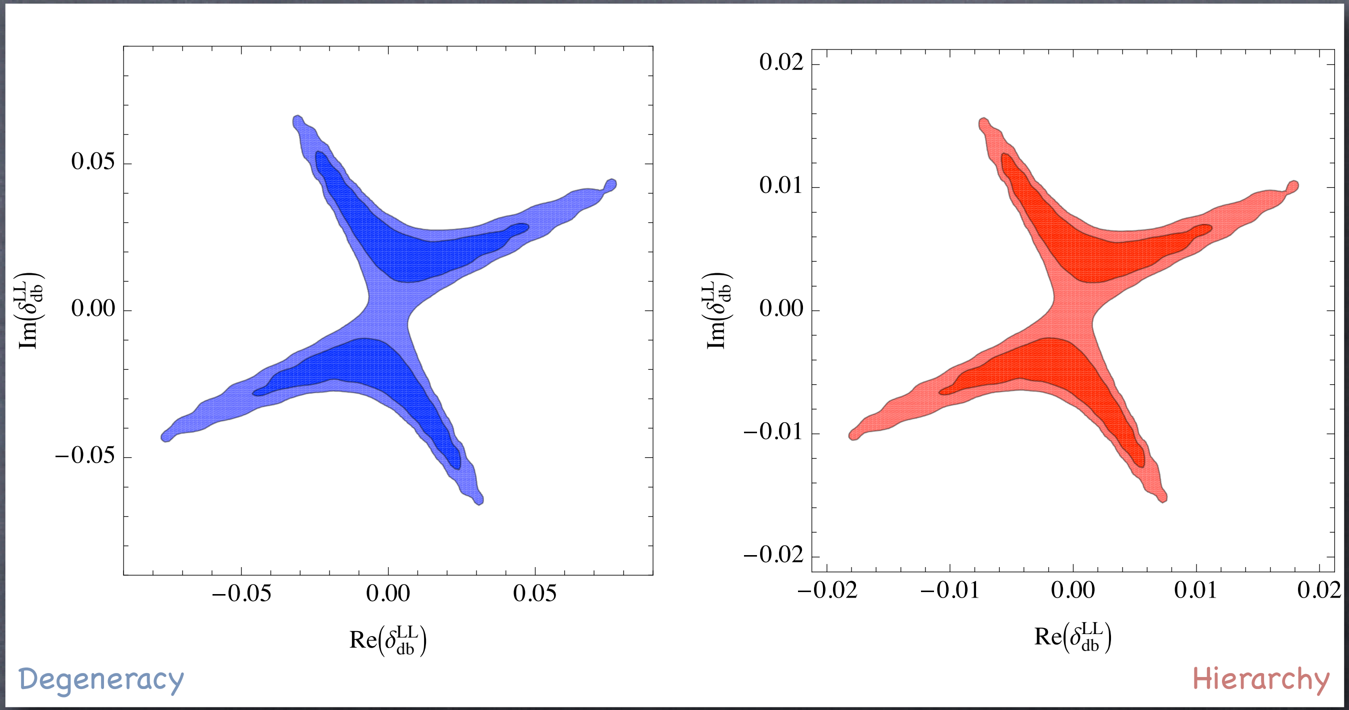
$u \leftrightarrow c$ transitions (D^0-$\bar{D}^0$)



68%, 95% C.L.

$\tilde{m} = M_3 = 350$ GeV

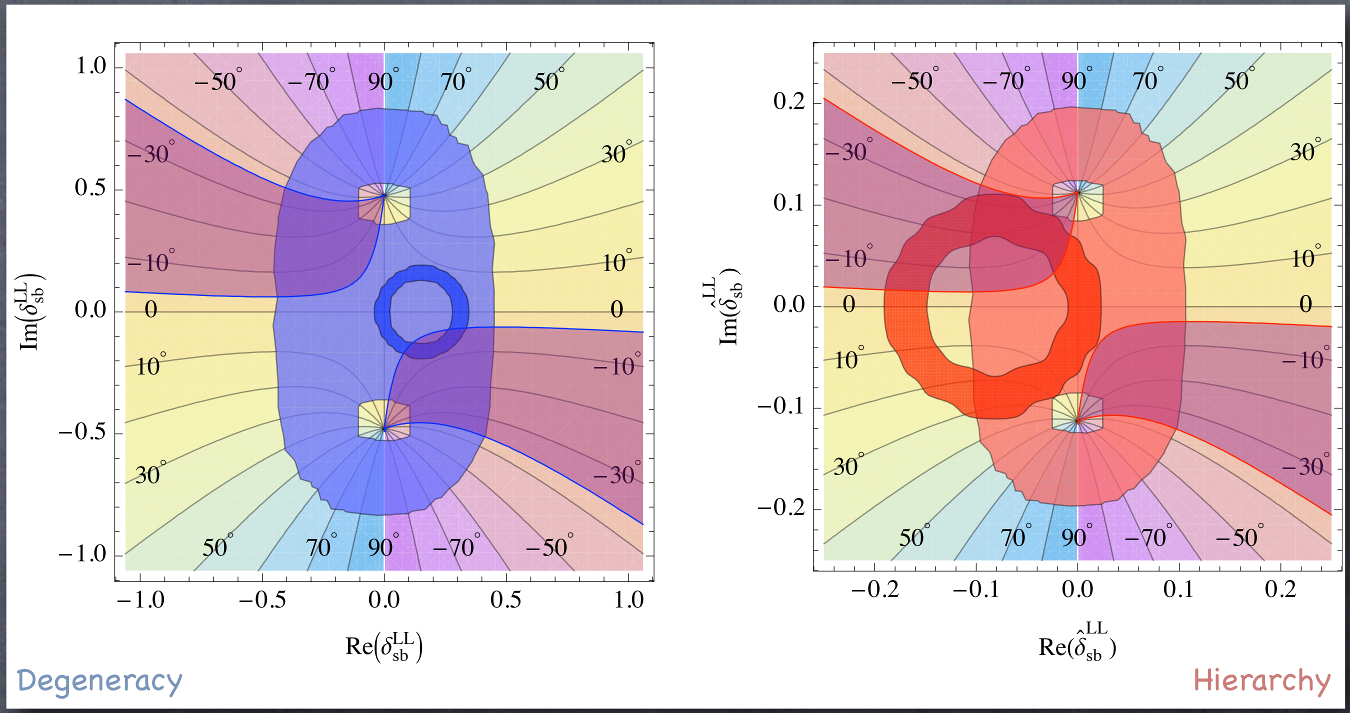
$b \leftrightarrow d$ transitions (Δm_{Bd}, $\sin 2\beta_{eff}$)



68%, 95% C.L.

$\tilde{m} = M_3 = 350$ GeV

b ↔ s transitions (Δm_{Bs}, $b \to s\gamma$, ϕ_{Bs})



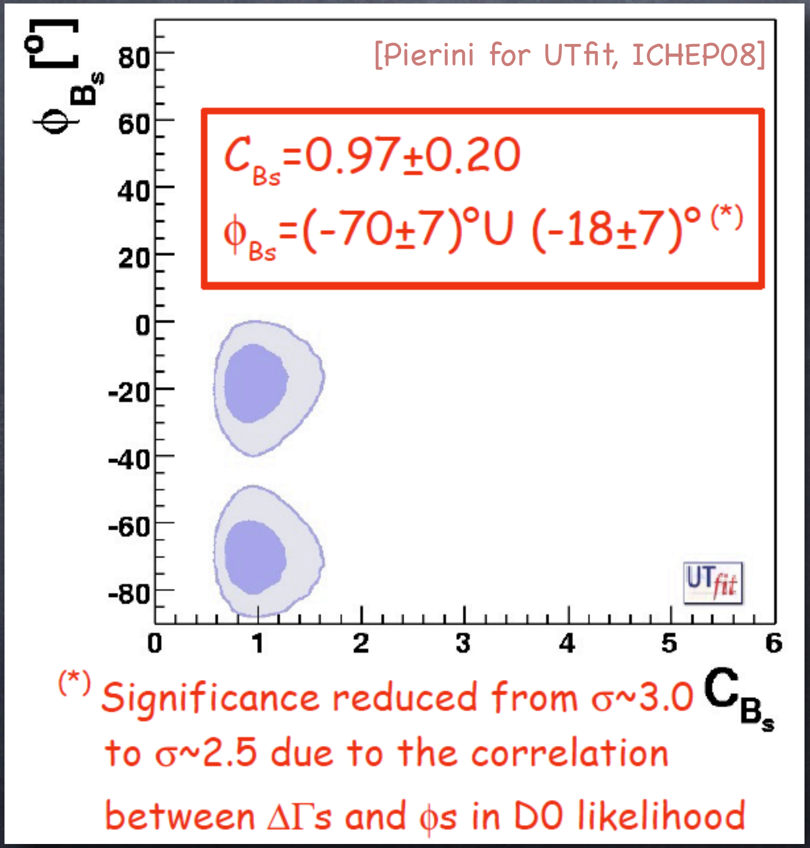
95% C.L.

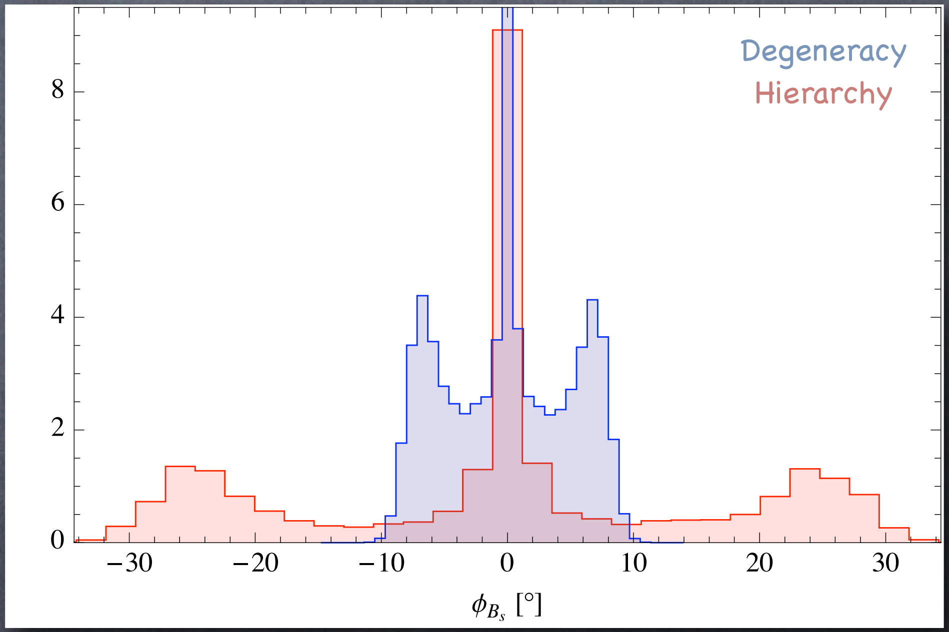
$\tilde{m} = M_3 = \mu = 350$ GeV, $\tan\beta = 10$, $A = 0$

ϕ_{Bs}

$$\langle B_s|H_{\rm eff}^{\rm full}|\bar{B}_s\rangle = C_{B_s} e^{2i\phi_{B_s}} \langle B_s|H_{\rm eff}^{\rm SM}|\bar{B}_s\rangle$$

$$\langle B_s|H_{\rm eff}^{\rm SM}|\bar{B}_s\rangle = A_s^{\rm SM} e^{-2i\beta_s} \qquad \beta_s = \arg(-(V_{ts}V_{tb}^*)/(V_{cs}V_{cb}^*)) = 0.018 \pm 0.001$$





Summary

- $\tilde{m}_3 \ll \tilde{m}_{1,2}$ not incompatible with naturalness
- Welcome to alleviate the SUSY flavour problem
- Useful to describe scenarios where the mass separation is moderate but sufficient to make the degeneracy assumption a poor starting point. Which is somewhat expected because of U(3) breaking by Y_t
- Complementary to the degeneracy assumption in a general study of SUSY flavour effects, with predictions different by factors O(1-10)
- Predictions
  - Less parameters than in the case of degeneracy: $d \leftrightarrow s$ via $d \leftrightarrow b \leftrightarrow s$
    $s_L \leftrightarrow b_R$ via $s_L \leftrightarrow b_L \leftrightarrow b_R$ (under assumptions)
  - Specific, distinct correlation $\Delta F = 1$ vs $\Delta F = 2$, with larger effects in $\Delta F = 2$ allowed
  - Experiment has begun to explore the range of expected $b \leftrightarrow s$ effects

Inputs

Parameter	Value	Gaussian (σ)	Uniform ($\frac{\Delta}{2}$)	Reference
$\lvert\varepsilon_K\rvert$	2.229×10^{-3}	0.012×10^{-3}	–	[19]
Δm_K (ps^{-1})	5.292×10^{-3}	0.009×10^{-3}	–	[19]
BR($B \to X_s\gamma$)	3.55×10^{-4}	0.26×10^{-4}	–	[20]
Δm_{B_s} (ps^{-1})	17.77	0.12	–	[19]
Δm_{B_d} (ps^{-1})	0.507	0.005	–	[19]
$\lvert M_{12}^D\rvert$ (ps^{-1})	7.7×10^{-3}	2.5×10^{-3}	–	[21]
$\bar{\rho}$	0.167	0.051	–	[22]
$\bar{\eta}$	0.386	0.035	–	[22]
λ	0.2255	0.010	–	[19]
$\lvert V_{cb}\rvert$	41.2×10^{-3}	1.1×10^{-3}	–	[19]
F_K (GeV)	0.160	–	–	[19]
F_{B_d} (MeV)	189	27	–	[23]
$F_{B_s}\sqrt{B_s}$ (MeV)	262	35	–	[23]
F_D (MeV)	201	3	17	[24]
$\hat{B}_K$	0.79	0.04	0.08	[24]
B_1^B	0.88	0.04	0.10	[24]
η_{cc}	0.47	0.04	–	[25]
η_{ct}	0.5765	0.0065	–	[25]
η_{tt}	1.43	0.23	–	[25]
$\overline{m}_t$ (GeV)	161.2	1.7	–	[24]
$\overline{m}_b$ (GeV)	4.21	0.08	–	[24]
$\overline{m}_c$ (GeV)	1.224	0.057	–	[26]