Quark and Glue Structure of the Nucleon

- Quark spin from anomalous Ward identity
- Momentum and angular momentum sum rules
- Glue spin
- πN σ term, strangeness
- Nucleon mass decomposition
- Strange quark magnetic moment

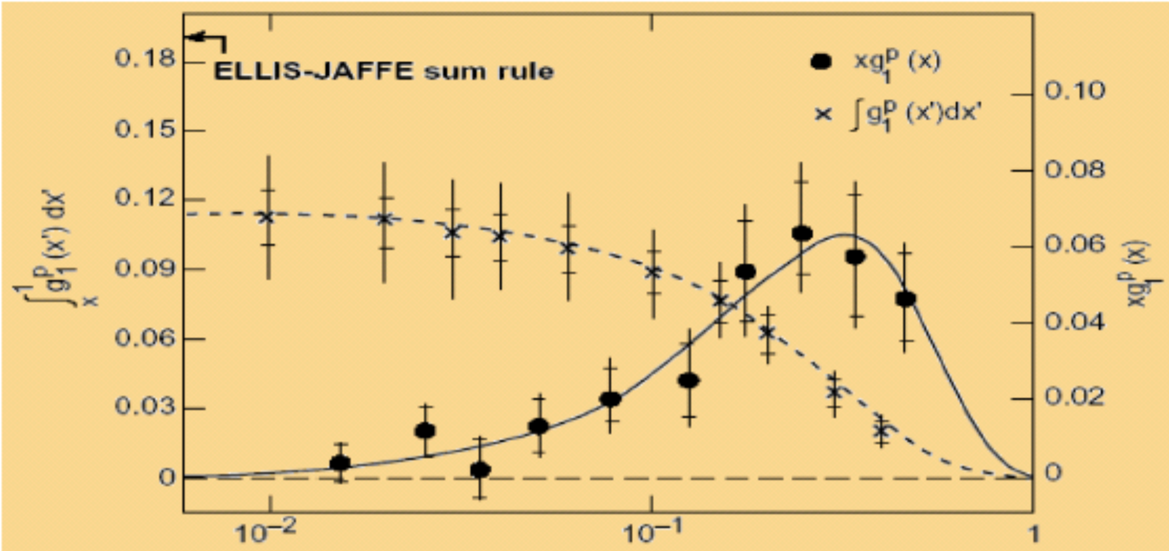
χ QCD Collaboration

CCNC, June 6, 2016

Where does the spin of the proton come from?

Twenty 7 years since the “spin crisis”

- ❑ **EMC experiment in 1988/1989 – “the plot”:**

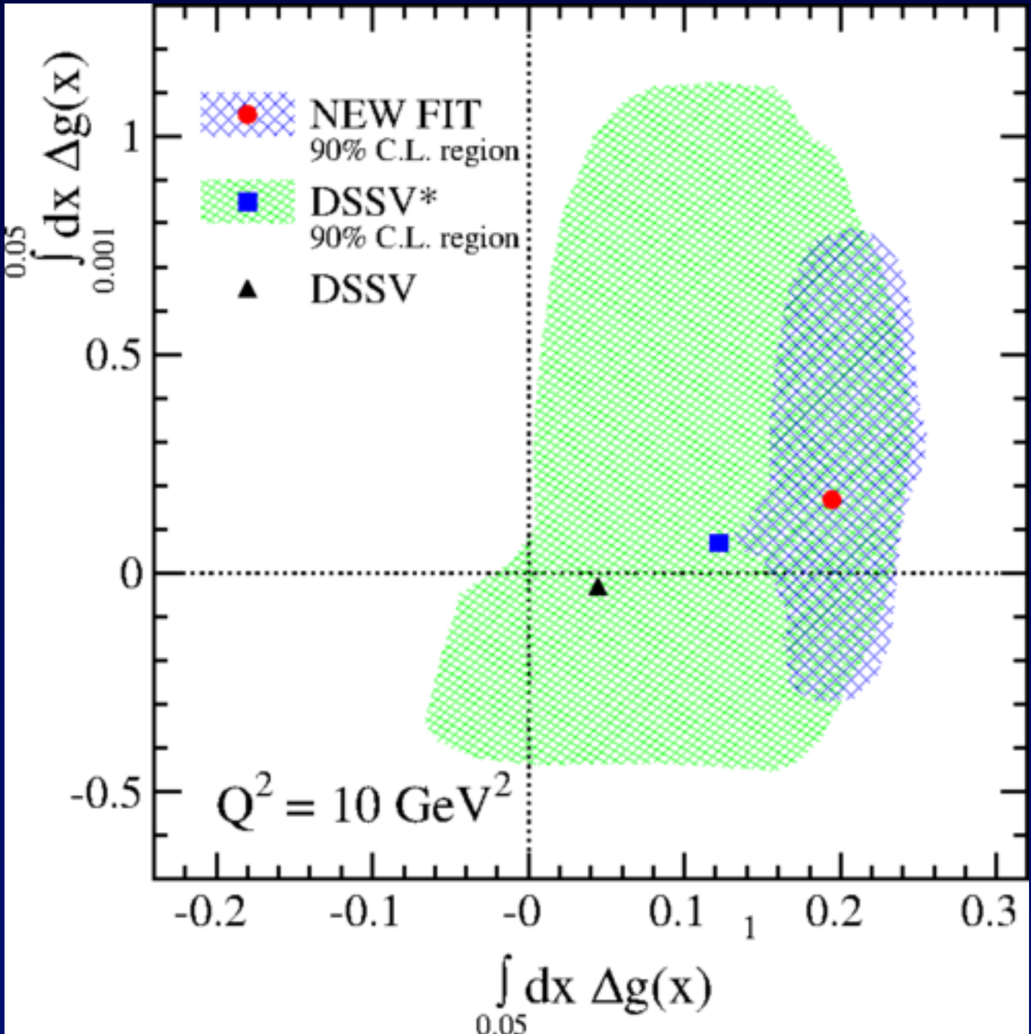


$$g_1(x) = \frac{1}{2}\sum_q e_q^2\left[\Delta q(x) + \Delta\bar{q}(x)\right] + \mathcal{O}(\alpha_s) + \mathcal{O}(1/Q)$$

$$\Delta q = \int_0^1 dx \Delta q(x) = \langle P, s_{\|}|\overline{\psi}_q(0)\gamma^+\gamma_5\psi_q(0)|P, s_{\|}\rangle$$

- ❑ **“Spin crisis” or puzzle:** $\Delta\Sigma = \sum_q \Delta q + \Delta\bar{q} = 0.2 - 0.3$

Glue Helicity ΔG



Experimental results from
STAR [1404.5134]
PHENIX [1402.6296]
COMPASS [1001.4654]

ΔG ~ 0.2 with large error

D. de Florian, R. Sassot, M. Stratmann, W. Vogelsang,
PRL 113, 012001 (2014)

Spin Sum Rules

- Jaffe and Manohar sum rule (1990)

$$J = \frac{\Sigma}{2} + L_q + S_G + L_G$$

$$\vec{J}_{Tot} = \int d^3x \ \psi^\dagger \frac{1}{2}\Sigma\psi + \int d^3x \ \vec{x} \times \psi^\dagger \vec{\nabla}\psi + \int d^3x \ \vec{E}^a \times \vec{A}^a + \int d^3x \ \vec{x} \times E^{aj} (\vec{x} \times \nabla) A^{aj}$$

  - Canonical EM tensor on light-cone with light-cone gauge
  - Not directly accessible on the lattice

- Ji sum rule (1997)

$$J = \frac{\Sigma}{2} + L_q + J_G$$

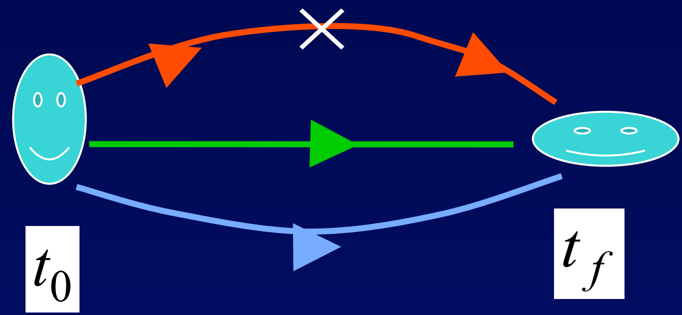
$$\vec{J}_{Tot} = \int d^3x \ \psi^\dagger \frac{1}{2}\Sigma\psi + \int d^3x \ \vec{x} \times \psi^\dagger \vec{D}\psi + \int d^3x \ \vec{x} \times \ (\vec{E}^a \times \vec{B}^a)$$

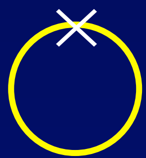
  - Symmetric EM tensor (Belinfante) → gauge invariant and frame independent.

Hadron Structure with Quarks and Glue

- Quark and Glue Momentum and Angular Momentum in the Nucleon

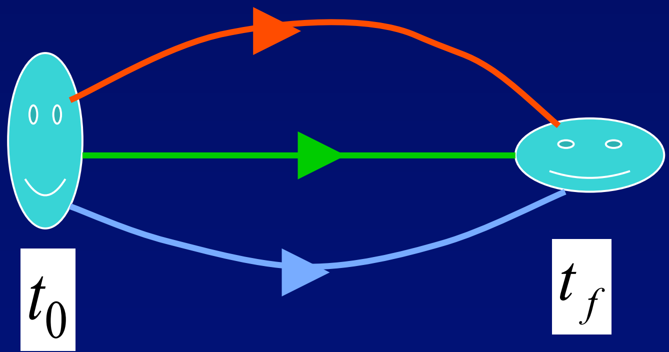
$$(\bar{u}\gamma_\mu D_\nu u + \bar{d}\gamma_\mu D_\nu d)(t)$$

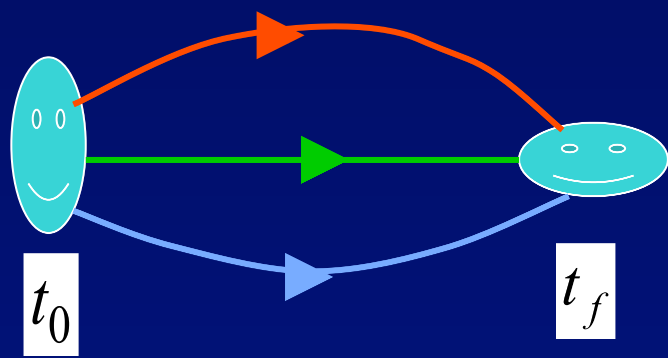




$$\bar{\Psi}\gamma_\mu D_\nu \Psi(t)(u,d,s)$$

$$F_{\mu\alpha}F_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}F^2$$





Quark Spin from Anomalous Ward Identify

- Calculation of the axial-vector in the DI is very noisy
- Instead, try AWI $\partial_\mu A_\mu^0 = i2mP - \frac{iN_f}{8\pi^2} G_{\mu\nu}\tilde{G}_{\mu\nu}$

$$\kappa_A \langle p',s | A_\mu | p,s \rangle = \lim_{q\to 0} \frac{i|s|}{\vec{q}\cdot\vec{s}} \langle p',s | 2\sum_{f=1}^{N_f} m_f \vec{q}_f i\gamma_5 q_f - 2iN_f q | p,s \rangle$$

  - P is totally dominated by small eigenmodes.
  - q(x) from overlap is exponentially local and captures the high modes from A_μ^0.
  - Direct check the origin of `proton spin crisis'.

Quark Spin from Anomalous Ward Identify

- Calculation of the point axial-vector in the DI is not sufficient.
- AWI needs to be satisfied. $\partial_\mu A_\mu^0 = i2mP - \frac{iN_f}{8\pi^2} G_{\mu\nu}\tilde{G}_{\mu\nu}$
- Unrenormalized AWI for overlap fermion for point current

$$\kappa_A \partial_\mu A_\mu^0 = i2mP - iN_f 2q(x)$$

Renormaliztion and mixing:

$$Z_A \kappa_A \partial_\mu A_\mu^0 = i2Z_m m Z_P P - iN_f 2(Z_q q(x) + \lambda \partial_\mu A_\mu^0)$$

  - Overlap fermion --> mP is RGI ($Z_m Z_P=1$)
  - Overlap operator for $q(x) = -1/2\ \mathrm{Tr}\gamma_5 D_{ov}(x,x)$ has no multiplicative renormalization.
  - Espriu and Tarrach (1982) $Z_A(2-\mathrm{loop}) = 1 - \left(\frac{\alpha_s}{\pi}\right)^2 \frac{3}{8} C_2(R) N_f \frac{1}{\varepsilon},$

$$\lambda = -\left(\frac{\alpha_s}{\pi}\right)^2 \frac{3}{16} C_2(R) \frac{1}{\varepsilon}$$

2+1 flavor DWF configurations (RBC-UKQCD)

La ~ 4.5 fm
m_π ~ 170 MeV

32^3 x 64, a =0.137 fm

($O(a^2)$ extrapolation)

La ~ 2.8 fm
m_π ~ 330 MeV

24^3 x 64, a =0.115 fm

→

La ~ 5.48 fm
m_π ~ 140 MeV

48^3 x 96, a =0.114 fm

La ~ 2.7 fm
m_π ~ 295 MeV

32^3 x 64, a =0.085 fm

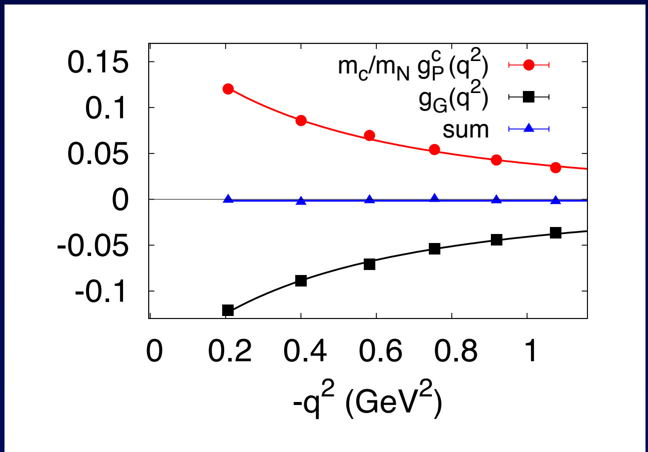
→

La ~ 5.35 fm
m_π ~ 140 MeV

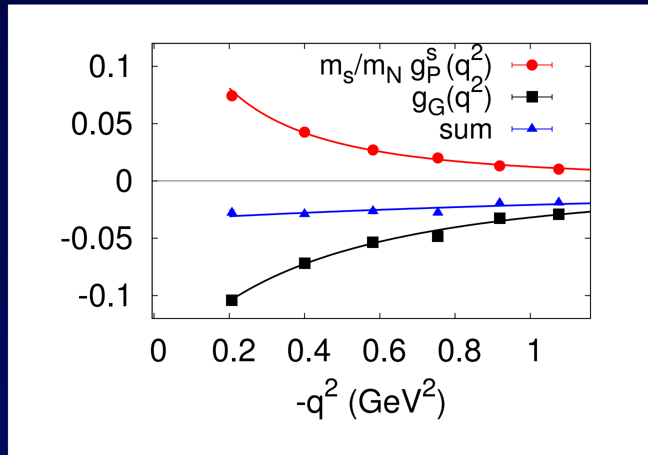
64^3 x 128, a =0.0837 fm

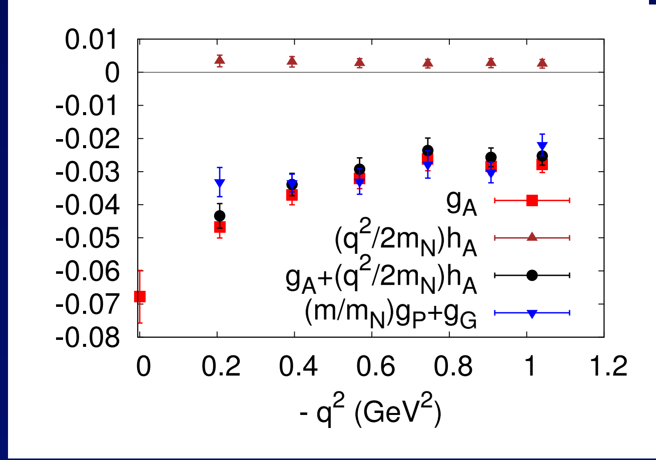
Disconnected Insertion

Charm

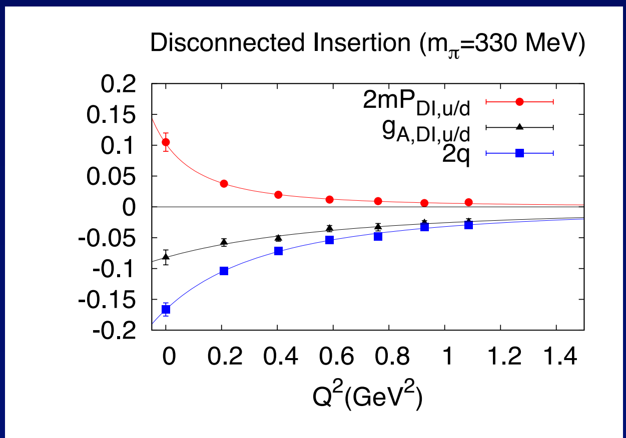


Strange



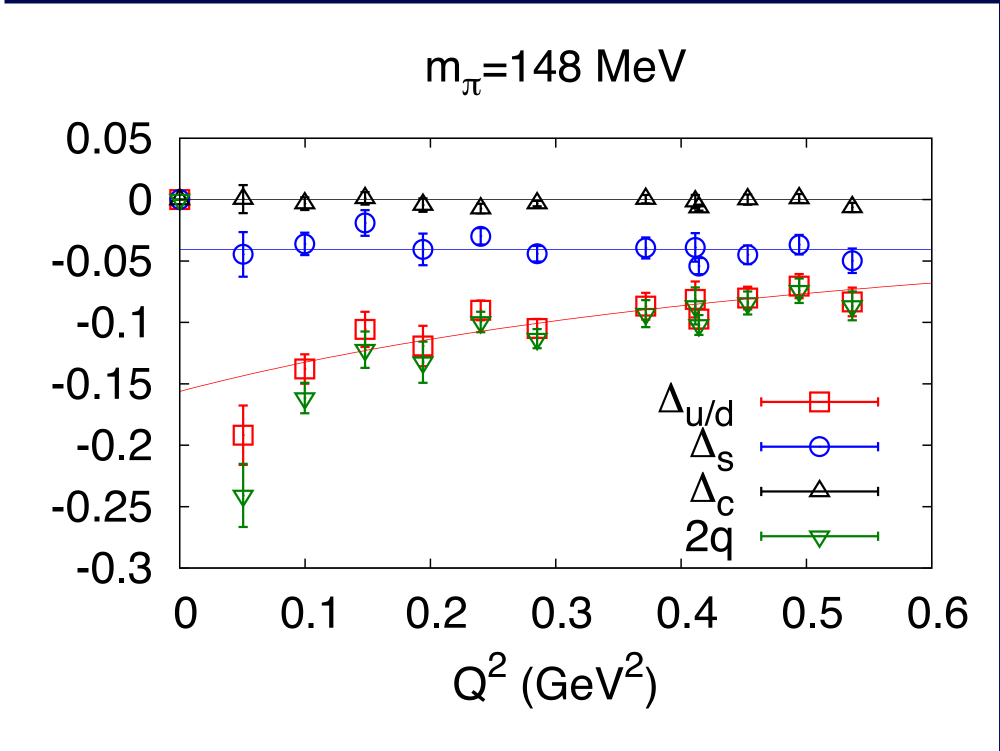


u/d



$$2M_N\kappa_A g_A^0(q^2)+q^2\kappa_{h_A}h_A^0(q^2)$$
$$=2mg_P^0(q^2)+2M_N g_G(q^2)$$

Quark Spin near Physical Pion Mass



preliminary

g_A^0	**m_π=148 MeV**
Δu+Δd (CI)	0.54(5)
Δc	~0
Δs	- 0.042(4)
Δu/Δd(DI)	- 0.16(2)
g_A^0	0.17(7)

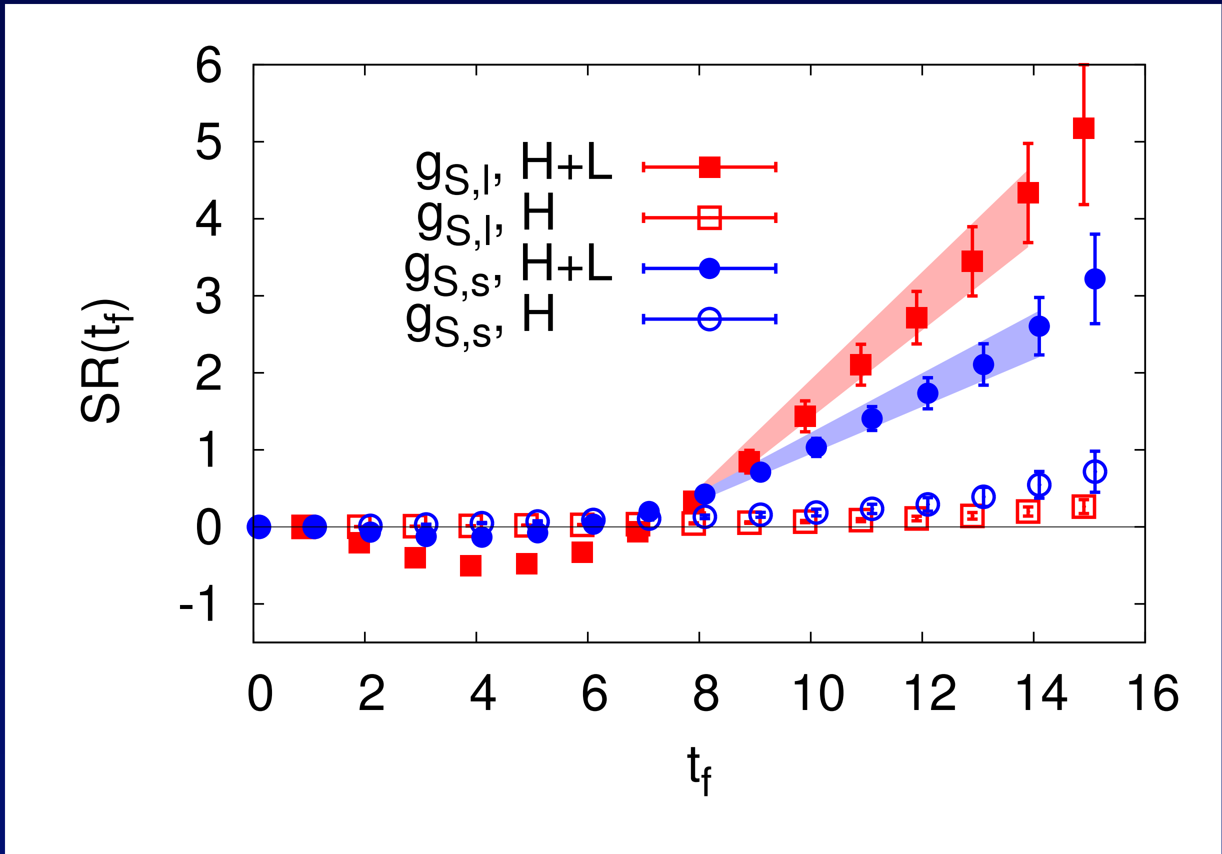
48^3 x 96 Lattice, L = 5.5 fm

triangle anomaly

Check with conserved current

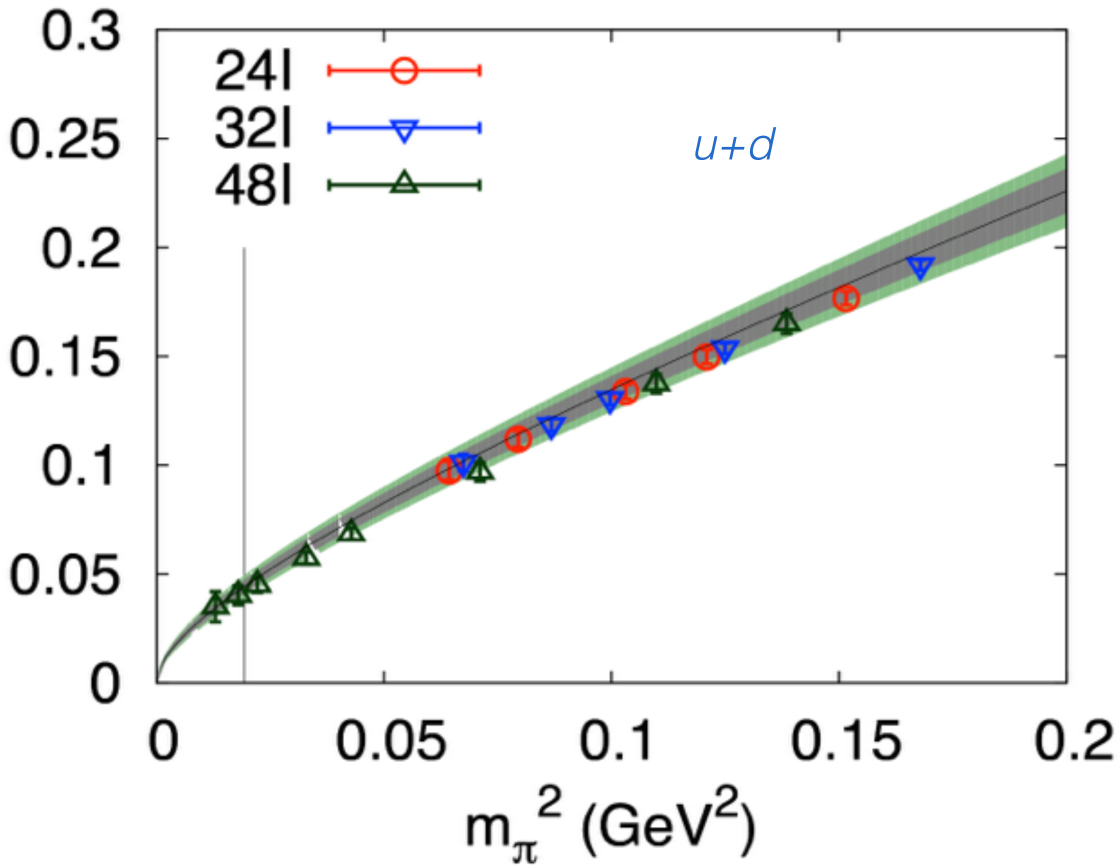
Pion-nucleon sigma term and strangeness content

$$\sigma_{\pi N} = \frac{m_u + m_d}{2} \langle N | \bar{u}u + \bar{d}d | N \rangle, \quad \sigma_{sN} = m_s \langle N | \bar{s}s | N \rangle$$



$$\mathrm{Tr\ loop} = \mathrm{Tr} \sum_n \frac{\phi_n^\dagger O \phi_n}{m + i\lambda_n} + \mathrm{Tr}\ \eta_H^\dagger O D^{-1} \eta_H$$

The πNσ term



$$\sigma_{\pi N}(m_\pi, a, L) = C_0^\pi m_\pi + C_1^\pi m_\pi^2 + C_2^\pi a^2 + C_3^\pi \left(\frac{m_\pi^2}{L} - m_\pi^3\right) e^{-m_\pi L},$$

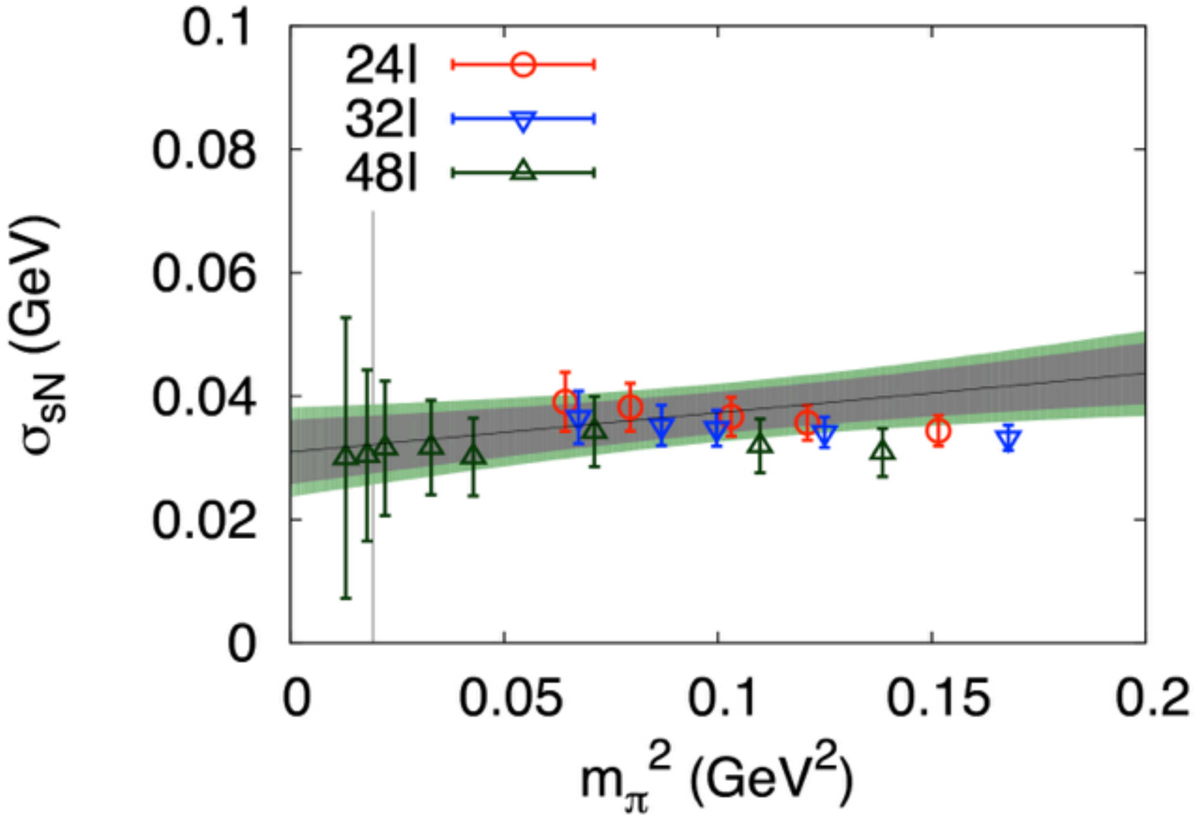
Y. Yang, et al. [χQCD], 1511.09089

- The plot shows the data points on three ensembles with different volume and lattice spacing.
- The data prefers *the ruler approximation*.
- The curve shows the quark mass dependence of the sigma term.
- The darker band for the statistic error and the lighter band for that combined with the systematic ones.
- The final prediction is,

$$\sigma_{\pi N} = 44.4(3.2)(4.5) \text{ MeV}$$

The strange σ term

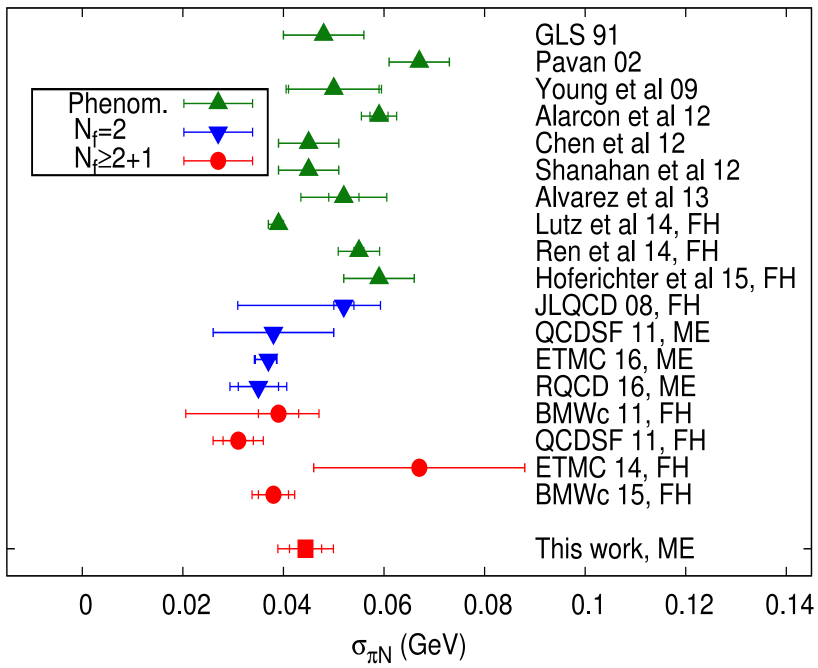
Y. Yang, et al. [χQCD], 1511.09089

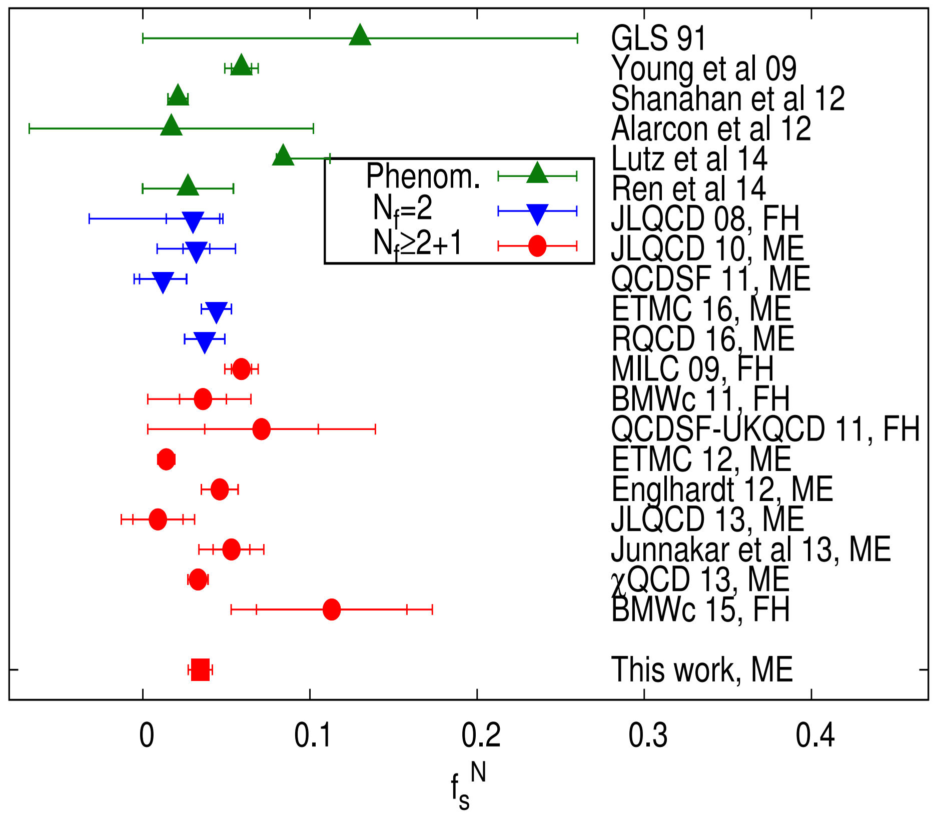


- Due the finite lattice spacing, volume and partially quenching effects, biases exist between the data points and curve.
- It can be cured by the global fit.
- The final prediction is,

$$\sigma_{sN} = 32.3(4.7)(4.8)\ \text{MeV}.$$

$$\sigma_{sN}(m_\pi, a, L) = C_0^s + C_1^s m_\pi^2 + C_2^s a^2 + C_3^s e^{-m_\pi L}.$$





Yibo Yang (χ QCD) et al.,
arXiv:1511.15089

$$\sigma_{\pi N} = 44.4(3.2)(4.5)\ \text{MeV}$$

$$\sigma_{sN} = 32.3(4.7)(4.9)\ \text{MeV}$$

$$H_m(u,d,s) / m_N = 8(1)\%$$

Momenta and Angular Momenta of Quarks and Glue

- Energy momentum tensor operators decomposed in quark and glue parts gauge invariantly --- Xiangdong Ji (1997)

$$T^q_{\mu\nu} = \frac{i}{4}\left[\bar{\psi}\gamma_\mu \overleftrightarrow{D}_\nu \psi + (\mu \leftrightarrow \nu)\right] \rightarrow \vec{\mathrm{J}}_q = \int d^3x \left[\frac{1}{2}\bar{\psi}\vec{\gamma}\gamma_5\psi + \vec{x}\times\bar{\psi}\gamma_4(-i\vec{D})\psi\right]$$

$$T^g_{\mu\nu} = F_{\mu\lambda}F_{\lambda\nu} - \frac{1}{4}\delta_{\mu\nu}F^2 \rightarrow \vec{\mathrm{J}}_g = \int d^3x\left[\vec{x}\times(\vec{E}\times\vec{B})\right]$$

- Nucleon form factors

$$\langle p,s \,|\, T_{\mu\nu} \,|\, p's'\rangle = \bar{u}(p,s)[T_1(q^2)\gamma_\mu\bar{p}_\nu - T_2(q^2)\bar{p}_\mu\sigma_{\nu\alpha}q_\alpha / 2m$$

$$\text{-i}\mathrm{T}_3(q^2)(q_\mu q_\nu - \delta_{\mu\nu}q^2)/m + T_4(q^2)\delta_{\mu\nu}m/2]u(p's')$$

- Momentum and Angular Momentum

$$Z_{q,g}T_1(0)_{q,g}\ [\mathrm{OPE}] \rightarrow \langle x\rangle_{q/g}(\mu,\overline{\mathrm{MS}}),\quad Z_{q,g}\left[\frac{T_1(0)+T_2(0)}{2}\right]_{q,g} \rightarrow J_{q/g}(\mu,\overline{\mathrm{MS}})$$

Renormalization and Quark-Glue Mixing

Momentum and Angular Momentum Sum Rules

$$\langle x\rangle_q^R = Z_q \langle x\rangle_q^L, \quad \langle x\rangle_g^R = Z_g \langle x\rangle_g^L,$$

$$J_q^R = Z_q J_q^L, \quad J_g^R = Z_g J_g^L,$$

$$Z_q \langle x\rangle_q^L + Z_g \langle x\rangle_g^L = 1,$$

$$Z_q J_q^L + Z_g J_g^L = \frac{1}{2}$$

$$\Rightarrow \begin{cases} Z_q T_1^q(0) + Z_g T_1^g(0) = 1, \\ Z_q (T_1^q + T_2^q)(0) + Z_g (T_1^g + T_2^g)(0) = 1, \\ Z_q T_2^q(0) + Z_g T_2^g(0) = 0 \end{cases}$$

Mixing

$$\begin{bmatrix} \langle x\rangle_q^{\overline{MS}}(\mu) \\ \langle x\rangle_g^{\overline{MS}}(\mu) \end{bmatrix} = \begin{bmatrix} C_{qq}(\mu) & C_{qg}(\mu) \\ C_{gq}(\mu) & C_{gg}(\mu) \end{bmatrix} \begin{bmatrix} \langle x\rangle_q^R \\ \langle x\rangle_g^R \end{bmatrix}$$

M. Glatzmaier, KFL
arXiv:1403.7211

Renormalized results: $Z_q = 1.05$, $Z_g = 1.05$ $\overline{MS}$ (2 GeV)

	CI(u)	CI(d)	CI(u+d)	DI(u/d)	DI(s)	Glue
<x>	0.416 (40)	0.151 (20)	0.567 (45)	0.037 (7)	0.023 (6)	0.334 (56)
$T_2(0)$	0.283 (112)	-.217 (80)	0.061 (22)	-0.002 (2)	-.001 (3)	-0.056 (52)
2J	0.704 (118)	-.070 (82)	0.629 (51)	0.035 (7)	0.022 (7)	0.278 (76)
g_A	0.91 (11)	-0.30 (12)	0.62 (9)	-0.12 (1)	-0.12 (1)	
2 L	-0.21 (16)	0.23 (15)	0.01 (10)	0.16 (1)	0.14 (1)	

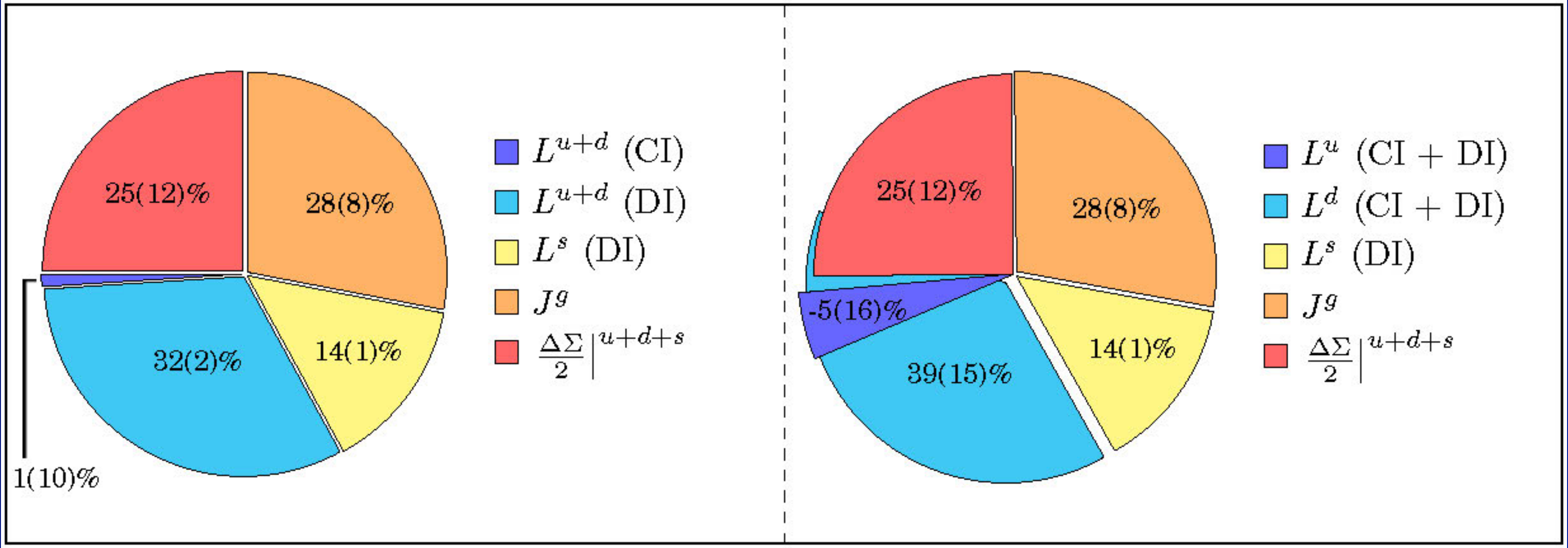
M. Deka et al., PRD (2015), 1312.4816
(χ QCD Collaboration)

$$T_2(0)^R_{CI} + T_2(0)^R_{DI} + T_2(0)^R_g = 0$$

I.Yu. Kobzarev, L.B. Okun, Zh. Eksp. Teor. Fiz. 43, 1904 (1962) [Sov. Phys. JETP 16, 1343 (1963);
S. Brodsky et al. NPB 593, 311(2001) → no anomalous gravitomagnetic moment

Quark Spin, Orbital Angular Momentum, and Gule Angular Momentum (M. Deka *et al*, 1312.4816, PRD)

pizza cinque stagioni



$\Delta q \approx 0.25;$

$2\, L_q \approx 0.47\ (0.01(\mathrm{CI}){+}0.46(\mathrm{DI}));$

$2\ \mathrm{J}_g \approx 0.28$

These are quenched results so far.

Summary of Quenched Lattice Calculations

- Complete calculation of momentum fractions of quarks (both valence and sea) and glue have been carried out for a quenched lattice:
  - Glue momentum fraction is ~ 33%.
  - g_A^0 ~ 0.25 in agreement with expt.
  - Glue angular momentum is ~ 28%.
  - Quark orbital angular momentum is large for the sea quarks (~ 47%).
- These are quenched results so far.

Orbital Angular Momentum

skyrmion

Trinacria, Erice

Glue Spin and Helicity ΔG

- Jaffe and Manohar -- spin sum rule on light cone

  $S_g = \int d^3x\, \vec{E} \times \vec{A}$ in light-cone gauge $(A^+ = 0)$ and IMF frame.

  - Not gauge invariant
  - Light cone not accessible on the Euclidean lattice.

- Manohar – gauge invariant light-cone distribution

  $$\Delta g(x)\, S^+ = \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi}\, e^{-ixP^+\xi^-} \langle PS \,|\, F_a^{+\alpha}(\xi^-) L^{ab}(\xi^-,0) \tilde{F}^+_{\alpha,b}(0) \,|\, PS \rangle$$

  - After integration of x, the glue helicity operator is

  $$\mathrm{H}_g(0) = \vec{E}^a(0) \times \left(\vec{A}^a(0) - \frac{1}{\nabla^+} \left(\vec{\nabla} A^{+,b} \right) L^{ba}(\xi^-,0) \right)$$

  - Non-local and on light cone

Glue Spin and Helicity ΔG

- X.S. Chen, T. Goldman, F. Wang; Wakamatsu; Hatta, etc.

Gauge invariant decomposition

$$\vec{J} = \vec{S}_q + \vec{L}_q + \vec{S}_G + \vec{L}_G$$

$$S_g = \int d^3x \; \mathrm{Tr}\,(\vec{E} \times \vec{A}_{phys}), \quad A^\mu = A^\mu_{phys} + A^\mu_{pure}, \quad F^{\mu\nu}_{pure} = 0;$$

$$A^\mu_{phys} \to g^\dagger A^\mu_{phys} g, \quad A^\mu_{pure} \to g^\dagger A^\mu_{pure} g - \frac{i}{g} g^\dagger \partial^\mu g$$

$$D^i A^i_{phys} = \partial^i A^i_{phys} - ig\,[A^i, A^i_{phys}] = 0$$

  – Gauge invariant but frame dependent

- X. Ji, J.H. Zhang, Y. Zhao; Y. Hatta, X. Ji, Y. Zhao

Infinite momentum frame

$$\vec{E}^a(0) \times \vec{A}^a_{phys} \xrightarrow[p_z \to \infty]{} \vec{E}^a(0) \times \left(\vec{A}^a(0) - \frac{1}{\nabla^+}\left(\vec{\nabla} A^{+,b}\right) L^{ba}(\xi^-, 0) \right)$$

Glue Spin and Helicity ΔG

- Large momentum limit

$$S_g = \frac{\langle PS | \int d^3x \ \mathrm{Tr}\,(\vec{E} \times \vec{A}_{phys})_z \,| PS \rangle}{2E_P} \xrightarrow[P_Z \to \infty]{} \Delta G$$

  - Calculate S_g at finite P_z
  - Match to MS-bar scheme at 2 GeV
  - Large momentum effective theory to match to IMF
  - Similar proof for the quark and glue orbital angular momenta which are related to form factors in generalized TMD (GTMD)

  (Y. Zhao, KFL, and Y. Yang, arXiv:1506.08832)

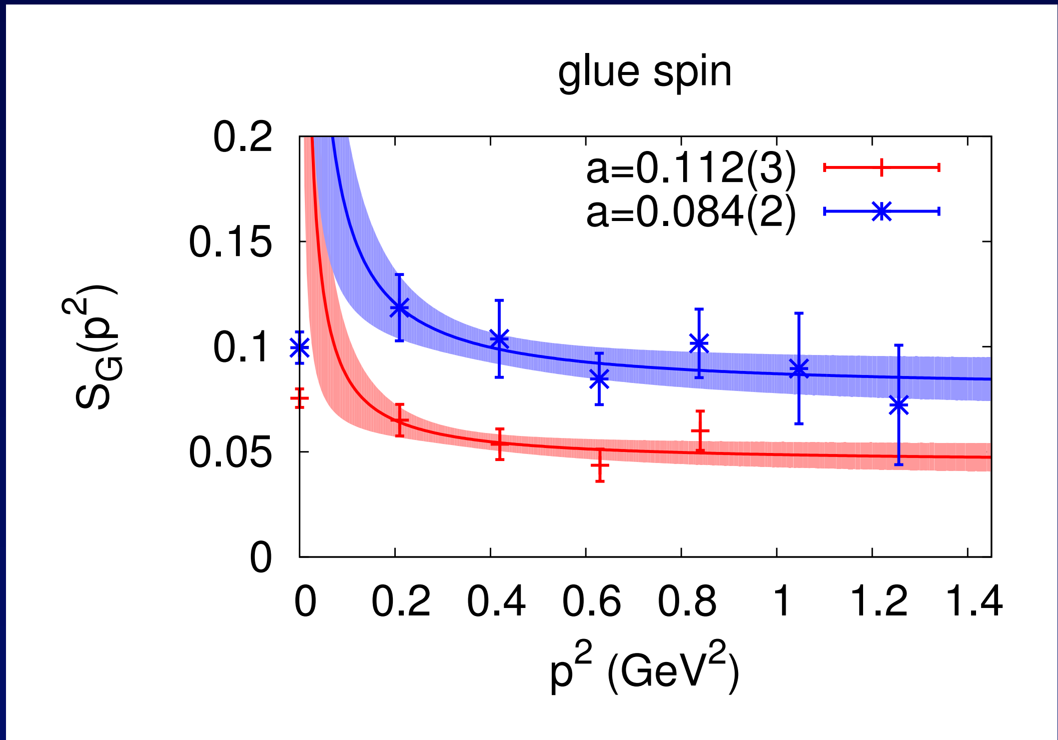
- Solution of A_{phys} -- related to A in Coulomb gauge

$$U^{\mu}(x) = g_c(x) U_c^{\mu}(x) g_c^{-1}(x + a\hat{\mu}),$$

$$U_{pure}^{\mu}(x) \equiv g_c(x) g_c^{-1}(x + a\hat{\mu}),$$

$$A_{phys}^{\mu}(x) \equiv \frac{i}{a g_0}\left(U^{\mu}(x) - U_{pure}^{\mu}(x)\right) = g_c(x) A_c(x) g_c^{-1}(x) + O(a).$$

S_G in Coulomb gauge at p^2 = 0 to 1.24 GeV^2 on the 24^3 x 64 and 32^3 x 64 lattices



$$a + \frac{b}{P^2}$$

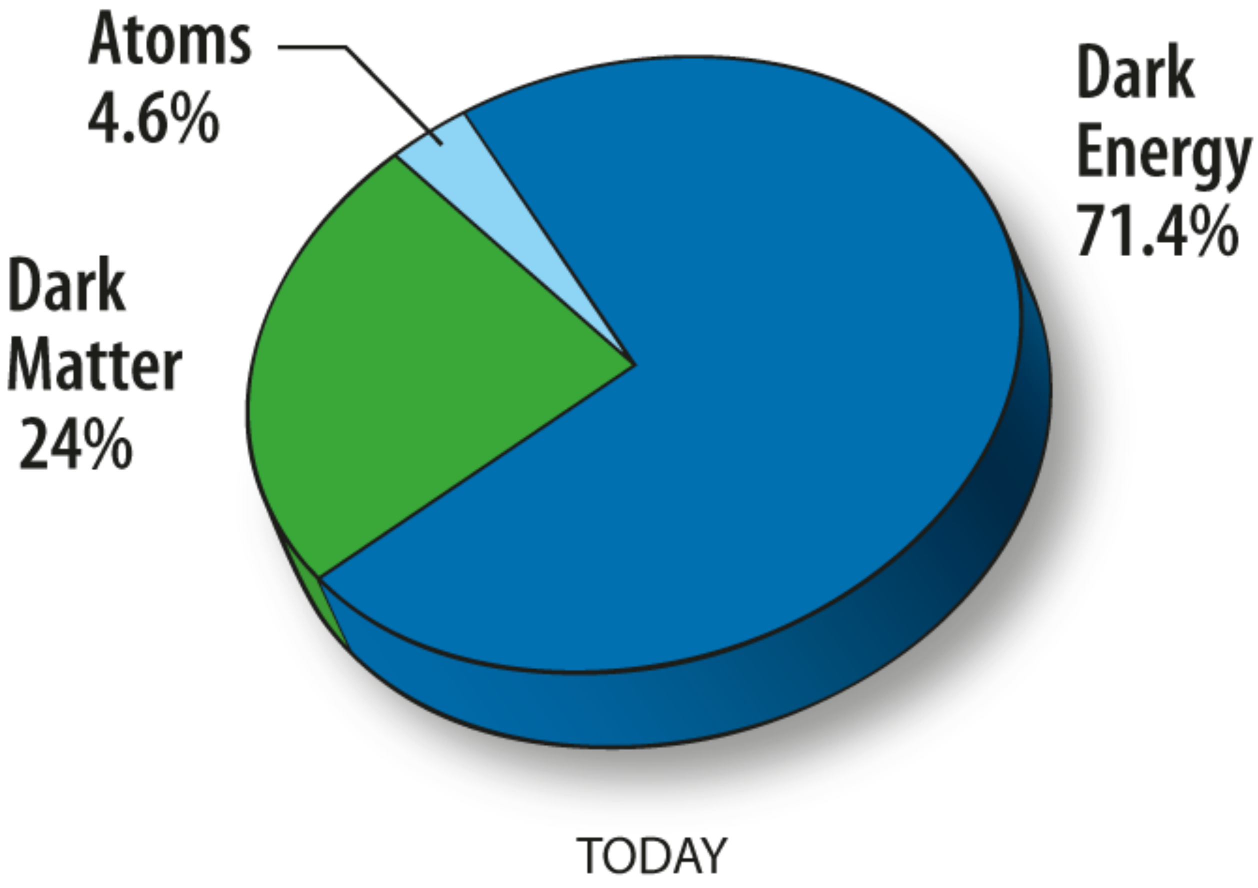
Y. Yang (χQCD), 1603.06256

S_G = 0.13(3)

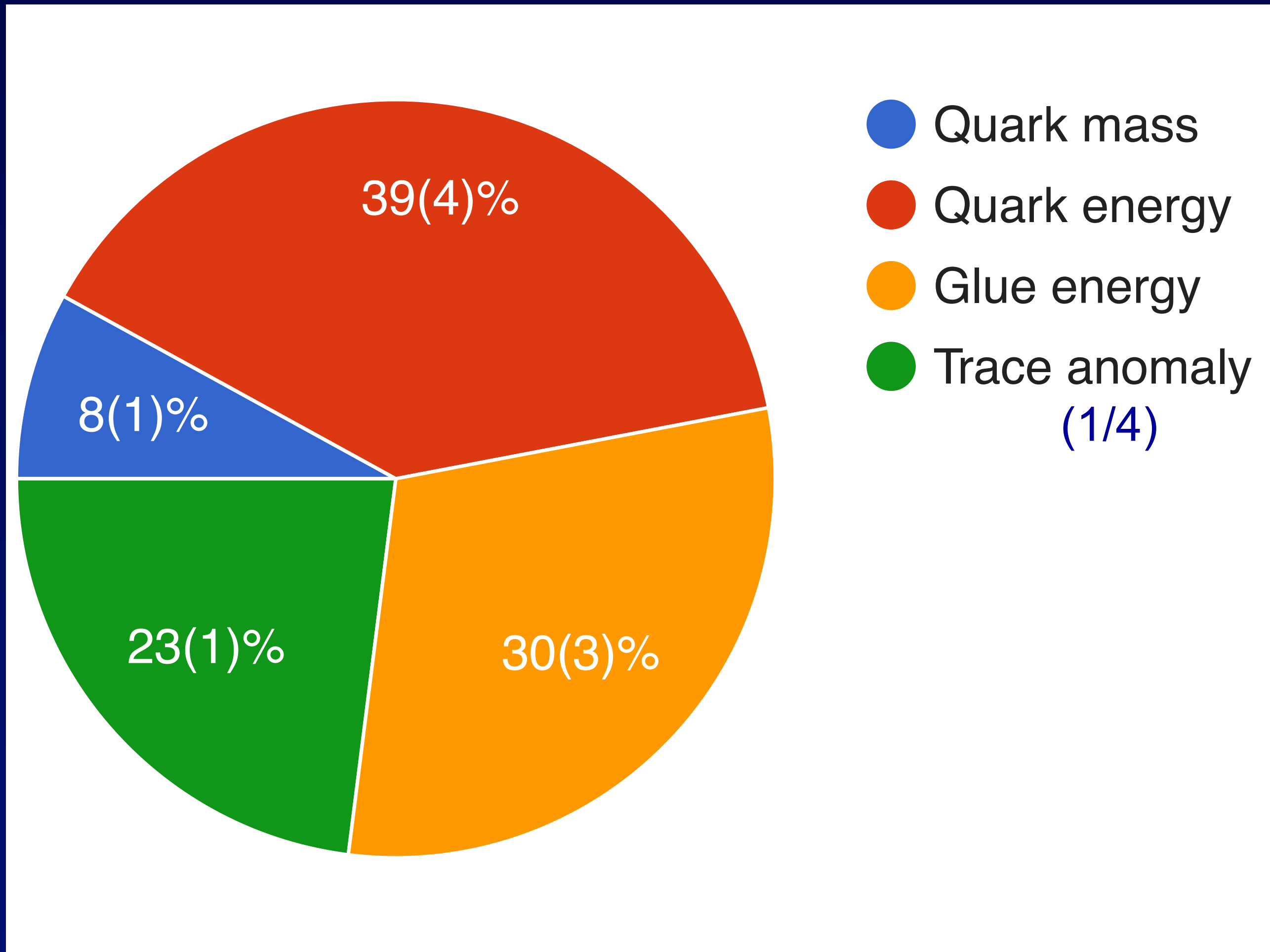
$$Tr(\vec{E} \times \vec{A}_{phys}) = Tr(\vec{E} \times g_C^{-1} \vec{A}_C g_C) = Tr(\vec{E}_C \times \vec{A}_C)$$

Motivation

Where does this observable 4.6% come from?

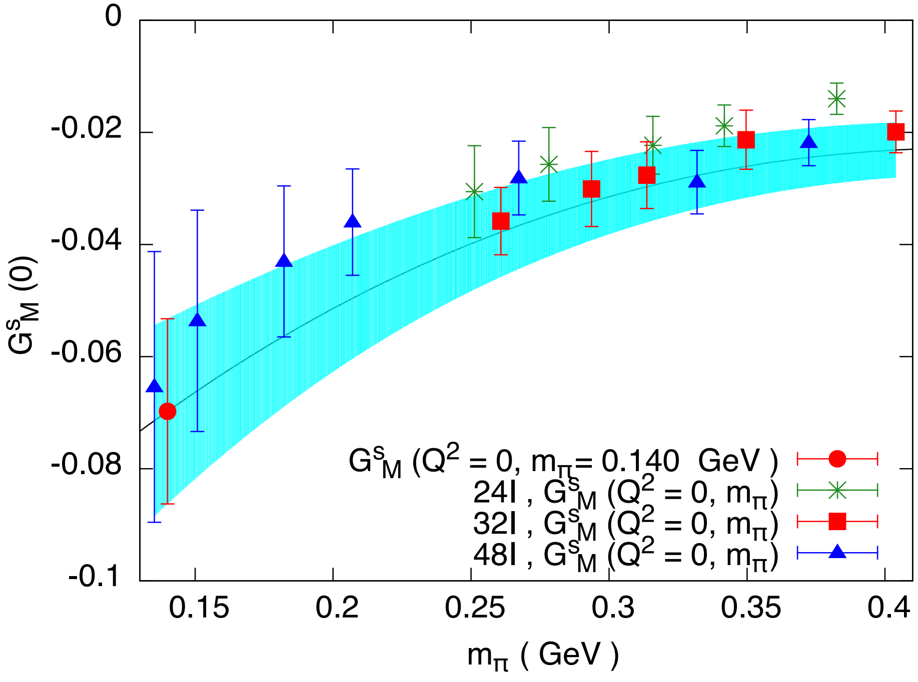


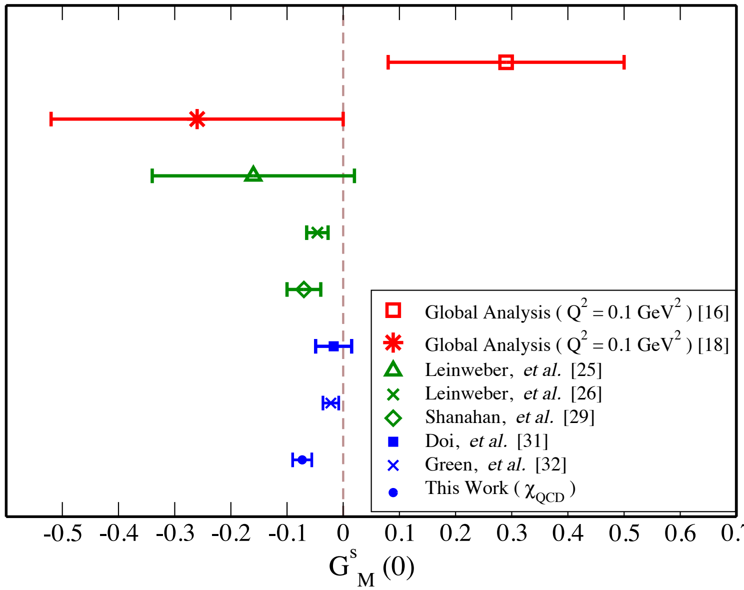
Nucleon Mass Components



Y. Yang

Strange quark magnetic moment





$G_M^S(0)$ = - 0.073(17) μ_N

R. Sufian

Summary and Challenges

- Decomposition of proton spin and hadron masses into quark and glue components on the lattice is feasible. Large momentum frame for the proton to calculate glue helicity remains a challenge.
- `Proton Spin Crisis' is likely to be the second (?) example of observable U(1) anomaly.
- Together with evolution, facorization, perturbative QCD, lattice QCD results with small enough statistical and systematic errors can compare directly with experiments and have an impact in further our understanding of the underline physics of the hadron structure (form factors, PDF, neutron electric dipole moment, muon g-2, etc).