Kosterlitz-Thouless transition and chiral rotation in external electromagnetic field

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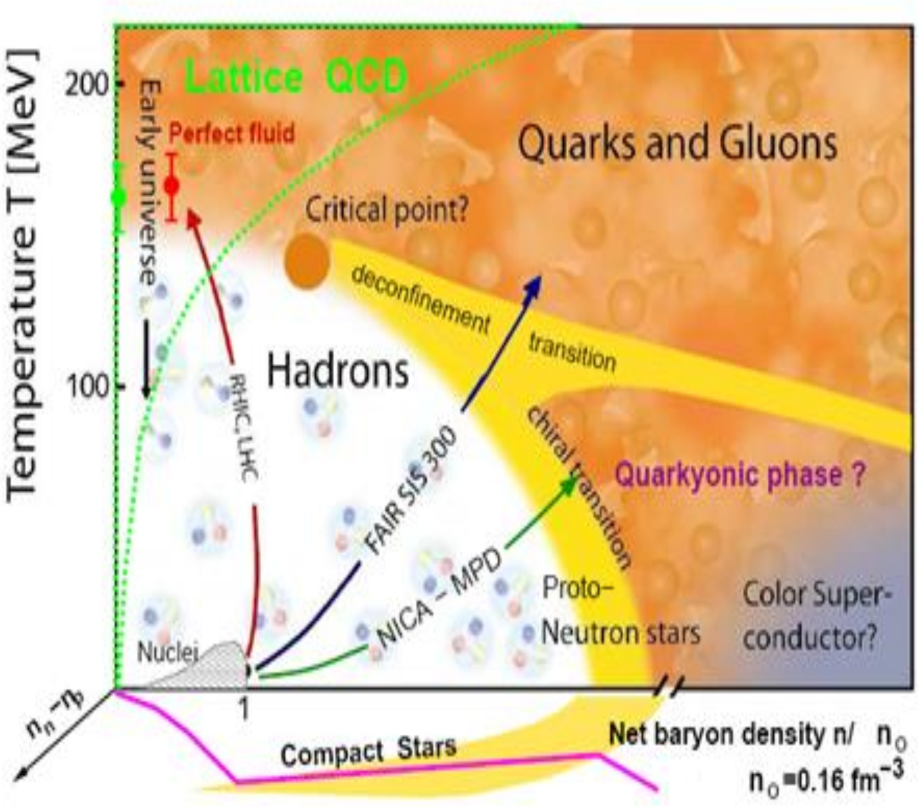
GC, L. He, P. Zhuang, PhysRevD.90.056005 (2014) ,
GC, XG Huang, Phys. Lett. B757.1 (2016).

Outline

- QCD transitions in strong EM field
- Nambu—Jona-Lasinio model
- KT transition and dHvA oscillation in pure magnetic field
- Chiral rotation and π_0 condensation in parallel EM field
- Conclusions

$T-\mu$ phase diagram of QCD



Crossover at low chemical potential--$T_c \sim 165 MeV$;

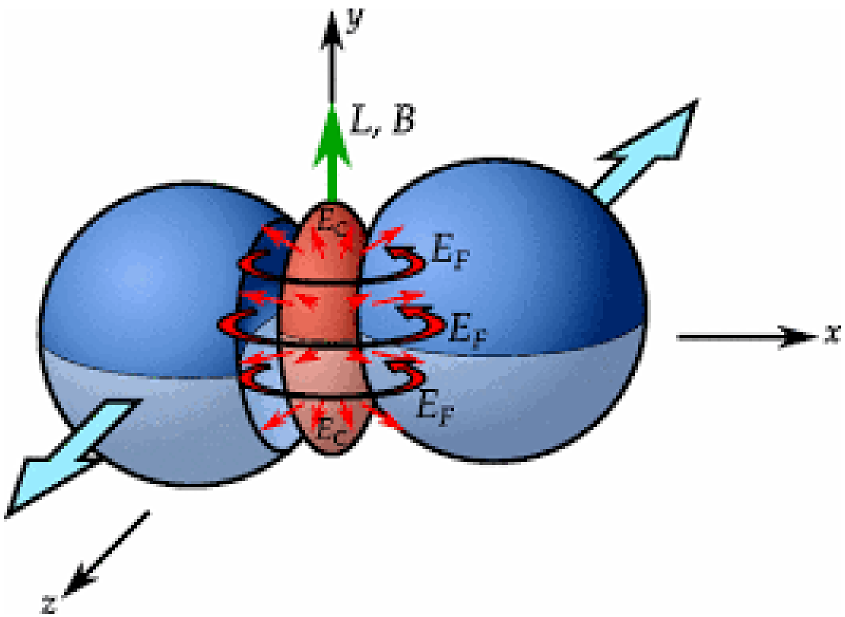
First-order transition at moderate chemical potential to quarkyonic matter--might be covered by inhomogeneous LOFF state;

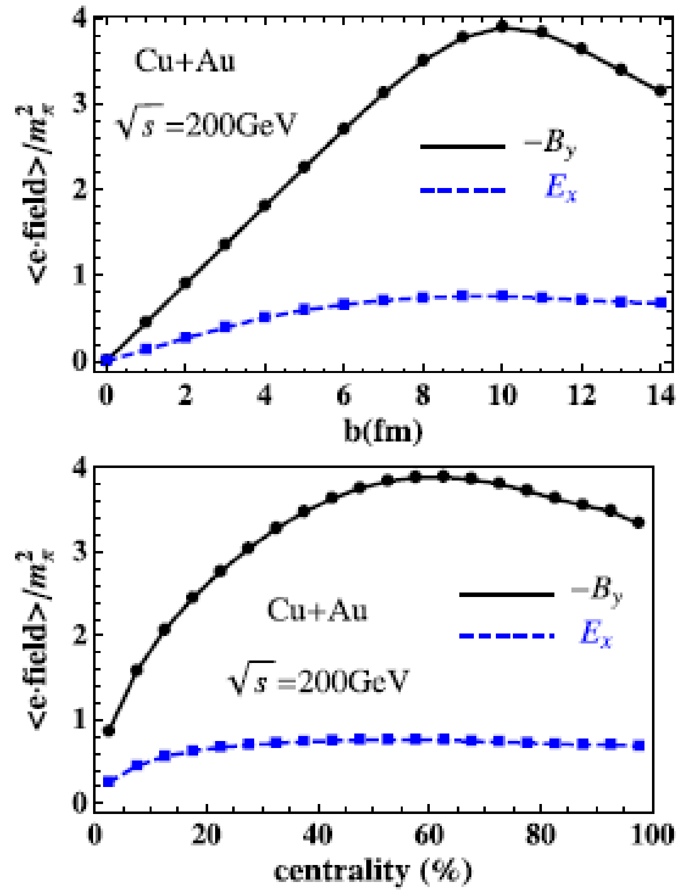
Color superconductor at high chemical potential– crystalline pairing structure.

New parameters, new features!

Pion superfluid at finite μ_I.

Strong EM field in HIC



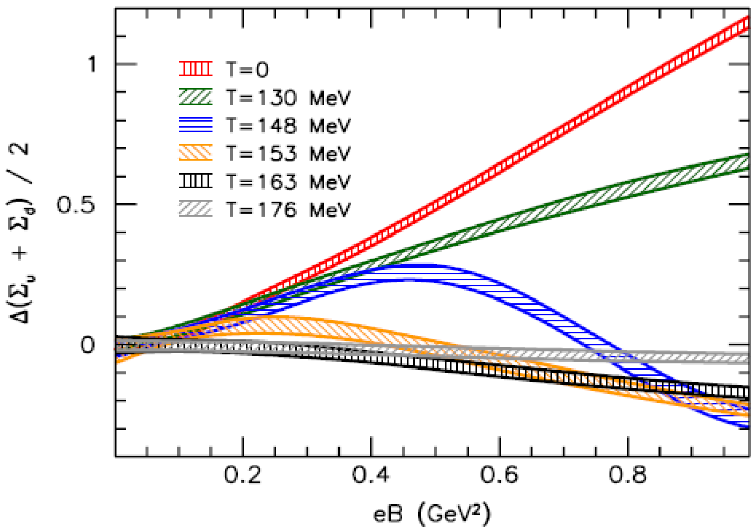


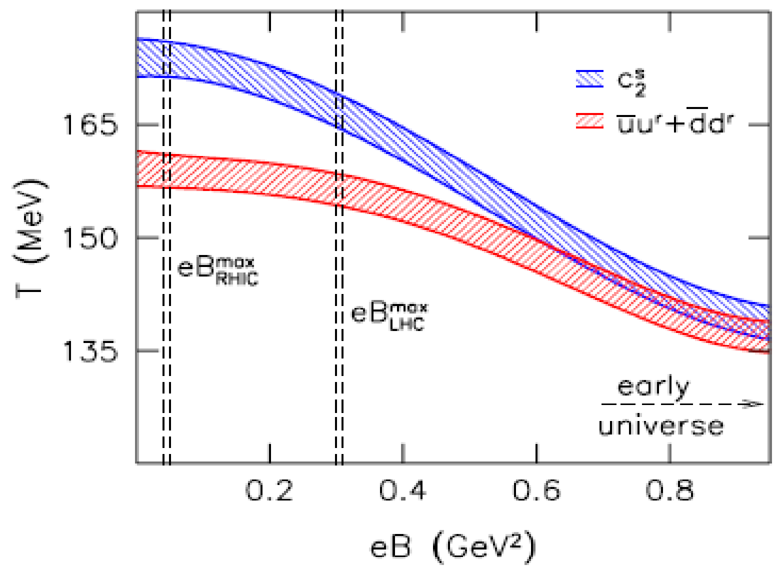
In LHC, magnetic field is even larger ——1 GeV^2.

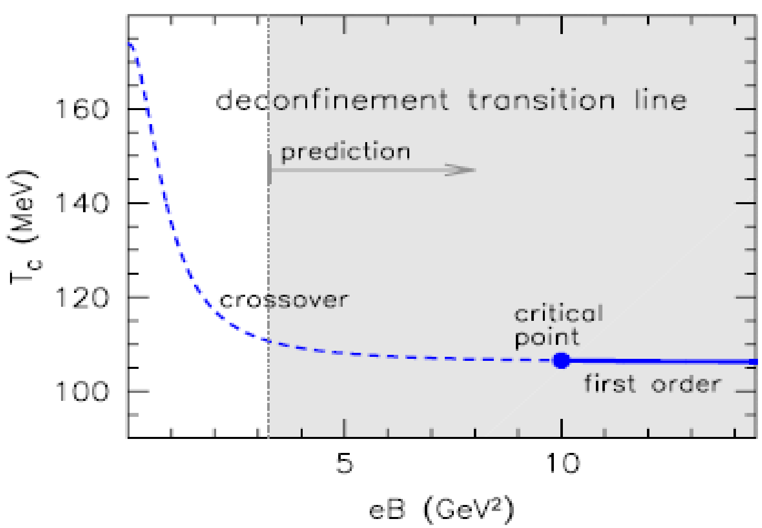
QCD phase diagram in strong EM field ?

W.-T. Deng, X.-G. Huang, PLB 742, 296 (2015).

Inverse magnetic catalysis (LQCD)







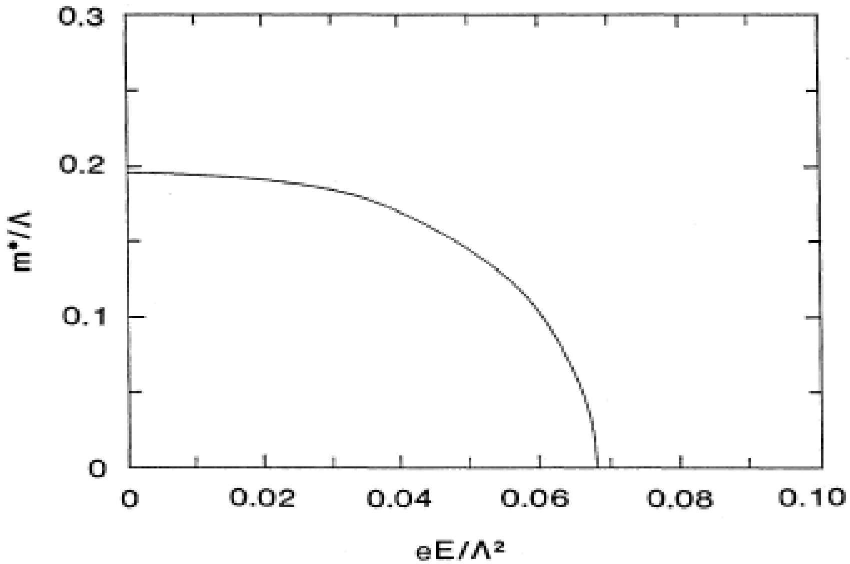
(1) At $T = 0$, Σ increases with B; around T_c, Σ decreases with B;

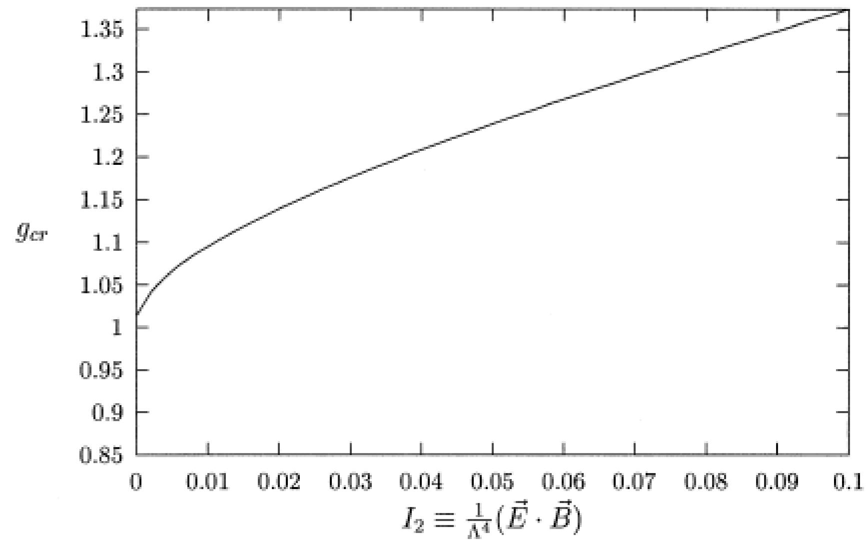
(2) T_c monotonically decreases with B;

(3) T_c converges to a finite value at large enough B and the crossover becomes first order.

G. Bali, et al., JHEP 1202 (2012) 044;
Phys.Rev. D86 (2012) 071502(R);
G. Endrődi, JHEP 1507 (2015) 173 .

The effect of electric field





(1) Mass gap decreases monotonically with E;

(2) Chiral transition is second order.

S. P. Klevansky and R. H. Lemmer, Phys. Rev. D 39, 3478 (1989).

(1) Critical coupling increases with I_2, the effect is more E-like;

(2) The transition is second order.

A. Y. Babansky, E. V. Gorbar and G. V. Shchepanyuk, Phys. Lett. B 419, 272 (1998).

Nambu—Jona-Lasinio model

The effective Lagrangian:

$$\mathcal{L} = \bar{\psi}(i\not{D} - m_0 - \mu\gamma^0)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2\right]$$

$$\psi(x) = (u(x), d(x))^T, D_\mu = \partial_\mu + iQA_\mu$$

Introducing auxiliary fields σ, π (Hubbard-Stratonovich transformation), the partition function becomes:

$$\mathcal{Z} = \int [\mathcal{D}\hat{\sigma}][\mathcal{D}\hat{\pi}_0][\mathcal{D}\hat{\pi}_\pm] \exp\left\{-\int dx[\frac{(m-m_0)^2+\hat{\sigma}^2+\hat{\pi}_0^2+\hat{\pi}_\pm^2}{4G}] + \mathrm{Tr}\ln\left[i\not{D} - m - \hat{\sigma} - i\gamma_5(\tau_3\hat{\pi}_0 + \tau_\pm\hat{\pi}_\pm) - i\mu\gamma^4\right]\right\},$$

In mean field approximation, the thermodynamic potential is

$$\Omega(m) = \frac{(m-m_0)^2}{4G} - \frac{1}{\beta V}\mathrm{Tr}\ln\left[i\not{D} - m - i\mu\gamma^4\right]$$

Nambu—Jona-Lasinio model

Gap equation

$$\frac{m-m_0}{2G}-\frac{1}{\beta V}\mathrm{Tr}\mathcal{S}(x,x')=0,$$

Collective modes give higher order contribution to Ω

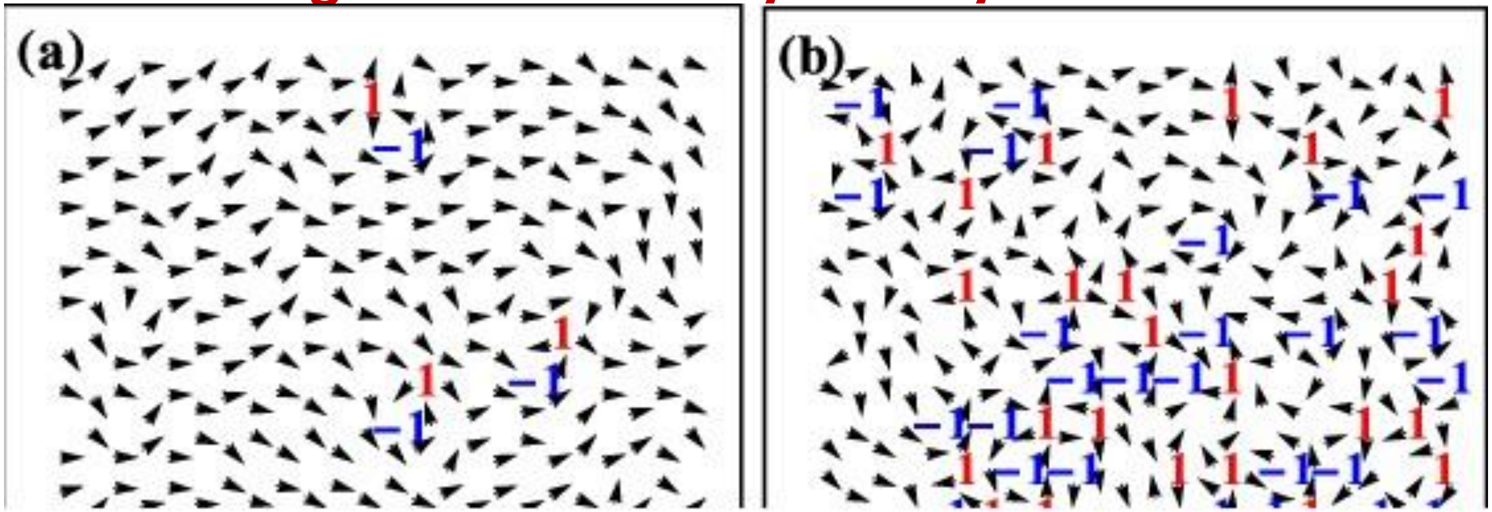
$$\begin{aligned} i\mathcal{D}_M^{-1}(x,x') &= \frac{\delta(x-x')}{2G}+\Pi_M(x,x') \\ &= \frac{e^{iq_M\int_{x'}^{x}Adx}}{2G}\delta(x-x')+\mathrm{Tr}\mathcal{S}(x,x')\Gamma_M\mathcal{S}(x',x)\Gamma_M^{*}. \end{aligned}$$

$$\Gamma_M=\begin{cases}1, & M=\hat{\sigma}\\ i\gamma_5\tau_+, & M=\hat{\pi}_+\\ i\gamma_5\tau_-, & M=\hat{\pi}_-\\ i\gamma_5\tau_3, & M=\hat{\pi}_0\end{cases}\;;\quad \Gamma_M^{*}=\begin{cases}1, & M=\hat{\sigma}\\ i\gamma_5\tau_-, & M=\hat{\pi}_+\\ i\gamma_5\tau_+, & M=\hat{\pi}_-\\ i\gamma_5\tau_3, & M=\hat{\pi}_0\end{cases}$$

For simplicity, we only study the properties of the Goldstone mode $\hat{\pi}_0$.

Kosterlitz-Thouless transition

Coleman-Mermin-Wagner theorem forbids spontaneous breaking of continuous symmetry at $T \neq 0$ in 2+1D



(a) Metallic State **(b) Insulator State**

KT transition temperature $$T_{KT} = \frac{\pi}{2} J$$

stiffness $$J = NM^2 \xi_1(M,T) = \frac{NM}{4\pi} \tanh \frac{M}{2T}$$

E. Babaev, Phys. Lett. B497, 323 (2001).

Kosterlitz-Thouless transition

Introduce chiral chemical potential:

$$\mathcal{L} = \bar{\psi}(i\not{D} + \mu_5\gamma^0\gamma^5)\psi + \frac{G}{2N}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2\right]$$

Utilizing Ritus's method， the Fermion propagator is

$$\begin{aligned} S_F(x,x') = & \sum_{s=\pm}\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}\frac{dp_0}{2\pi}\int_{-\infty}^{\infty}\frac{dp_2}{2\pi}\frac{i}{2[p_0^2-(\varepsilon_n^s)^2]}e^{-ip_0(x_0-x_0')+ip_2(x_2-x_2')} \\ & \times\left\{G_n\left(x_1-\frac{p_2}{eB}\right)G_n\left(x_1'-\frac{p_2}{eB}\right)\left[p_0\gamma^0+(\mu_5+s\lambda_n)\gamma^0\gamma^5+M\right]\right. \\ & \left.+G_n\left(x_1-\frac{p_2}{eB}\right)\gamma^1 G_n\left(x_1'-\frac{p_2}{eB}\right)\left(\gamma^1\right)^\dagger\left[p_0\gamma^0-(\mu_5+s\lambda_n)\gamma^0\gamma^5-M\right](-is\gamma^1\gamma^3)\right\}. \end{aligned}$$

Excitation energy $\varepsilon_n^s = \sqrt{(\lambda_n + s\mu_5)^2 + M^2}$.

Kosterlitz-Thouless transition

Gap equation can be derived to

$$\frac{M^2}{\pi} - \frac{MM_0}{\pi}\mathrm{sgn}(G - G_c) - \frac{eB}{2\pi}\frac{\eta f_{3/2}(\eta)}{\sqrt{\pi}} - \frac{eB}{4\pi}\sum_{s=\pm}\sum_{n=0}^{\infty}\alpha_n M\left[\frac{1-2n_{\mathrm{F}}(\varepsilon_n^s)}{\varepsilon_n^s} - \frac{1}{\varepsilon_n}\right] = 0$$

$$f_n(\eta) = \int_0^\infty \frac{dx}{x^n} e^{-\eta^2 x}\left(\frac{x}{\tanh x} - 1\right) \qquad \eta = M/(eB)^{1/2}$$

The stiffness is complex:

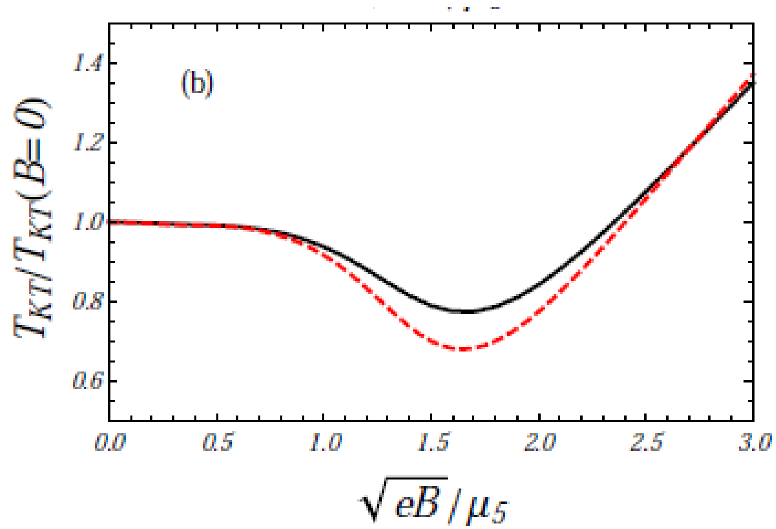
The properties of the eigenfunctions of harmonic oscillator are used

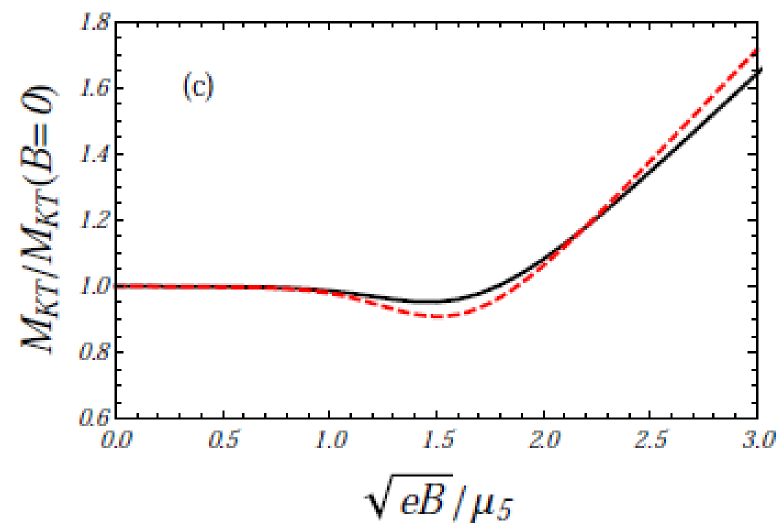
$$\frac{J}{N} = \frac{M^2}{4\pi}\frac{\mu_5^2 - eB}{2\mu_5^2 - eB}\frac{1}{\varepsilon_0}\tanh\frac{\varepsilon_0}{2T} - \frac{M^2}{8\pi}\sum_{s=\pm}\frac{\lambda_1 + s\mu_5}{\lambda_1 + 2s\mu_5}\frac{1}{\varepsilon_1^s}\tanh\frac{\varepsilon_1^s}{2T}$$

$$+\frac{M^2}{16\pi}\sum_{s=\pm}\sum_{n=1}^{\infty}\left\{\frac{2n(eB)^2 + eB\mu_5(2\mu_5 + s\lambda_n) - 8n\mu_5^2(\mu_5 + s\lambda_n)^2}{(eB)^2 - 4\mu_5^2(\mu_5 + s\lambda_n)^2}\frac{2}{\varepsilon_n^s}\tanh\frac{\varepsilon_n^s}{2T}\right.$$

$$\left.-\frac{(\mu_5 + s\lambda_{n+1})[2(2n+1)\mu_5 + s\lambda_{n+1}]}{2\mu_5^2 + 2s\mu_5\lambda_{n+1} + eB}\frac{1}{\varepsilon_{n+1}^s}\tanh\frac{\varepsilon_{n+1}^s}{2T} - \frac{(\mu_5 + s\lambda_{n-1})[2(2n-1)\mu_5 - s\lambda_{n-1}]}{2\mu_5^2 + 2s\mu_5\lambda_{n-1} - eB}\frac{1}{\varepsilon_{n-1}^s}\tanh\frac{\varepsilon_{n-1}^s}{2T}\right\}.$$

Kosterlitz-Thouless transition





1. **Weak B ($0 < \sqrt{eB}/\mu_5 < 1/\sqrt{2}$)， determined by μ_5;**
2. **Intermediate B ($1/\sqrt{2} < \sqrt{eB}/\mu_5 < 2$) ,a valley shows up——de Hass-van Alphen Effect;**
3. **Strong B ($\sqrt{eB}/\mu_5 > 2$)， T and M increases linearly with B——lowest Landau level approximation;**
4. **The inverse magnetic catalysis effect shows up in the lower B.**

π_0 condensation in parallel EM field

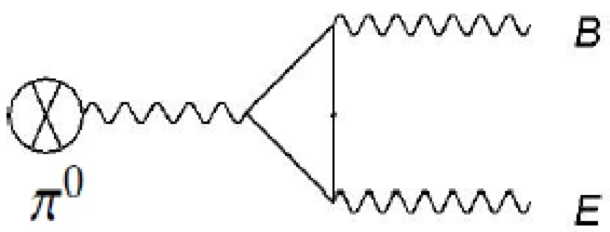
(1) π_0 condensate can inhomogeneously exist in the vacuum through chiral density wave:

$$\langle\bar{\psi}\psi\rangle = \Delta\cos(\mathbf{q}\cdot\mathbf{r}), \qquad \langle\bar{\psi}i\gamma_5\tau_3\psi\rangle = \Delta\sin(\mathbf{q}\cdot\mathbf{r}),$$

Can homogeneous π_0 condensate exist?

(2) π_0 is a neutral pseudo-scalar field and the second Lorentz invariant $I_2 = B\cdot E$ is also a neutral pseudo-scalar.

In principal, π_0 can be generated through triangle anomaly for nonzero I_2:



NJL model calculation

Considering both σ and π_0 condensation, Ω becomes:

$$\Omega = \frac{(m-m_0)^2 + (\pi^0)^2}{4G} - \frac{1}{V_4}\mathrm{Tr}\ln S^{-1},$$

Within Schwinger approach, the fermion propagators are modified:

$$\begin{aligned} \mathcal{S}_f(x,x') &= \frac{-i}{(4\pi)^2}\int_0^\infty \frac{ds}{s^2}\, e^{-iq_f\int_{x'}^{x} A\cdot dx}\Big[-\frac{1}{2}\gamma\big(q_f F\coth(q_f F s) - q_f F\big)(x-x') + m - \mathrm{sgn}(q_f)i\gamma^5\pi^0\Big] \\ &\times \exp\Big\{-i\big[m^2 + (\pi^0)^2\big]s + \frac{i}{4}(x-x')q_f F\coth(q_f F s)(x-x') + \frac{i}{2}q_f\sigma F s\Big\}\frac{-(q_f s)^2 I_2}{\mathrm{Im}\cosh\big(iq_f s(I_1+2iI_2)^{1/2}\big)}, \end{aligned}$$

The coupled gap equations

$$\frac{m-m_0}{2G} = \frac{mN_c}{4\pi^2}\sum_{f=u,d}\int_0^\infty ds\, e^{-[m^2+(\pi^0)^2]s}\,\frac{q_f^2 I_2\mathrm{Re}\cosh[q_f s\sqrt{I_1+2iI_2}]}{\mathrm{Im}\cosh[q_f s\sqrt{I_1+2iI_2}]} - \frac{N_c}{4\pi^2}\frac{\pi^0}{m^2+(\pi^0)^2}(q_u^2 - q_d^2)I_2,$$

$$\frac{\pi^0}{2G} = \frac{\pi^0 N_c}{4\pi^2}\sum_{f=u,d}\int_0^\infty ds\, e^{-[m^2+(\pi^0)^2]s}\,\frac{q_f^2 I_2\mathrm{Re}\cosh[q_f s\sqrt{I_1+2iI_2}]}{\mathrm{Im}\cosh[q_f s\sqrt{I_1+2iI_2}]} + \frac{N_c}{4\pi^2}\frac{m}{m^2+(\pi^0)^2}(q_u^2 - q_d^2)I_2.$$

NJL model calculation

An universal relation for small I_2:

$$\pi_0 = \frac{N_c}{4\pi^2}\frac{2G}{m_0}(q_u^2 - q_d^2)I_2.$$

Gell-Mann—Oakes—Renner relation: $m_\pi^2 f_\pi^2 = m_0 m^* (2G)^{-1}$

The final result for the whole range of I_2 ($m^* = \sqrt{(\pi^0)^2 + m^2}$):

$$\frac{\pi^0}{m^*} = \begin{cases} \frac{N_c}{4\pi^2 f_\pi^2 m_\pi^2}(q_u^2 - q_d^2)\boldsymbol{E}\cdot\boldsymbol{B} & \text{for } |I_2| < I_2^c, \\ \mathrm{sgn}(I_2) & \text{for } |I_2| > I_2^c, \end{cases}$$

I_2^c **is the point where chiral rotation ends.**

Chiral perturbation theory calculation

The Lagrangian with Wess-Zumino-Witten term $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\rm WZW}$,

$$\mathcal{L}_0 = \frac{f_\pi^2}{4}{\rm tr}\left[D_\mu U^\dagger D^\mu U + m_\pi^2(U + U^\dagger)\right] \qquad \mathcal{L}_{\rm WZW} = \frac{N_c}{48\pi^2}A_\mu\epsilon^{\mu\nu\alpha\beta}[{\rm tr}\,(QL_\nu L_\alpha L_\beta + QR_\nu R_\alpha R_\beta) - iF_{\alpha\beta}T_\nu]$$

Choosing Weinberg parameterization $U = \frac{1}{f_\pi}(s + i\tau\cdot t)$ **and setting**

$$s = f_\pi\cos\phi, t_3 = f_\pi\sin\phi, t_1 = t_2 = 0,$$

the thermodynamic potential becomes

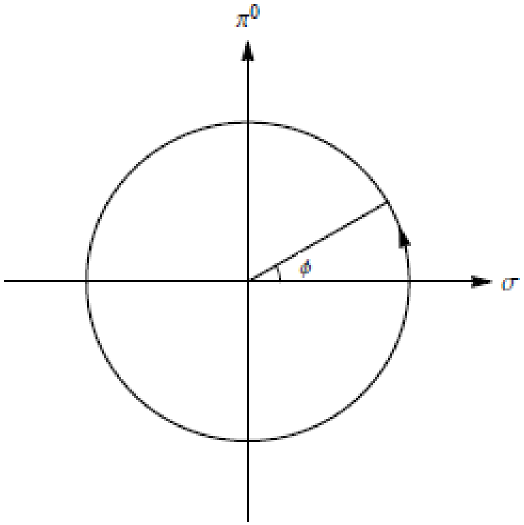
$$-\Omega(\phi) \;=\; f_\pi^2 m_\pi^2\cos\phi + \frac{N_c I_2}{4\pi^2}{\rm tr}(Q^2\tau_3)\phi$$

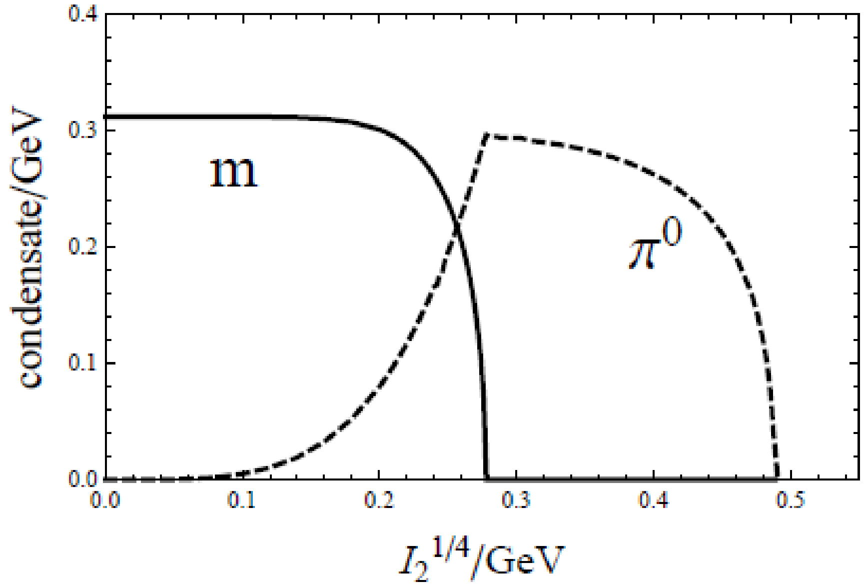
The minimal point gives the same result as in NJL model.

Consistent with the linear sigma model in study of DCC

M. Asakawa et. al., Phys. Rev. D **58**, 094011 (1998).

Chiral rotation in parallel EM field





(1) π_0 condensate initially increases with I_2 and the decreasing feature is due to chiral restoration;

(2) During the chiral rotation, m^* is almost a constant;

(3) A jump in chiral limit.

The robustness of π_0 condensation

(1) Invalidation of Vafa-Witten Theorem:

$$\mathcal{D} = \gamma_\mu(\partial_\mu - igA_\mu) + Q\gamma_4 A_0 + iQ\gamma_i A_i + M,$$

The static electric field is given by A_0 which acts like μ and the positivity of Det D is lost.

(2) The stability related to Schwinger mechanism (mainly $E \to \pi^+ + \pi^-$):

The QCD vacuum is like insulator and QED system lies on top of it.

Stable QED system $|eE| \ll m_\pi^2 + |eB|,$ ➡ $m_{\pi^\pm}^2$

Conclusions

- Kosterlitz-Thouless transition and chiral rotation are studied in NJL model with Ritus and Schwinger approach, respectively;
- When μ_5 is introduced, inverse magnetic catalysis will show up in the KT transition temperature.
- Neutral pion condensation is found in parallel EM field due to triangle anomaly and is robust to Vafa-Witten theorem and Schwinger mechanism.

Thank you very much!

Schwinger approach

J. Schwinger, Phys. Rev. 82, 664 (1951)

EM current interaction

$$\delta W^{(1)} = \int (dx)\delta A_\mu(x)\langle j_\mu(x)\rangle = ie\,\mathrm{Tr}\gamma\delta AG$$

$$= -\mathrm{Tr}\delta(\gamma\Pi)\gamma\Pi\int_0^\infty ds\,\exp[-i(m^2-(\gamma\Pi)^2)s]$$

$$= \delta\left[\tfrac{1}{2}i\int_0^\infty ds s^{-1}\exp[-i(m^2-(\gamma\Pi)^2)s]\right],$$

Equivalent to introduce an effective Lagrangian

$$\mathfrak{L}^{(1)}(x) = \tfrac{1}{2}i\int_0^\infty ds s^{-1}\exp(-im^2s)\,\mathrm{tr}(x|U(s)|x),$$

$$U(s) = \exp(-i\mathcal{H}s),$$

U(s) is the time-evolution operator $(x'|U(s)|x'') = (x(s)'|x(0)'')$

Schwinger approach

For an uniform field , the fermion propagator is

$$G(x', x'') = i\int_0^\infty ds \exp(-im^2 s)[-(x(s)'|\Pi_\mu(0)|x(0)'')\gamma_\mu + m(x(s)'|x(0)'')]$$

The evolution of the operators are

$$(x(s)'|\Pi(0)|x(0)'') = \tfrac{1}{2}[eF\coth(eFs) - eF] \times (x' - x'')(x(s)'|x(0)'')$$

$$(x(s)'|x(0)'') = -i(4\pi)^{-2}\Phi(x', x'')e^{-L(s)}s^{-2} \times \exp[i\tfrac{1}{4}(x'-x'')eF\coth(eFs)(x'-x'')]\cdot\exp[i\tfrac{1}{2}e\sigma Fs]$$

Ritus approach

V. I. Ritus, Ann. Phys. (Berlin) 69, 555 (1972);
V. I. Ritus, Zh. Eksp. Teor. Fiz. 75, 1560 (1978).

For simplicity， we refer to
J. Warringa, Phys. Rev. D86, 085029 (2012).

Dirac Equation $(i\gamma^{\mu} D_{\mu} - m)\psi(x) = 0$, $A_{\mu} = (0, 0, B_z x, A_z(t))$

Set the solutions to have the forms

$$\psi^{+}_{ps}(x) = F^{+}_{p}(t) G_{p}(\mathrm{x}) e^{i p_y y + i p_z z} u_s(\tilde{p}_{+}),$$

$$\psi^{-}_{ps}(x) = F^{-}_{p}(t) G_{p}(\mathrm{x}) e^{i p_y y + i p_z z} v_s(\tilde{p}_{-}).$$

and satisfy

$$[i\gamma^0 \partial_t - \gamma^3(p_z - qA_z(t))] F^{\pm}_{p}(t) = \pm \kappa F^{\pm}_{p}(t) \gamma^0,$$

$$[i\gamma^1 \partial_{\mathrm{x}} - \gamma^2(p_y - qB_z \mathrm{x})] G_{p}(\mathrm{x}) = \lambda G_{p}(\mathrm{x}) \gamma^2.$$

Ritus approach

Dirac equation becomes a free-Fermion equation

$$(\tilde{\not p}_+ - m)u_s(\tilde{p}_+) = 0, \quad (\tilde{\not p}_- + m)v_s(\tilde{p}_-) = 0. \quad \tilde{p}^\mu_\pm = (\kappa, 0, \mp\lambda, 0)$$

Final solution of $\mathrm{G_p}(\boldsymbol{x})$

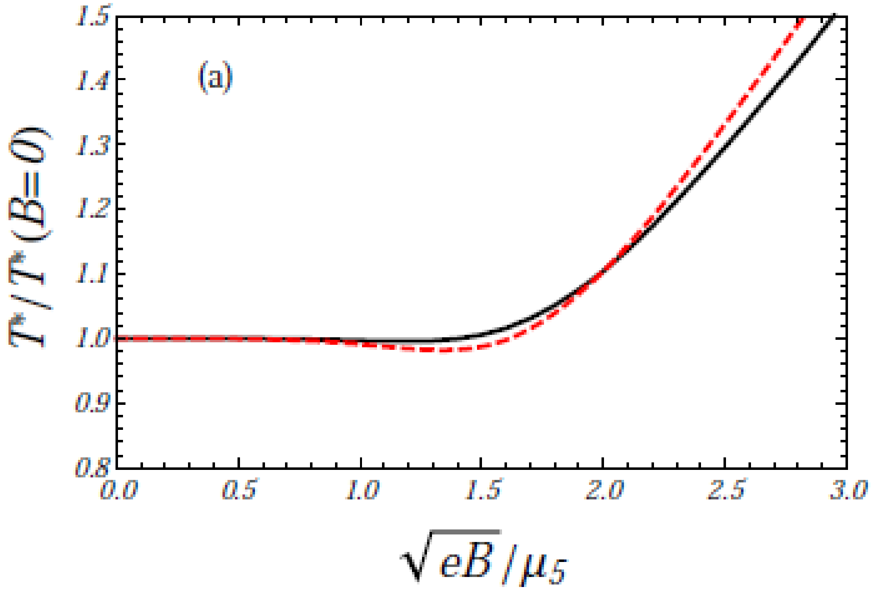
$$G_p(x) = \frac{1 + i\mathrm{sgn}(qB_z)\gamma^1\gamma^2}{2}\phi_k(x - \frac{p_y}{qB_z}) + \frac{1 - i\mathrm{sgn}(qB_z)\gamma^1\gamma^2}{2}\phi_{k-1}(x - \frac{p_y}{qB_z})$$

Second quantization

$$\Psi(x) = \sum_{s=\pm}\sum_p \frac{1}{\sqrt{2\kappa_p}}\left[b_{ps}\psi^+_{ps}(x) + d^\dagger_{-ps}\psi^-_{ps}(x)\right],$$

The propagators of particle and antiparticle

$$S^\pm(x, x') = \sum_p \frac{1}{2\kappa_p} \mathrm{e}^{\mathrm{i}p_y(\mathrm{y}-\mathrm{y}')+\mathrm{i}p_z(\mathrm{z}-\mathrm{z}')} F^\pm_p(t) G_p(\mathrm{x}) \times (\tilde{\not p}_\pm \pm m)\gamma^0 F^\pm_p(t')^\dagger \gamma^0 G_p(\mathrm{x}').$$



$\boldsymbol{T}^*$—the temperature at which M vanishes show a similar behavior.