Exploring the QCD Phase Diagram for a Signature of the Critical Point

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arXiv: 1410:5454(PRD), 1509.04968(PRD), 1603.05198

QCD Phase Structure III, CCNU June 6-9,2016

Outlines

1. Motivation (from QCD to QGP)

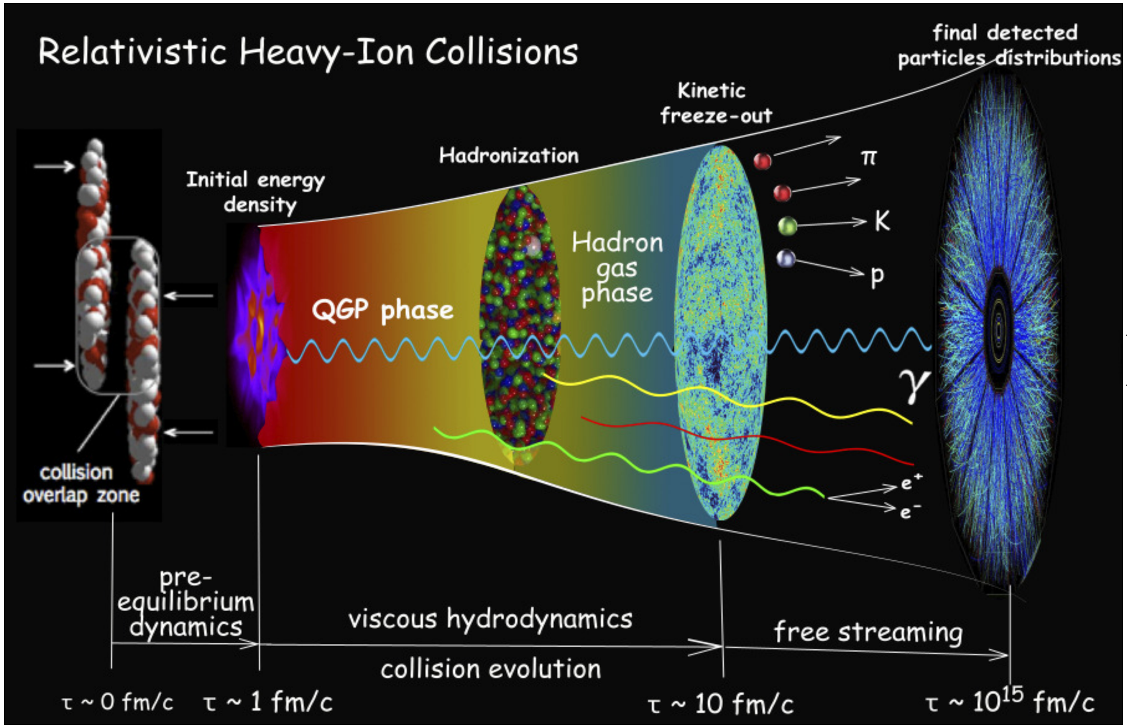
2. Phase diagram and CP(from CP to BES)

3. A signature of the criticality

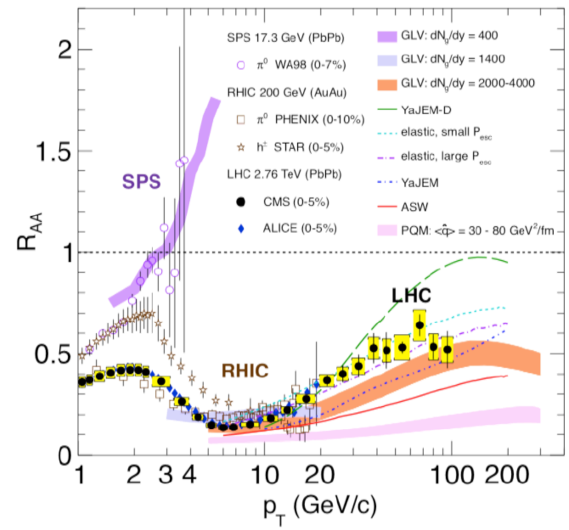
4. Observables: calculation vs. Data

5. Summary

QGP with Little Bang

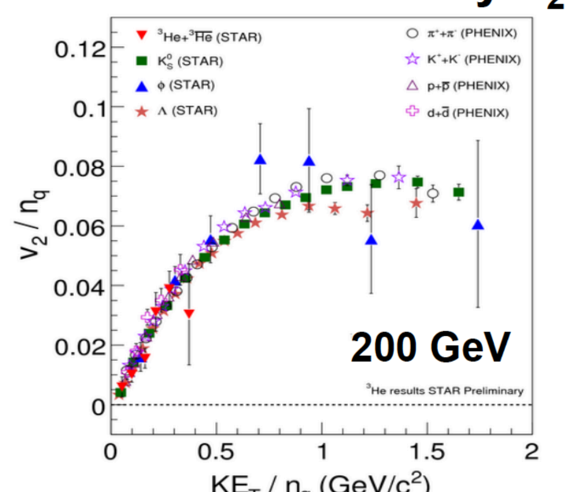


Jet Quenching



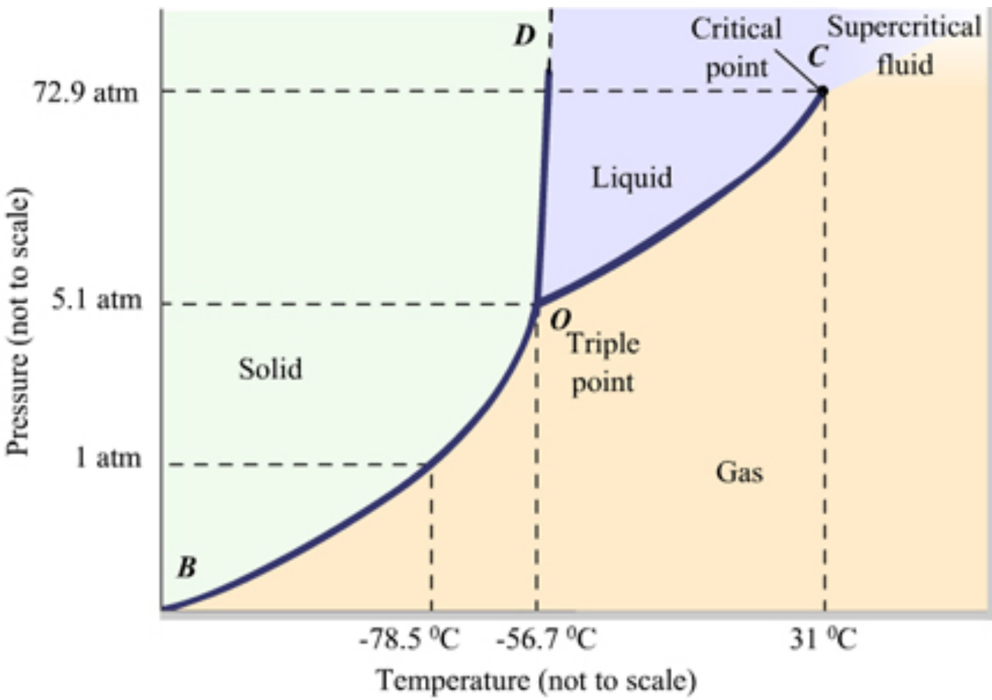
Eur.Phys.J. C72, 1945 (2012).

Partonic Collectivity v_2



- QGP and phase diagram studied in heavy ion collisions since 1987 at AGS(5GeV), 1996 at SPS(17GeV), since 2000 at RHIC (200GeV), since 2010 at LHC(2.76TeV).
- Indirect evidence for strongly coupled and liquid like QGP formed in HIC.
- Trying to search for the direct pictures about phase transition. Strong and special !

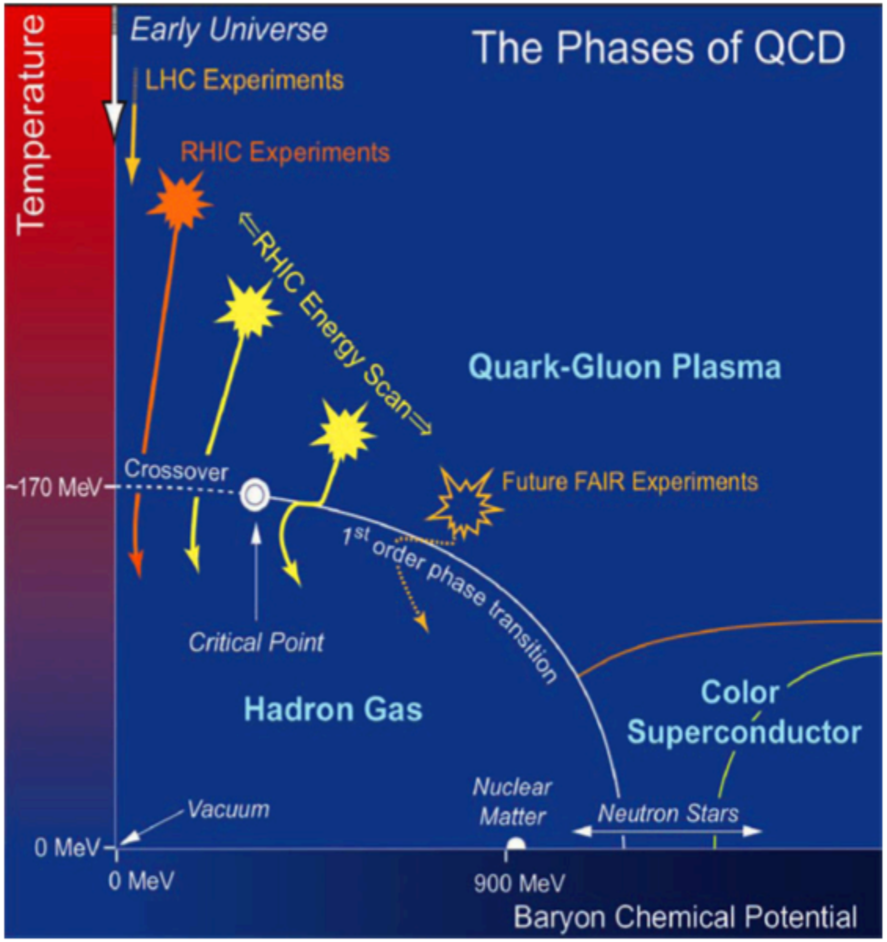
Critical Point and Critical Opalescence



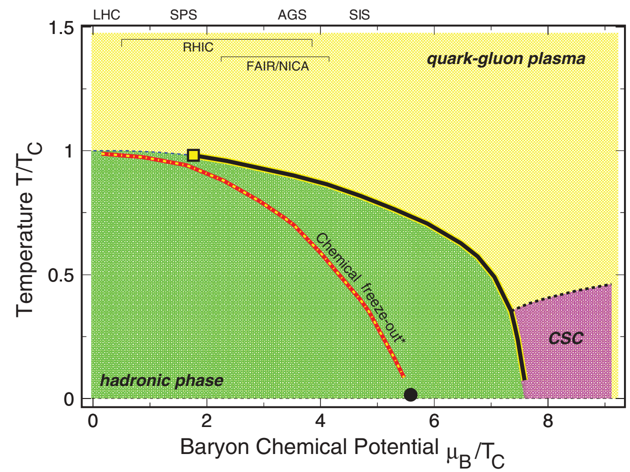
Critical point: End of a first order phase transition line + crossover

Critical opalescence: As the CP approached, the density begin to fluctuate over a large length scales, comparable to the wave length of light. (for CO_2 gas, 0.3nm ➔ 600nm)

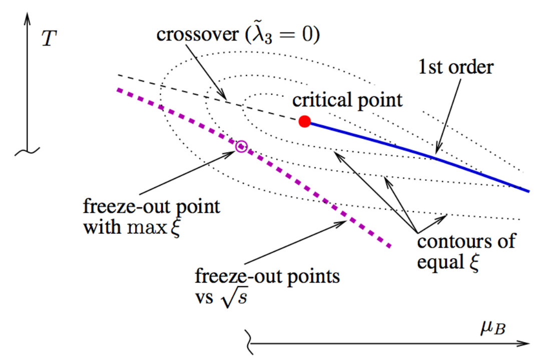
QCD Phase Diagram, Beam Energy Scan



STAR white paper 2014, Studying the phase diagram of QCD matter at RHIC

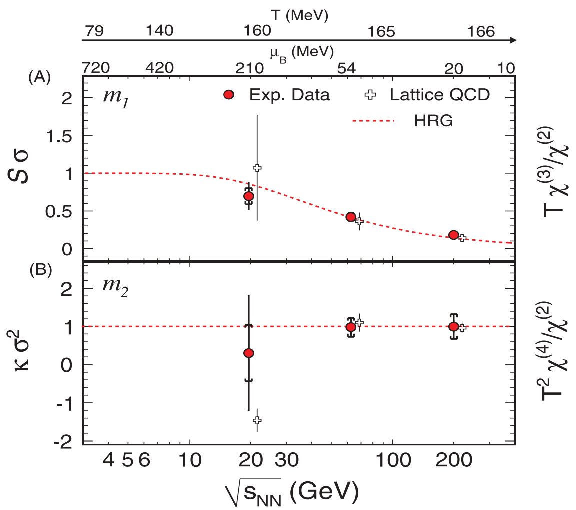


Chemical freeze-out line VS. QCD phase boundary, mapping



measurement VS. thermal equilibrium, singularity

Lattice and Experimental approaches



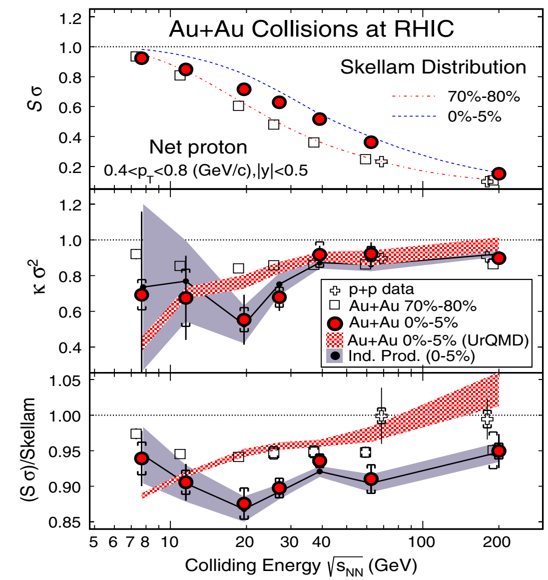
$$T^{n-4}\chi_B^{(n)}\left(\frac{T}{T_c},\frac{\mu_B}{T}\right) = \frac{1}{T^4}\frac{\partial^n}{\partial(\mu_B/T)^n}P\left(\frac{T}{T_c},\frac{\mu_B}{T}\right)\Big|_{T/T_c}$$

$$(m_1): S\sigma = \frac{[B^3]}{[B^2]} = \frac{T\chi_B^{(3)}}{\chi_B^{(2)}},$$

$$(m_2): \kappa\sigma^2 = \frac{[B^4]}{[B^2]} = \frac{T^2\chi_B^{(4)}}{\chi_B^{(2)}}$$

$$(m_3): \frac{\kappa\sigma}{S} = \frac{[B^4]}{[B^3]} = \frac{T\chi_B^{(4)}}{\chi_B^{(3)}}$$

RHIC BES STAR PRL 2014



Scale for the Phase Diagram of QCD

S. Gupta, X. Luo, B. Mohanty, H. G. Ritter, N. Xu, Science 2011

Sign Problem in Lattice: $\det(D+m+\mu\gamma_0)^* = \det(D+m-\mu^*\gamma_0)$, H-T Ding, et al, Int. J. Mod. Phys. E 2015

Extrapolate from mu=0: $P(T,\mu) = P(T) + \frac{\mu^2}{2!}\chi^{(2)}(T) + \frac{\mu^4}{4!}\chi^{(4)}(T) + \cdots$ C.R.Allton, et al, PRD 2002

Shoot from imaginary: $\log Z(\mu_I) = a_0 - a_2\mu_I^2 + a_4\mu_I^4 + O(\mu_I^6).$ M. Alford, et al, PRD 2003

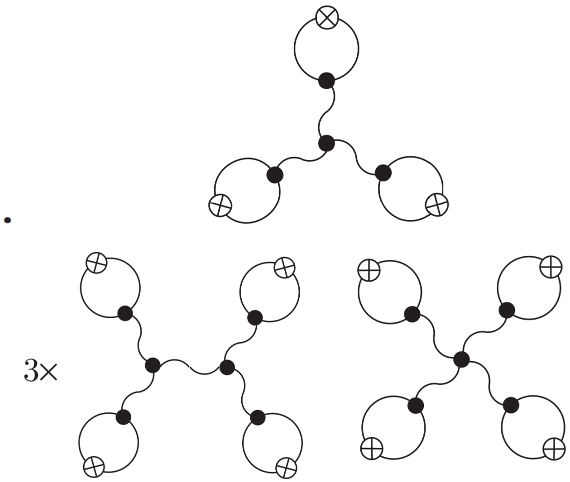
Spin imbalanced Fermi gas on a lattice: J. Braun, J-W Chen, JD, et al. PRL 2013

Higher moments are crucial in HIC

Maximum correlation length 2~3 fm (dynamical evolution, freeze out…)

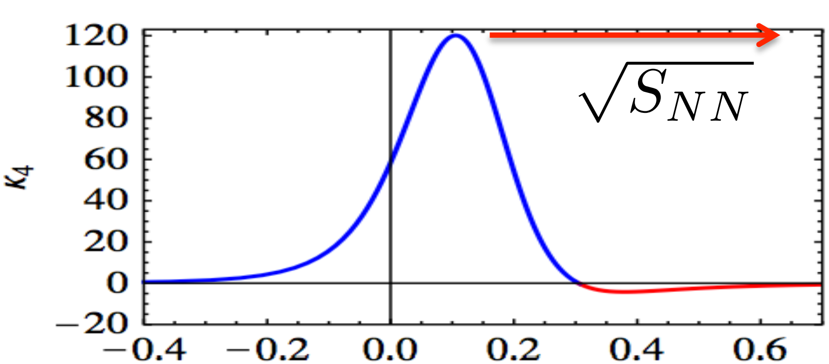
$$\kappa_2 = \langle \sigma_0^2 \rangle = \frac{T}{V}\xi^2; \qquad \kappa_3 = \langle \sigma_0^3 \rangle = \frac{2\lambda_3 T}{V}\xi^6;$$

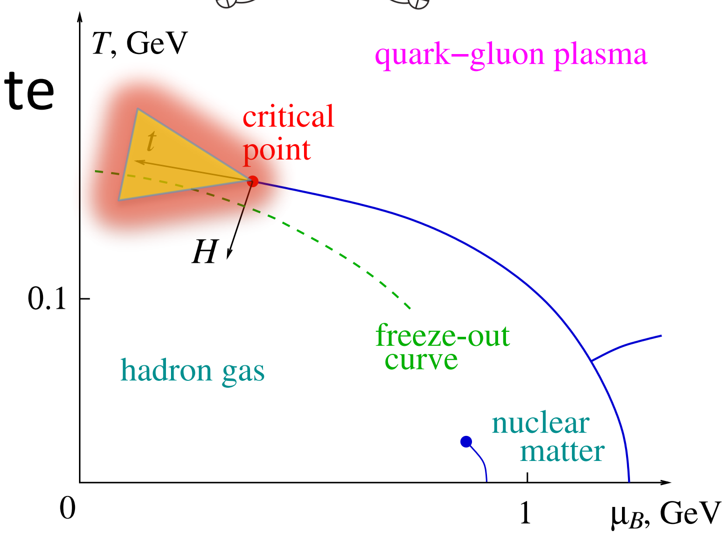
$$\kappa_4 = \langle \sigma_0^4 \rangle_c \equiv \langle \sigma_0^4 \rangle - 3\langle \sigma_0^2 \rangle^2 = \frac{6T}{V}[2(\lambda_3\xi)^2 - \lambda_4]\xi^8.$$



Non-monotonic functions of the collision energy, higher moments more sensitive signature of CP. M. Stephanov, PRL 2009

Universally, sign change of Kurtosis indicate that CP is close. M. Stephanov, PRL 2011





From phase diagram to observables

Susceptibilities = Cumulants

$$T\frac{\partial}{\partial\mu}\log Z = <N>=\bar{N}$$

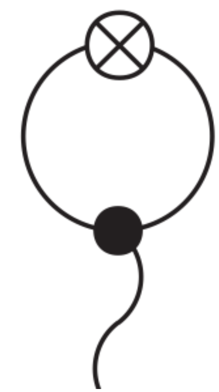
$$T^2\frac{\partial^2}{\partial\mu^2}\log Z = <(N-\bar{N})^2>$$

$$T^3\frac{\partial^3}{\partial\mu^3}\log Z = <(N-\bar{N})^3>$$

$$T^4\frac{\partial^4}{\partial\mu^4}\log Z = <(N-\bar{N})^4> -3<(N-\bar{N})^2>^2$$

Partition function:

$$Z = \mathrm{Tr}\left[\exp\left(-\frac{H-\mu N}{T}\right)\right]$$



By coupling the critical sigma field with the number density, the fluctuation and the criticality will be transferred to the measurements

M. Stephanov, PRL 2011

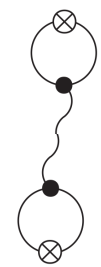
$$\langle(\delta N)^4\rangle_c = \langle N\rangle + \boxed{\langle\sigma_V^4\rangle_c\left(\frac{gd}{T}\int_p\frac{n_p}{\gamma_p}\right)^4} + \cdots,$$

Most singular part

Susceptibility

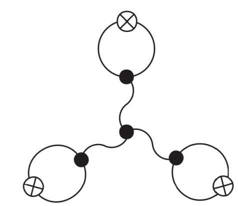
Effective potential: $\Omega(\mu, T, \sigma)$ Grand Canonical ensemble

Partition function: $Z = \int [d\sigma] \exp\left(-\frac{\Omega(T, \mu, \sigma)}{T}\right)$



$$T\frac{\partial}{\partial\mu}\log Z = -\langle\frac{\partial\Omega[\mu, T, \sigma]}{\partial\mu}\rangle = -\langle\Omega'\rangle$$

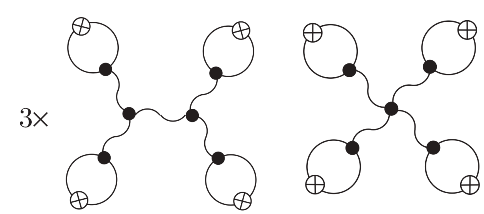
$$T^2\frac{\partial^2}{\partial\mu^2}\log Z = -T < \Omega'' > + \left(\frac{\partial^2\Omega(\mu, T, \sigma_0)}{\partial\mu\partial\sigma}\right)^2 \langle\delta\sigma^2\rangle$$



$$T^3\frac{\partial^3}{\partial\mu^3}\log Z = -T^2 < \Omega''' > + \left(\frac{\partial^2\Omega(\mu, T, \sigma_0)}{\partial\mu\partial\sigma}\right)^3 \langle\delta\sigma^3\rangle + \cdots$$

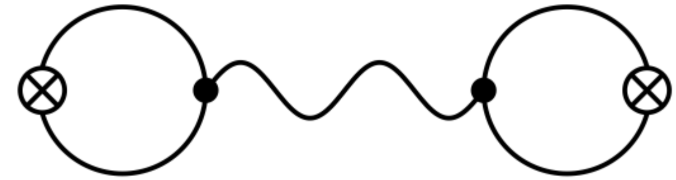
$$T^4\frac{\partial^4}{\partial\mu^4}\log Z = -T^3 < \Omega'''' > + \left(\frac{\partial^2\Omega(\mu, T, \sigma_0)}{\partial\mu\partial\sigma}\right)^4 (\langle\delta\sigma^4\rangle - 3\langle\delta\sigma^2\rangle^2) + \cdots$$

Sign change ➔ near zeros ➔ small
how about other terms?



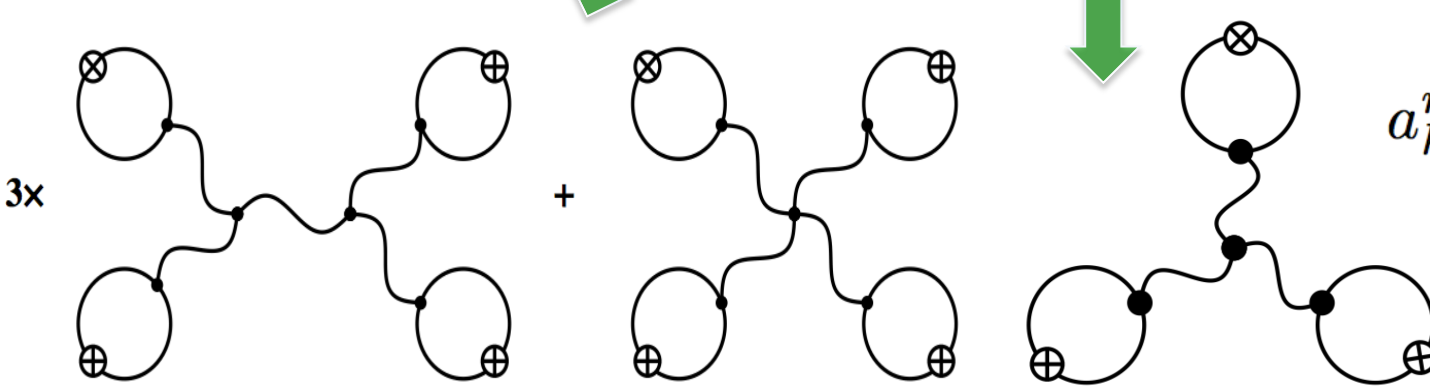
Tree level contribution

$$T^2 \frac{d^2}{d\mu^2} \log Z = -T a_2^0 + (a_1^1)^2 < \delta\sigma^2 >$$



$$T^3 \frac{d^3}{d\mu^3} \log Z = -T^2 a_3^0 + 3T a_1^1 a_2^1 < \delta\sigma^2 > -(a_1^1)^3 < \delta\sigma^3 > -6(a_1^1)^2 a_1^2 < \delta\sigma^2 >^2$$

$$\begin{aligned} T^4 \frac{d^4}{d\mu^4} \log Z = & -T^3 a_4^0 + 3T^2 (a_2^1)^2 < \delta\sigma^2 > +4T^2 a_1^1 a_3^1 < \delta\sigma^2 > -6T (a_1^1)^2 a_2^1 < \delta\sigma^3 > \\ & + (a_1^1)^4 \left(< \delta\sigma^4 > -3 < \delta\sigma^2 > \right) -12T \left[(a_1^1)^2 a_2^2 + 2 a_1^1 a_1^2 a_2^1 \right] < \delta\sigma^2 >^2 \\ & + 24 \left[(a_1^1)^3 a_1^3 + 2 (a_1^1)^2 (a_1^2)^2 \right] < \delta\sigma^2 >^3 + 24 (a_1^1)^3 a_1^2 < \delta\sigma^2 >< \delta\sigma^3 > \end{aligned}$$



$$a_k^n = \frac{V}{n!} \frac{\partial^{k+n} \Omega}{\partial \mu^k \partial \sigma^n}$$

Tree level contribution

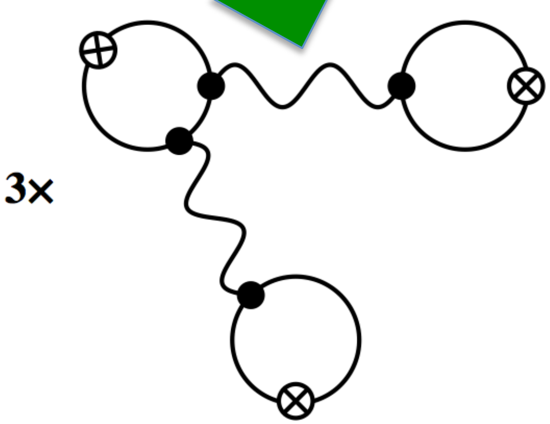
$$T^2 \frac{d^2}{d\mu^2}\log Z = -Ta_2^0 + (a_1^1)^2 < \delta\sigma^2 >$$

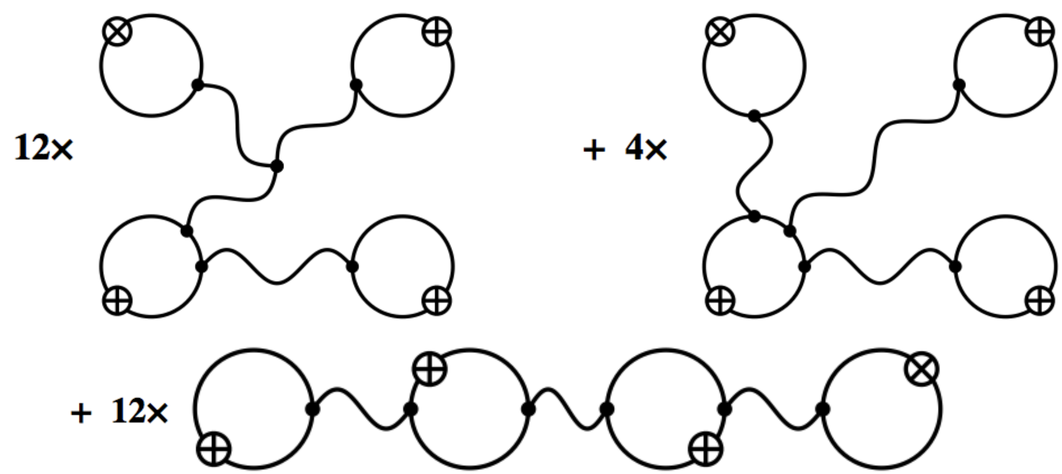
$$T^3 \frac{d^3}{d\mu^3}\log Z = -T^2a_3^0 + 3Ta_1^1a_2^1 < \delta\sigma^2 > -(a_1^1)^3 < \delta\sigma^3 > -6(a_1^1)^2a_1^2 < \delta\sigma^2 >^2$$

$$T^4 \frac{d^4}{d\mu^4}\log Z = -T^3a_4^0 + 3T^2(a_2^1)^2 < \delta\sigma^2 > +4T^2a_1^1a_3^1 < \delta\sigma^2 > -6T(a_1^1)^2a_2^1 < \delta\sigma^3 >$$

$$+(a_1^1)^4\left(< \delta\sigma^4 > -3 < \delta\sigma^2 >\right) -12T\left[(a_1^1)^2a_2^2 + 2a_1^1a_1^2a_2^1\right] < \delta\sigma^2 >^2$$

$$24\left[(a_1^1)^3a_1^3 + 2(a_1^1)^2(a_1^2)^2\right] < \delta\sigma^2 >^3 +24(a_1^1)^3a_1^2 < \delta\sigma^2 >< \delta\sigma^3 >$$

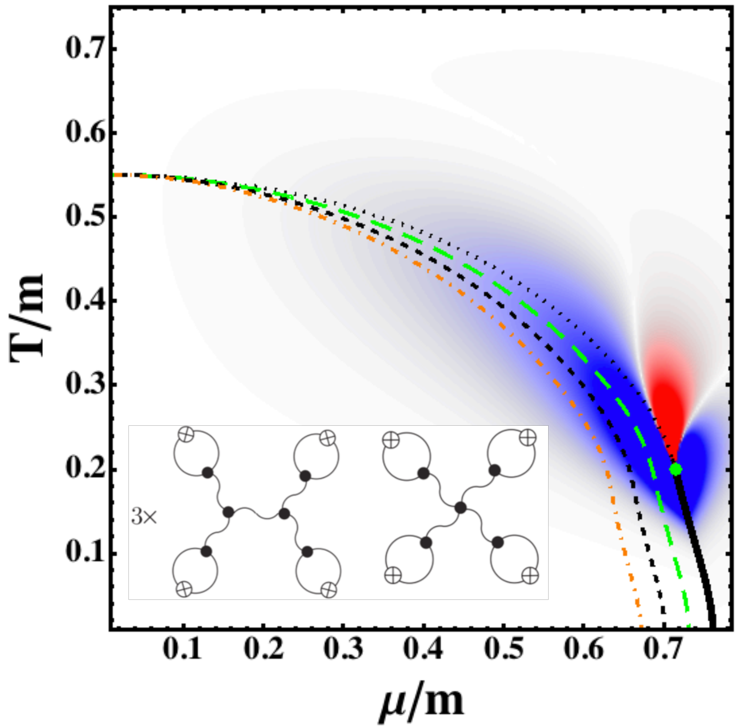




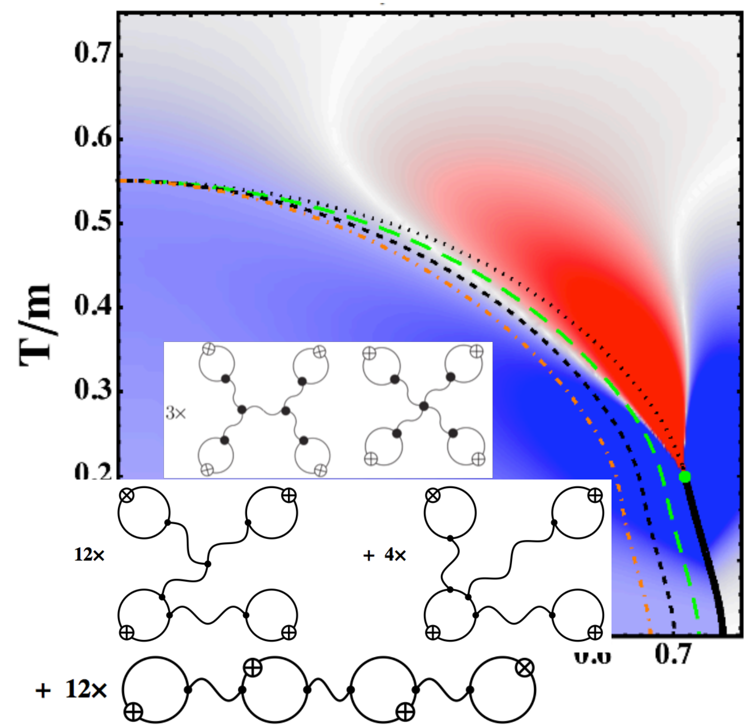
Ratio of the number susceptibilities

$$\kappa_4 = \langle \sigma_V^4 \rangle_c = 6VT^3[2(\lambda_3 \xi)^2 - \lambda_4]\xi^8$$

$$m_2 = \frac{T^4 \frac{d^4}{d\mu^4} \log Z}{T^2 \frac{d^2}{d\mu^2} \log Z}$$

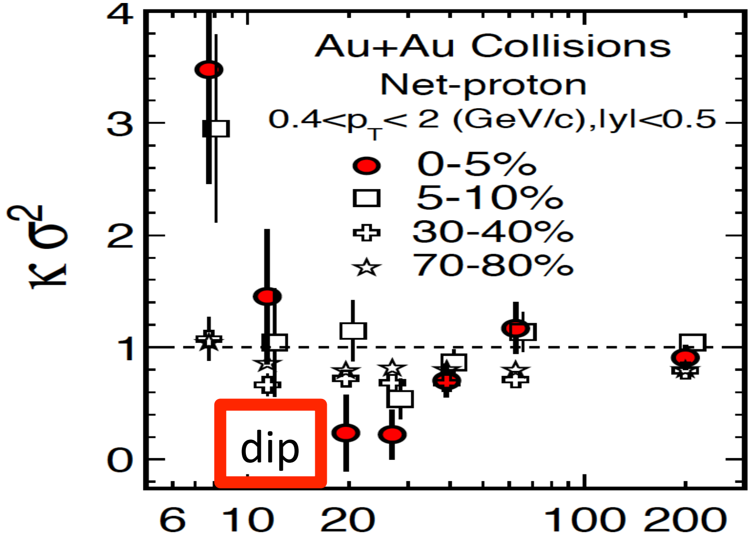


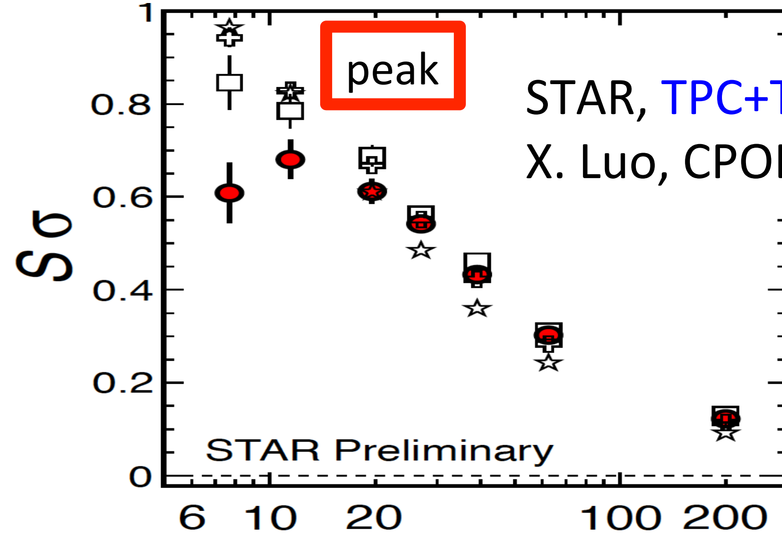
Kurtosis from order parameter fluctuation



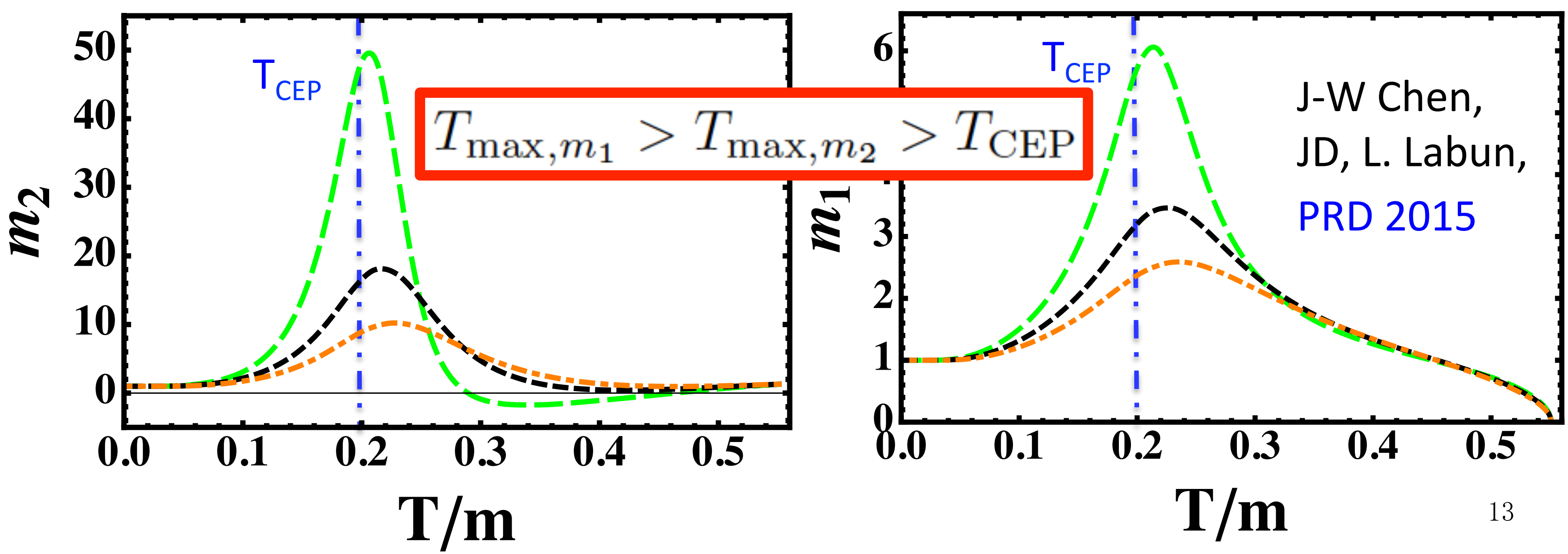
There is a large region of negative m_2, beginning at the critical point and opening up into the crossover region. The negative m_2 region overlaps with the "hadronic" phase near the critical point ➔ non-monotonic feature, sign change!

Comparing with STAR new data





STAR, TPC+TOF
X. Luo, CPOD2014



Further study with 3f-NJL model

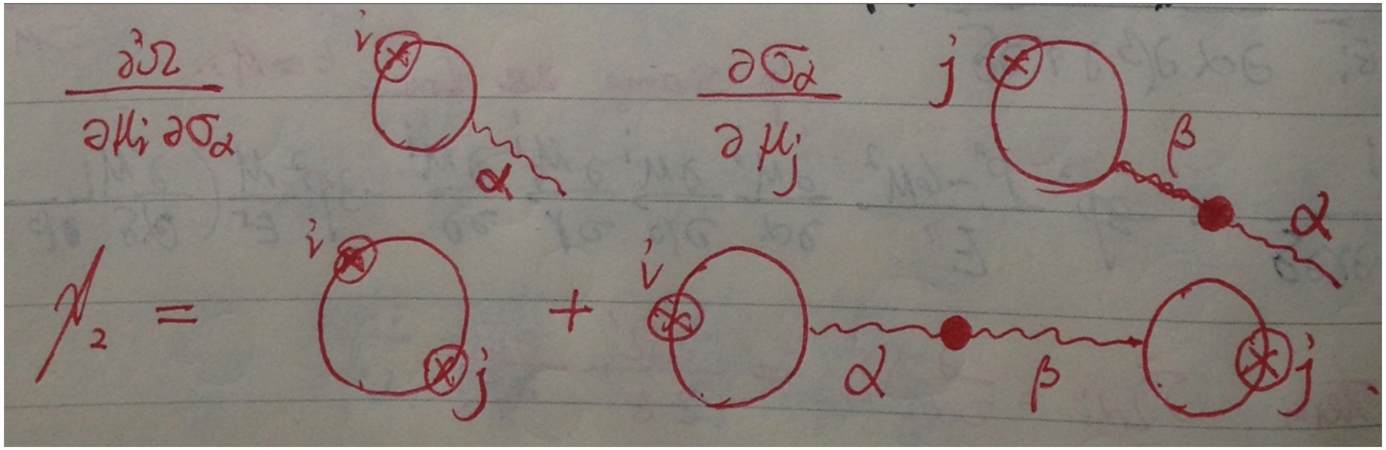
Effective potential:

$$\Omega(T,\mu_u,\mu_d,\mu_s) = 2G\left(\sigma_u^2+\sigma_d^2+\sigma_s^2\right) - 4K\sigma_u\sigma_d\sigma_s + \sum_{f=u,d,s}\Omega_f(T,\mu_f;m_f),$$

$$\Omega_f(T,\mu_f;m_f) = -2N_c\int\frac{d^3p}{(2\pi)^3}\left[E_f\Theta(\Lambda^2-\vec{p}^{\,2}) + T\ln\left[1+e^{-(E_f-\mu_f)/T}\right] + T\ln\left[1+e^{-(E_f+\mu_f)/T}\right]\right].$$

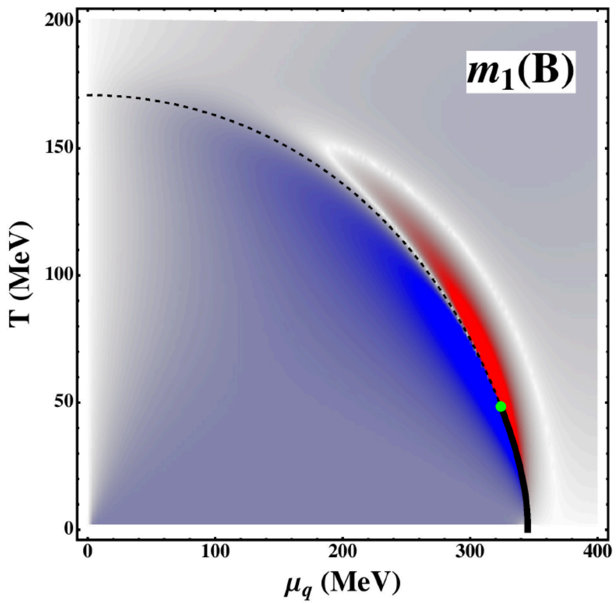
Flavor coupled:

$$E_f=\sqrt{m_f^2+p^2},\quad m_f=m_f^0-4G\sigma_f+2K\sigma_{f'}\sigma_{f''},\qquad f\neq f'\neq f''\in\{u,d,s\}$$



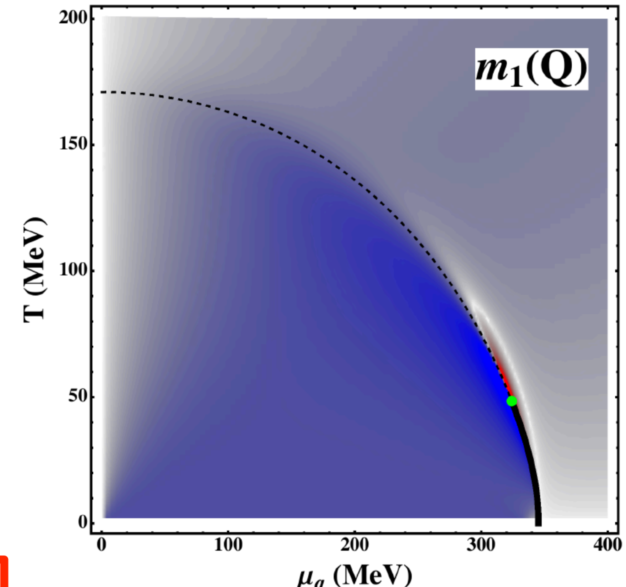
$$\chi_2^{ij} = \left.\frac{\partial^2\Omega}{\partial\mu_i\partial\mu_j}\right|_{\vec{\sigma}_0} - \sum_\alpha \left.\frac{\partial^2\Omega}{\partial\mu_i\partial\sigma_\alpha}\right|_{\vec{\sigma}_0}\left[\left.\frac{\partial^2\Omega}{\partial\sigma_\beta\partial\sigma_\alpha}\right|_{\vec{\sigma}_0}\right]^{-1}\left.\frac{\partial^2\Omega}{\partial\sigma_\beta\partial\mu_j}\right|_{\vec{\sigma}_0}$$

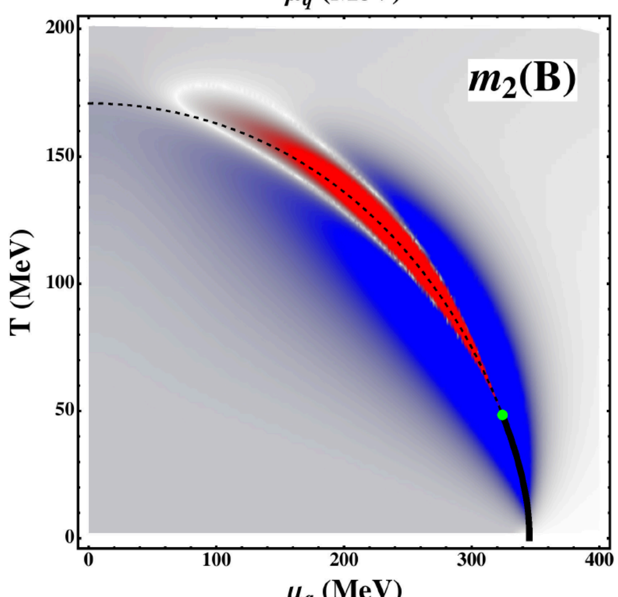
Susceptibilities with 3f-NJL model



Net Baryon

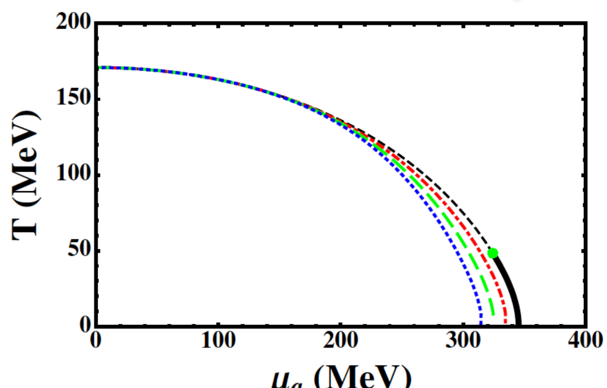
$$N_B = \frac{1}{3}(N_u + N_d + N_s)$$

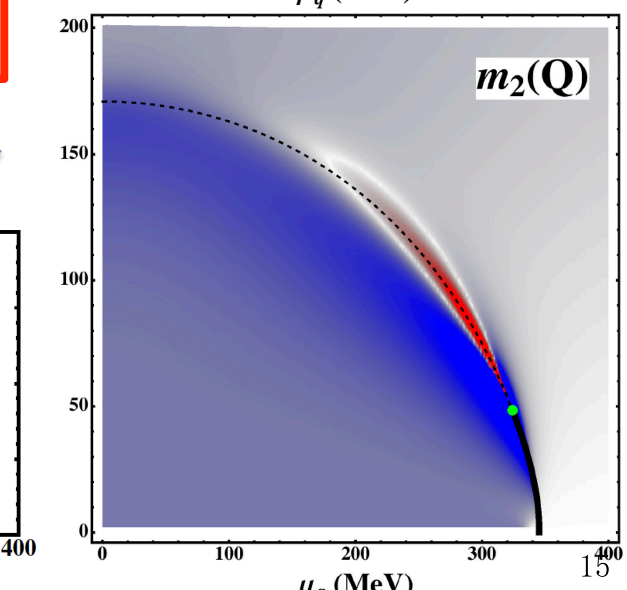




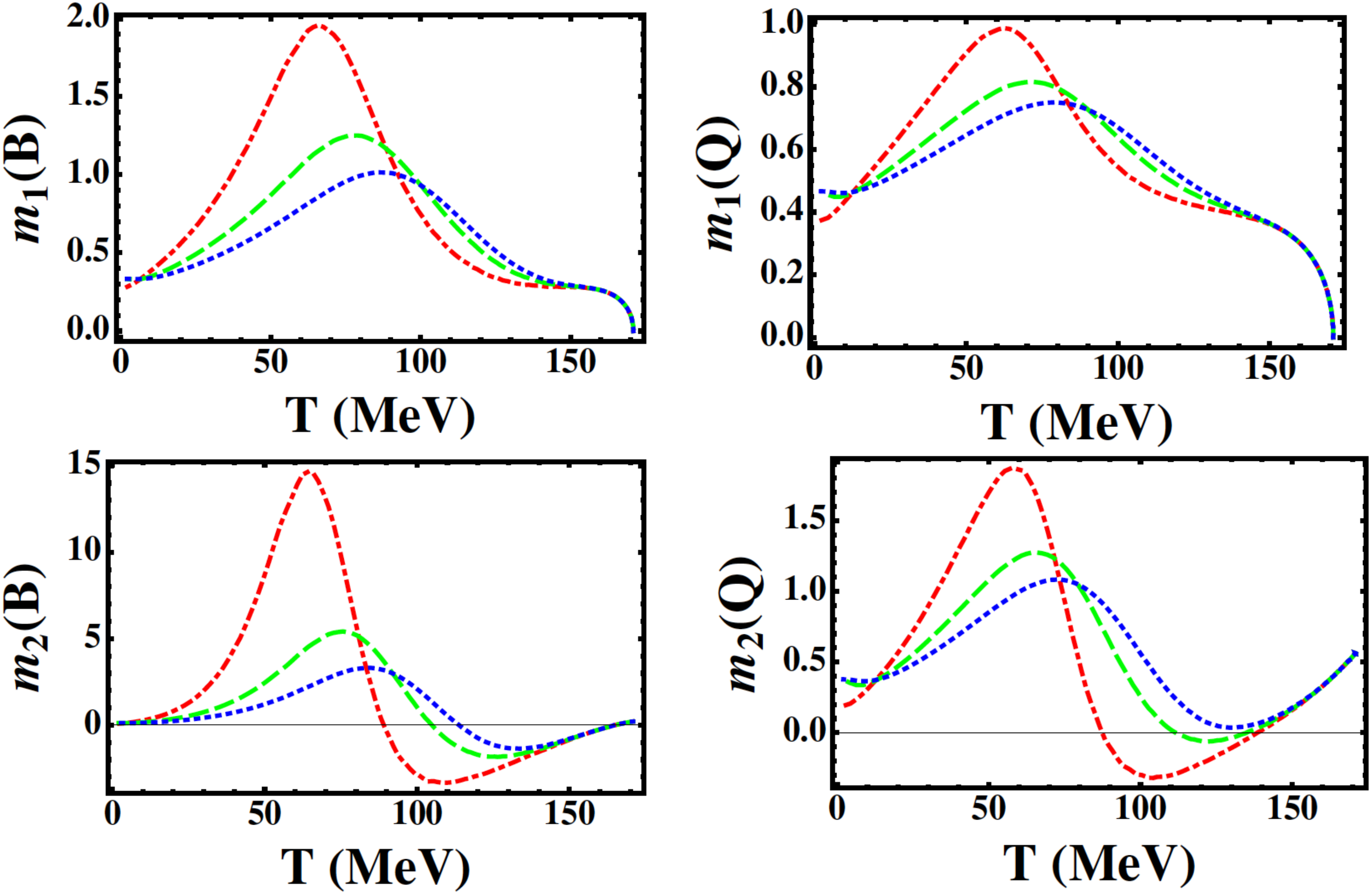
Net Charge

$$N_Q = \frac{1}{3}(2N_u - N_d - N_s)$$

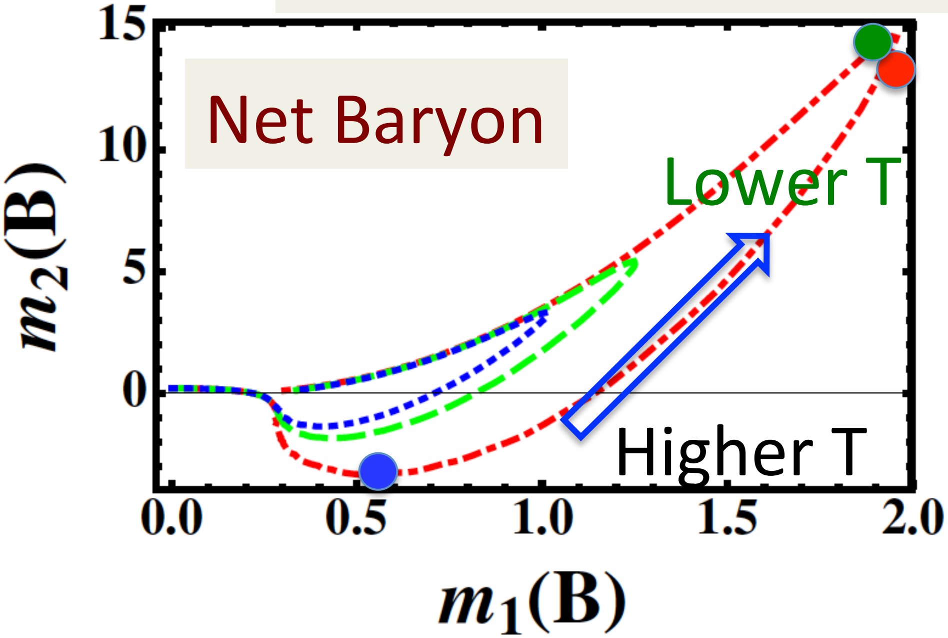


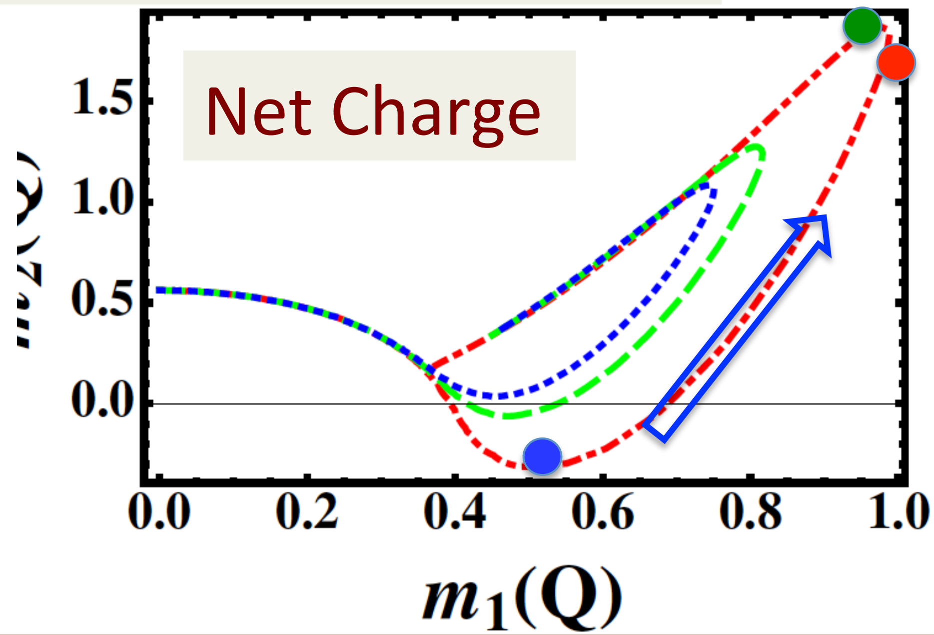


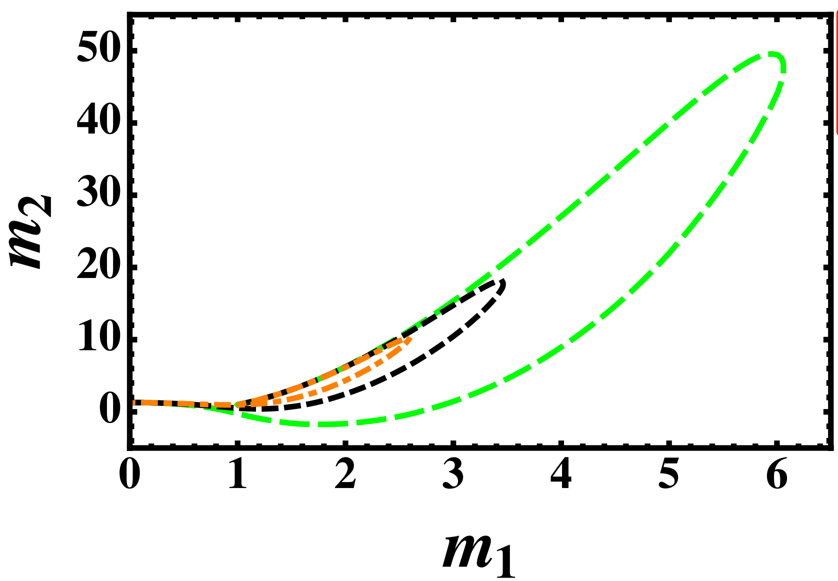
Along hypothetical Freeze-out lines



Combination of m_1 and m_2, common?



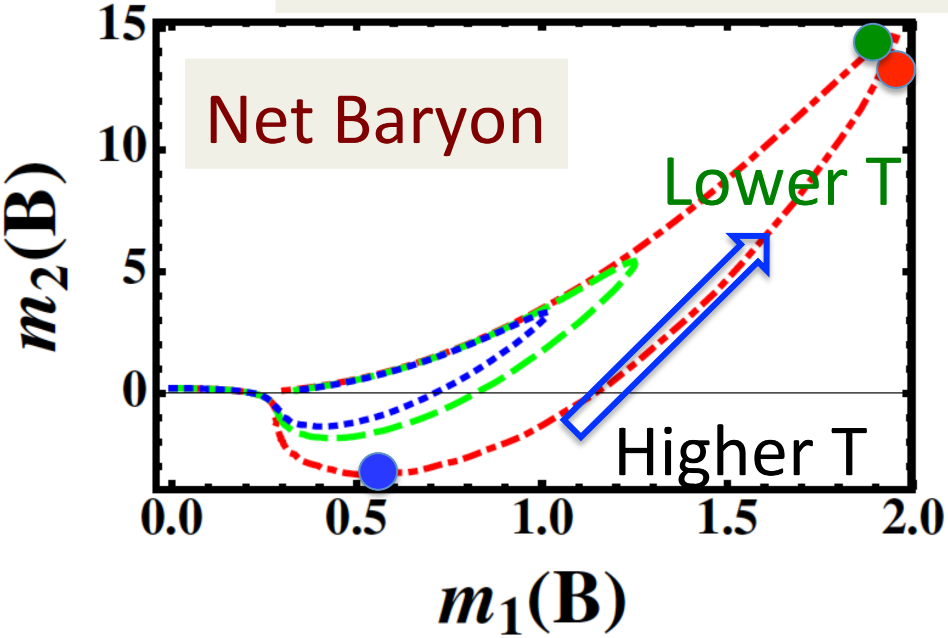


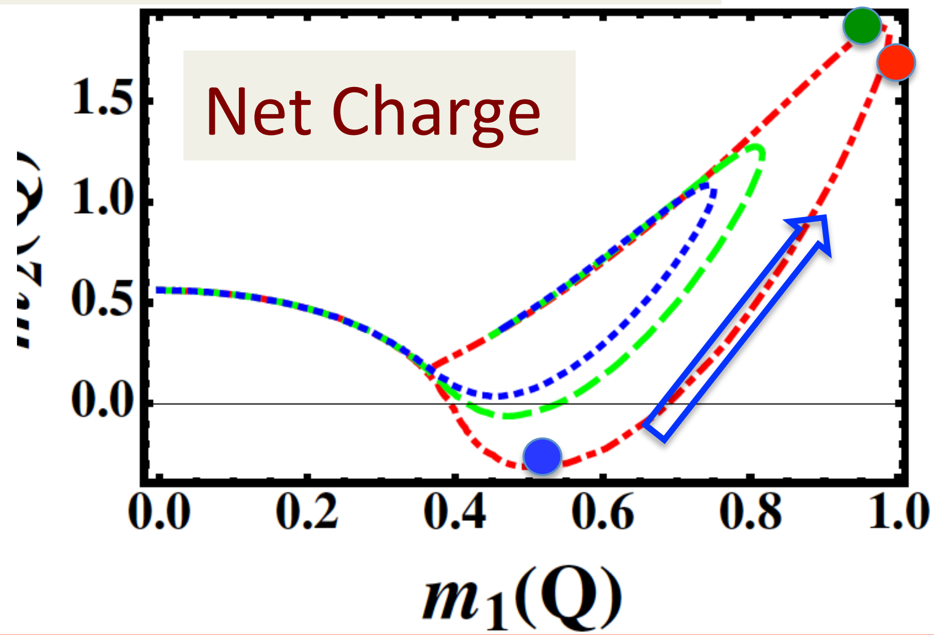


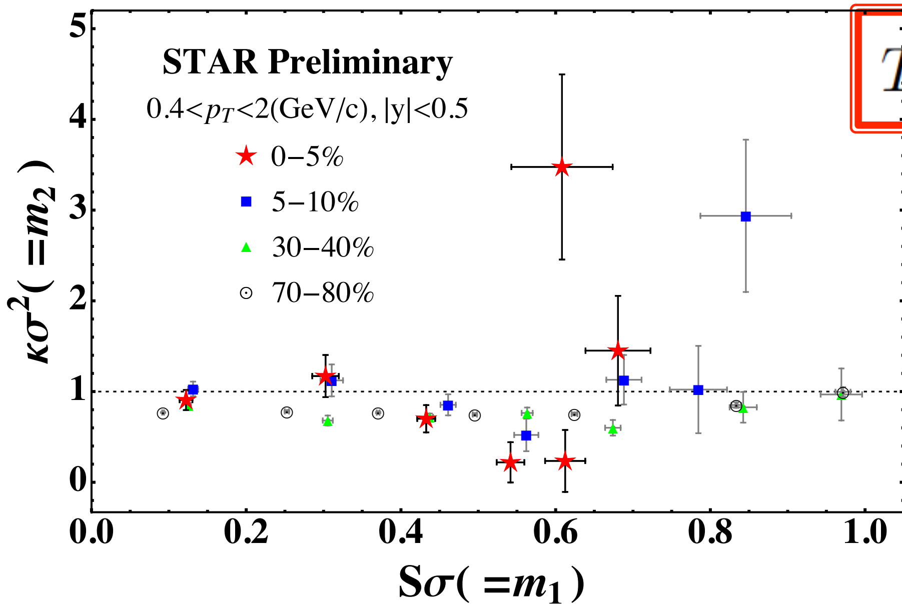
$$T_{\mathrm{min},m_2} > T_{\mathrm{max},m_1} > T_{\mathrm{max},m_2} > T_{\mathrm{CEP}}$$

- also seen with Ising model
- robust features of CP
- survive after non-thermal effects

Combination of m_1 and m_2, common?



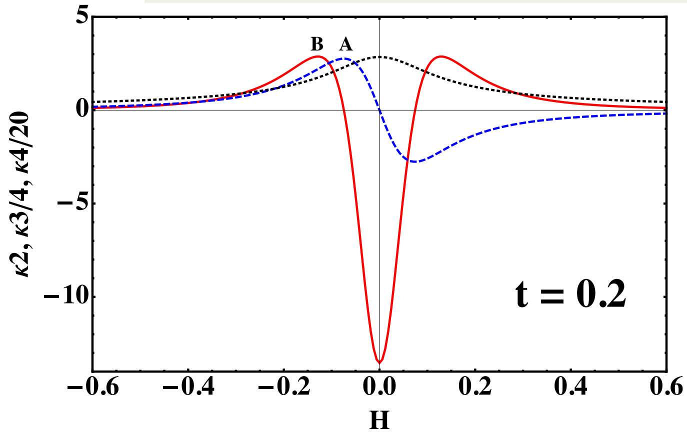


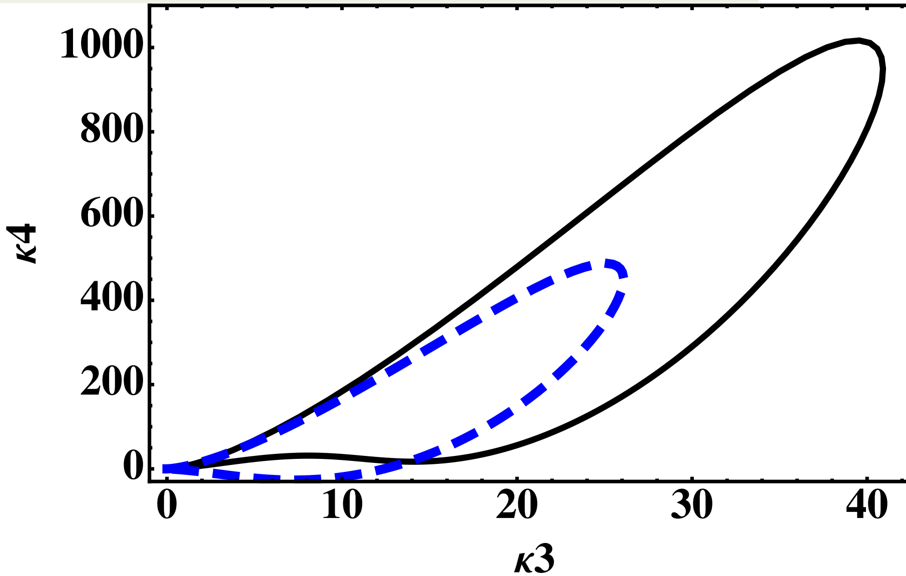


$$T_{\min,m_2} > T_{\max,m_1} > T_{\max,m_2} > T_{\mathrm{CEP}}$$

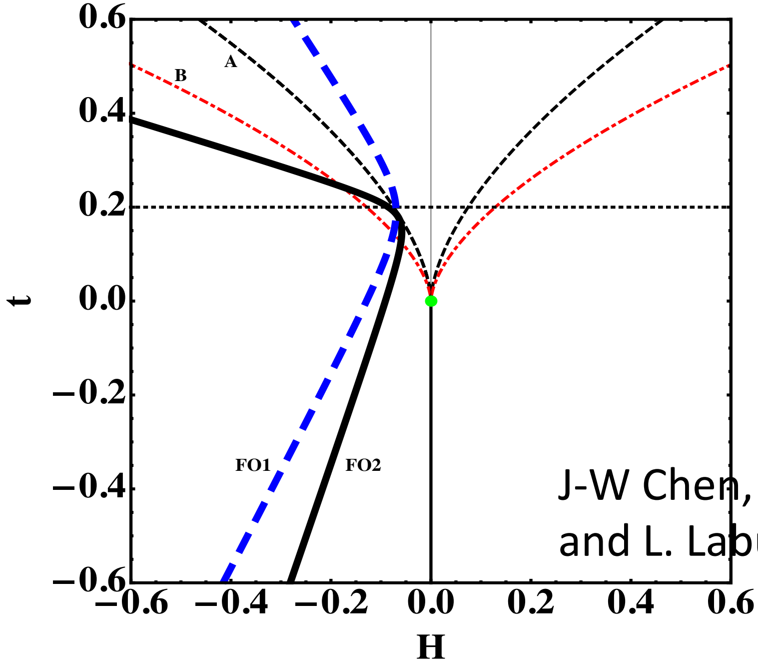
- also seen with Ising model
- robust features of CP
- survive after non-thermal effects

Reasons for T ordering

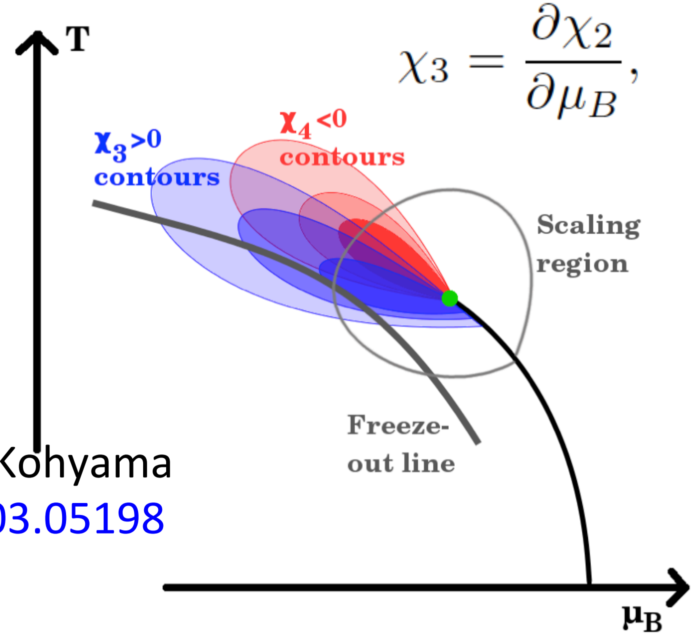




$$\kappa_2 \equiv \frac{\partial M}{\partial H}, \quad \kappa_3 = \frac{\partial \kappa_2}{\partial H}, \quad \kappa_4 = \frac{\partial \kappa_3}{\partial H},$$



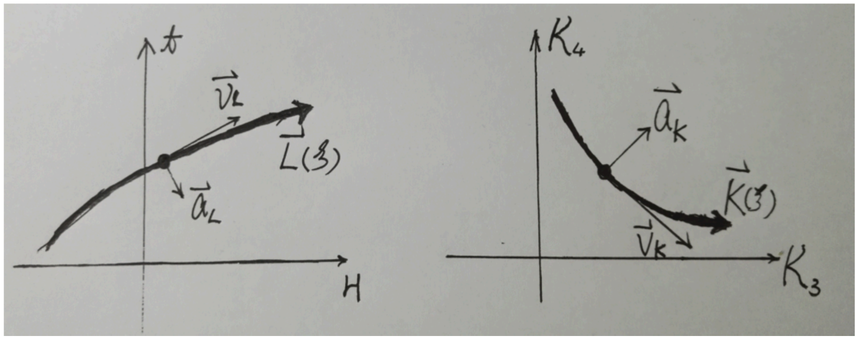
$$\chi_3 = \frac{\partial \chi_2}{\partial \mu_B}, \quad \chi_4 = \frac{\partial \chi_3}{\partial \mu_B}$$



J-W Chen, JD, H. Kohyama
and L. Labun, 1603.05198

Reasons for T ordering

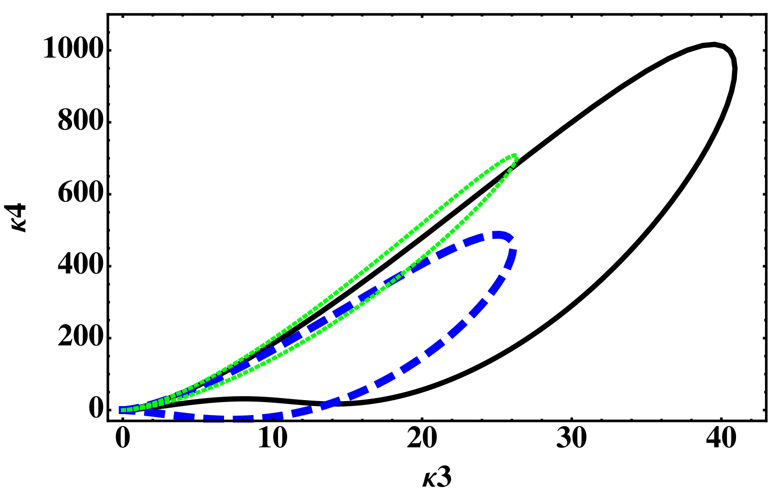
$$(\vec{v}_K \times \vec{a}_K)_z = (\vec{\nabla}\kappa_3 \cdot \vec{v}_L)(\vec{v}_L \cdot \Box\kappa_4 \cdot \vec{v}_L) - (\vec{\nabla}\kappa_4 \cdot \vec{v}_L)(\vec{v}_L \cdot \Box\kappa_3 \cdot \vec{v}_L) + (\vec{\nabla}\kappa_3 \times \vec{\nabla}\kappa_4) \cdot (\vec{v}_L \times \vec{a}_L)$$

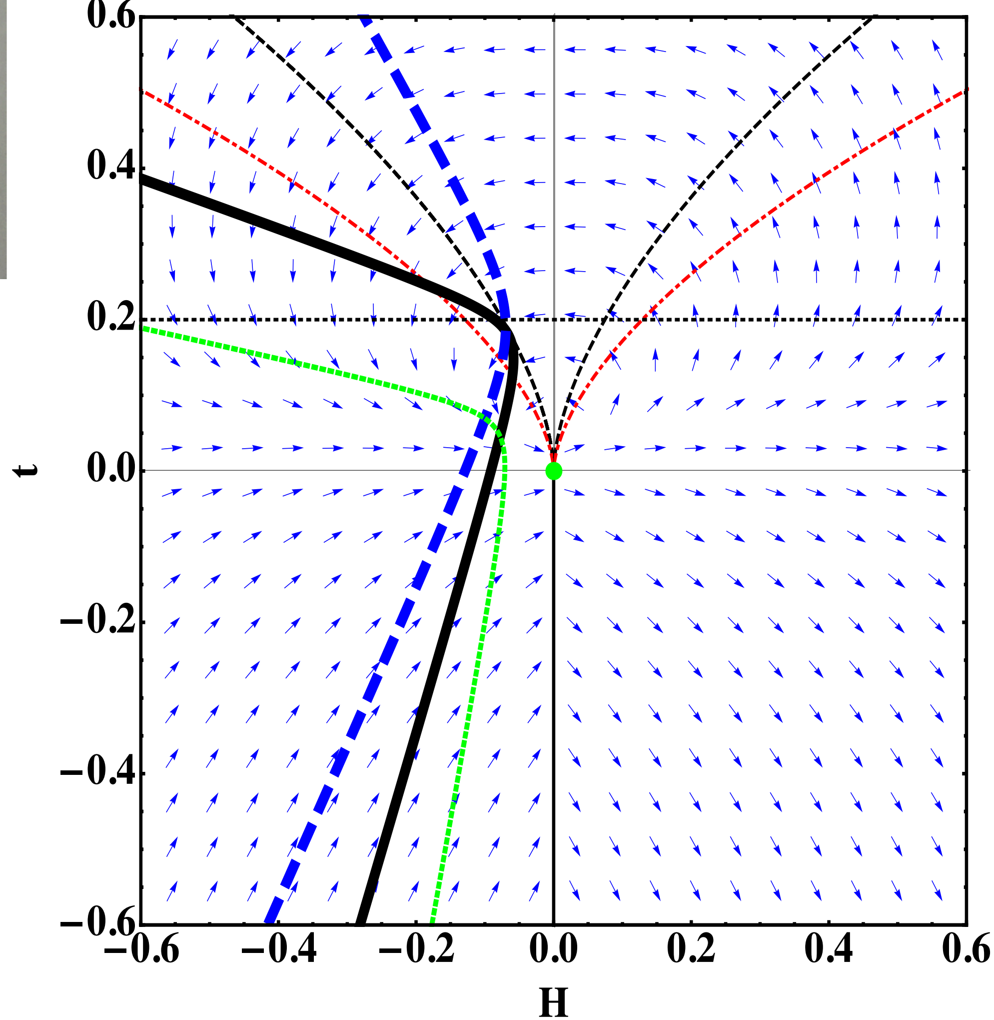


$$\vec{L}(\xi) = \vec{L}_0 + \vec{v}_L \Delta\xi + \frac{1}{2}\vec{a}_L \Delta\xi^2 + \cdots ,$$

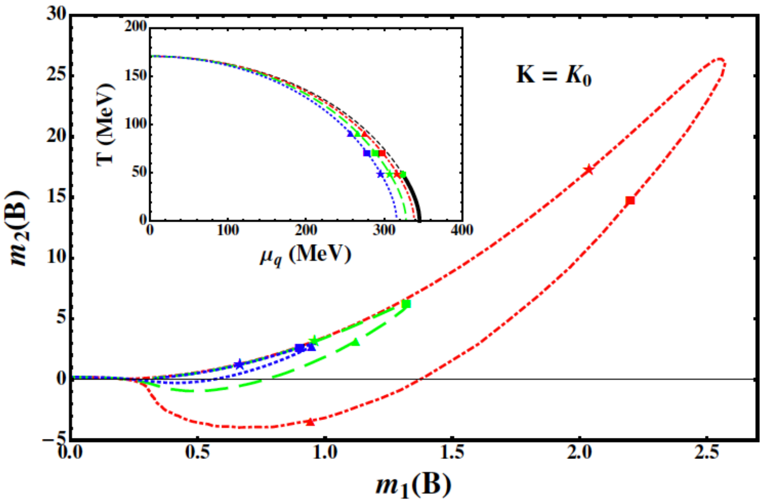
$$\vec{K}(\xi) = \vec{K}_0 + \vec{v}_K \Delta\xi + \frac{1}{2}\vec{a}_K \Delta\xi^2 + \cdots ,$$

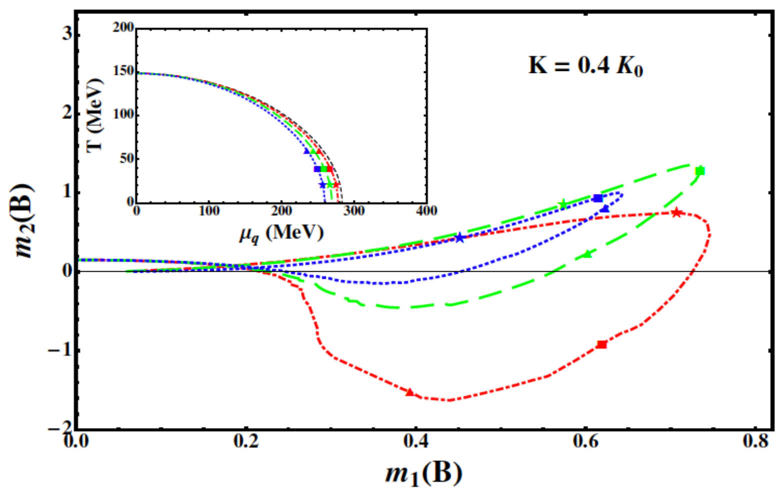
$$\vec{\nabla}\kappa_i = \left(\frac{\partial\kappa_i}{\partial H}, \frac{\partial\kappa_i}{\partial t}, 0\right), \Box\kappa_i = \begin{pmatrix} \frac{\partial^2\kappa_i}{\partial H^2} & \frac{\partial^2\kappa_i}{\partial H\partial t} & 0 \\ \frac{\partial^2\kappa_i}{\partial H\partial t} & \frac{\partial^2\kappa_i}{\partial t^2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

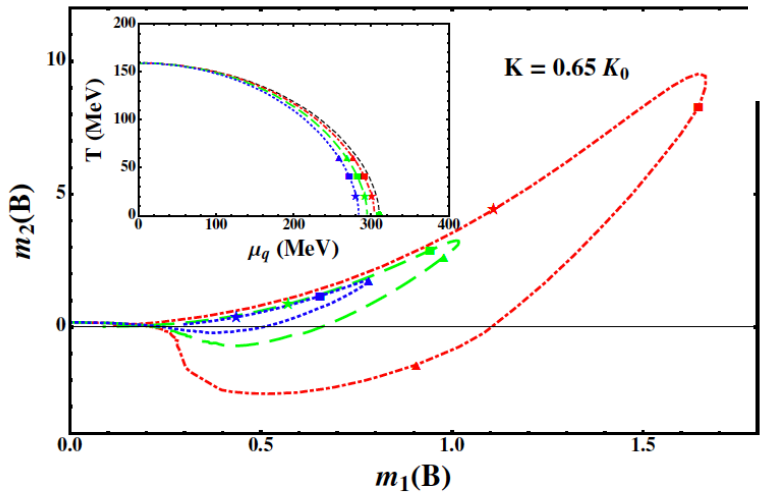




CP and the loops







$$T_{\min,m_2} > T_{\max,m_1} > T_{\max,m_2} > T_{\mathrm{CEP}}$$

- CP ➔ Banana ➔ T ordering
- $m_2 >> m_1 \geq 1$ ➔ FO near CP
- Serve as a first check for CP
- BES-II, $\sqrt{s_{NN}} < 20$ GeV , can answer

J-W Chen, JD, H. Kohyama and L. Labun, 1603.05198

Summary

1. QCD phase diagram with CP are explored. Higher order susceptibilities and relations between observables are crucial.

2. Sign change of m_2, and peak in m_1 indicate non-monotonic behaviors. The banana shape of m_2 vs. m_1, and ordering $T_{min,m2} > T_{max,m1} > T_{max,m2} > T_{CEP}$ help to indicate the location of CP.

3. Data comparison shows QCD criticality. CP is needed to explain the data. BES-II will answer ...

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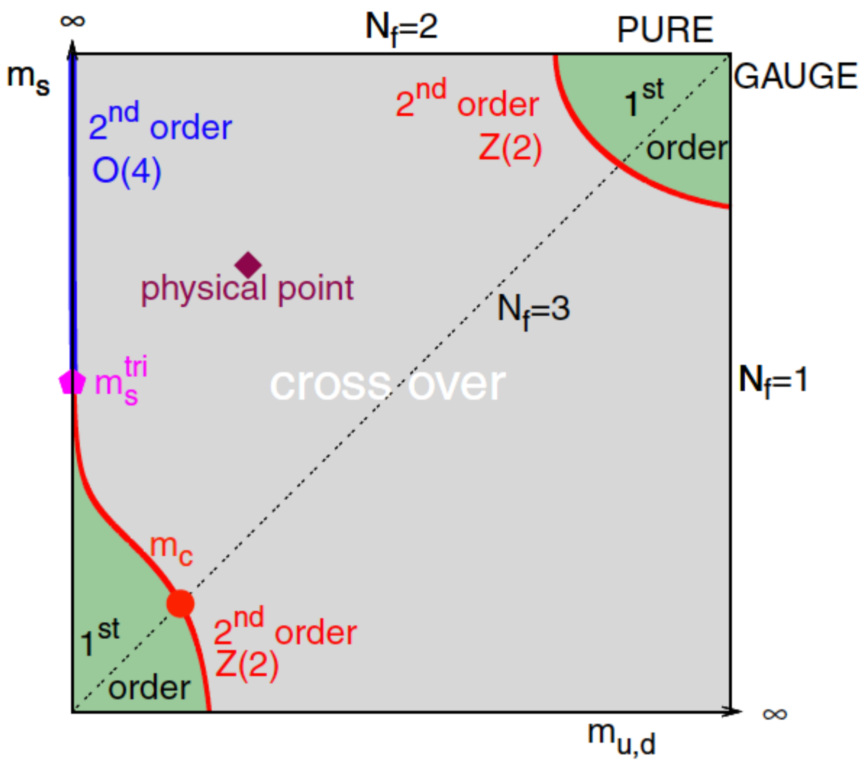
Thanks

Back up

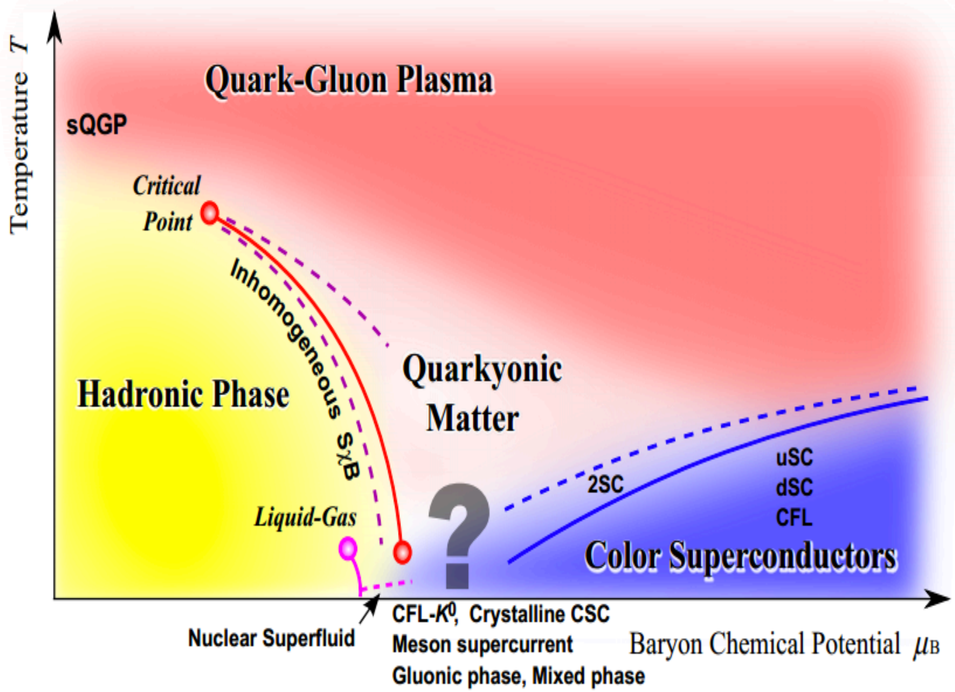
Phase transition and CP within QCD



F. R. Brown, et al. PRL 1990

Full lattice QCD simulation

Because the relevant symmetry is explicitly broken by quark mass, symmetry arguments no longer imply the existence of a finite temperature phase transition.



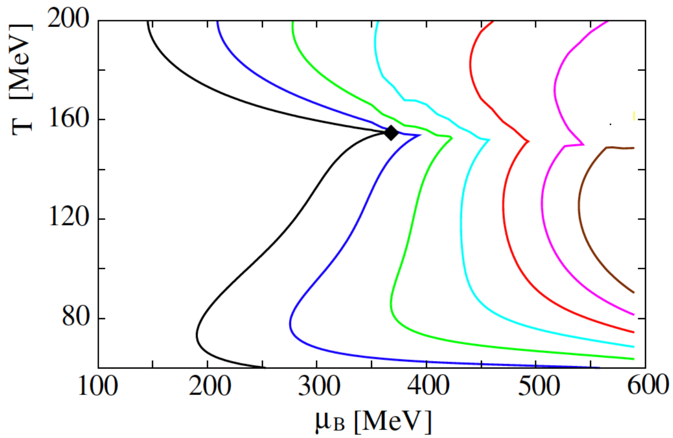
K. Fukushima and T. Hatsuda

Rep. Prog. Phys. 2010

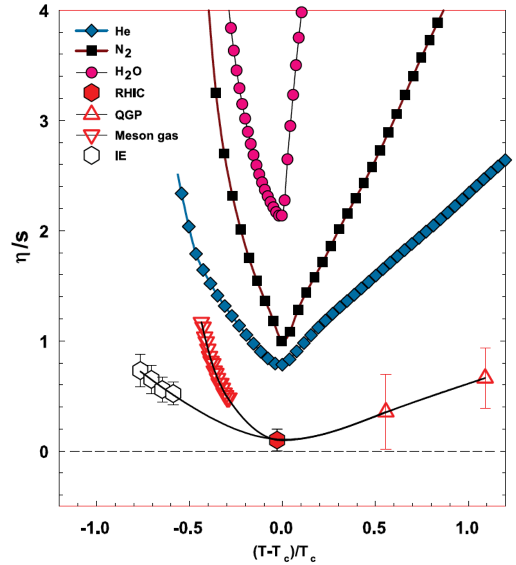
Even no reliable information from the first-principle LQCD calculation, effective chiral models suggest a first order chiral phase transition in the large density region.

CEP in HIC

Crucial in diagram, test QCD in non-perturbative region

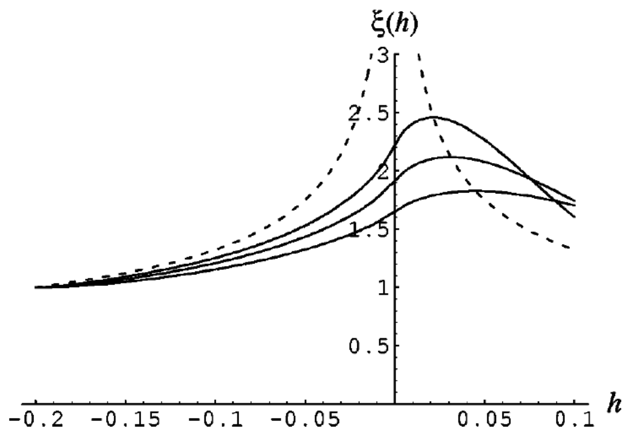


Attractor for thermodynamic trajectories in HIC. C. Nonaka and M. Asakawa, PRC 2005



Most difficult for momentum transport near CEP

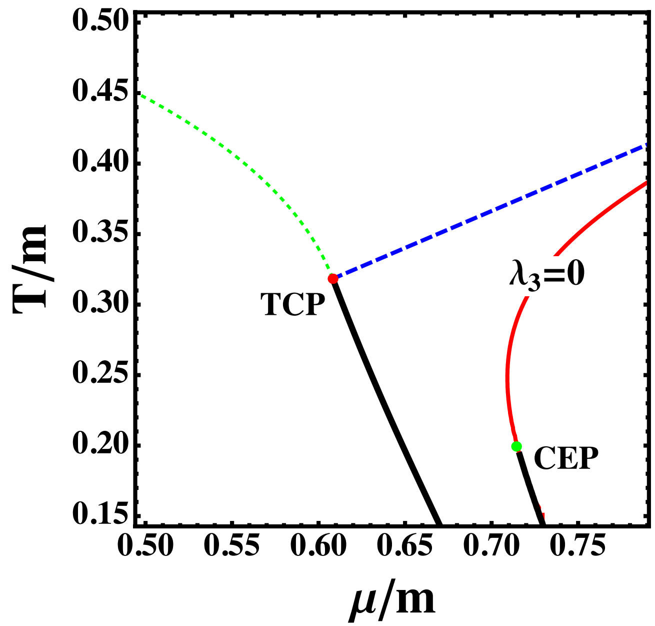
Roy A. Lacey, et al. PRL 2007

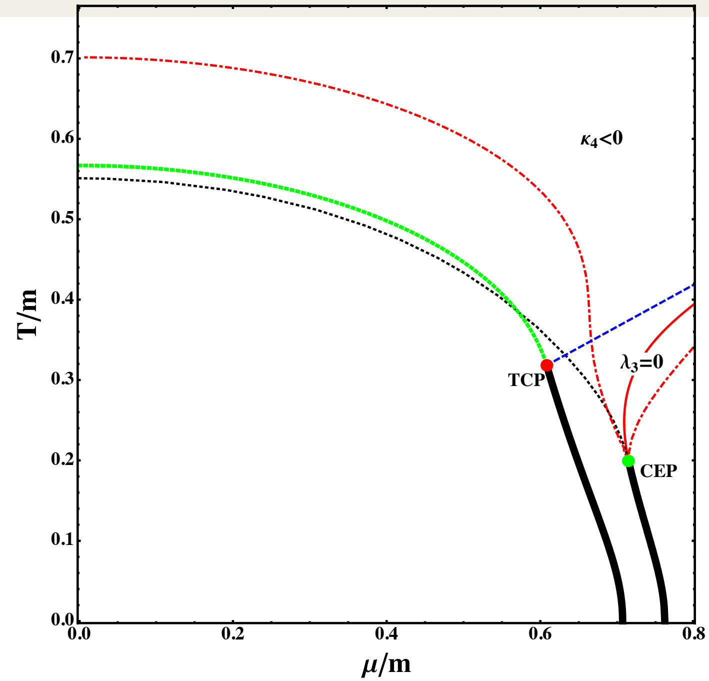


Slowing out of equilibrium near CEP

B. Berdnikov and K. Rajagopal, PRD 2000

Key features with Gross-Neveu model





J-W Chen,
JD, L. Labun,
PRD 2015

$$\kappa_3 = \langle \sigma_0^3 \rangle = \frac{2\lambda_3 T}{V} \xi^6; \; \kappa_4 = \langle \sigma_V^4 \rangle_c = 6VT^3[2(\lambda_3 \xi)^2 - \lambda_4]\xi^8,$$

The line $\lambda_3 = 0$ does NOT lead the crossover line!
but separates the positive and negative region of skewness,
guides the negative region of kurtosis of the sigma field.

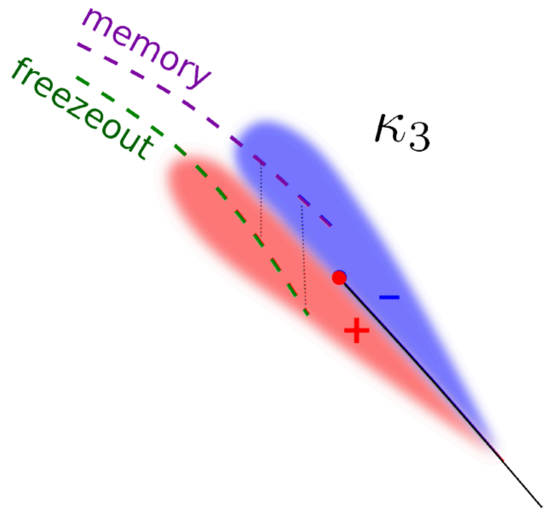
Time evolution of cumulants (menory)

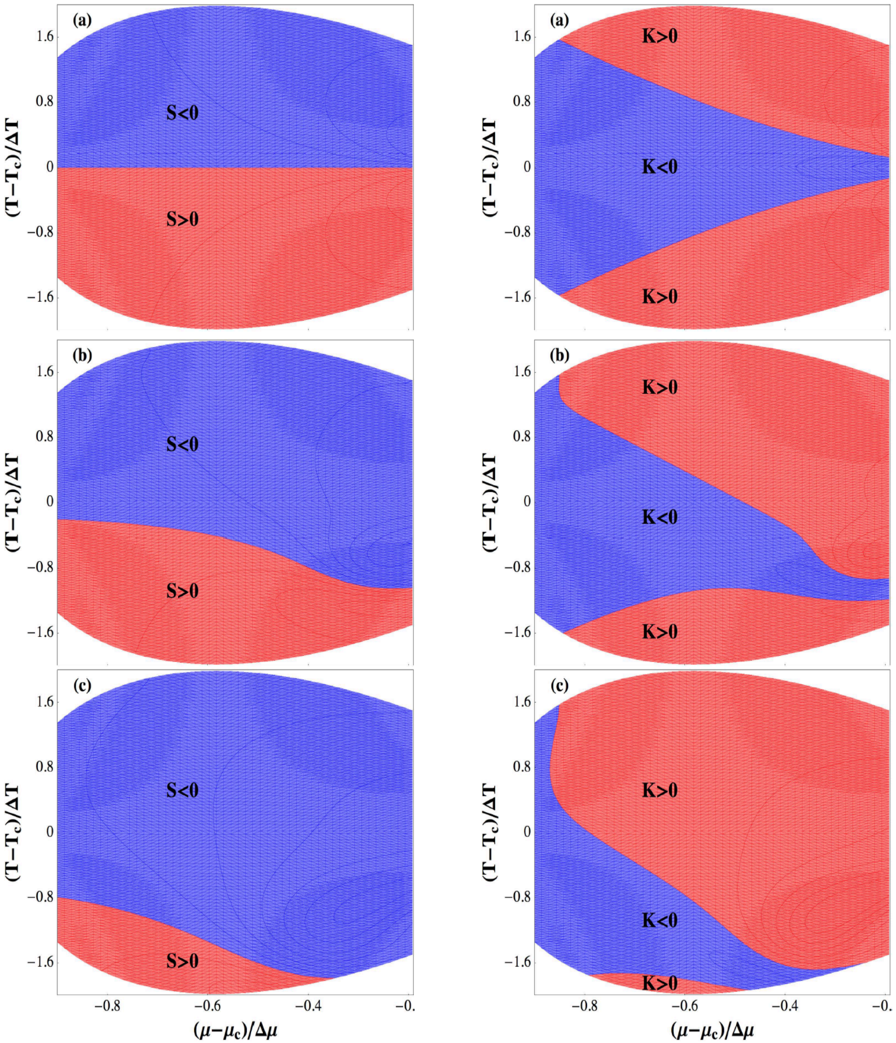
Critical slowing down

$$\frac{dP}{d\tau} = F[P]$$

$$\Downarrow$$

$$\frac{d\kappa_n}{d\tau} = L[\kappa_n, \kappa_{n-1}, \ldots]$$





S. Mukherjee, et.al. arXiv:1506.00645