Fluctuations of conserved charges from Lattice QCD and BES

Heng-Tong Ding (丁亨通)
Central China Normal University

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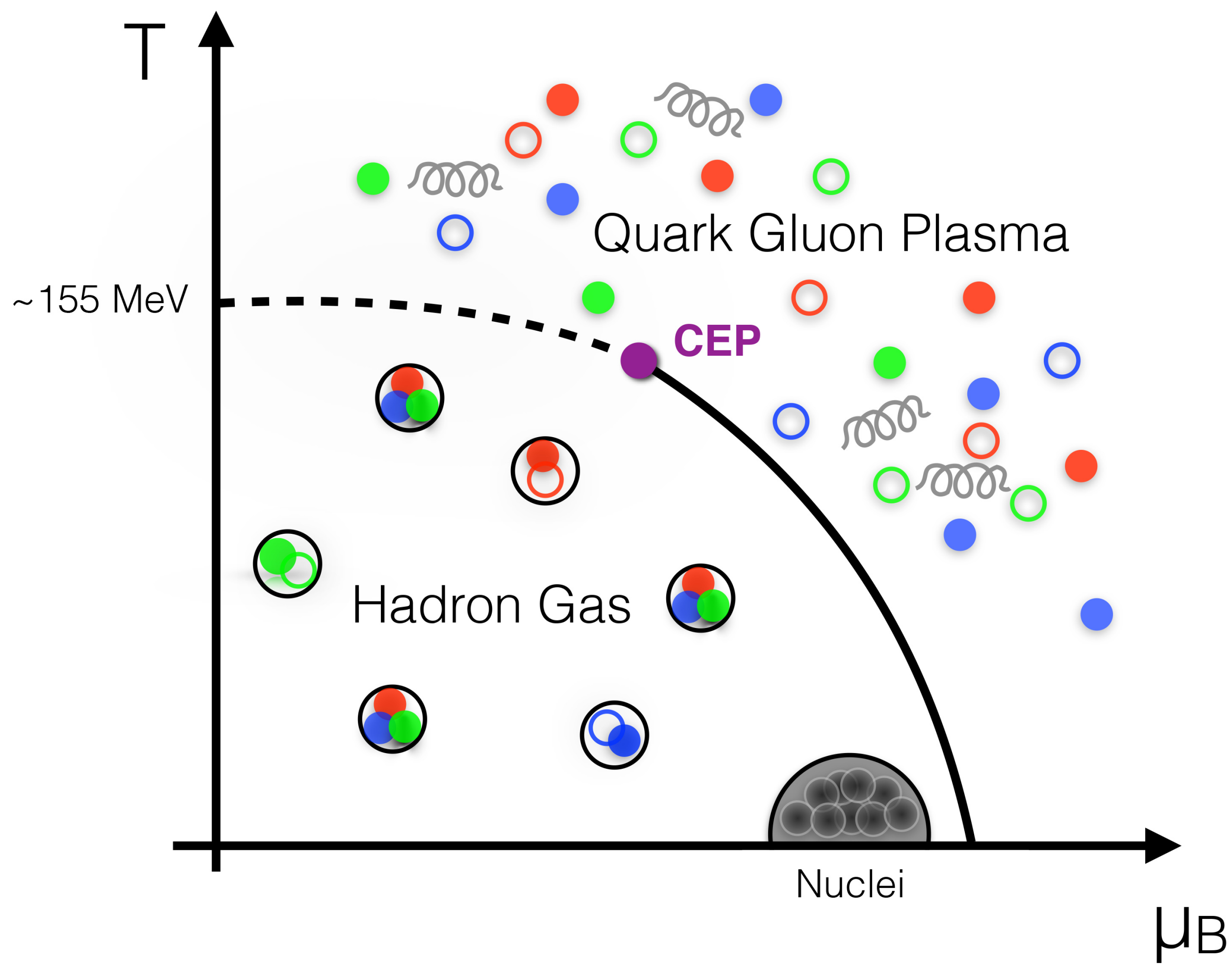
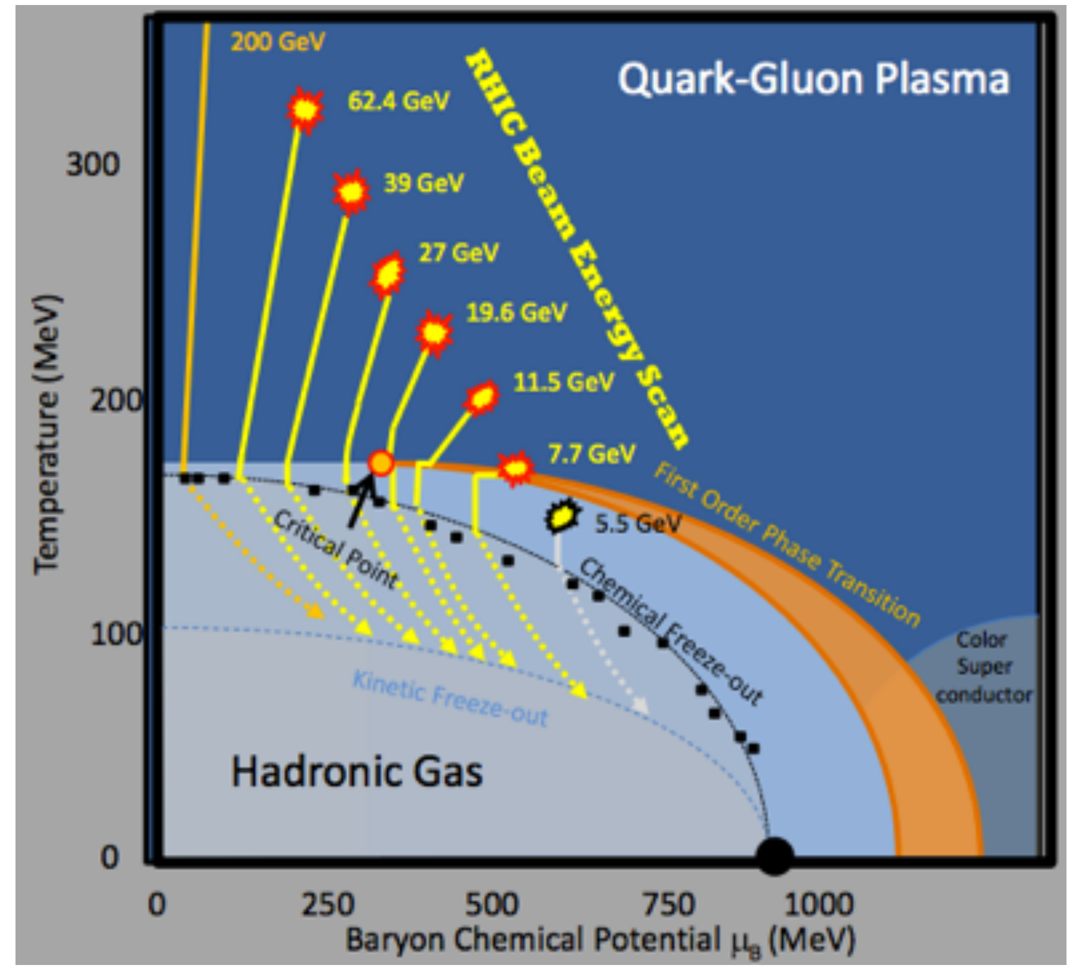
QCD phase diagram

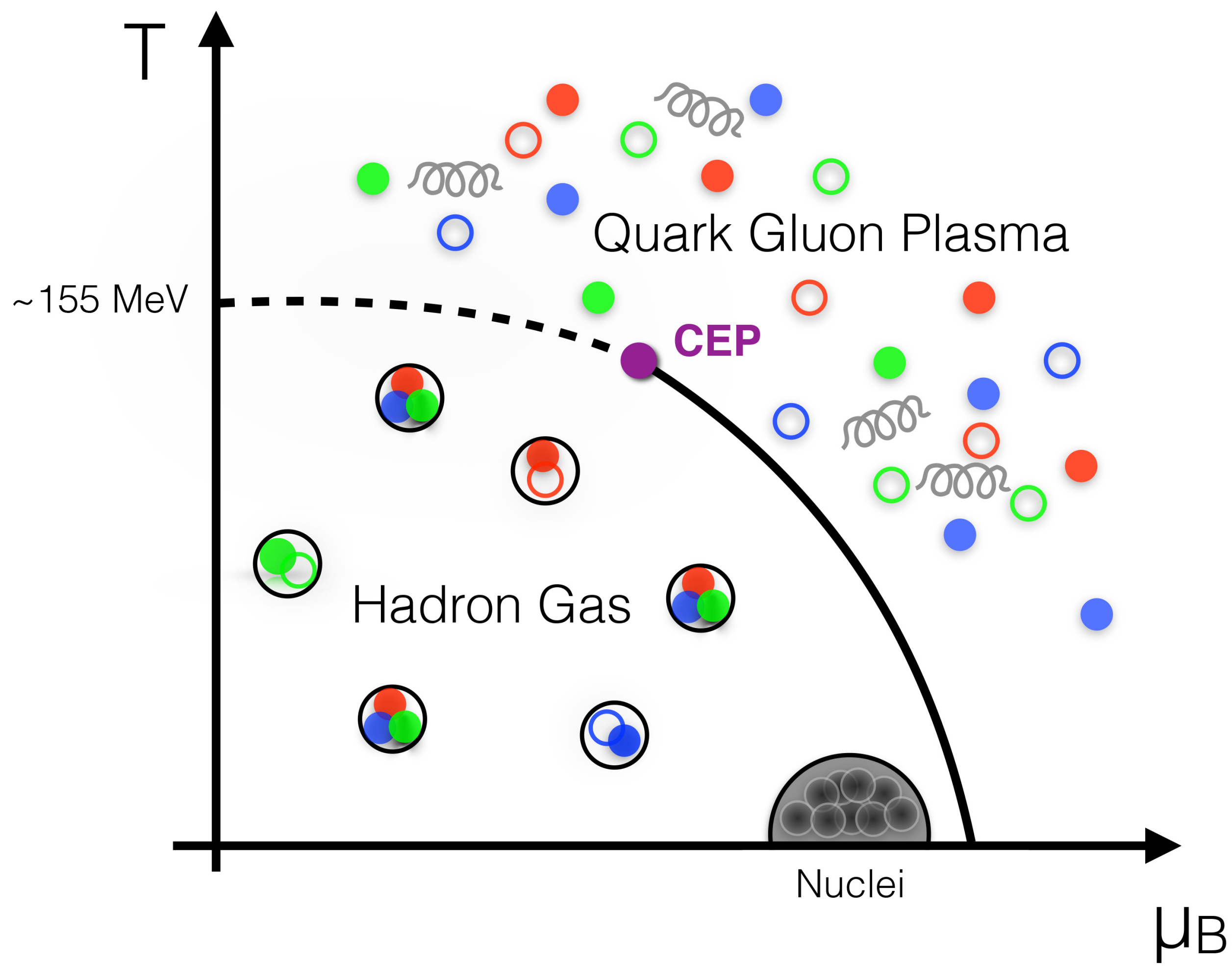


HTD, F. Karsch, S. Mukherjee, arXiv:1504.0527



- EOS at finite mu
- Freeze-out line
- Chiral phase transition line

QCD phase diagram



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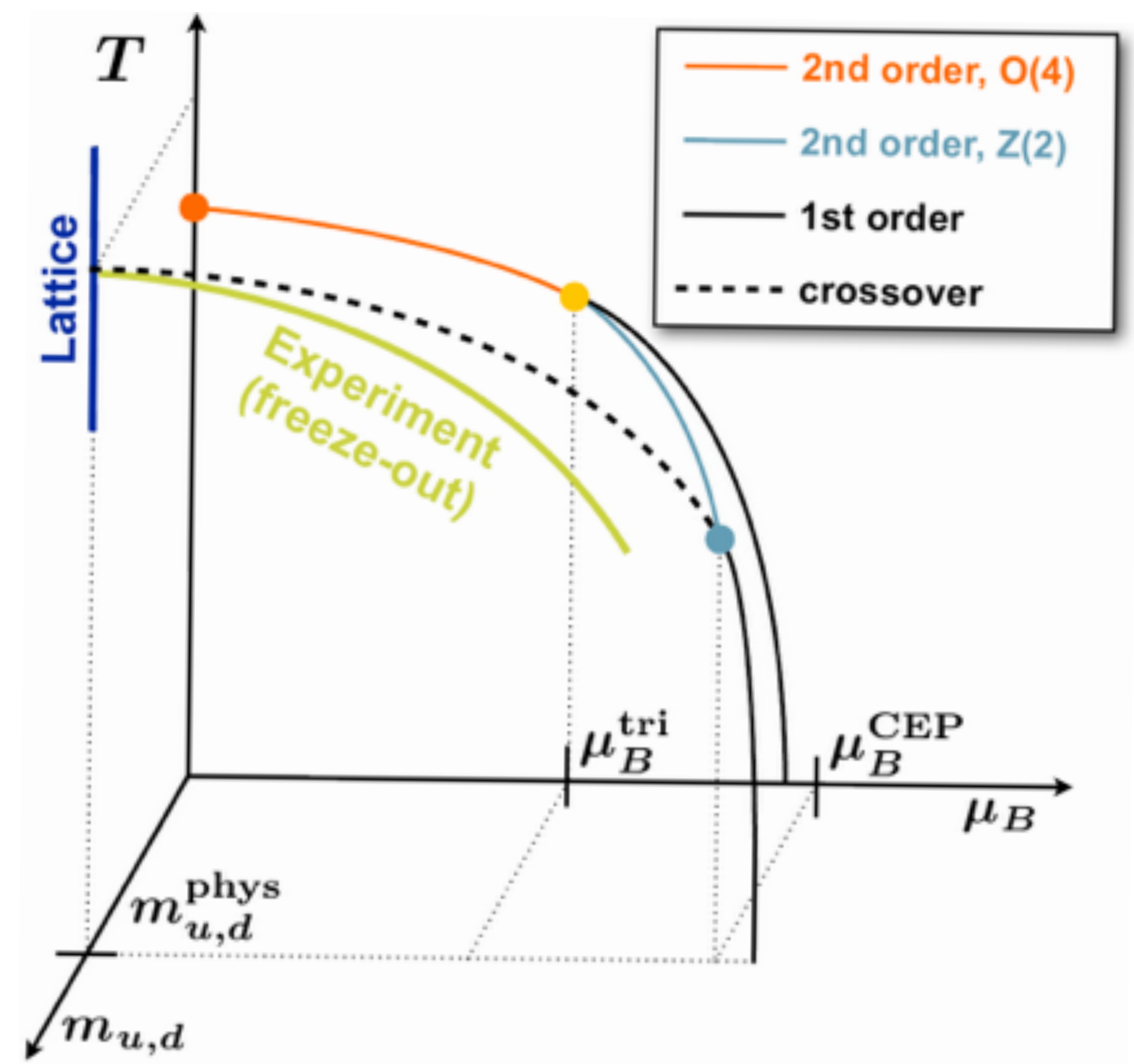

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Fluctuations of conserved charges

Taylor expansion of the QCD pressure:

Allton et al., Phys.Rev. D66 (2002) 074507

Gavai & Gupta et al., Phys.Rev. D68 (2003) 034506

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln \mathcal{Z}(T, V, \hat{\mu}_u, \hat{\mu}_d, \hat{\mu}_s) = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{BQS}}{i!j!k!} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

- Calculate the Taylor expansion coefficients at μ=0

$$\chi_{ijk}^{BQS} \equiv \chi_{ijk}^{BQS}(T) = \frac{1}{VT^3} \frac{\partial P(T,\hat{\mu})/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k}\Big|_{\hat{\mu}=0}$$

- Obtain other quantities using thermodynamic relations, e.g.

$$\frac{\epsilon - 3p}{T^4} = T \frac{\partial P/T^4}{\partial T} = \sum_{i,j,k=0}^{\infty} \frac{T\, \mathrm{d}\chi_{ijk}^{BQS}/\mathrm{d}T}{i!j!k!} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

Pressure of hadron resonance gas (HRG)

$$\frac{p}{T^4} = \sum_{m \in meson, baryon} \ln Z(T, V, \mu) \sim \exp(-m_H/T)\, \exp((B\mu_B + S\mu_s + Q\mu_Q)/T)$$

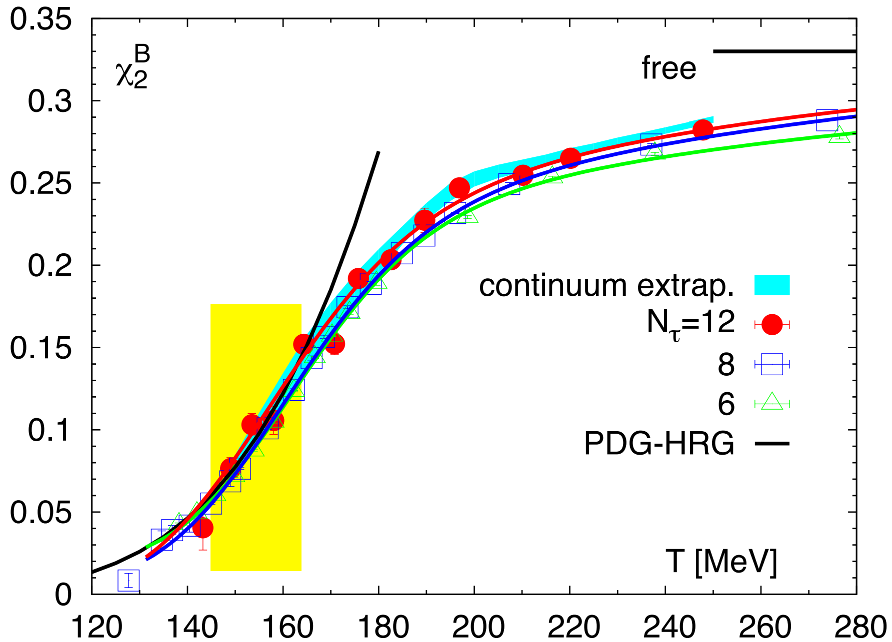
Pressure of QCD at nonzero muB

$$\Delta(P/T^4) = \frac{P(T,\mu_B) - P(T,0)}{T^4} = \sum_{n=1}^{\infty} \frac{\chi_{2n}^B(T)}{(2n)!} \left(\frac{\mu_B}{T}\right)^{2n}$$

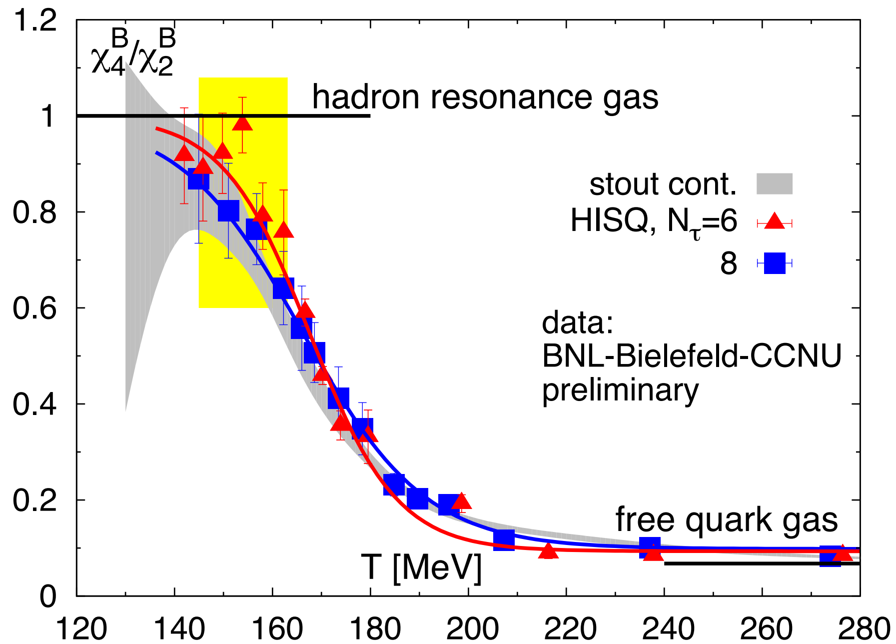
$$= \frac{1}{2}\chi_2^B(T)\hat{\mu}_B^2 \left(1 + \frac{1}{12}\frac{\chi_4^B(T)}{\chi_2^B(T)}\hat{\mu}_B^2 + \frac{1}{360}\frac{\chi_6^B(T)}{\chi_2^B(T)}\hat{\mu}_B^4 + \cdots\right)$$

$\mu_Q = \mu_S = 0$

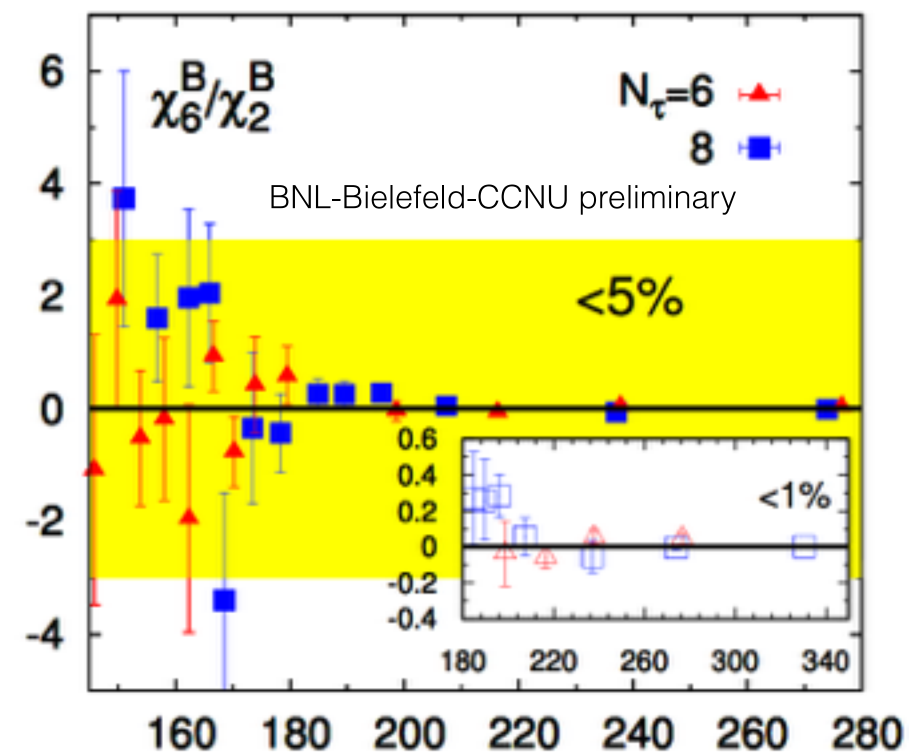
LO expansion coefficient
variance of net-baryon number distribution



NLO expansion coefficient
kurtosis * variance



NNLO expansion coefficient

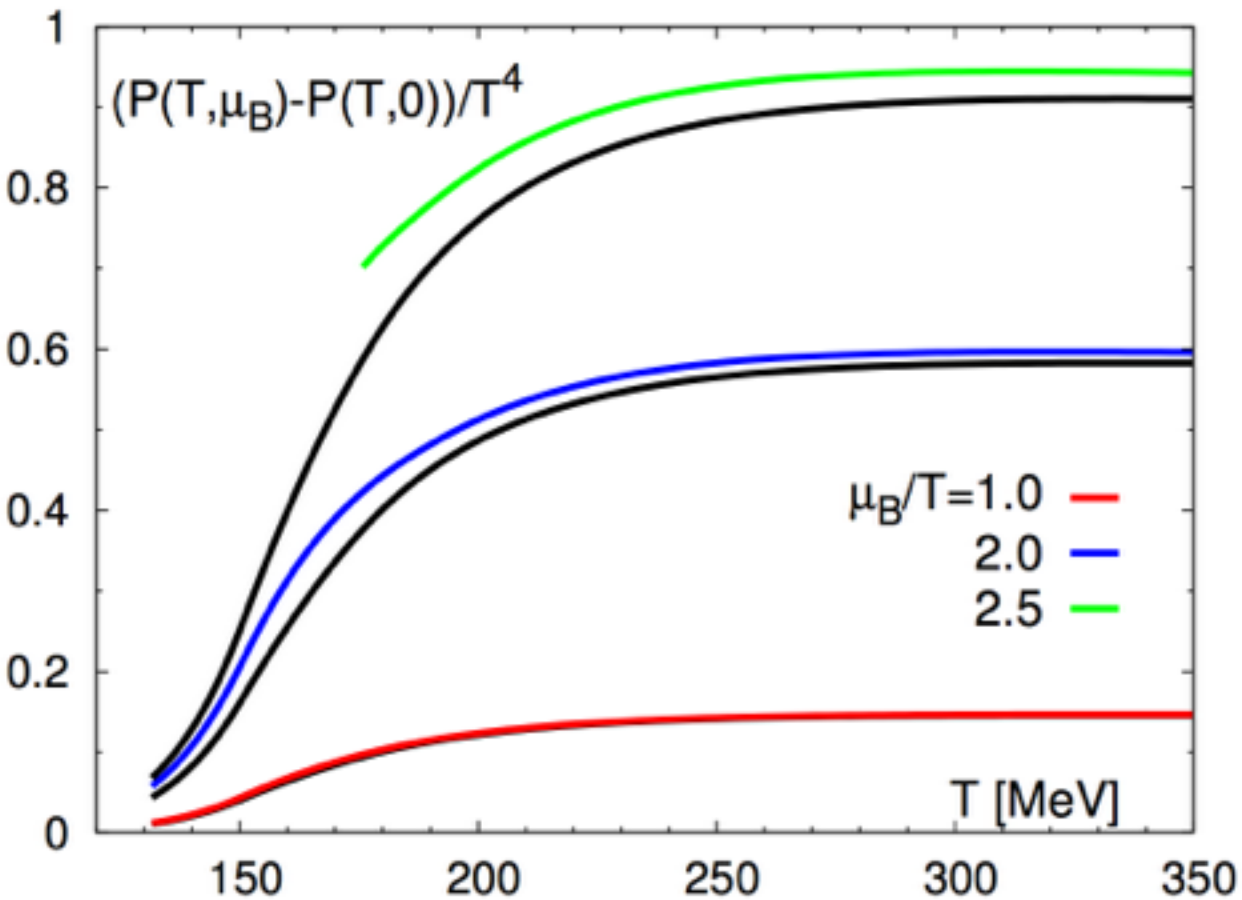


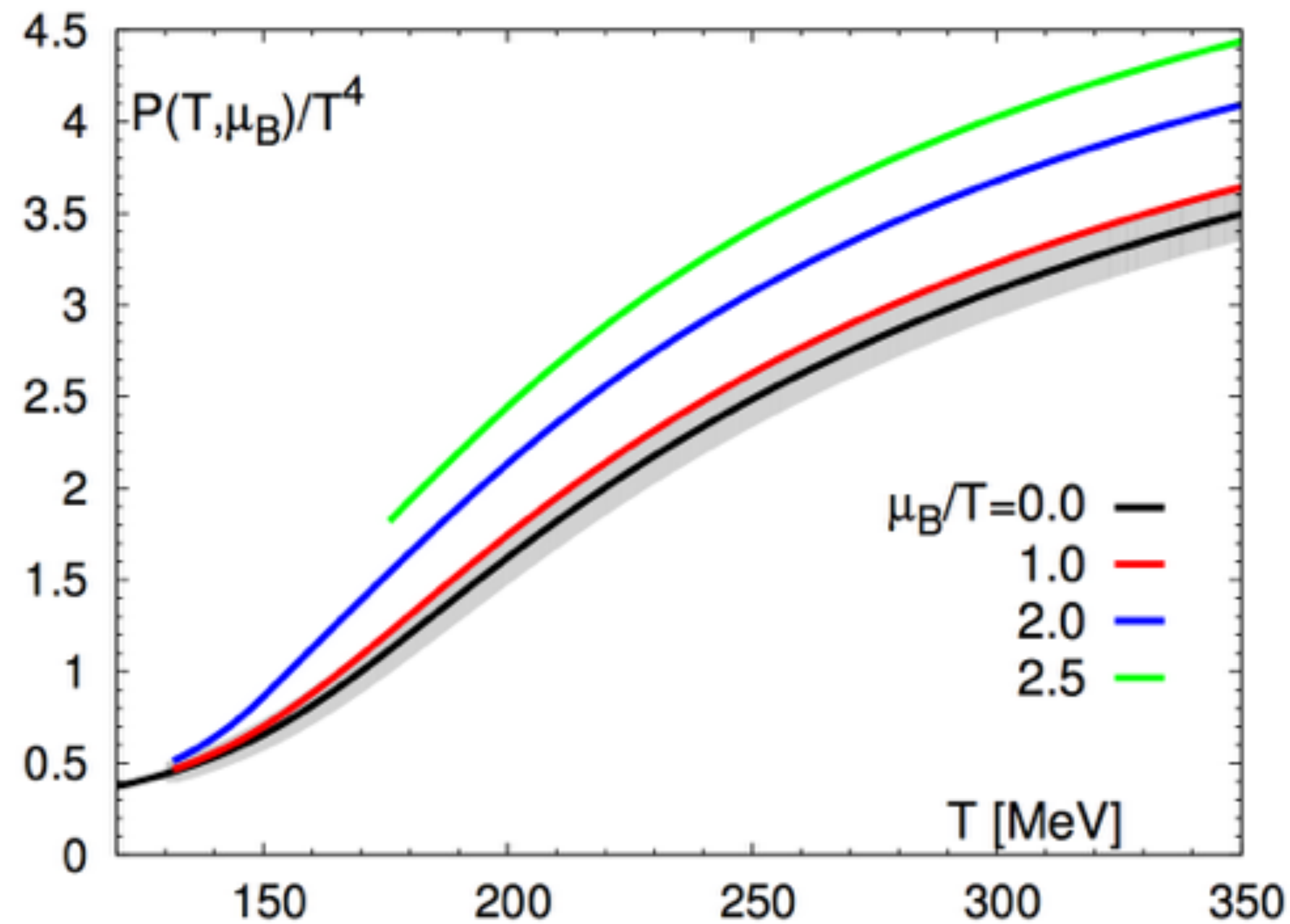
HTD, Nucl. Phys. A 931 (2014) 52-62
HTD, F. Karsch, S. Mukherjee, arXiv:1504.0527

- HRG describes well on the LO expansion coefficient up to ~160 MeV while it deviates from NLO expansion coefficient ~ 40% in the crossover region

- For small muB/T the LO contribution dominates

Pressure with $\mu_Q = \mu_s = 0$





BNL-Bielefeld-CCNU, arXiv:1412.6727

- Leading order corrections dominates at small μ_B/T
- Higher order corrections becomes significant at $\mu_B/T \gtrsim 2$

Conditions meet in heavy ion collisions

- Zero net strangeness $n_S=0$, and $n_Q/n_B=r=0.4$ as in PbPb collision systems

$$\frac{n_X}{T^3} = \frac{\partial P/T^4}{\partial \hat{\mu}_X} \quad , X=B,Q,S$$

$$n_S = n_S^{(1)}\mu_B + n_S^{(3)}\mu_B^3 + \cdots = 0\,, \quad n_Q = n_Q^{(1)}\mu_B + n_Q^{(3)}\mu_B^3 + \cdots\,,$$
$$n_I = n_I^{(1)}\mu_B + n_I^{(3)}\mu_B^3 + \cdots = (\frac{1}{r} - 2)n_Q$$

E.g. 1st order coefficient in n_S: $n_S^{(1)} = \chi_2^S \frac{\mu_S}{\mu_B} + \chi_{11}^{QS}\frac{\mu_Q}{\mu_B} + \chi_{11}^{BS}$

- Expand μ_Q and μ_S in terms of μ_B

$$\frac{\mu_Q}{T} = q_1\frac{\mu_B}{T} + q_3\left(\frac{\mu_B}{T}\right)^3 + \cdots \;, \quad \frac{\mu_S}{T} = s_1\frac{\mu_B}{T} + s_3\left(\frac{\mu_B}{T}\right)^3 + \cdots$$

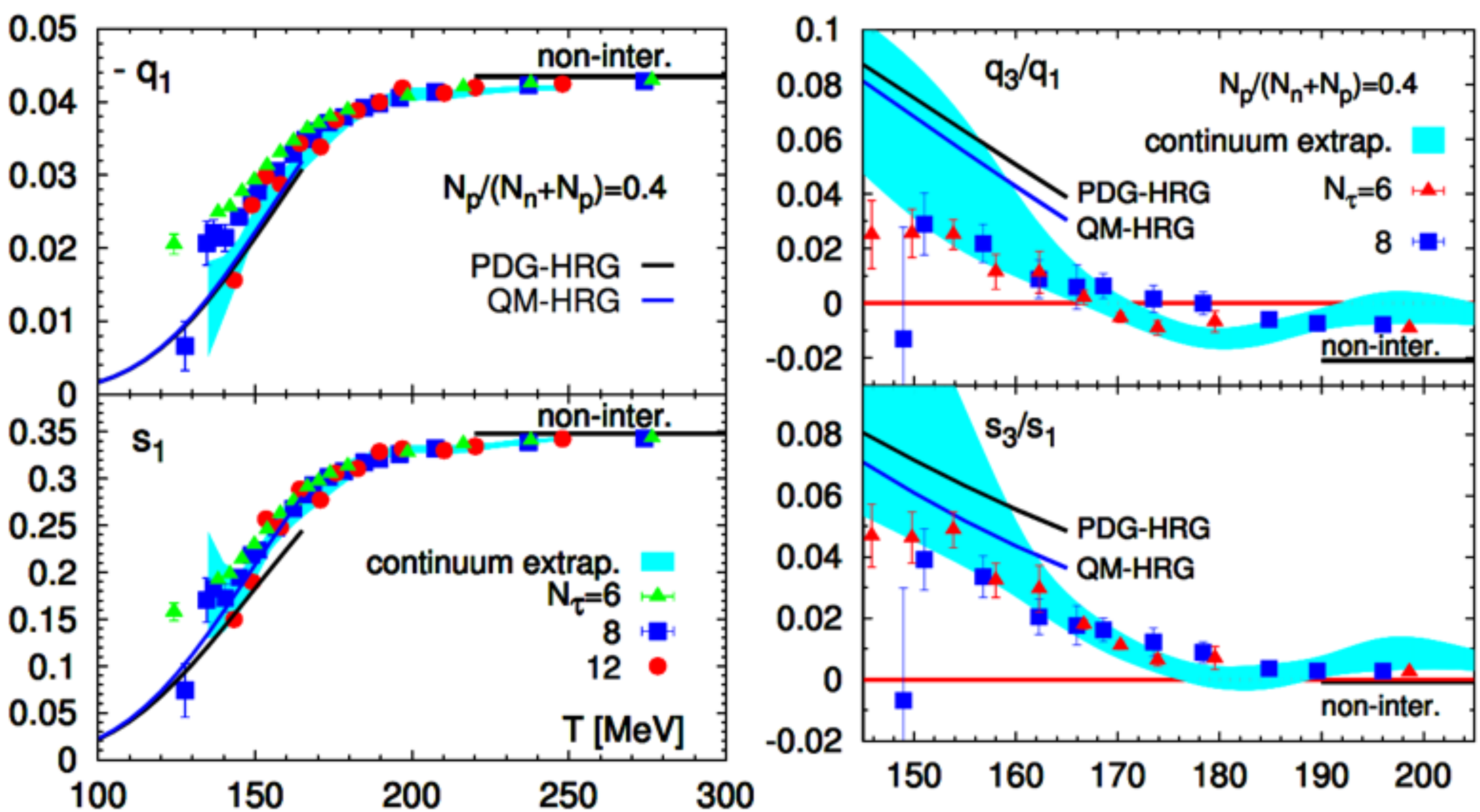
- With constrains from isospin symmetry etc., one can derive q_i and s_i order by order and then the pressure etc.

A. Bazavov, HTD et al., Phys. Rev. Lett. 109 (2012)192302

Conditions meet in heavy ion collisions

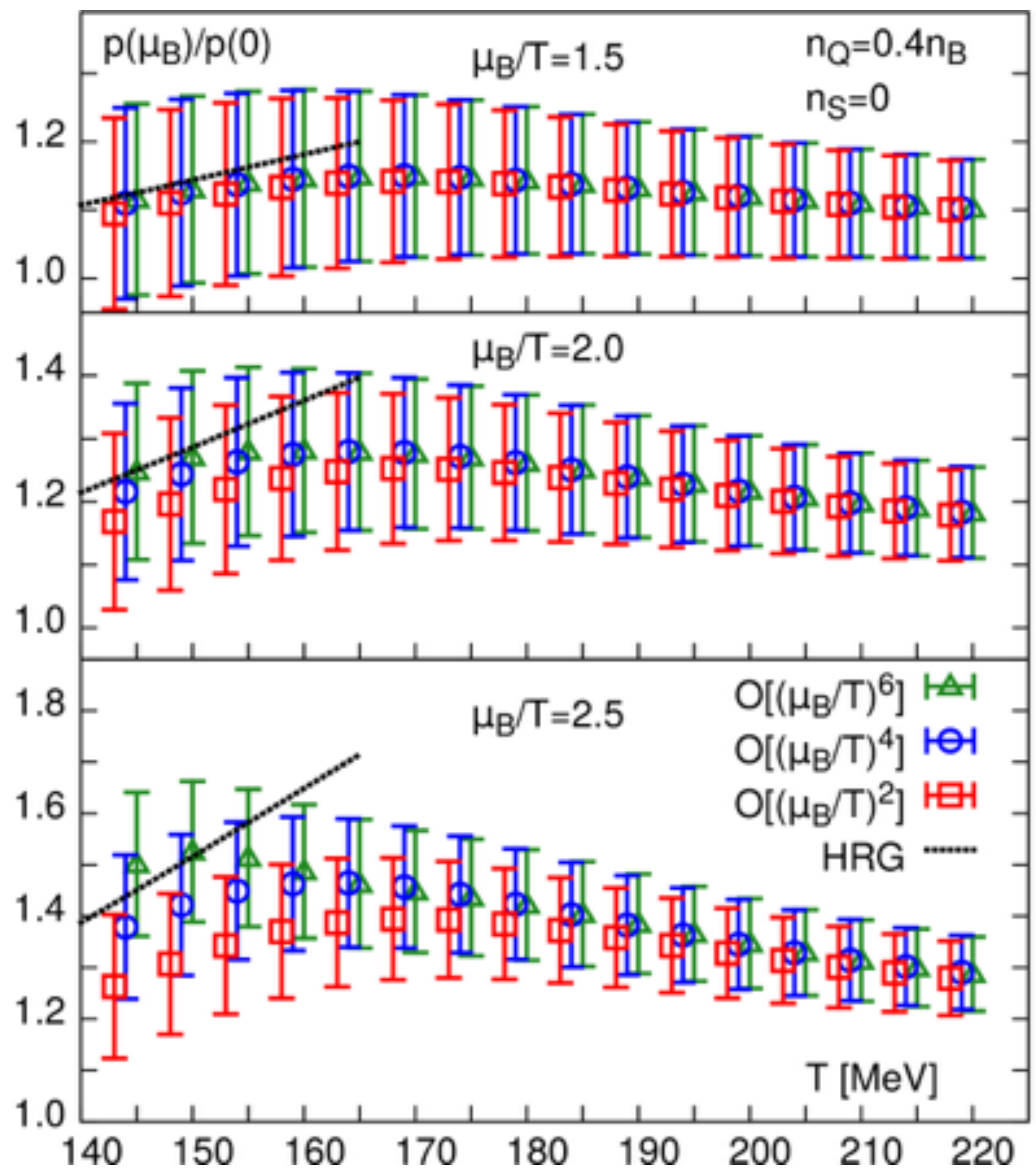
- Zero net strangeness $n_S=0$, and n_Q/n_B = r=0.4 as in PbPb collision systems

$$\frac{\mu_Q}{T} = q_1 \frac{\mu_B}{T} + q_3 \left(\frac{\mu_B}{T}\right)^3 + \cdots \ , \ \frac{\mu_S}{T} = s_1 \frac{\mu_B}{T} + s_3 \left(\frac{\mu_B}{T}\right)^3 + \cdots$$

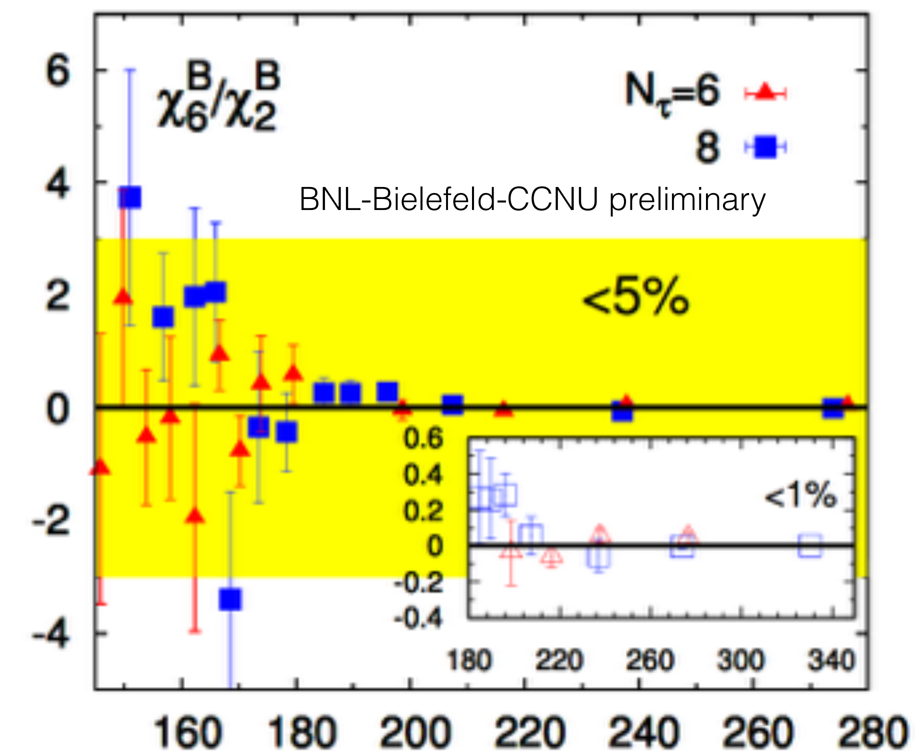


A. Bazavov, HTD et al., Phys. Rev. Lett. 109 (2012)192302

Pressure in the strangeness neutral system



NNLO expansion coefficient

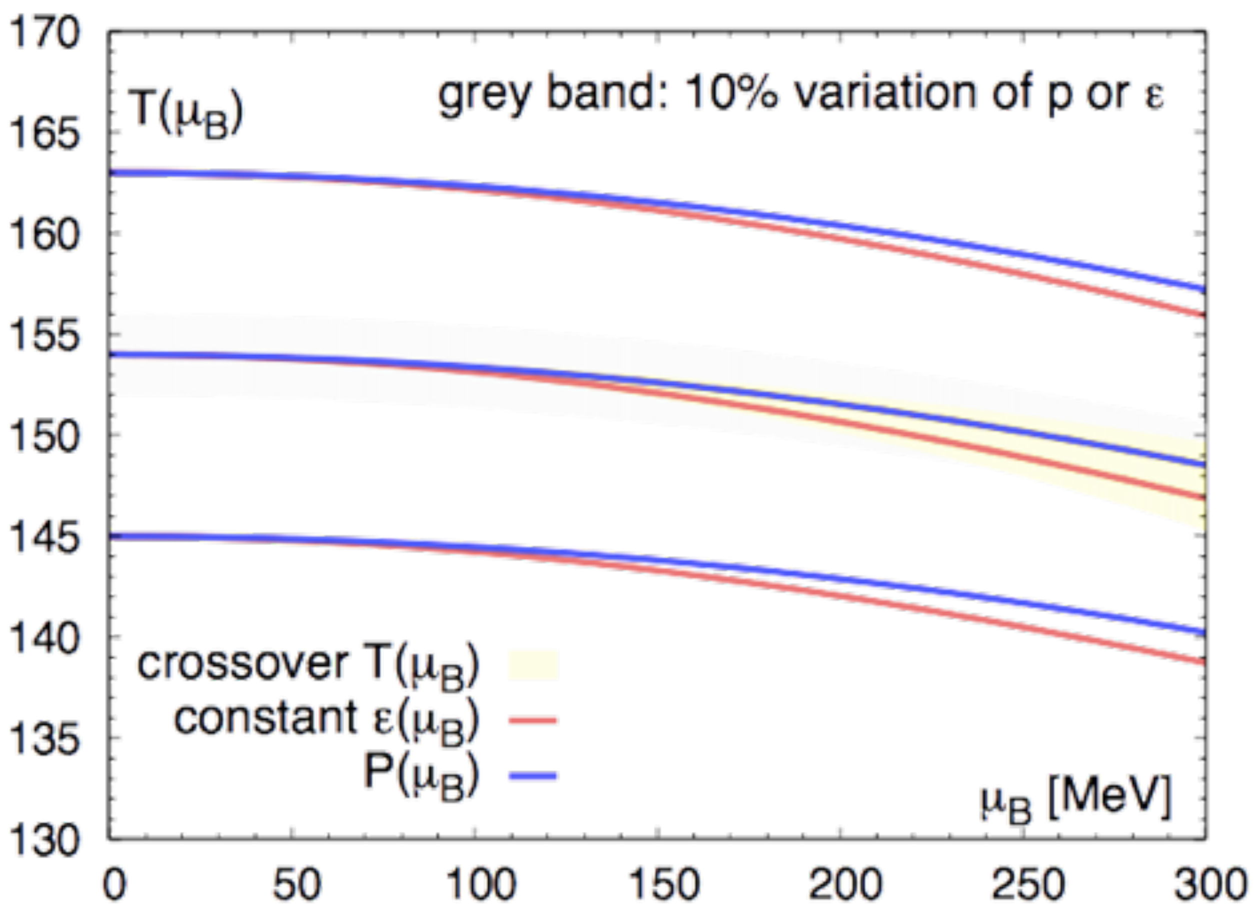


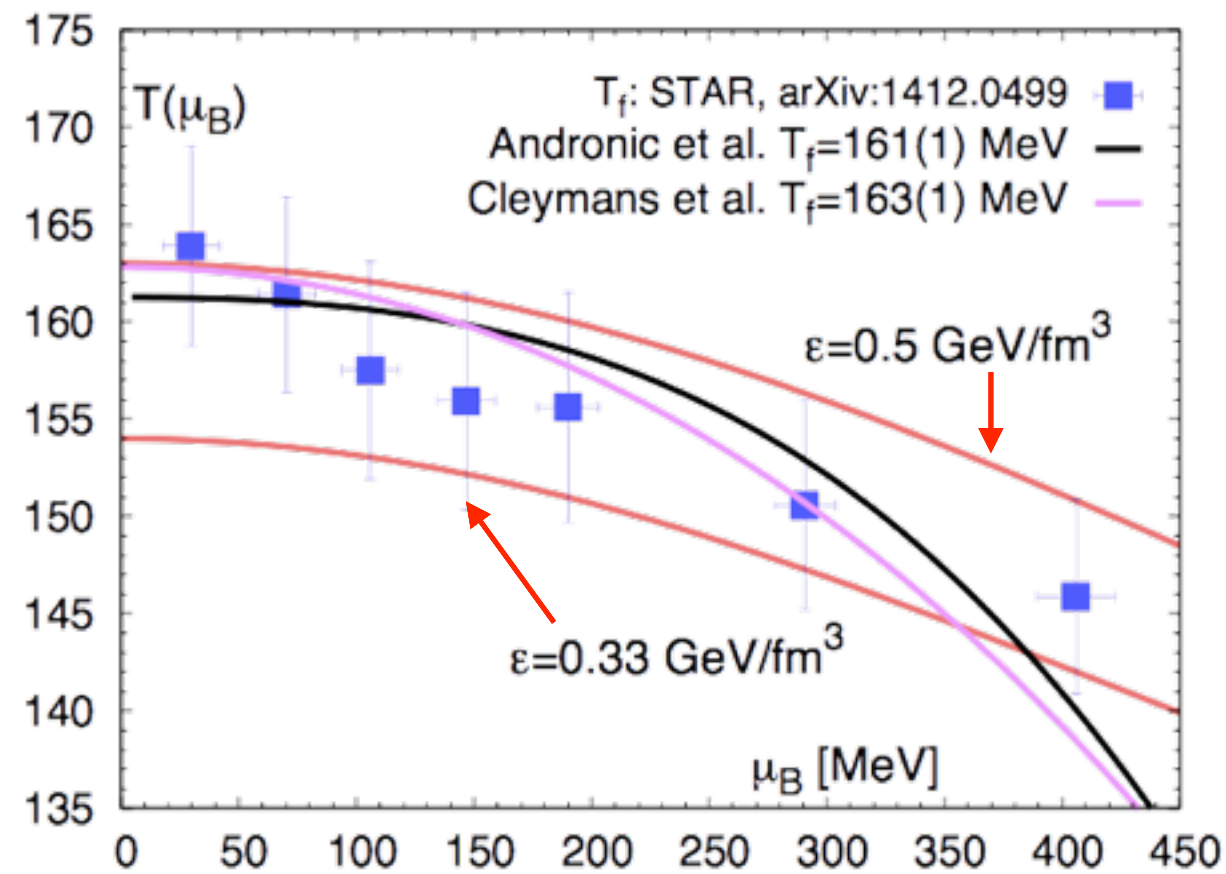
$$\frac{1}{2}\chi_2^B(T)\,\frac{1}{360}\frac{\chi_6^B(T)}{\chi_2^B(T)}\hat{\mu}_B^6$$

The EoS is well under control at $\mu_B/T \lesssim 2$ or $\sqrt{s_{NN}} \gtrsim 20$ GeV

BNL-Bielefeld-CCNU, arXiv:1412.6727

Line of constant physics and freeze-out





Parameterization $T(\mu_B) = T(0)(1 - \kappa_2 \hat{\mu}_B^2 + \mathcal{O}(\hat{\mu}_B^4))$

curvature at constant pressure: $\kappa_{2,p} \approx 0.011$

curvature at constant energy: $\kappa_{2,\epsilon} \approx 0.013$

curvature on the crossover line: $\kappa_{2,c} \approx 0.006 - 0.013$

Explore the QCD phase diagram through fluctuations of conserved charges

Comparison of experimentally measured higher order cumulants of conserved charges to those from LQCD, e.g.:

$$\frac{M_Q(\sqrt{s})}{\sigma_Q^2(\sqrt{s})} = \frac{\langle N_Q \rangle}{\langle (\delta N_Q)^2 \rangle} = \frac{\chi_1^Q(T,\mu_B)}{\chi_2^Q(T,\mu_B)} = R_{12}^Q(T,\mu_B)$$

$$\frac{S_Q(\sqrt{s})\,\sigma_Q^3(\sqrt{s})}{M_Q(\sqrt{s})} = \frac{\langle (\delta N_Q)^3 \rangle}{\langle N_Q \rangle} = \frac{\chi_3^Q(T,\mu_B)}{\chi_1^Q(T,\mu_B)} = R_{31}^Q(T,\mu_B)$$

HIC

mean: M_Q

variance: σ_Q^2

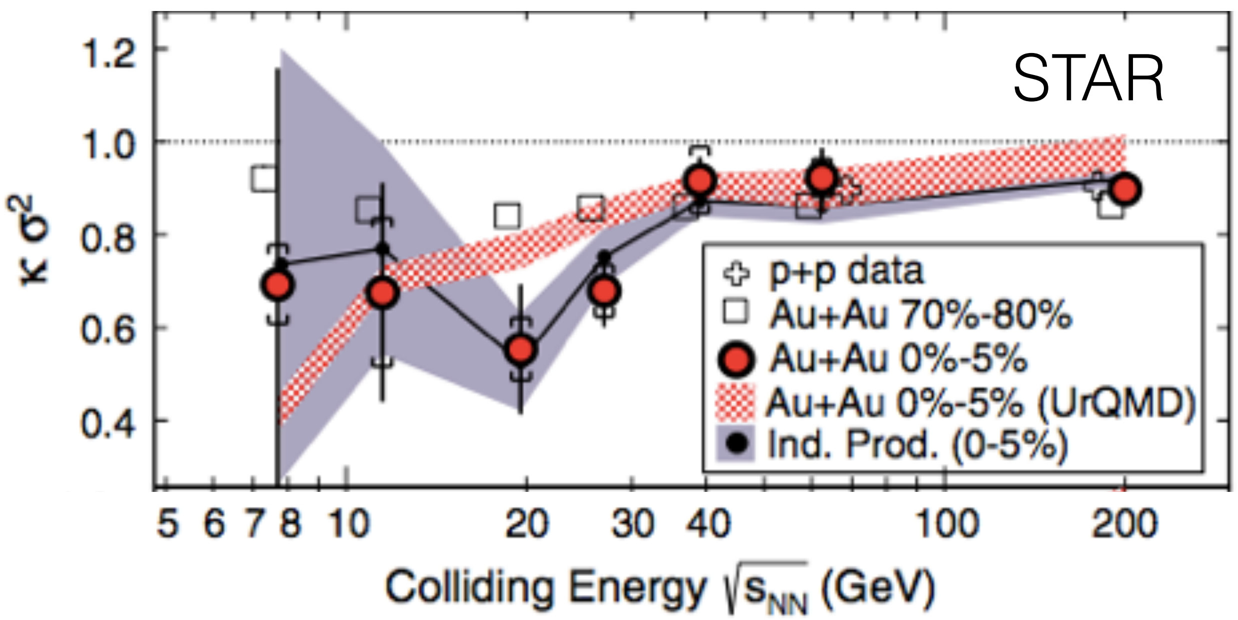
skewness: S_Q

LQCD

generalized susceptibilities

$$\chi_n^Q(T,\vec{\mu}) = \frac{1}{VT^3}\frac{\partial^n \ln Z(T,\vec{\mu})}{\partial(\mu_Q/T)^n}$$

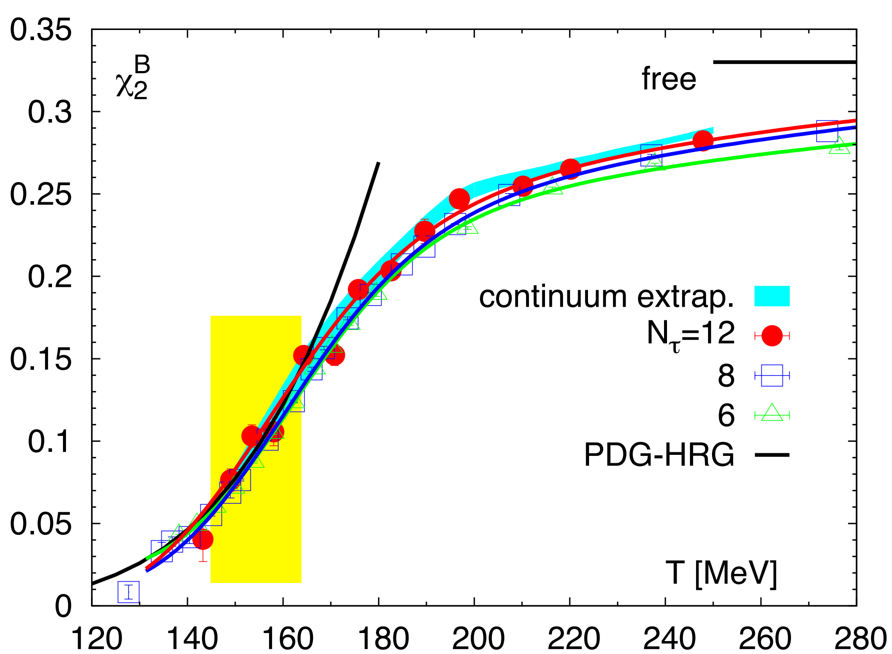
Explore the QCD phase diagram

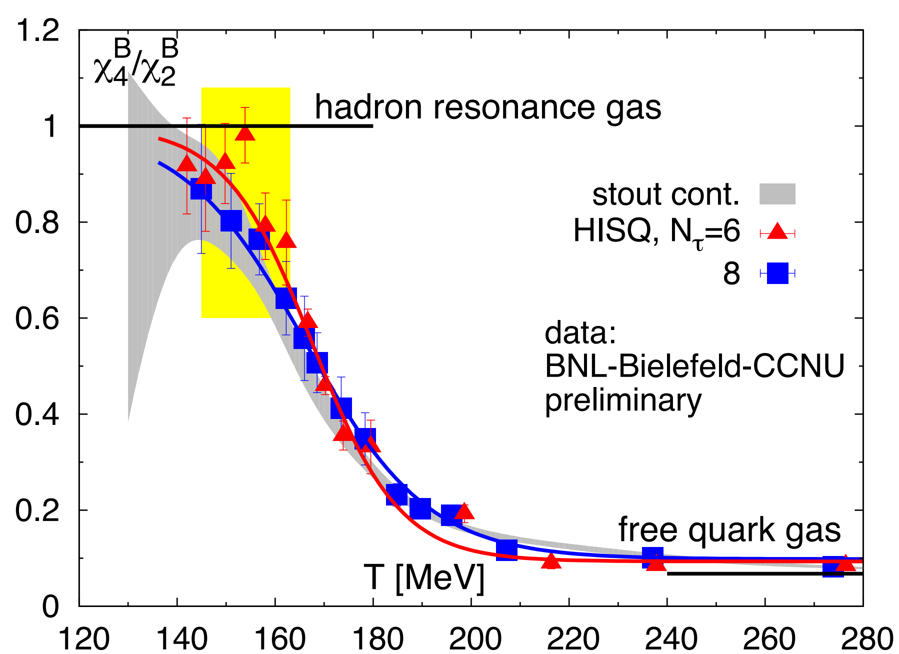


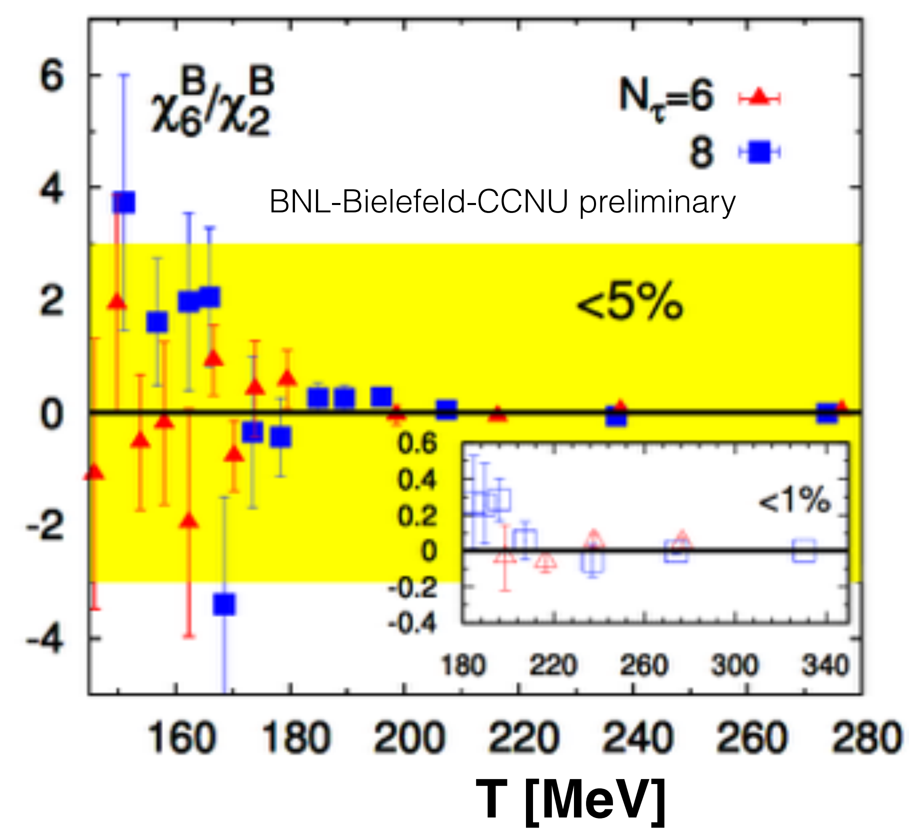
$$(\kappa\sigma^2)_B = \frac{\chi^B_{4,\mu}}{\chi^B_{2,\mu}} = \frac{\chi^B_4}{\chi^B_2}\left[1+\left(\frac{\chi^B_6}{\chi^B_4}-\frac{\chi^B_4}{\chi^B_2}\right)\left(\frac{\mu_B}{T}\right)^2+\cdots\right]$$

In the O(4) universality class:

$$\chi^B_6 < 0\,, \qquad T \sim T_c$$







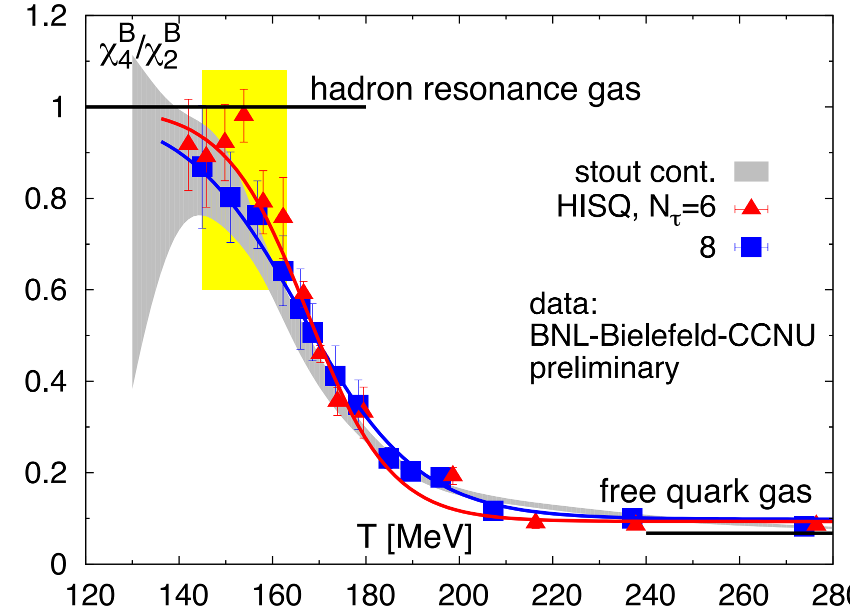
HTD, Nucl. Phys. A 931 (2014) 52-62
HTD, F. Karsch, S. Mukherjee, arXiv:1504.0527

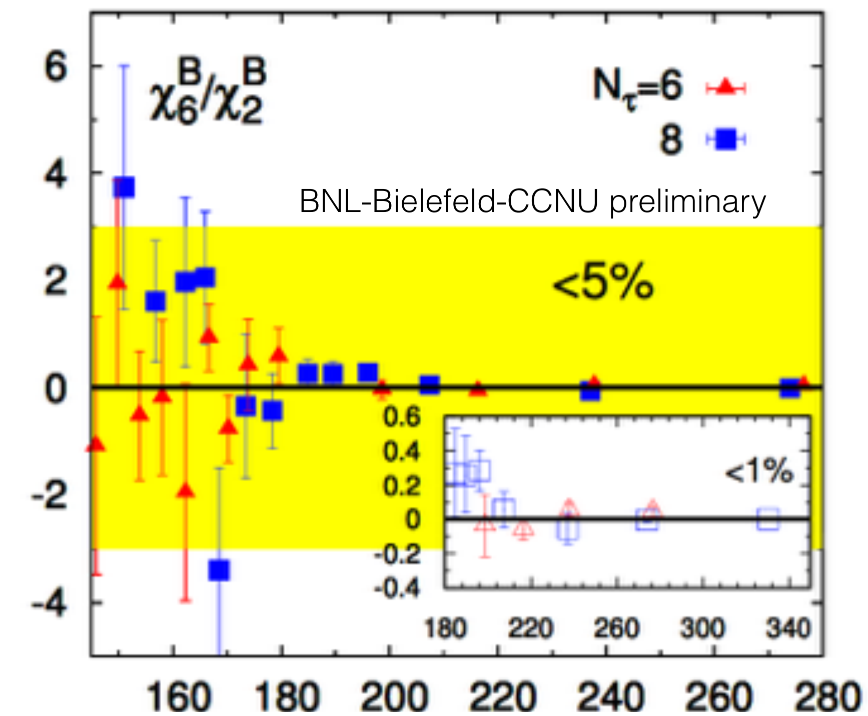
conserved charge fluctuations & freeze-out

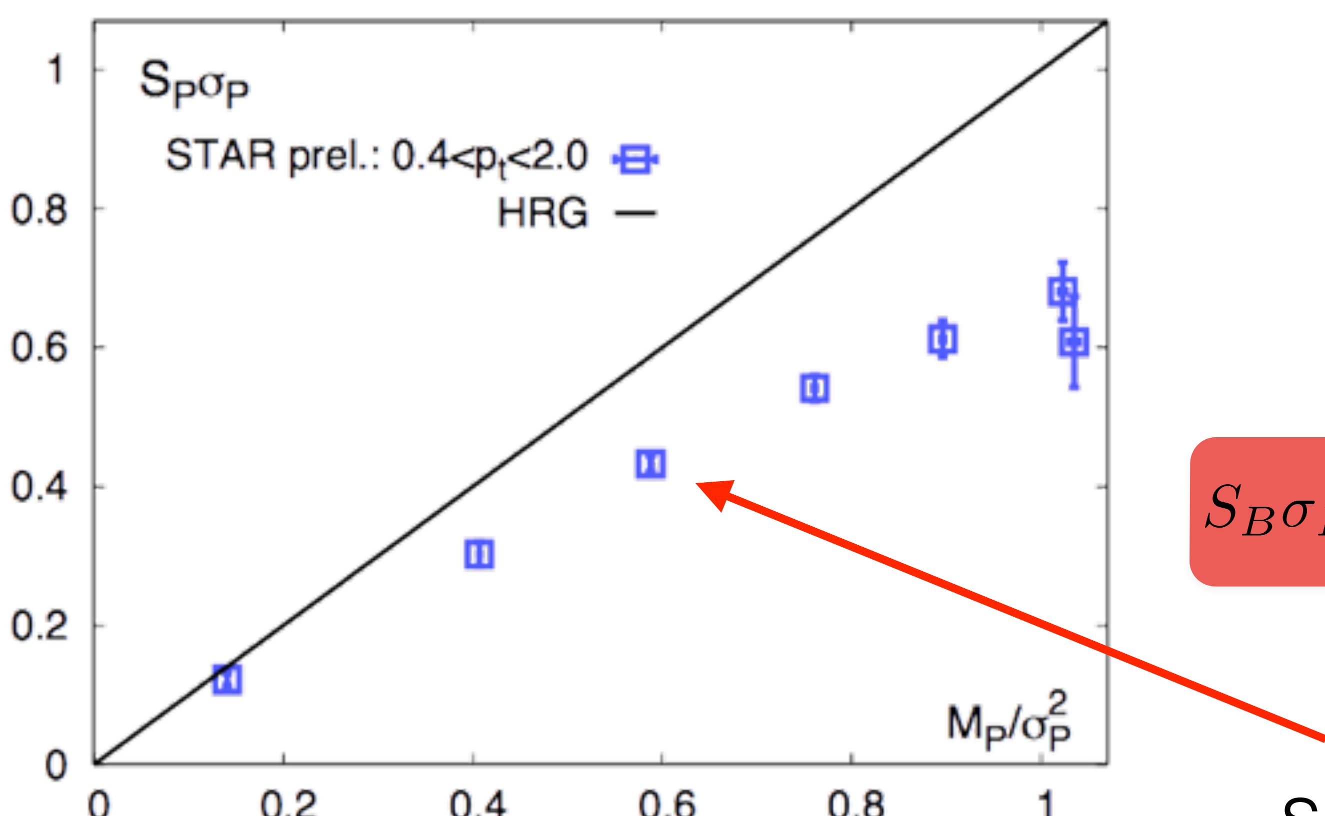
$\mu_Q=\mu_S=0$:

$$\frac{M_B}{\sigma_B^2} = \frac{\chi^B_{1,\mu}}{\chi^B_{2,\mu}} = \frac{\mu_B}{T}\,\frac{1+\frac{1}{6}\frac{\chi_4^B}{\chi_2^B}(\frac{\mu_B}{T})^2+\cdots}{1+\frac{1}{2}\frac{\chi_4^B}{\chi_2^B}(\frac{\mu_B}{T})^2+\cdots}$$

$$S_B\sigma_B = \frac{\chi^B_{3,\mu}}{\chi^B_{2,\mu}} = \frac{\mu_B}{T}\frac{\chi_4^B}{\chi_2^B}\,\frac{1+\frac{1}{6}\frac{\chi_6^B}{\chi_4^B}(\frac{\mu_B}{T})^2+\cdots}{1+\frac{1}{2}\frac{\chi_4^B}{\chi_2^B}(\frac{\mu_B}{T})^2+\cdots}$$







STAR data from X.F. Luo, 1503.02558

$$S_B\sigma_B = \frac{\mu_B}{T}\frac{\chi_4^B}{\chi_2^B} + \mathcal{O}((\frac{\mu_B}{T})^3) = \frac{M_B}{\sigma_B^2}\frac{\chi_4^B}{\chi_2^B} + \mathcal{O}((\frac{\mu_B}{T})^3)$$

slope is smaller than 1 as $\frac{\chi_4^B}{\chi_2^B} < 1$

Ratio on charge fluctuations on the freeze-out line

In heavy ion collisions $M_S=0$ and $M_Q/M_B=r$

ratios of mean to variance:

$$R_{12}^X(T,\mu) \equiv \frac{M_X}{\sigma_X^2} = \frac{\chi_1^X(T,\mu)}{\chi_2^X(T,\mu)} \ , \qquad X=B,Q$$

ratio of electrical charge to baryon ratio :

$$\Sigma_r^{QB} \equiv R_{12}^Q/R_{12}^B = r\,\sigma_B^2/\sigma_Q^2$$

Expand the ratio around $\mu_B=0$:

$$\Sigma_r^{QB}(T,\hat{\mu}_B) = \Sigma_r^{QB}(T,\hat{\mu}_B=0) + \frac{1}{2!}\frac{\partial^2\Sigma_r^{QB}(T,\hat{\mu}_B)}{\partial\hat{\mu}_B^2}\Big|_{\hat{\mu}_B=0}\hat{\mu}_B^2$$

Expand the ratio around $T_f(\mu_B)=T_f(\mu_B=0)$:

$$\Sigma_r^{QB}(T_f,\hat{\mu}_B) = \Sigma_r^{QB}(T_f=T_{f,0},\hat{\mu}_B) + \frac{\mathrm{d}\Sigma_r^{QB}(T_f,\hat{\mu}_B)}{\mathrm{d}T}\Big|_{T_{f,0}}(T_f-T_{f,0})$$

BNL-Bielefeld-CCNU, PRD 93 (2016)014512

Parameterization of freeze-out line

Parameterization of $T_f(\mu_B)$: works well in HRG models

$$T_f(\mu_B) = T_{f,0}\left(1 - \kappa_2^f \left(\mu_B/T_{f,0}\right)^2\right)$$

Cleymans et al., PRC 73(2006)034905
Andronic, Braun-Munzinger & Stachel, NPA 772(2006)167

Taylor expansion of the ratio at T= $T_f(\mu_B=0)$ and $\mu_B=0$

BNL-Bielefeld-CCNU, PRD 93 (2016)014512

$$\Sigma_r^{QB}(T,\hat{\mu}_B) = \Sigma_r^{QB}(T,\hat{\mu}_B=0) + \frac{1}{2!}\frac{\partial^2 \Sigma_r^{QB}(T,\hat{\mu}_B)}{\partial \hat{\mu}_B^2}\Big|_{\hat{\mu}_B=0}\hat{\mu}_B^2$$

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$$\Sigma_r^{QB}(T_f,\hat{\mu}_B) = \Sigma_r^{QB}(T_f=T_{f,0},\hat{\mu}_B) + \frac{\mathrm{d}\Sigma_r^{QB}(T_f,\hat{\mu}_B)}{\mathrm{d}T}\bigg|_{T_{f,0}}(T_f - T_{f,0})$$

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Ratio of $(M_Q/\sigma_Q^2)/(M_B/\sigma_B^2)$ can be expressed in terms of κ_2^f:

$$\Sigma_r^{QB}(T_f,\hat{\mu}_B) = \Sigma_r^{QB}(T_{f,0},\hat{\mu}_B=0) + \left(\frac{1}{2!}\frac{\partial^2\Sigma_r^{QB}(T_f,\hat{\mu}_B)}{\partial\hat{\mu}_B^2} - \kappa_2^f\,T_{f,0}\frac{\mathrm{d}\Sigma_r^{QB}(T_f,\hat{\mu}_B)}{\mathrm{d}T}\right)\Big|_{T_{f,0},\hat{\mu}_B=0}\hat{\mu}_B^2$$

Experimentally accessible

LQCD computable

To be determined

Parameterization of freeze-out line

Parameterization of $T_f(\mu_B)$: works well in HRG models

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Andronic, Braun-Munzinger & Stachel, NPA 772(2006)167

Taylor expansion of the ratio at T= $T_f(\mu_B=0)$ and $\mu_B=0$

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$$\Sigma_r^{QB}(T_f,\hat{\mu}_B) = \Sigma_r^{QB}(T_f,\hat{\mu}_B=0) + \frac{1}{2!}\frac{\partial^2\Sigma_r^{QB}(T_f,\hat{\mu}_B)}{\partial\hat{\mu}_B^2}\bigg|_{\hat{\mu}_B=0}\hat{\mu}_B^2$$

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Experimentally accessible

LQCD computable

To be determined

$\hat{\mu}_B$ above can replaced:

$$R_{12}^B(T_f,\mu_B) \equiv \frac{M_B}{\sigma_B^2}(T_f,\mu_B) = \frac{\partial R_{12}^B}{\partial\hat{\mu}_B}\bigg|_{\hat{\mu}_B=0}\hat{\mu}_B + \mathcal{O}(\hat{\mu}_B^3)$$

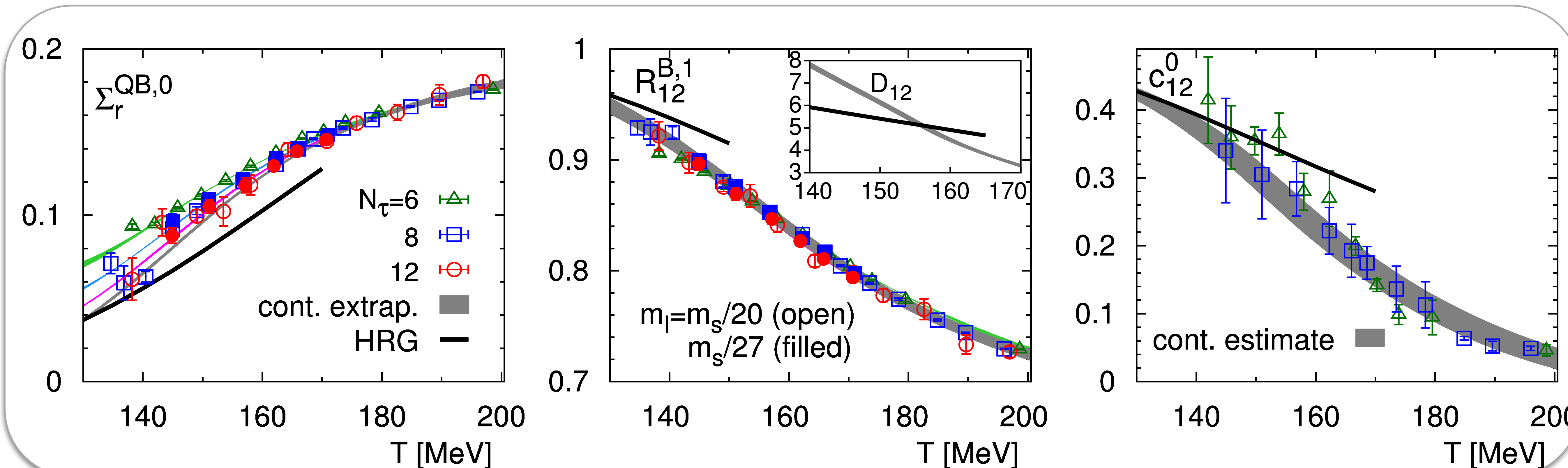
$$\frac{\partial R_{12}^B}{\partial\hat{\mu}_B} \equiv R_{12}^{B,1}$$

Temperature dependence of (N)LO expansion coefficients

NLO expansion of $M_Q/\sigma_Q^2/(M_B/\sigma_B^2) \equiv R_{12}^Q/R_{12}^B$:

$$\Sigma_r^{QB} = a_{12}\left[1 + \left(c_{12}^0 T_{f,0} - \kappa_2^f D_{12}(T_{f,0})\right)\left(R_{12}^B\right)^2\right] + \mathcal{O}\left((R_{12}^B)^4\right)$$

$$a_{12} = \Sigma_r^{QB,0} \qquad D_{12}^0(T) = \left(\frac{1}{R_{12}^{B,1}}\right)^2 T\,\frac{\mathrm{d}\ln\Sigma_r^{QB,0}}{\mathrm{d}T} \qquad c_{12}^0(T) = \left(\frac{1}{R_{12}^{B,1}}\right)^2 \frac{\Sigma_r^{QB,2}}{\Sigma_r^{QB,0}}$$



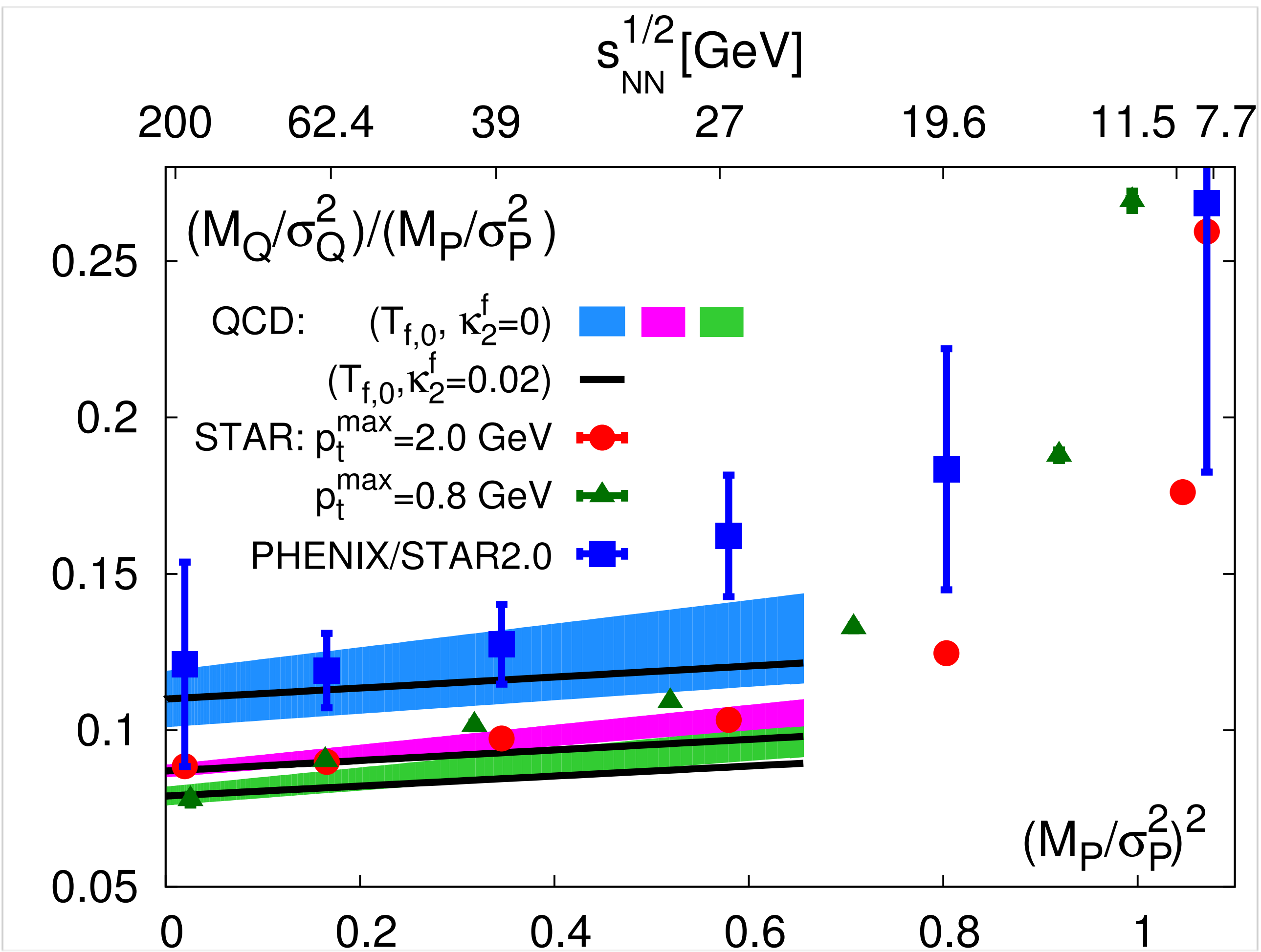
BNL-Bielefeld-CCNU, PRD 93 (2016)014512

r=M_Q/M_B≃0.4 for describing AuAu or PbPb collision system

Comparison to experiment data

NLO expansion of $M_Q/\sigma_Q^2/(M_B/\sigma_B^2) \equiv R_{12}^Q/R_{12}^B$:

$$\Sigma_r^{QB} = a_{12}\left[1 + \left(c_{12}^0 T_{f,0} - \kappa_2^f D_{12}(T_{f,0})\right)\left(R_{12}^B\right)^2\right] + \mathcal{O}\left((R_{12}^B)^4\right)$$



Upper bound on the curvature of the freeze-out line

$$\kappa_2^f \lesssim 0.011$$

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chiral phase transition and universal scaling

Behavior of the free energy close to critical lines

$$f(m,T)=h^{1+1/\delta} f_s(z) + f_{reg}(m,T), \qquad z=t/h^{1/\beta\delta}$$

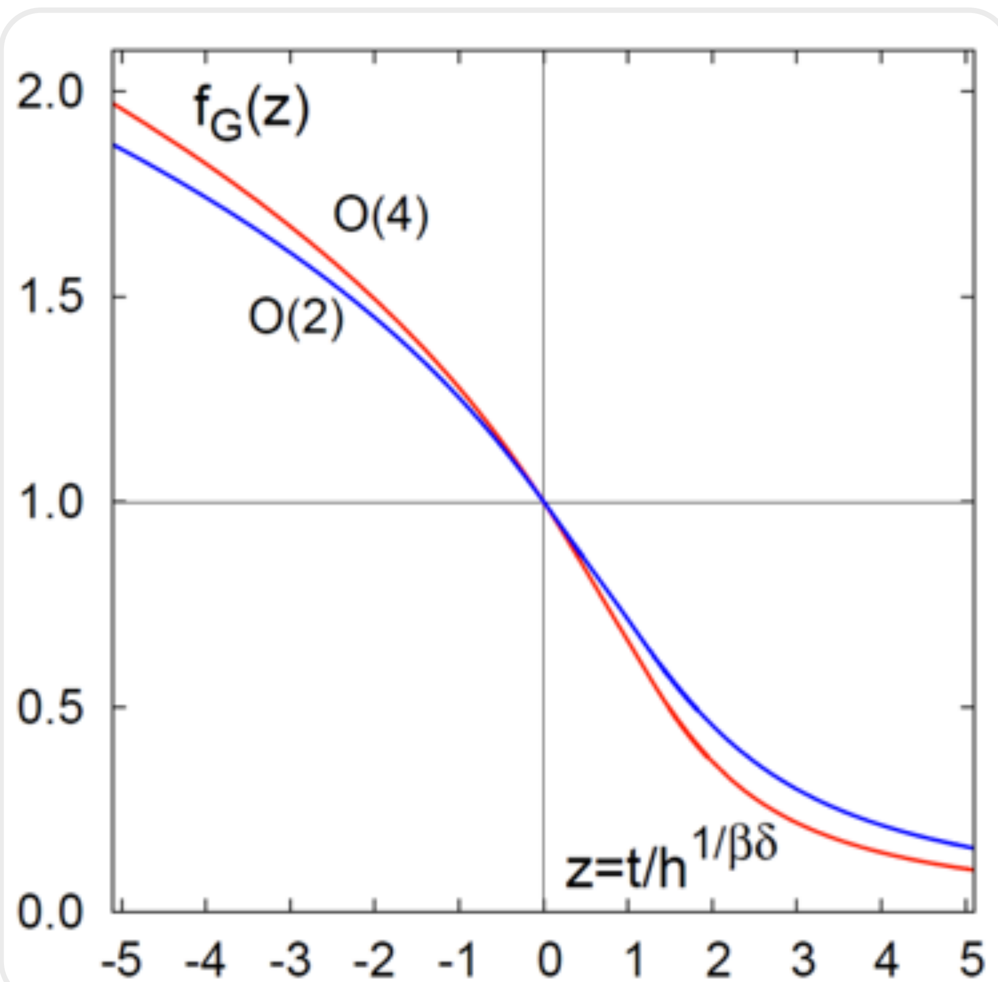
h: external field, t: reduced temperature, β,δ: universal critical exponents

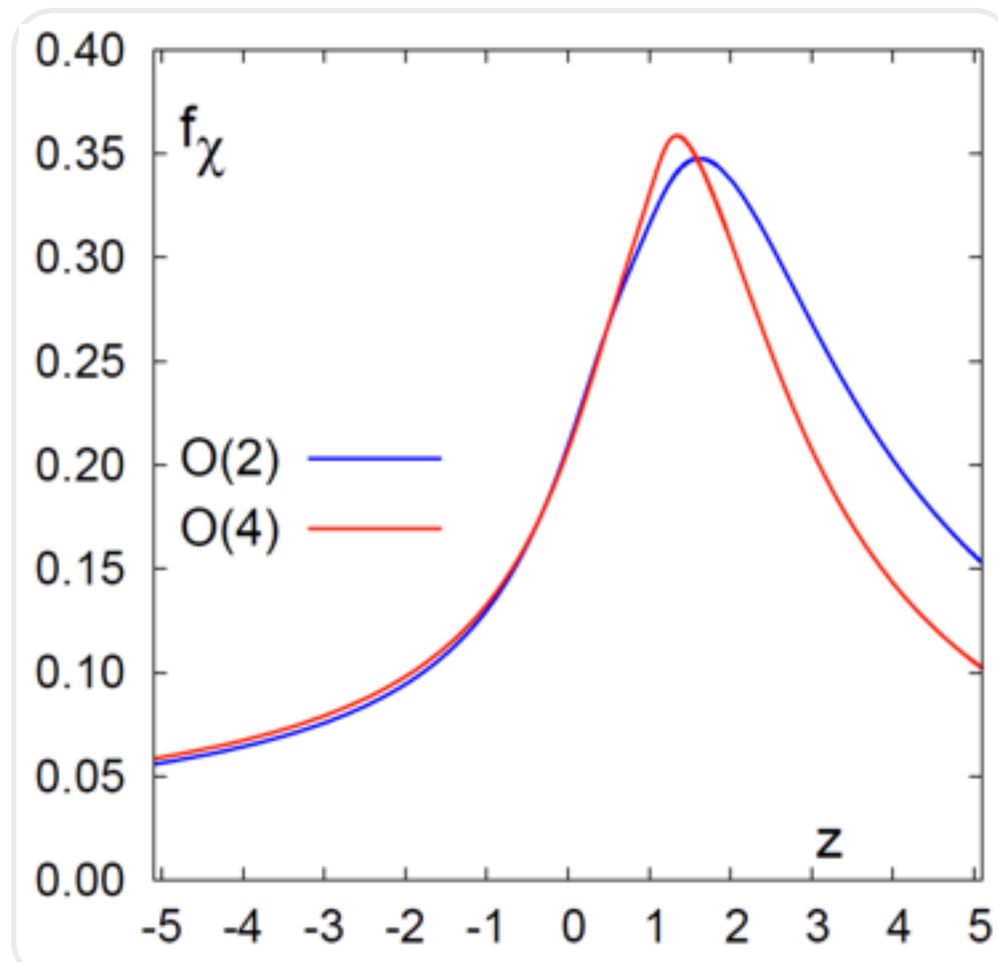
$f_s(z)$: universal scaling function, O(N) etc.

$$h=\frac{1}{h_0}\frac{m_l}{m_s} \qquad t=\frac{1}{t_0}\frac{T-T_c}{T_c}$$

Magnetic Equation of State (MEoS):

$$M = -\partial f_s(t,h)/\partial h = h^{1/\delta} f_G(z) \qquad f_\chi(z)=h_0^{1/\delta}(m_l/m_s)^{1-1/\delta}\partial M/\partial h$$





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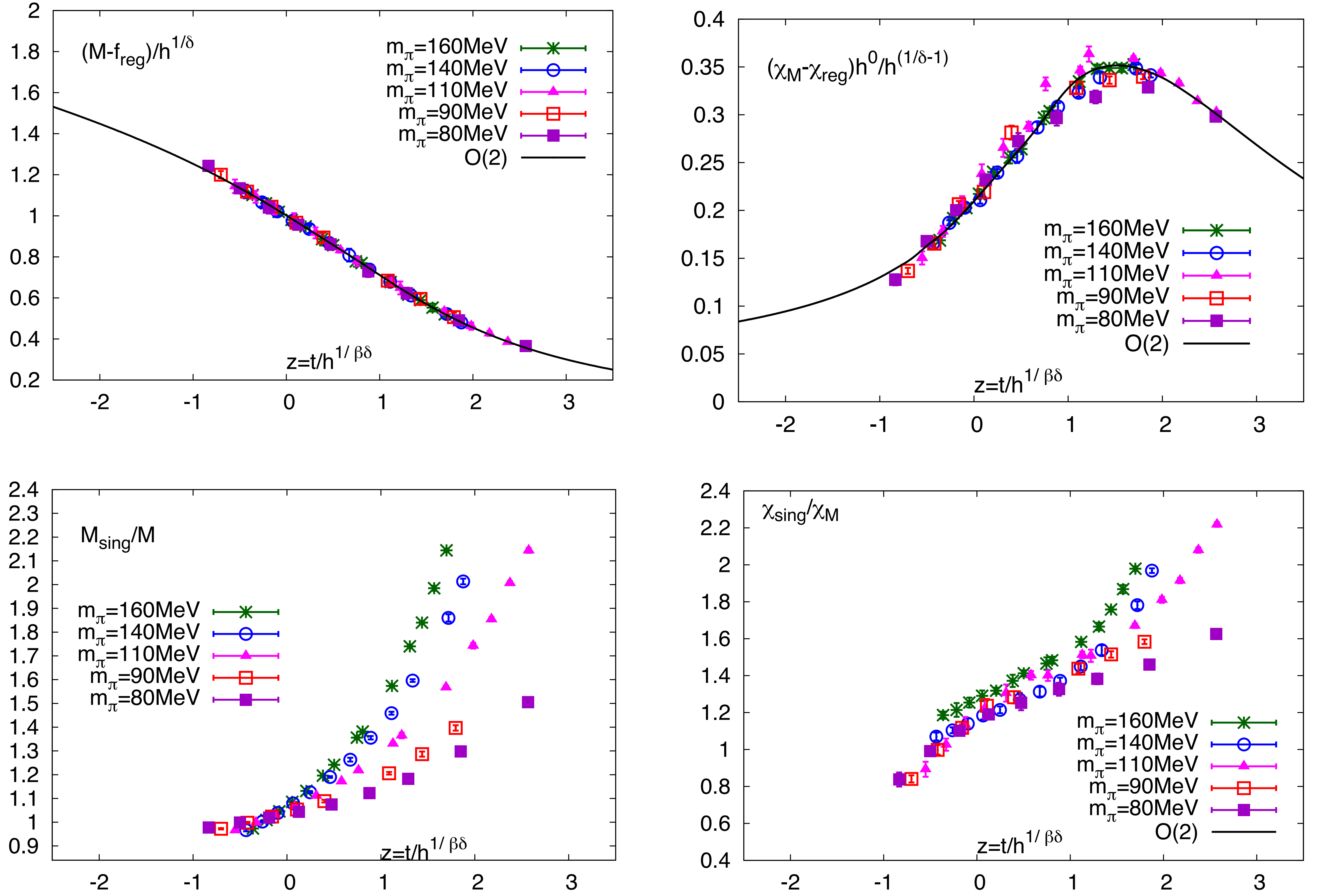
- Comparison with QCD

$$M = m_s \langle\bar{\psi}\psi\rangle_l/T^4\,, \qquad \chi_M = m_s^2\chi_{tot}/T^4$$

- Contributions from the regular term

$$M = h^{1/\delta} f_G(z) + f_{reg} \quad , \quad \chi_M = h_0^{-1} h^{1/\delta-1} f_\chi(z) + f'_{reg}$$

O(2) scaling fit to chiral susceptibilities & condensates



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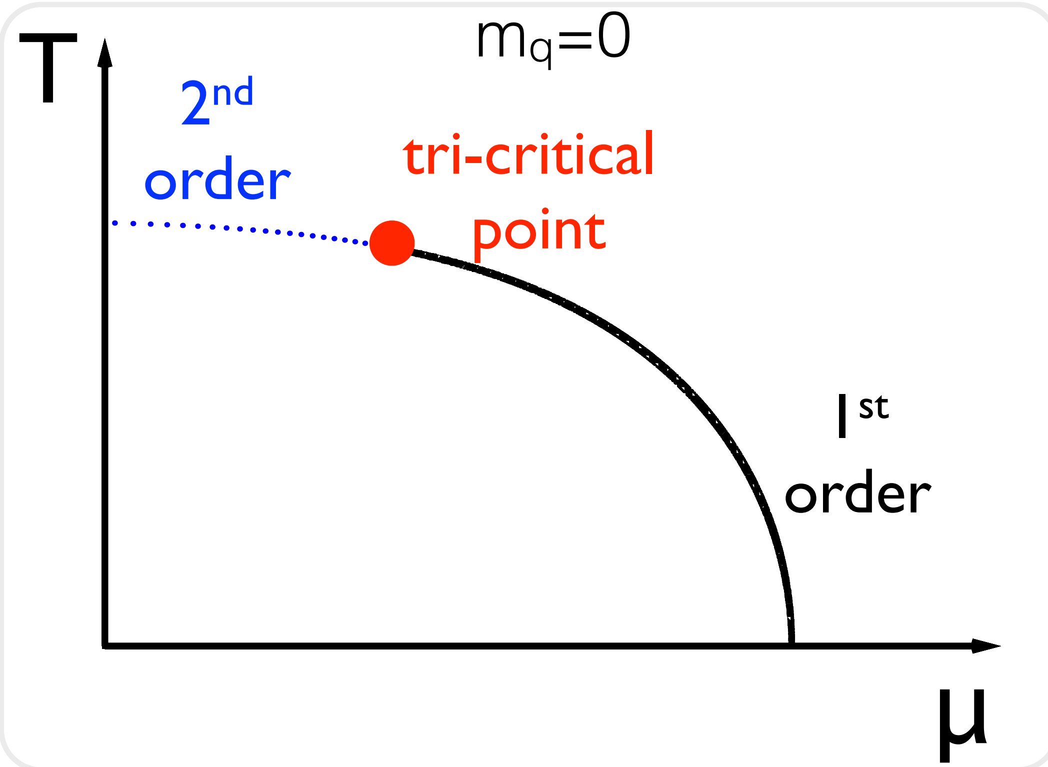
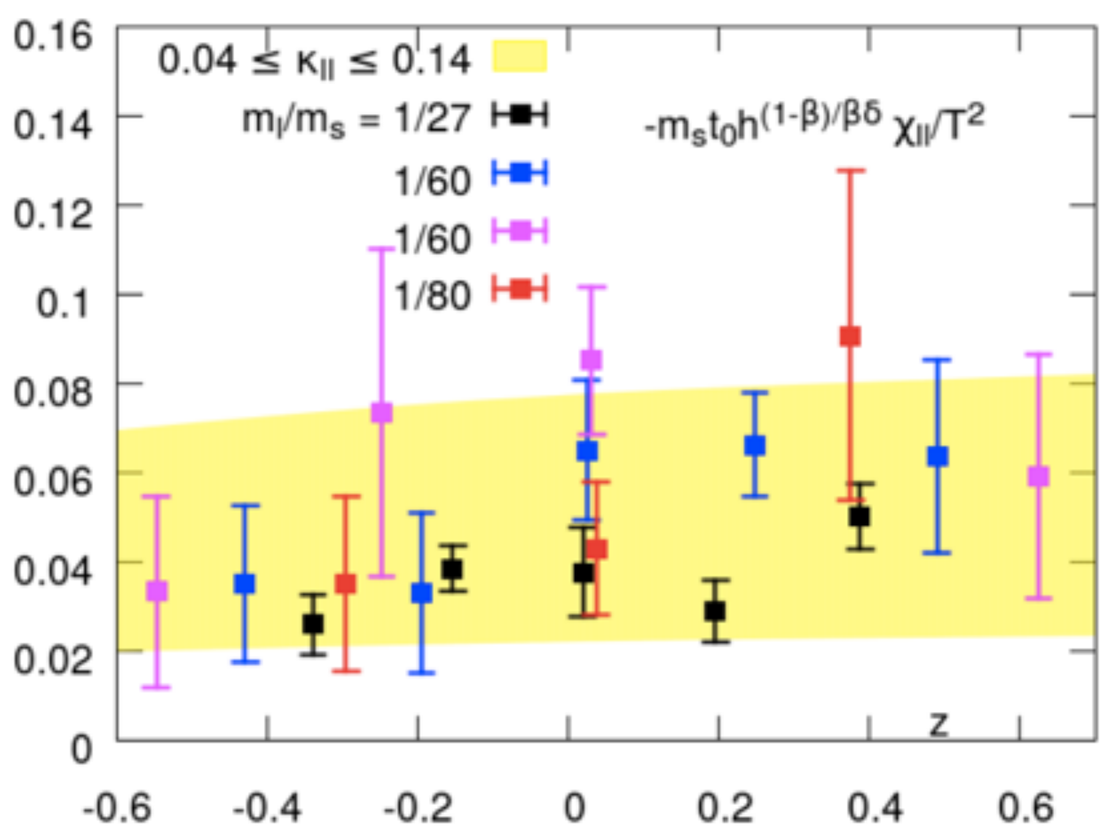
universality at small baryon chemical potential





BNL-Bielefeld-CCNU preliminary

The curvature of chiral phase transition line: κ_q

$$\frac{T_c(\mu_q)}{T_c} = 1 - \kappa_q \left(\frac{\mu_q}{T}\right)^2 + \mathcal{O}\left(\left(\frac{\mu_q}{T}\right)^4\right)$$

Taylor expansion of chiral condensate about μ=0

$$\frac{\langle\bar\psi\psi\rangle_l}{T^3} = \left(\frac{\langle\bar\psi\psi\rangle_l}{T^3}\right)_{\mu_q=0} + \frac{\chi_{m,q}}{2T}\left(\frac{\mu_q}{T}\right)^2 + \mathcal{O}((\mu_q/T)^4)$$

Universal scaling

$$\frac{\chi_{m,q}}{T} = \frac{\partial^2\langle\bar\psi\psi\rangle_l/T^3}{\partial(\mu_q/T)^2} = \frac{2\kappa_q T}{t_0 m_s} h^{-(1-\beta)/\beta\delta} f'_G(z)$$

The curvature of the chiral phase transition line:

$$\kappa_B \approx 0.004 - 0.015$$

Conclusion & Summary

- The EoS is well controlled at $\mu_B/T \lesssim 2$ or $\sqrt{s_{NN}} \gtrsim 20$ GeV
- We provided a framework that allows to determine the curvature of the freeze-out line through the direct comparison between experimental data and lattice QCD calculations of cumulant ratios
- At least for collision energy larger than 27 GeV it suggests that freeze-out happens close to the cross over & chiral phase transition line