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A microscopic implementation scheme of hadronization with first-order phase transition

Zhe Xu

with Bohao Feng and Carsten Greiner

清華大學
Tsinghua University

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Outline

- Motivations
- Macroscopic volume change
- Microscopic implementation scheme
- Numerical results with comparisons
- Summary and Outlook

Motivations

A dynamical description of the QCD phase transition is needed to have a complete physical picture and understand phenomena in relativistic heavy ion collisions.

To understand the contribution of gluons to the buildup of hadron collective flow and examine the coalescence models used to explain the quark number scaling.

To study the viscous effect during the phase transition and examine the Cooper-Frye prescription.

A further development of BAMPS towards a MultiPhase transport model.

Motivations

The very first step of the development

to be improved:

- No correlations
- Only from gluons to pions
- Simple EoS

advantages:

- Self-consistent
- Applicable for any hydrodynamic systems

Macroscopic volume change

EoS:

gluonic phase: MIT bag model

$$e_g = 3n_g T_g + B, \qquad P_g = \frac{1}{3}\left(e_g - 4B\right) = n_g T_g - B$$

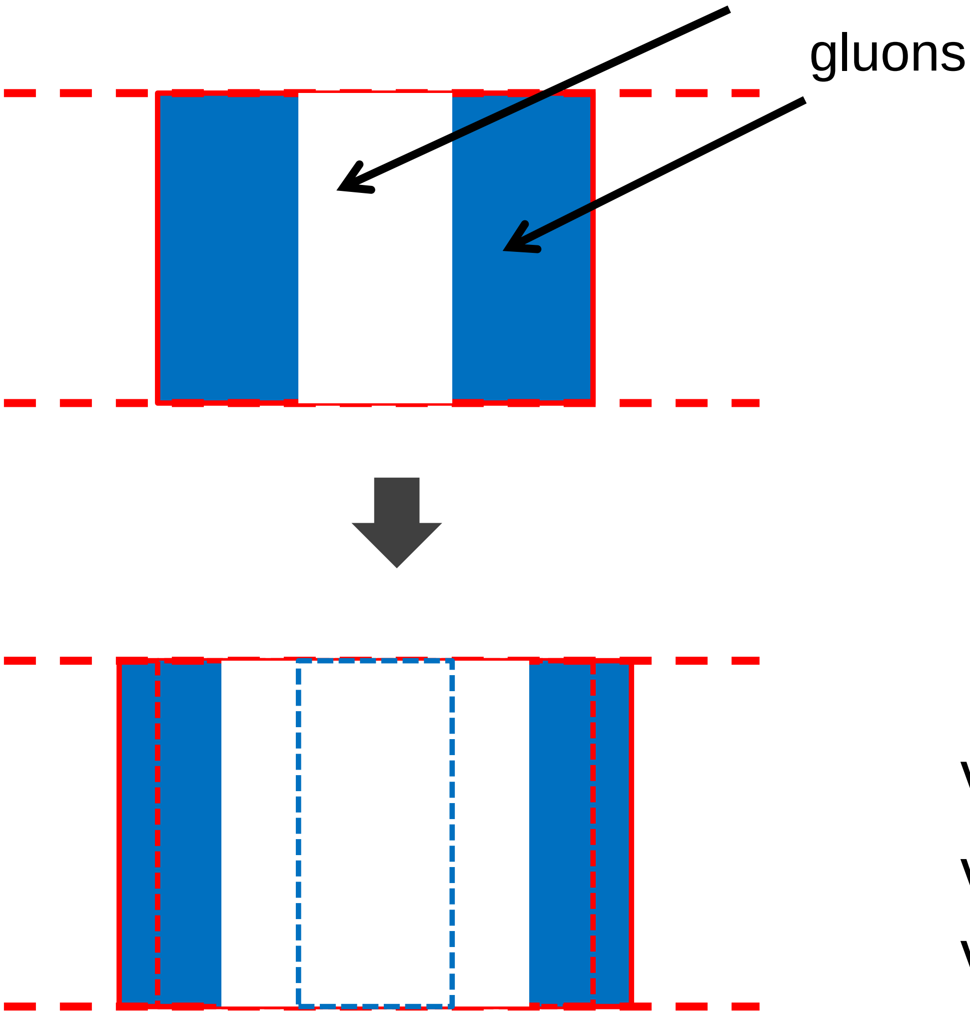
pionic phase:

$$e_\pi = 3n_\pi T_\pi, \qquad P_\pi = \frac{1}{3} e_\pi = n_\pi T_\pi$$

Gibbs condition for first-order phase transition:

$$P_g = P_\pi = P_c, \qquad T_g = T_\pi = T_c, \qquad \mu_g = \mu_\pi = \mu_c$$

Macroscopic volume change



at time τ

volume element: V
volume of gluons: V_g
volume of pions: V_π

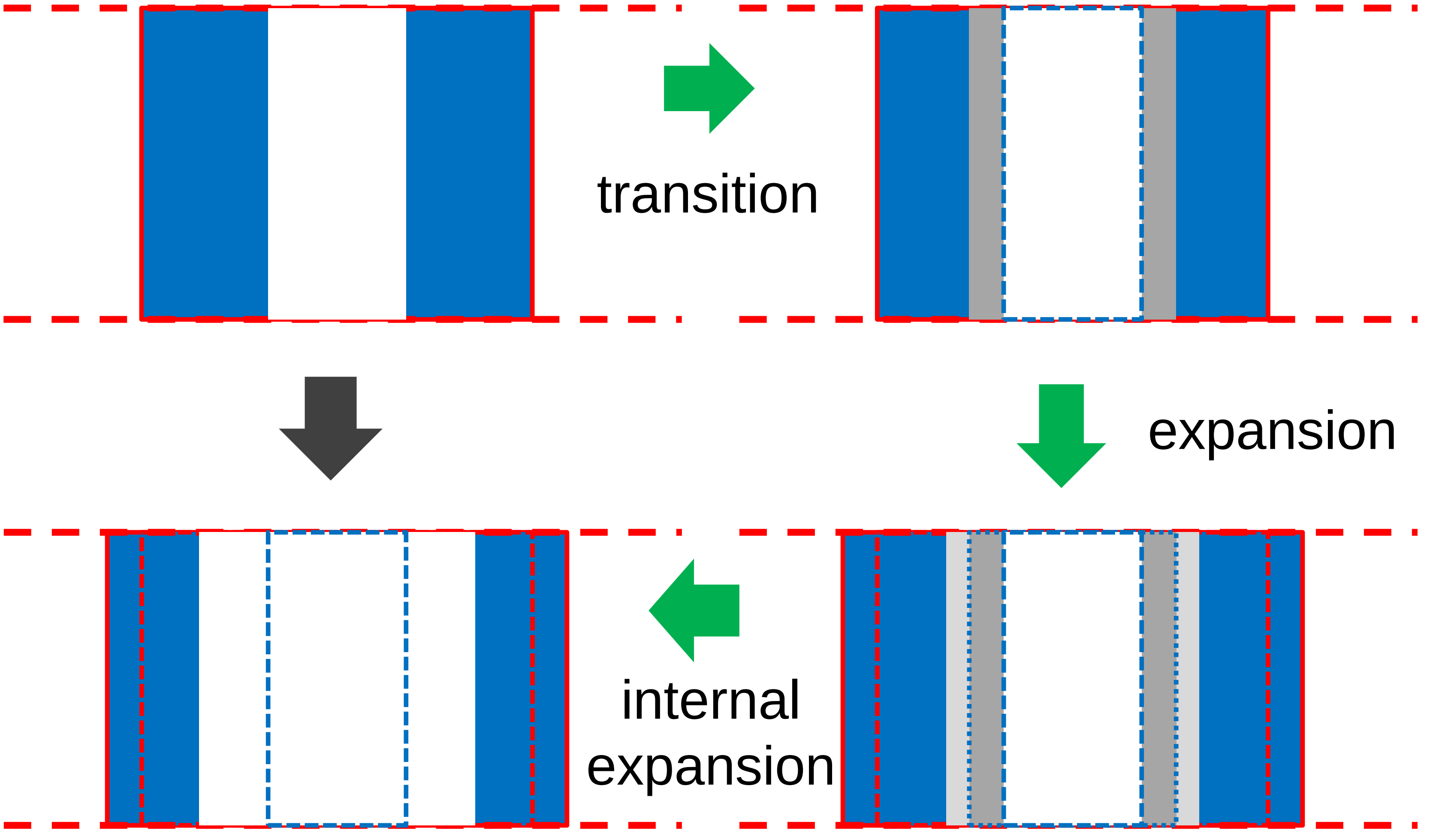
$$f_g = V_g/V$$

at time $\tau+d\tau$

volume element: V+dV

volume of gluons: V_g - **dV_g ?**
volume of pions: $V_\pi+dV_\pi$

Macroscopic volume change



Macroscopic volume change

energy balance:

$$e_g^c dV_g - (P_c + \tilde{\pi}_g) f_g dV - (P_c + \tilde{\pi}_\pi)(1 - f_g) dV = e_\pi^c dV_\pi$$

$$\Rightarrow \quad \boxed{dV_g = \frac{e_\pi^c + P_c + \tilde{\pi}_m}{e_g^c - e_\pi^c} dV} \qquad \tilde{\pi}_m = f_g \tilde{\pi}_g + (1 - f_g)\, \tilde{\pi}_\pi$$

dissipation obtained from hydrodynamic equation:

$$De = -(e+P)\nabla_\mu U^\mu + \pi^{\mu\nu}\nabla_{<\mu}U_{\nu>} = -(e+P+\tilde{\pi})\nabla_\mu U^\mu \,,$$

$$\tilde{\pi} = -\frac{\pi^{\mu\nu}\nabla_{<\mu}U_{\nu>}}{\nabla_\mu U^\mu} \qquad \frac{1}{V}\frac{dV}{d\tau} = \nabla_\mu U^\mu$$

Macroscopic volume change

note:

- The flow velocity and shear tensor are obtained from transport calculations.

- The dynamical picture for the energy balance is correct, since mathematically

$$e = f_g e_g^c + (1 - f_g) e_\pi^c$$

$$De \rightarrow \frac{df_g}{d\tau} = \frac{d}{d\tau} \frac{V_g}{V} \rightarrow dV_g,$$

which agrees with the derivation before.

- can be applied for transitions from pions to gluons.

Macroscopic volume change

Latent heat from the bag pressure:

$$e_g^c dV_g \; -(P_c+\tilde{\pi}_g) f_g dV \; -(P_c+\tilde{\pi}_\pi)(1-f_g) dV \; = e_\pi^c dV_\pi$$

$$= (3n_g^c T_c + B) dV_g \; -(n_g^c T_c - B + \tilde{\pi}_g) f_g dV \; -(P_c+\tilde{\pi}_\pi)(1-f_g) dV$$

$$= 3n_g^c T_c dV_g \; -(n_g^c T_c + \tilde{\pi}_g) f_g dV \; -(P_c+\tilde{\pi}_\pi)(1-f_g) dV$$

$$+B(dV_g + f_g dV)$$

Microscopic implementation scheme

Since

$$n_\pi^c dV_\pi - n_g^c dV_g = -\frac{\tilde{\pi}_m}{4T_c} dV$$

we consider the following microscopic processes:

$$g + g \leftrightarrow \pi + \pi, \qquad g + g \leftrightarrow \pi + \pi + \pi$$

Because of the latent heat from the bag pressure,

in $g + g \rightarrow \pi + \pi, \qquad g + g \rightarrow \pi + \pi + \pi$

the total momentum and kinetic energy are NOT conserved,

but

$$x \sum p_g = \sum p_\pi\,, \qquad x > 1$$

Microscopic implementation scheme

In back reactions

$$\pi + \pi \to g + g, \qquad \pi + \pi + \pi \to g + g$$

the total momentum and kinetic energy are conserved.

Pions should be those newly produced.

We combine back reactions to

$$g + g \to \pi + \pi \to g^* + g^*,$$

$$g + g \to \pi + \pi + \pi \to g^* + g^*,$$

$$g + g \to \pi + \pi + \pi \to g^* + g^* + \pi$$

Microscopic implementation scheme

We have finally

$$g + g \to \pi + \pi, \quad g + g \to \pi + \pi + \pi,$$

$$g + g \to g^* + g^*, \quad g + g \to g^* + g^* + \pi$$

with the corresponding probabilities $P_{22}, P_{23}, P_{22b}, P_{23b}$.

Microscopic implementation scheme

The number of gluons in the volume element V: $N_g = n_g^c f_g V$

number of lost gluons in V during dτ:

$$\frac{1}{2} N_g (N_g - 1) \left(2P_{23} + 2P_{22}\right) = n_g^c dV_g \, ,$$

number of gained pions in V during dτ:

$$\frac{1}{2} N_g (N_g - 1) \left(3P_{23} + 2P_{22} + P_{23b}\right) = n_\pi^c dV_\pi$$

energy of gained pions in V during dτ:

$$\frac{1}{2} N_g (N_g - 1) \left(P_{23} + P_{22} + \frac{1}{3} P_{23b}\right) 6 T_c x$$
$$= \; e_\pi^c dV_\pi + (n_\pi^c T_c + \tilde{\pi}_\pi)(1 - f_g) dV \, .$$

Microscopic implementation scheme

number of gained gluons (g*) in V during dτ:

$$\frac{1}{2}N_g(N_g-1)\,(2P_{22b}+2P_{23b})=N_{g^*}\ ,$$

energy of gained gluons (g*) in V during dτ:

$$\frac{1}{2}N_g(N_g-1)\left(P_{22b}+\frac{2}{3}P_{23b}\right)6T_c x$$
$$=\ 3N_{g^*}T_c+(n_g^c T_c+\tilde{\pi}_g)f_g dV\ .$$

eliminate the number of gained gluons (g*) :

$$\frac{1}{2}N_g(N_g-1)\left[P_{22b}(6T_c x-6T_c)+P_{23b}(4T_c x-6T_c)\right]$$
$$=\ (n_g^c T_c+\tilde{\pi}_g)f_g dV\ .$$

Microscopic implementation scheme

5 unknowns $P_{22}, P_{23}, P_{22b}, P_{23b}, x$, but 4 equations.

We set $P_{23} = 0$ and obtain

$$P_{22} = \frac{e_\pi^c + P_c + \tilde{\pi}_m}{n_g^c f_g^2 (e_g^c - e_\pi^c)} \nabla_\mu U^\mu \frac{d\tau}{V}$$

$$P_{23b} = -\frac{\tilde{\pi}_m}{2(n_g^c f_g)^2 T_c} \nabla_\mu U^\mu \frac{d\tau}{V}$$

$$x = \frac{e_\pi^c dV_\pi + (n_\pi^c T_c + \tilde{\pi}_\pi)(1 - f_g) dV}{T_c (2 n_\pi^c dV_\pi + n_g^c dV_g)}$$

$$P_{22b} = \frac{(n_g^c T_c + \tilde{\pi}_g) f_g + (x - \frac{3}{2}) \tilde{\pi}_m}{3 (n_g^c f_g)^2 T_c (x - 1)} \nabla_\mu U^\mu \frac{d\tau}{V}$$

Numerical results with comparisons

We simulate the transition from gluons to pions in a one dimensional expansion with Bjorken boost invariance.

$$U^{\mu} = \frac{1}{\tau}(t, 0, 0, z)$$

$$\Rightarrow \quad \frac{1}{V}\frac{dV}{d\tau} = \frac{1}{\tau}, \quad \frac{\nabla^{<\mu}U^{\nu>}\nabla_{<\mu}U_{\nu>}}{\nabla_{\mu}U^{\mu}} = \frac{2}{3\tau} \qquad \pi = -\frac{4}{3}\eta\frac{1}{\tau}$$

$\Rightarrow$ We can analytically solve $T_c,\ \mu_c,\ dV_g,\ f_g,\ n, e, s$ and also the times of the begin and end of the phase transition in each piece of expanding volume.

Comparisons with these values will prove our numerical scheme.

Numerical results with comparisons

Numerical extractions of the four-flow and energy-momentum tensor

$$N^\mu = \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{p^0} f = \frac{1}{V_{slice}} \frac{1}{N_{test}} \sum_i \frac{p_i^\mu}{p_i^0} \,,$$

$$T^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu p^\nu}{p^0} f = \frac{1}{V_{slice}} \frac{1}{N_{test}} \sum_i \frac{p_i^\mu p_i^\nu}{p_i^0}$$

flow velocity:
$$U^\mu = \frac{N_g^\mu + N_\pi^\mu}{\sqrt{(N_g^\nu + N_\pi^\nu)(N_{g\nu} + N_{\pi\nu})}}$$

densities:
$$n_i' = N_i^\mu U_\mu \,, \quad e_i' = U_\mu T_i^{\mu\nu} U_\nu$$

$$n_g = n_g'/f_g \,, \quad e_g = e_g'/f_g + B \,,$$

$$n_\pi = n_\pi'/(1 - f_g) \,, \quad e_\pi = e_\pi'/(1 - f_g)$$

Numerical results with comparisons

temperatures: $T_g = \frac{e_g - B}{3n_g} = \frac{e'_g}{3n'_g}\,, \quad T_\pi = \frac{e_\pi}{3n_\pi} = \frac{e'_\pi}{3n'_\pi}$

chemical potentials: $e^{\frac{\mu_i}{T_i}} = \frac{n_i}{n_i^{eq}}\,, \quad n_i^{eq} = \frac{d_i}{\pi^2} T_i^3$

Assuming $\mu_g/T_g = \mu_\pi/T_\pi$, we obtain

$$f_g(\tau) = \left(1 + \frac{d_g T_g^3}{d_\pi T_\pi^3} \frac{n'_\pi}{n'_g}\right)^{-1}$$

entropy densities:

$$s_i = \frac{e_i + P_i - \mu_i n_i}{T_i} = \left(4 - \frac{\mu_i}{T_i}\right) n_i$$

Numerical results with comparisons

We consider, for simplicity,

$$g + g \to g + g, \qquad \pi + \pi \to \pi + \pi$$

with constant cross section and isotropic distribution of the scattering angle.

shear viscosity: $\eta_i = \dfrac{6T_i}{5\sigma_i}$ $\qquad i = g, \pi$

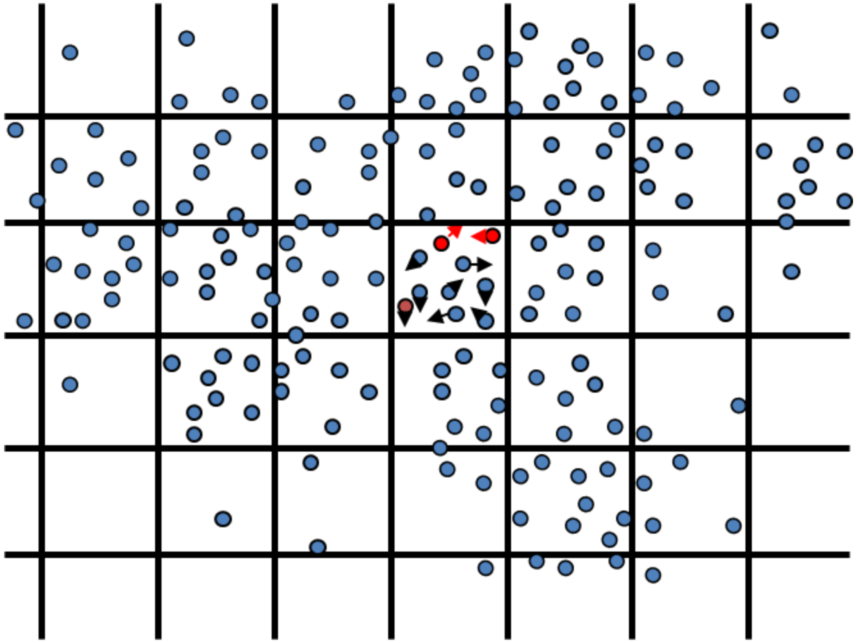
η_i / s_i are constant and same during the phase transition.

Numerical results with comparisons

BAMPS: Boltzmann Approach of MultiParton Scatterings

solves the **semi-classical**, **relativistic** Boltzmann equation in the framework of **pQCD** by **Monte Carlo** simulations.

ZX and C. Greiner, PRC 71, 064901 (2005)



$$\left(\partial_t + \frac{\vec{p}_1}{E_1} \cdot \vec{\nabla}\right) f_1(x, p_1) = C$$

test particle representation of f

stochastic interpretation of the collision rates

$$P_g = v_{rel} \frac{\sigma_g}{N_{test}} \frac{\Delta t}{f_g V_r}$$

Numerical results with comparisons

setups:

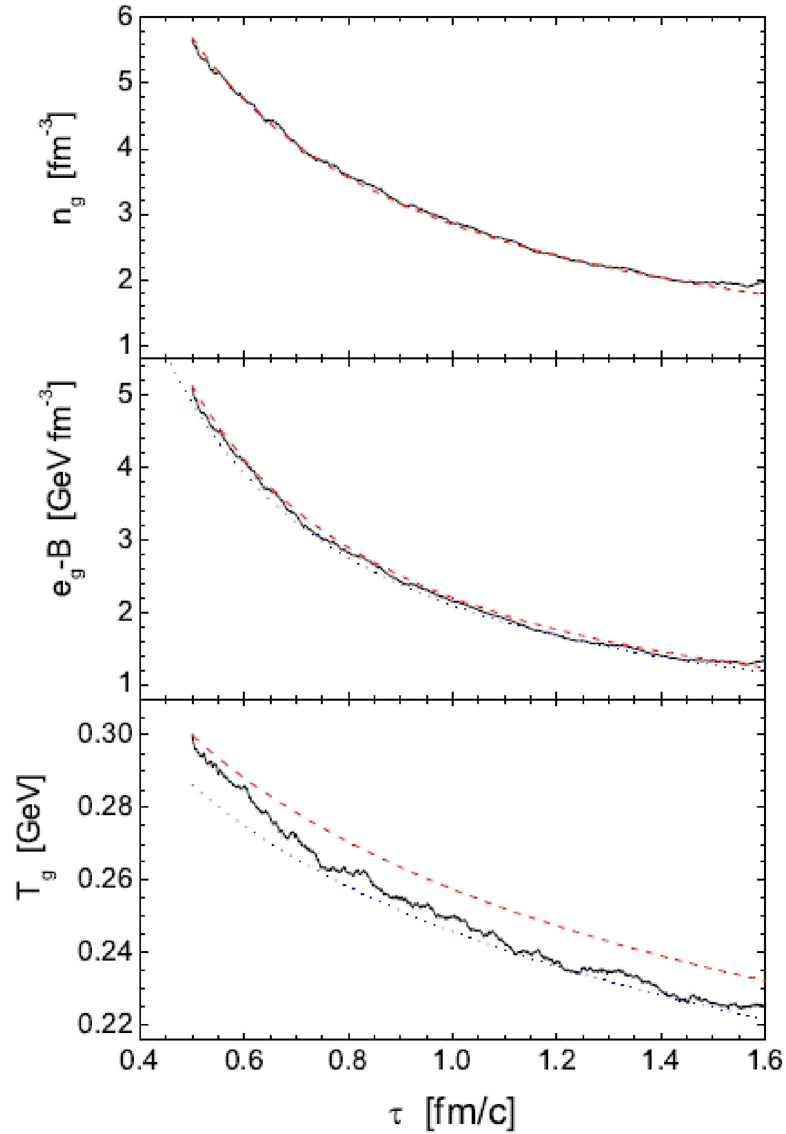
$T_0 = 0.3\ GeV$
$\tau_0 = 0.5\ fm/c$
$\sigma_g = 16.5\ mb$

analytical solutions:

$T_c = 0.2357\ GeV$
$\tau_c = 1.4979\ fm/c$
$\mu_c/T_c = -0.3735$
$\eta_g/s_g = 0.1045$

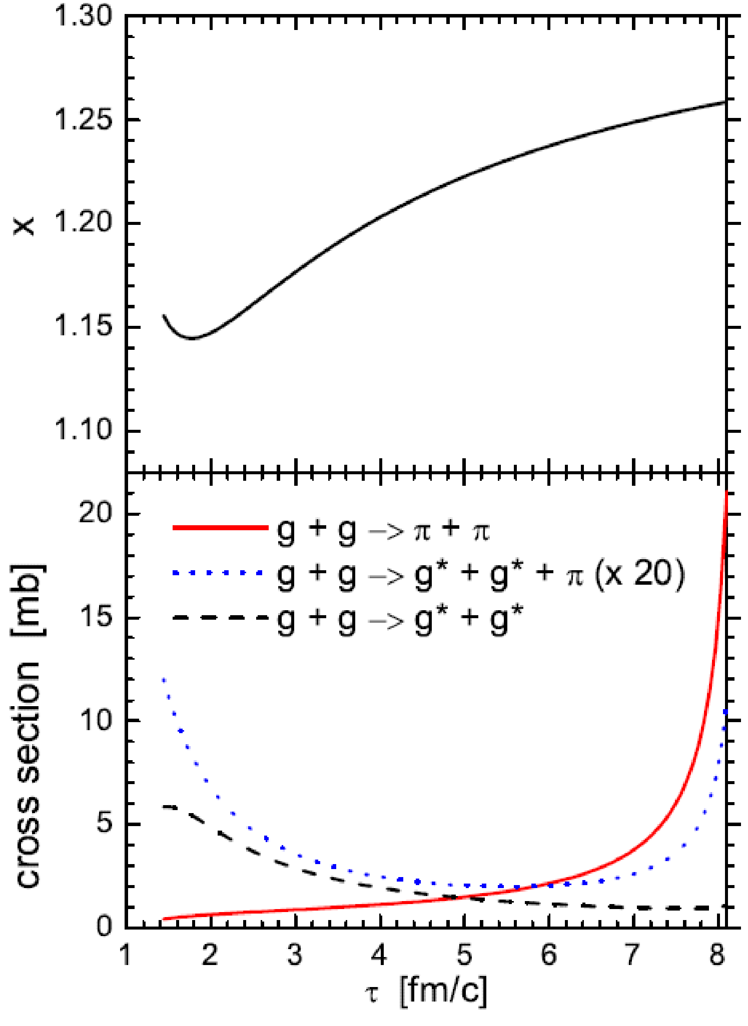
numerical results:

$T_c = 0.2269\ GeV$
$\tau_c = 1.4412\ fm/c$
$\mu_c/T_c = -0.224$
$\eta_g/s_g = 0.1004$



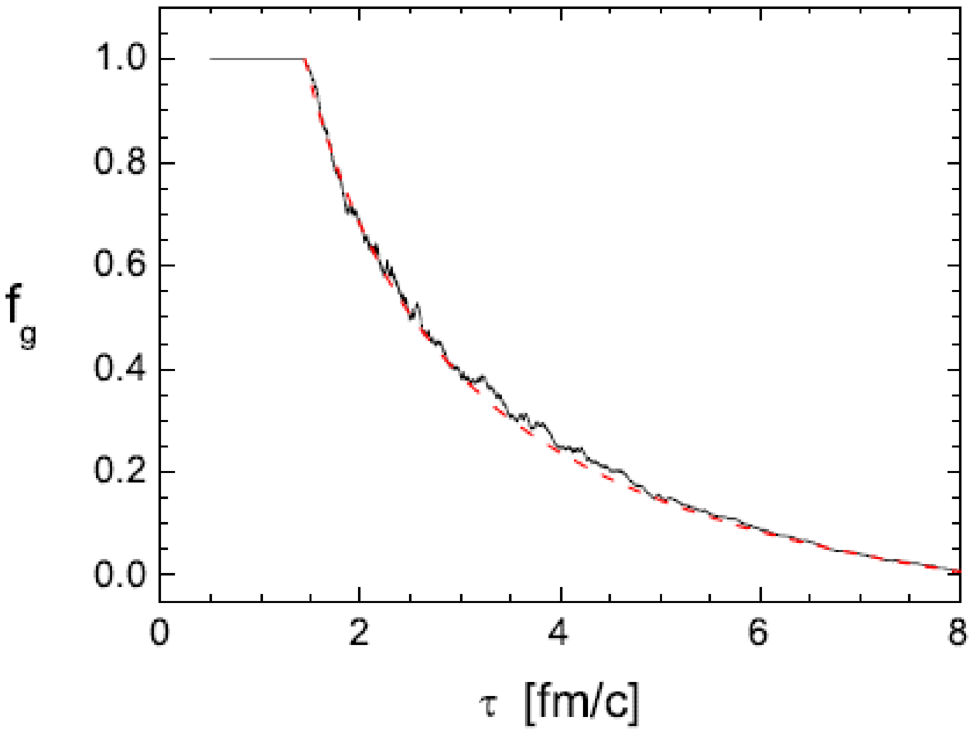
Numerical results with comparisons

Cross sections of the transition processes and the factor x

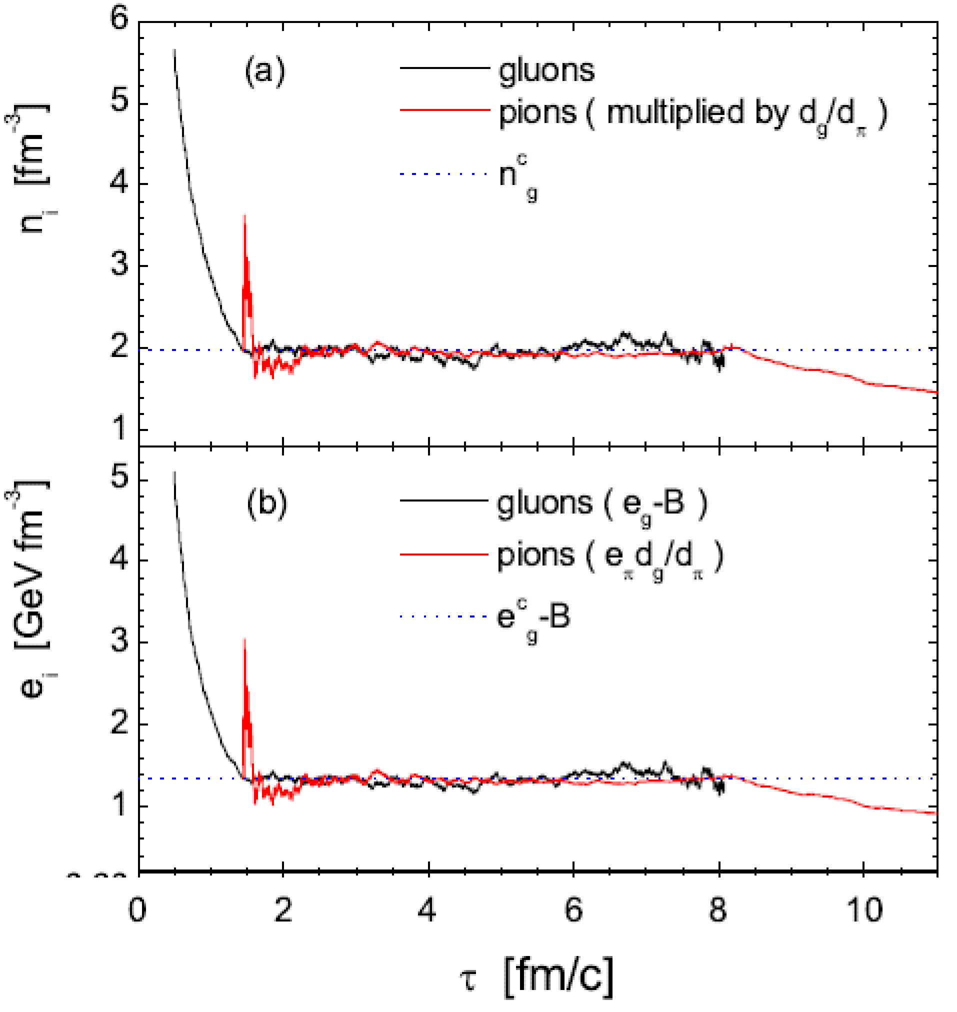


Numerical results with comparisons

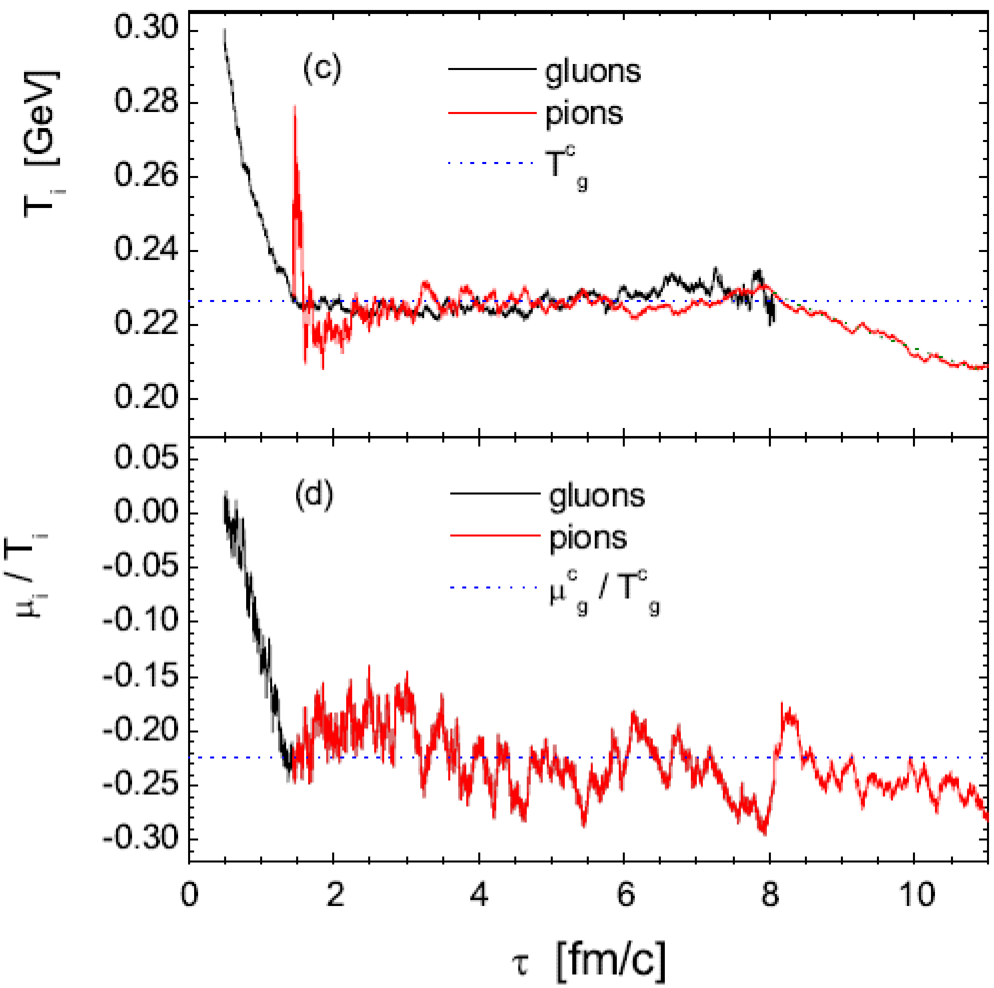
gluonic fraction



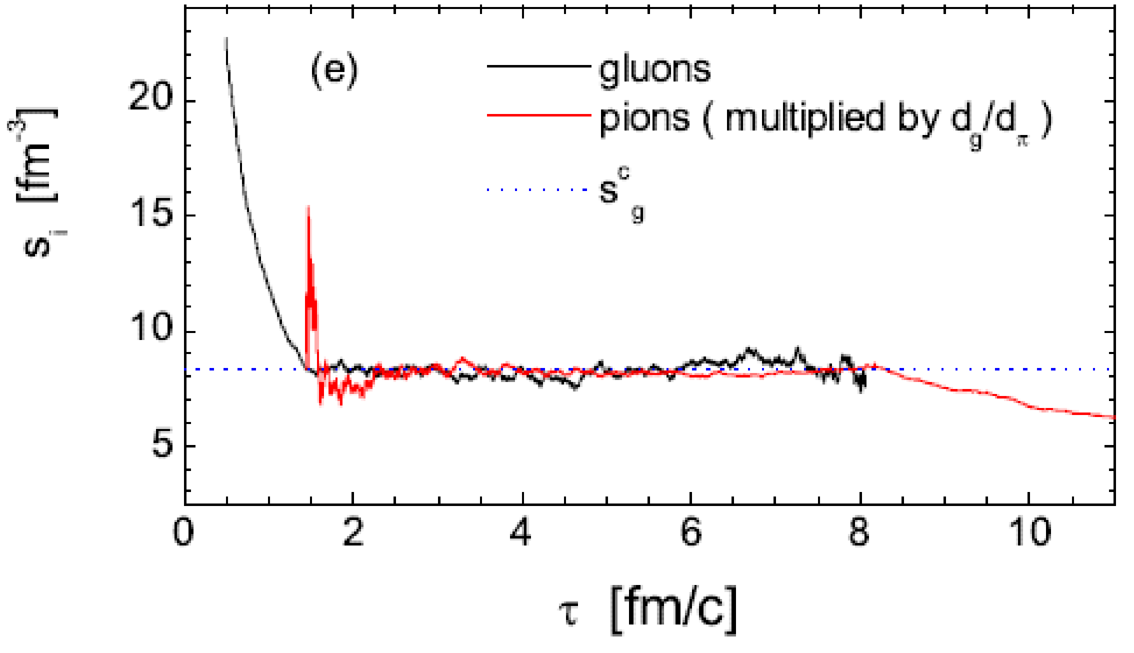
Numerical results with comparisons



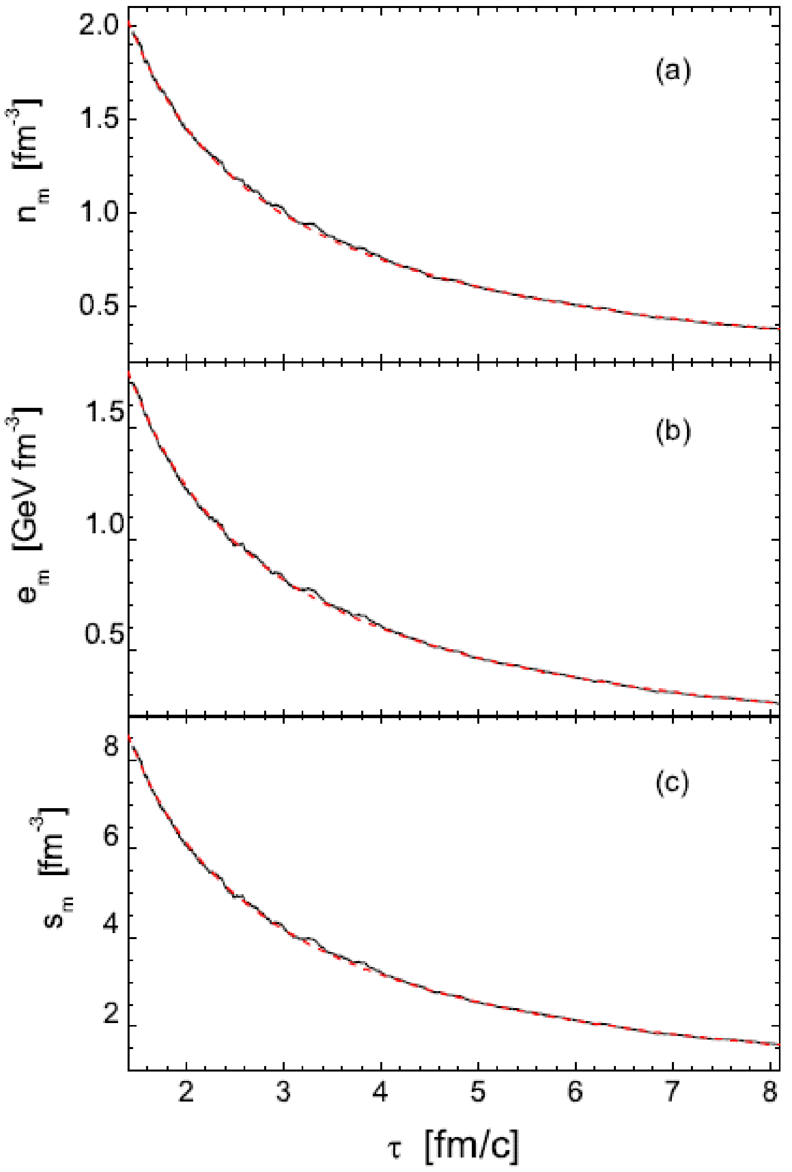
Numerical results with comparisons



Numerical results with comparisons



Numerical results with comparisons



$$n_m = n_g f_g + n_\pi (1 - f_g)$$
$$e_m = e_g f_g + e_\pi (1 - f_g)$$
$$s_m = s_g f_g + s_\pi (1 - f_g)$$

Summary and outlook

- establish a microscopic implementation scheme of hadronization with first-order phase transition
- show the preliminary results and comparisons

Further studies:

- limit of dissipation for the occurrence of QCD first-order phase transition (suggested by Pengfei)
- add quarks and more hadron species
- buildup of elliptic flow during the phase transition
- viscous effect during the phase transition

Theoretical description

energy balance:

$$e_g^c dV_g - (P_c + \tilde{\pi}_g) f_g dV - (P_c + \tilde{\pi}_\pi)(1 - f_g) dV = e_\pi^c dV_\pi$$

$$\Rightarrow \boxed{dV_g = \frac{e_\pi^c + P_c + \tilde{\pi}_m}{e_g^c - e_\pi^c} dV} \qquad \tilde{\pi}_m = f_g \tilde{\pi}_g + (1 - f_g)\, \tilde{\pi}_\pi$$

dissipation obtained from hydrodynamic equation:

$$De = -(e+P)\nabla_\mu U^\mu + \pi^{\mu\nu}\nabla_{<\mu}U_{\nu>} = -(e+P+\tilde{\pi})\nabla_\mu U^\mu \,,$$

$$\tilde{\pi} = -\frac{\pi^{\mu\nu}\nabla_{<\mu}U_{\nu>}}{\nabla_\mu U^\mu}$$