AdS/CFT & Some of its Applications

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QCD Phase structure III，wuhan June 6-9

Outlines

- **Introduction and motivation**
- **Heavy quark potential from AdS/CFT**
- **Jet quenching parameter from AdS/CFT**
- **G-L free energy of Holographic Superconductor**
- **Summary**

Motivations

Many interesting phenomena in QCD lie in strongly coulped region

Experiments aspect:

Heavy-ion Collisions@ RHIC & LHC:

sQGP-- the almost perfect fluid known $\eta/s >= .1\text{-}.2 << 1$

- **CDM : High T_c superconductor**

  Cold atoms .....

New theoretical techniques needed!

Lattice QCD

Continuum

(1) Effective Models: (p)NJL、(p)QMC…

(2) Field Theory: DS E , FRGE,   HT(d)L ,
Chiral Perturbation, Sum rules ….

(3) AdS/CFT,   AdS/QCD

AdS/CFT has been applied widely

* **Viscosity ratio, η/s.** $\frac{\eta}{s} = \frac{1}{4\pi}$ Policastro, Son and Starinet
* **Thermodynamics.** $s = \frac{3}{4} s^{(0)}$ Gubser
* **Jet quenching** Liu, Rajagopal and Wiederman
* **Photon production** Yaffe et al
* **Heavy quarkonium (hard probe)** Maldacena
* **Thermalization , phase transition**
* **Hardron spectrum (AdS/QCD) (M. Huang's talk)**
* **AdS/CDM** Herzog, Gubser, Hartnoll

AdS/CFT corerspondence

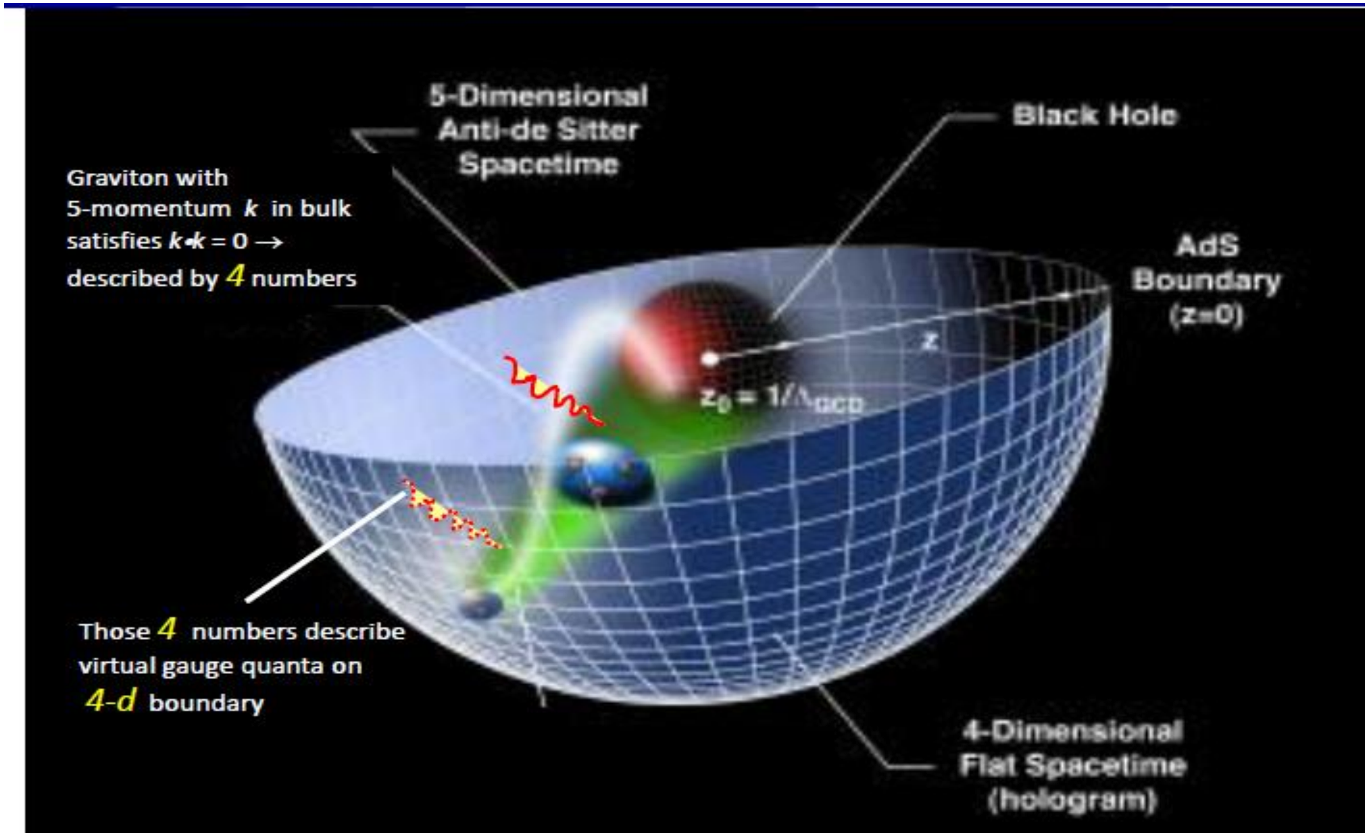
4dim. Large-Nc strongly coupled SU(Nc) N=4 SYM (finite T).

Maldacena '97 ↕ **conjecture** Witten '98

Type II B Super String theory on AdS5-BH×S5

Basic Idea For AdS/CFT

- Certain configuration in Higher Dimensional "AdS" Space dual to a Quantum (Conformal) Field Theory on some surface in that space
- Some complicated Field theory calculations become simple "geometric' problems in higher dimensions



(Adopted from S. Brodsky figure)

Maldacena conjecture: *Maldacena, Witten*

$$N=4 \text{ SUSY YM on the boundary} \quad \Leftrightarrow \quad \text{Type IIB string theory in the bulk}$$

't Hooft coupling $\lambda \equiv N_c g_{YM}^2 \quad = \quad \frac{1}{\alpha'^2} \qquad (\text{string tension} = \frac{1}{2\pi\alpha'})$

$$\frac{\lambda}{N_c} \quad = \quad 4\pi g_s$$

$$< e^{\int d^4x \phi_0(x) O(x)} > \quad = \quad Z_{\text{string}}[\phi(x,0) = \phi_0(\text{x})]$$

In the limit $N_c \to \infty$ and $\lambda \to \infty$

$$Z_{\text{string}}[\phi(x,0) = \phi_0(x)] = e^{-I_{\text{sugra}}[\phi]} \,|_{\phi(x,0)=\phi_0(x)}$$

$$I_{\text{sugra}}[\phi] = \text{classical supergravity action}$$

Heavy quark potential

The gravity dual of a Wilson loop at large N_c and large λ

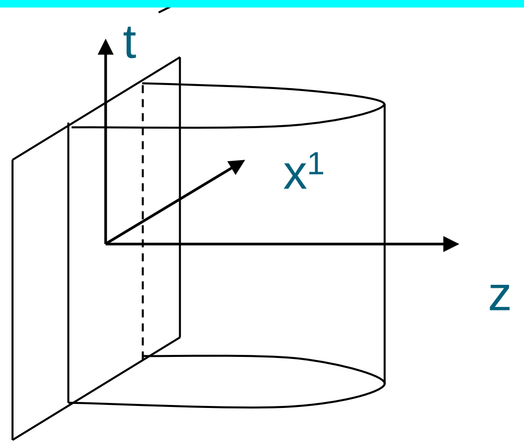
$$\mathrm{tr} < W(C) > = e^{-\sqrt{\lambda} S_{\min}[C]}$$

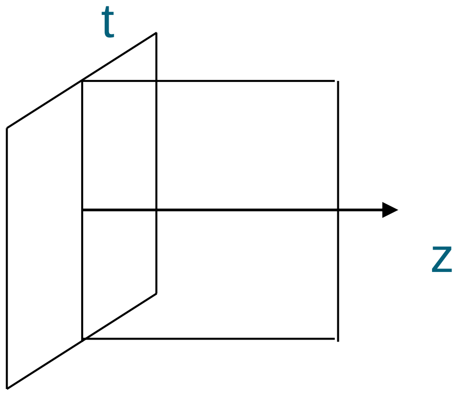
the min.area of string world sheet in the AdS_5

$$W(C) = P e^{-i \oint_C dx^\mu A_\mu(x)}$$

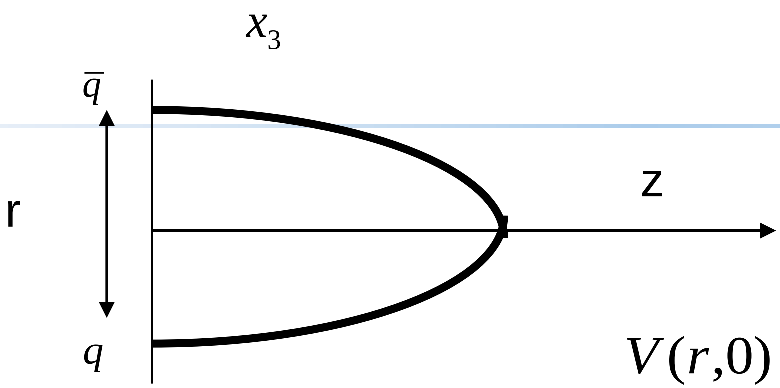
Heavy quark potential probes confinement hadronic phase and meson melting in plasma

$$F(r,T) = T\left(S_{\min}[\text{parallel lines}] - 2S_{\min}[\text{single line}]\right)$$





Heavy quark potential at zero temperature

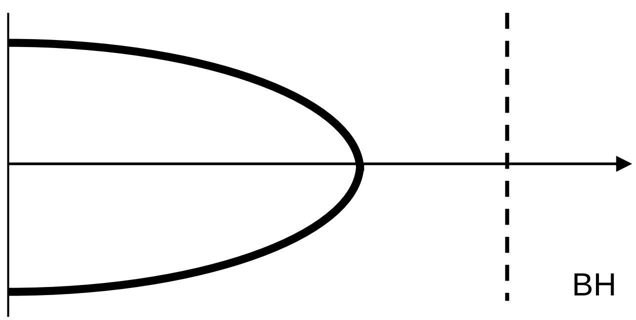


Maldacena 98

$$V(r,0) = F(r,0) = -\frac{4\pi^2\sqrt{2N_c g_{YM}^2}}{\Gamma^4\left(\frac{1}{4}\right)r}$$

world sheet at the minimum

Heavy quark potential at a nonzero temperature



Rey, Theisen and Yee

r_c

Free energy:

$$F(r,T) = -\frac{4\pi^2\sqrt{\lambda}}{\Gamma^4\left(\frac{1}{4}\right)r}\phi(\pi Tr)\theta(r_c - r)$$

$$\phi(\pi T r_c) = 0$$

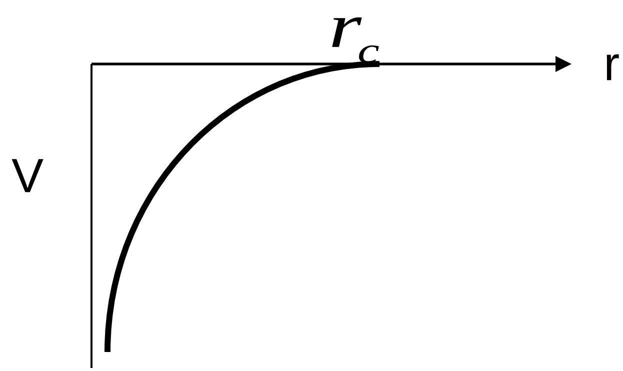
$$\phi(\rho) = 1 - \frac{\Gamma^4\left(\frac{1}{4}\right)}{4\pi^3}\rho + \frac{3\Gamma^8\left(\frac{1}{4}\right)}{640\pi^6}\rho^4 + O(\rho^8).$$

$$r_c \cong \frac{0.7541}{\pi T}$$

Potential:

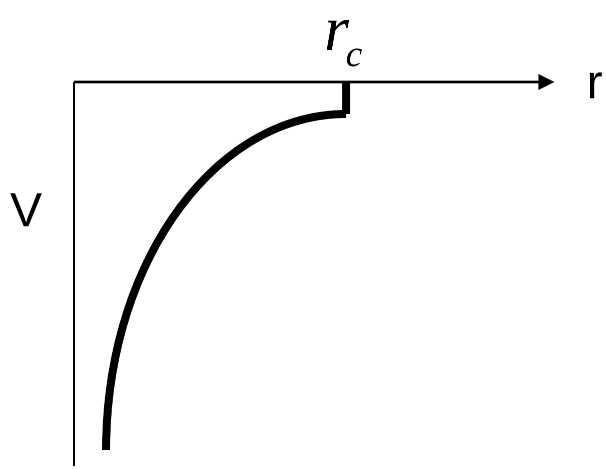
F-ansatz

$$V(r,T) = F(r,T)$$



U-ansatz

$$V(r,T) = F(r,T) + TS(r,T) = -T^2\frac{\partial}{\partial T}\left(\frac{F}{T}\right)$$



Non Yukawa screening!

Heavy quarkonium Dissociate Temperature

Hou , Ren, JHEP0801:029

ansatz	$J/\psi(1S)$	$J/\psi(2S)$	$J/\psi(1P)$	$\Upsilon(1S)$	$\Upsilon(2S)$	$\Upsilon(1P)$
F	67-124	15-28	13-25	197-364	44-81	40-73
U	143-265	27-50	31-58	421-780	80-148	92-171

With deformed metric

ansatz	J/ψ	Υ
F	NA	235-385
U	219-322	459-780

T_d/T_c

ansatz	J/ψ(holographic)	J/ψ(lattice)	Υ(holographic)	Υ(lattice)
F	NA	1.1	1.3-2.1	2.3
U	1.2-1.7	2.0	2.5-4.2	4.5

Relativistic correction

Guo, Shi, Zhuang ,PLB718 (2012)

Wu, Hou , Ren , PRC 87 (2013),025203

	$c\bar{c}$		$b\bar{b}$	
	$\lambda = 5.5$	$\lambda = 6\pi$	$\lambda = 5.5$	$\lambda = 6\pi$
$1s$	162.54	387.54	478.76	1139.11
$2s$	29.15	62.75	85.67	184.44
$1p$	32.04	62.14	94.18	182.66

This lists the results of $T_0 + \delta_1 T$ in MeV's, that we just considered the correction of the p^4 term, which increased the dissociation temperature.

Relativistic correction

Wu, Hou , Ren , PRC 87 (2013),025203

	$c\bar{c}$		$b\bar{b}$	
	$\lambda = 5.5$	$\lambda = 6\pi$	$\lambda = 5.5$	$\lambda = 6\pi$
$1s_0^1$	130.79	188.65	385.63	555.58
$1s_1^3$	130.79	188.65	385.63	555.58
$2s_0^1$	26.71	48.16	79.15	142.59
$2s_1^3$	26.71	48.16	79.15	142.59
$1p_1^1$	31.53	61.33	93.54	180.79
$1p_0^3$	32.65	68.48	96.85	201.80
$1p_1^3$	32.09	64.90	95.20	191.30
$1p_2^3$	30.96	57.76	91.89	170.29

We wrote the state as: nL_J^{2S+1}

For J/Psi,the magnitude of the correction ranges from 8% to 30%!

Higher order corrections

Leading orders are strictly valid when $N_c \to \infty$, $\lambda \to \infty$

- **For real QCD. The t'Hooft coupling is not infinity**

$$5.5 < \lambda < 6\pi.$$

- **The super gravity correction to the AdS-Schwarschild metric is of order** $O(\lambda^{-\frac{3}{2}})$

,

- **The fluctuation around the minimum world sheet presents at all T, and is of order** $O(\lambda^{-\frac{1}{2}})$ **(more important)**

Semi-classical expansion

Strong coupling expansion

The type ⅡB superstring

N=4 SYM

The path integral of the string-sigma action

Wilson loop expectation

Classical limit

Leading order

One loop correction

Sub-leading order

Gravity dual of a Wilson loop at finite coupling

Metsaev and Tseytlin

$$W[C] \equiv < \exp\left(i \oint_C dx^\mu A_\mu \right) > = \int [dX][d\theta] \exp\left[\frac{i}{2\pi\alpha'} S(X,\theta) \right]$$

Strong coupling expansion ⇄ Semi-classical expansion

$$\ln W[C] = i\sqrt{\lambda}\left[S\left(\bar{X}, 0\right) + \frac{b[C]}{\sqrt{\lambda}} + \ldots \right]$$

$$X^\mu = \bar{X}^\mu + \delta X^\mu, \qquad \theta \neq 0 \qquad g_{ij} = \bar{g}_{ij} + \delta g_{ij}$$

$\bar{X}$

Partition function at finite T with fluctuations

Hou, Liu, Ren, PRD80,2009

Straight line：

$$Z = Z_B Z_F = \frac{\det^2\left(-\nabla_+^2 + 1 + \frac{1}{4}R^{(2)}\right)\det^2\left(-\nabla_-^2 + 1 + \frac{1}{4}R^{(2)}\right)}{\det^{\frac{3}{2}}\left(-\nabla^2 + \frac{8}{3} + \frac{1}{2}R^{(2)}\right)\det^{\frac{5}{2}}(-\nabla^2)}$$

Parallel lines：

$$Z = \frac{\det^2\left(-\nabla_+^2 + 1 + \frac{1}{4}R^{(2)}\right)\det^2\left(-\nabla_-^2 + 1 + \frac{1}{4}R^{(2)}\right)}{\det^{\frac{1}{2}}\left(-\nabla^2 + 4 + R^{(2)} - 2\delta\right)\det(-\nabla^2 + 2 + \delta)\det^{\frac{5}{2}}(-\nabla^2)}$$

Next Leading order Results

Chu, Hou, Ren, JHEP0908, (2009)

$$V(r) \approx -\frac{4\pi^2}{\Gamma^4(\frac{1}{4})}\frac{\sqrt{\lambda}}{r}[1-\frac{1.33460}{\sqrt{\lambda}}+O(\frac{1}{\lambda})] \quad \text{for } \lambda \gg 1$$

Confirmed by Forini JHEP 1011 (2010) 079

$$\begin{aligned} a_1 &= \frac{5\pi}{12} - 3\ln 2 + \frac{2\mathbb{K}}{\pi}\left(\mathbb{K} - \sqrt{2}(\pi + \ln 2) + \mathcal{I}^{\text{num}}\right) \\ &= -1.3345953052806007 7364\ldots , \end{aligned}$$

-1.33460

$$V_{\text{ladder}}(r) = -\frac{\sqrt{\lambda}}{\pi r}\left(1 - \frac{\pi}{\sqrt{\lambda}}\right).$$

Erickson etc. NPB582, 2000

Next leading order potential at finite T

Zhang,Hou, Ren,Yin JHEP07:035（2011）

$$V(r) \simeq -\frac{4\pi^2}{\Gamma^4\left(\frac{1}{4}\right)}\frac{\sqrt{\lambda}}{r}\left[g_0(rT) - \frac{1.33460 g_1(rT)}{\sqrt{\lambda}} + O\left(\frac{1}{\lambda}\right)\right]$$

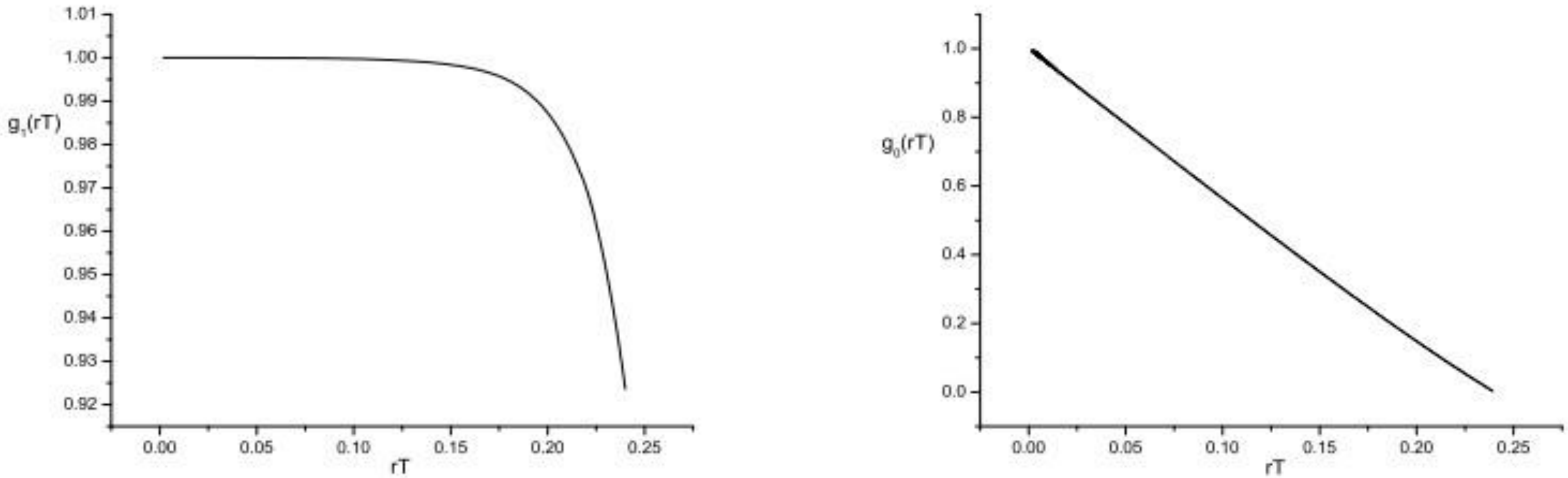
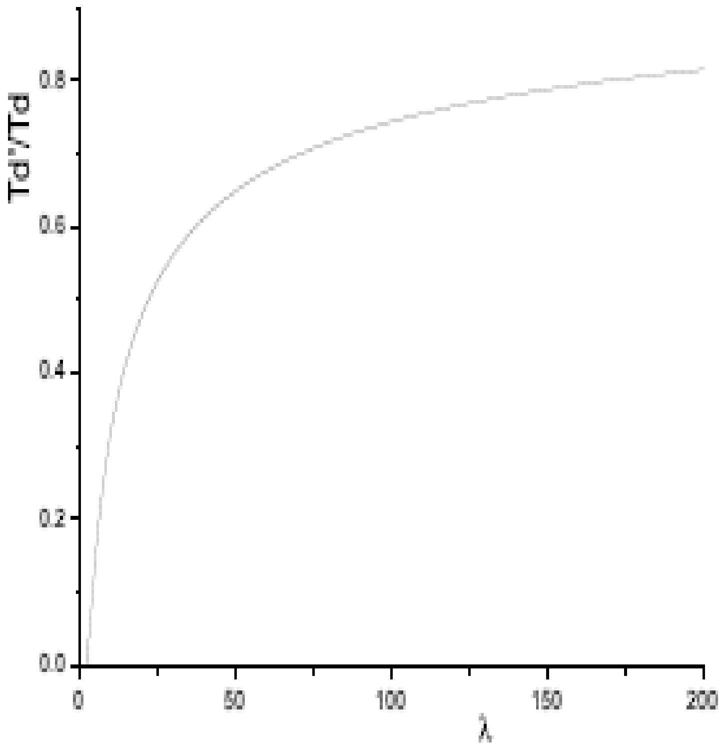


Figure 3. The left curve represents $g_1(rT)$, while the right represents $g_0(rT)$.

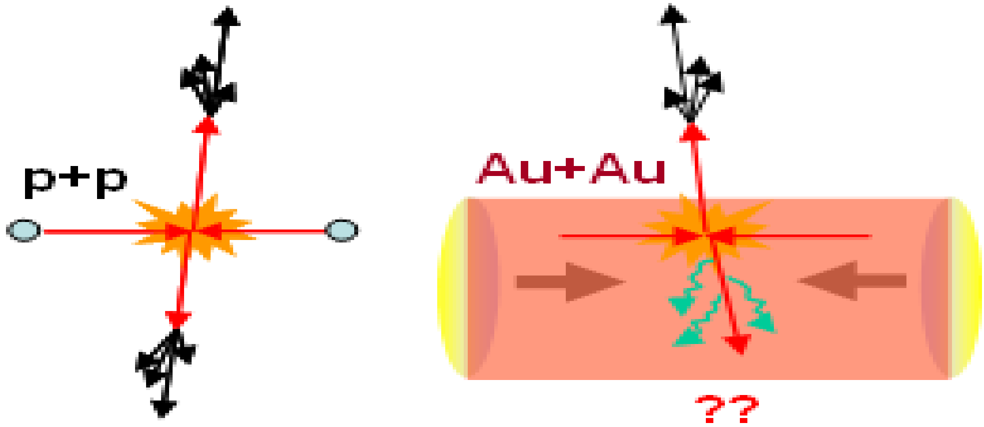
Heavy quarkonium Dissociate Td with NL potential

ZQ Zhang , Yan Wu, Defu Hou,2015

	$T_d(\lambda = 5.5)$	$T_d(\lambda = 6\pi)$
$J/\Psi(1s)$	143	265
$J/\Psi(2s)$	27	50
$J/\Psi(1p)$	31	58
$\Upsilon(1s)$	421	780
$\Upsilon(2s)$	80	148
$\Upsilon(1p)$	92	171



Jet quenching in QGP

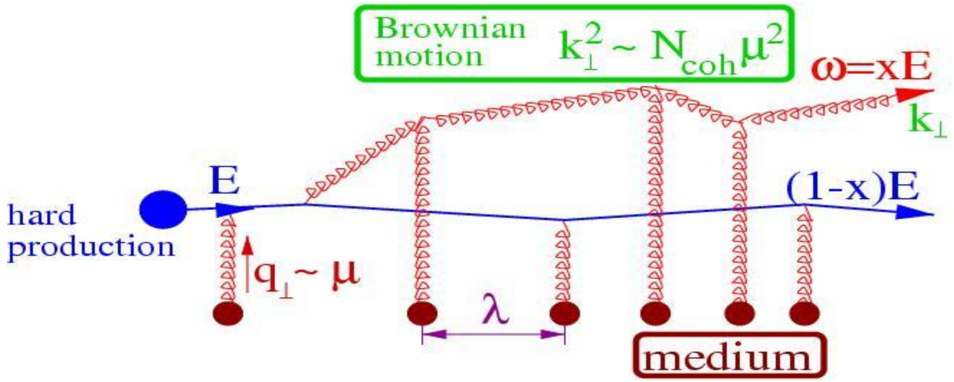


$$\Delta E \approx -\frac{\alpha_S}{2\pi} N_C \, \hat{q} \, L^2$$

Baier, Dokshitzer, Mueller, Peigne, Schiff (1996):

$\hat{q}$ **reflects the ability of the medium to "quench" jets.**

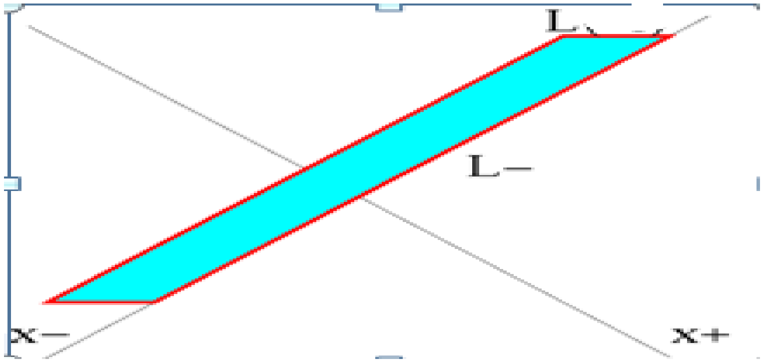
A non-perturbative definition of $\hat{q}$



Wiedemann 2000

$$W^A[C] = \exp\left(-\frac{\hat{q}\, L_- L^2}{4\sqrt{2}}\right)$$

$$L^- >> 1/T >> L$$

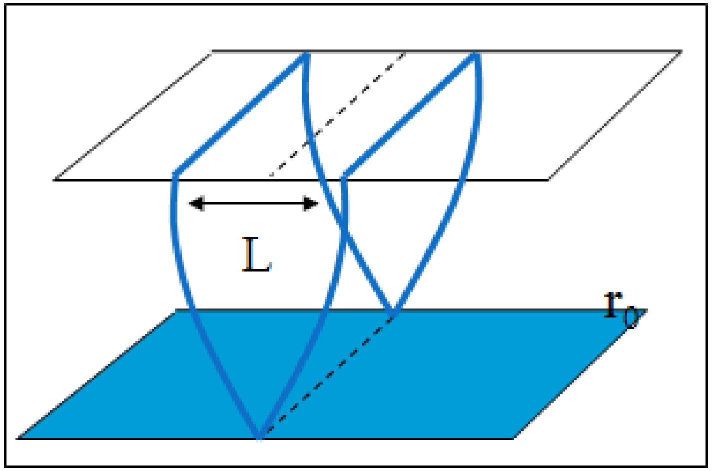


Leading order jet quenching parameter from AdS/CFT

Liu, Rajagopal & Wiedemann, PRL,97,182301(2006)

$$\hat{q}_0 = \frac{\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})}\sqrt{\lambda}T^3$$

Dipole amplitude: two parallel Wilson lines in the light cone:

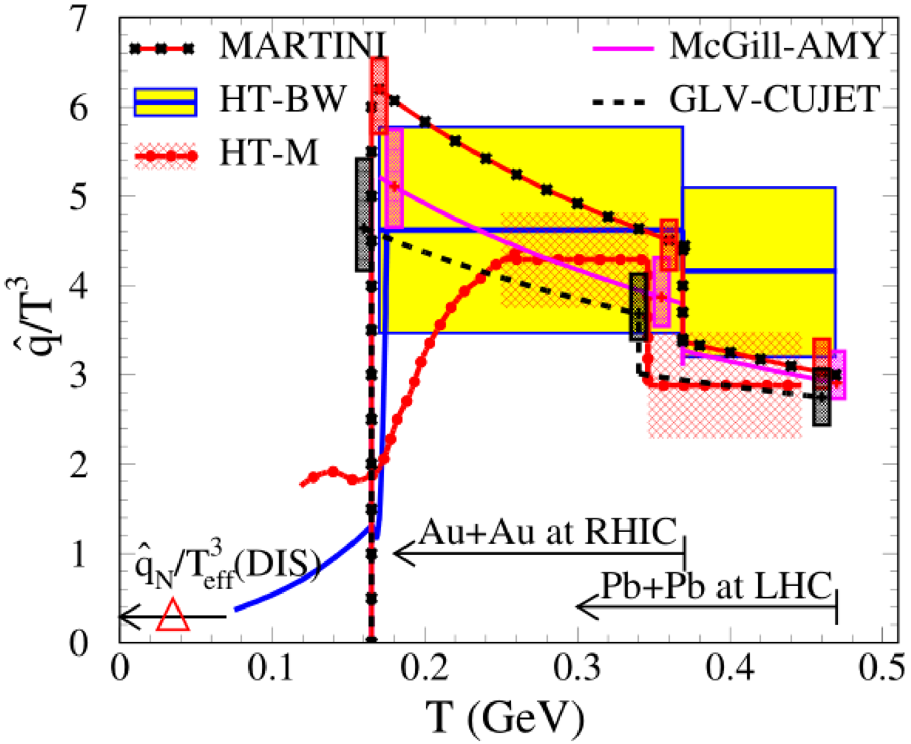


NL correction to jet quenching parameter

Zhang, Hou, Ren, JHEP1301 (2013) 032

$$\hat{q} = \frac{\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})}\sqrt{\lambda}T^3[1-1.97\lambda^{-1/2}+O(\lambda^{-1})]$$

Agrees with that from data



Jet Collaboration, PRC 90,014909(2014)

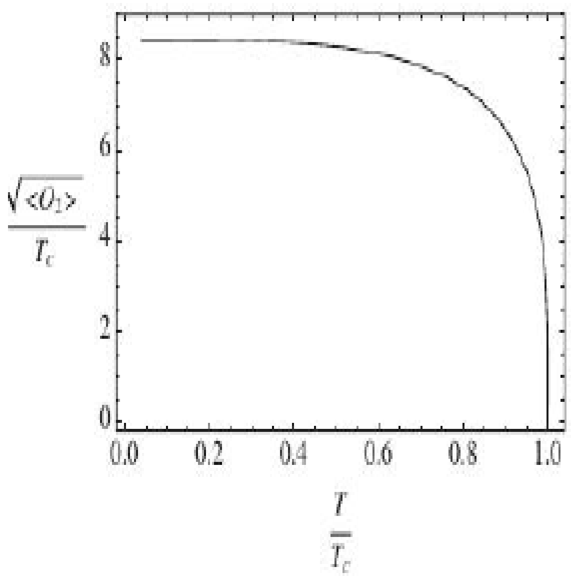
G-L free energy of HSC

Gubser, Horowitz, Hartnoll, Herzog PRD78 (2008); PRL101 (2008)

$$S_{\mathrm{HSC}} = \int d^{d+1}x\sqrt{|g|}\left(R - 2\Lambda - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |\nabla\psi - iqA\psi|^2 - V(|\psi|)\right)$$

$$V(|\Psi|) = m^2\Psi^2$$

Boundary	Bulk
Thermodynamic Temperature	Hawking Temperature
Order-Parameter	Hairy Black Hole



G-L free energy of HSC

Yin, Hou, Ren, PRD91 (2015)

$$\omega = -\frac{\mu^2}{2z_s} + a_{GCE}\ \langle \mathcal{O}_\Delta \rangle^2 + \frac{1}{2}\, b_{GCE}\ \langle \mathcal{O}_\Delta \rangle^4$$

$\triangle$	Grand Canonical Ensemble	Canonical Ensemble
1	$\langle \mathcal{O}_1 \rangle = 16.37\, T_c \sqrt{1 - \frac{T}{T_c}}$ $a_{GCE} = 1.696\ (T - T_c);$ $b_{GCE} = 6.33\ \times 10^{-3} \frac{1}{T_c}$ $\omega_{on\text{-}shell} = -45.568\, T_c^3 - 227.204\, T_c (T_c - T)$	$\langle \mathcal{O}_1 \rangle = 9.462\, T_c \sqrt{1 - \frac{T}{T_c}}$ $a_{CE} = 3.391\ (T - T_c);$ $b_{CE} = 3.788 \times 10^{-2} \frac{1}{T_c}$ $f_{on\text{-}shell} = 45.568\, T_c^3 - 512.398\, T_c (T_c - T)^2$
2	$\langle \mathcal{O}_2 \rangle = 163.68\, T_c^2 \sqrt{1 - \frac{T}{T_c}}$ $a_{GCE} = 0.0725\, \frac{1}{T_c} \left(\frac{T}{T_c} - 1\right);$ $b_{GCE} = 2.706\ \times 10^{-6} \frac{1}{T_c^3}$ $\omega_{on\text{-}shell} = -606.896\, T_c^3 - 971.034\, T_c (T_c - T)^2$	$\langle \mathcal{O}_2 \rangle = 143.574\, T_c^2 \sqrt{1 - \frac{T}{T_c}}$ $a_{CE} = 0.145\, \frac{1}{T_c} \left(\frac{T}{T_c} - 1\right)$ $b_{CE} = 7.0346 \times 10^{-6} \frac{1}{T_c^3}$ $f_{on\text{-}shell} = 606.896\, T_c^3 - 1493.898\, T_c (T_c - T)^2$

BCS vs Holographic Superconductor

Yin, Hou, Ren, PRD91 (2015)

Holographic Superconductor

Grand Canonical

$$\langle \mathcal{O}_1 \rangle = 16.37 T_c \sqrt{1 - \frac{T}{T_c}}$$

Canonical

$$\langle O_1 \rangle = 9.462 T_c \sqrt{1 - \frac{T}{T_c}}$$

BCS theory

$$\Delta = \sqrt{-\frac{a}{b}} = T_c \sqrt{\frac{8\pi^2}{7\zeta(3)} \left(1 - \frac{T}{T_c}\right)} = 3.0633 T_c \sqrt{1 - \frac{T}{T_c}}$$

Summary and discussion

AdS/CFT provides a useful way to address the physics at strong coupling .

Heavy quark potential & Jet quenching parameter are computed up to sub-leading order from AdS/CFT

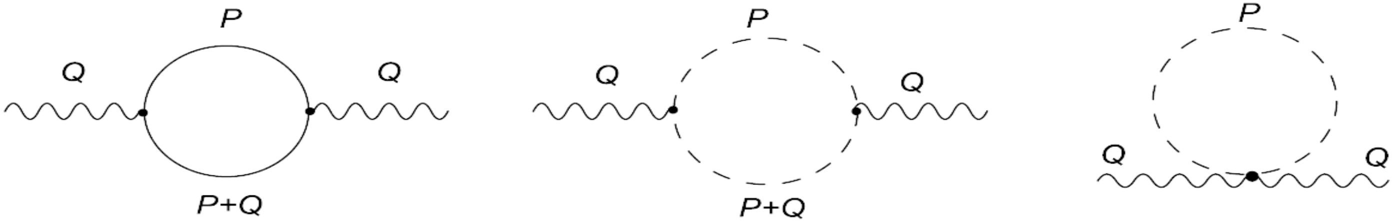
We estimated the melting T with holographic potential and its relativitic correction

The GL free-energy of HSC are derived

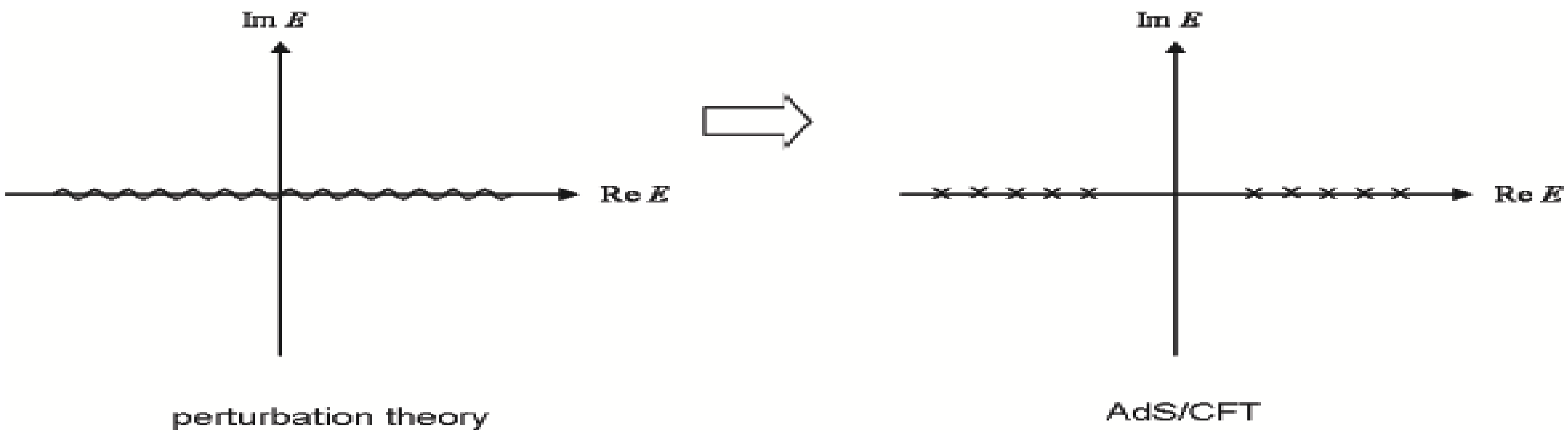
The applicability of those results demands phenomenological work to explain them in a way which can be translated to real QCD.

Thanks

2-point correlators from perturbation



(a) Fermion loop (b) Scalar loop (c) Scalar self-coupling



Continuum spectrum

Bound sates

JHEP 1007:042,2010, Hou, Liu , Li , Ren

Shear viscosity from AdS/CFT

$$\eta = \lim_{\omega\to 0} \frac{1}{2\omega} \int dt\, dx e^{i\omega t} \langle \left[T_{xy}(t,x), T_{xy}(0,0) \right] \rangle$$

$$\sigma_{abs} = -\frac{16\pi G}{\omega} \text{Im}\, G^R(\omega)$$

$$= \frac{8\pi G}{\omega} \int dt\, dx e^{i\omega t} \langle \left[T_{xy}(t,x), T_{xy}(0,0) \right] \rangle$$

$$\eta = \frac{\sigma_{abs}(0)}{16\pi G}$$

Graviton's component h_y^x obeys equation for a minimally coupled massless scalar. But then $\sigma_{abs}(0) = A_H$.

Since the entropy (density) is $s = A_H/4G$

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

D. Son, P. Kovtun, A.S

Correlation function from AdS/CFT

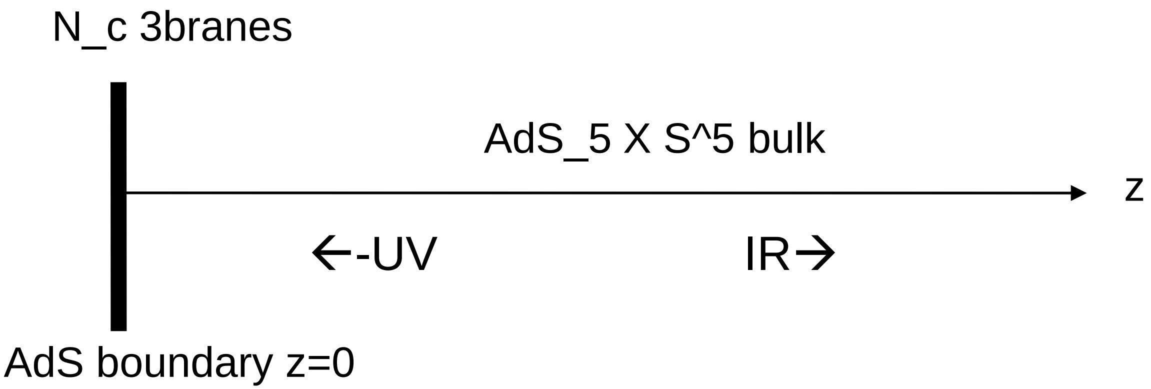
- Solving the Maxwell equation and the linearized Einstein equation subject to the boundary conditions

$$S_{\rm sugr} = S^{(0)}_{\rm sugr} + \frac{1}{2}\int_{u=0} d^4x \int_{u=0} d^4y \left[\mathcal{C}_{\mu\nu}(x-y)\bar{A}^{\mu}(x)\bar{A}^{\nu}(y) + \frac{1}{4}\mathcal{C}_{\mu\nu,\rho\lambda}(x-y)\bar{h}^{\mu\nu}(x)\bar{h}^{\rho\lambda}(y)\right]$$
$$= \frac{1}{2}\int \frac{d^4\vec{Q}}{(2\pi)^4}\left[\mathcal{C}_{\mu\nu}(Q)\bar{A}^{\mu *}(Q)\bar{A}^{\nu}(Q) + \frac{1}{4}\mathcal{C}_{\mu\nu,\rho\lambda}(Q)\bar{h}^{\mu\nu *}(Q)\bar{h}^{\rho\lambda}(Q)\right]$$

- The coefficients $C_{\mu\nu}$, give rise to the R-photon self-energy tensor

$$F(q) \equiv \mathcal{C}_{00}(0,q) = -\frac{N_c^2 T^2}{8}\frac{A_0'(\varepsilon|q)}{A_0(\varepsilon|q)}$$

Policastro, Son & Starinets, JHEP0209(02)043



The metric at T=0

$$ds^2 = \frac{1}{z^2}\left(-dt^2 + d\mathbf{x}^2 + dz^2\right) + d\Omega_5^2$$

The metric at T>0

$$ds^2 = \frac{1}{z^2}\left(-fdt^2 + d\mathbf{x}^2 + \frac{dz^2}{f}\right) + d\Omega_5^2$$

$$f = 1 - \frac{z^4}{z_h^4} \qquad z_h = \frac{1}{\pi T}$$