EM fields in conducting medium and properties of chiral fermions

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Contents

- **Analytic solutions for EM fields of point charges in electrically and chirally conducting medium (σ and σ_χ)**
- **Properties of chiral fermions from Wigner function: magnetic energy and spin-vorticity coupling**

Quark-gluon plasma: from big bang to little bang

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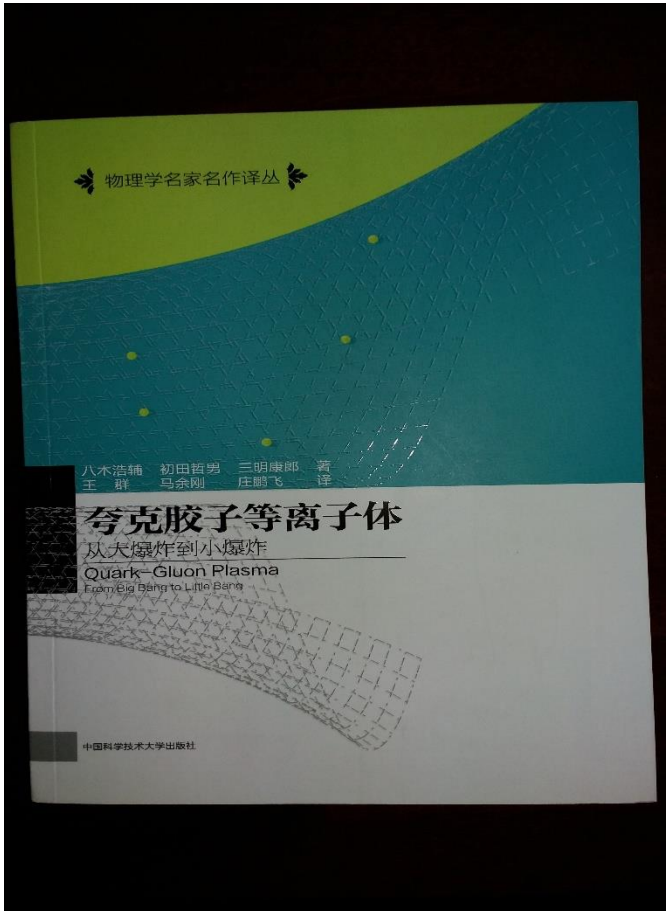
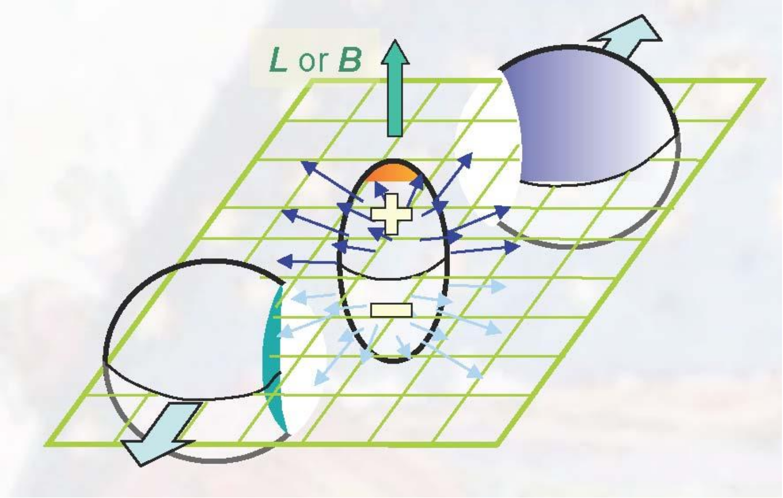


EM field in HIC



- **High energy HIC**

$$v = \sqrt{(s - m_n^2)/s} \sim 1 - \frac{m_n^2}{2s}$$
$$\gamma = 1/\sqrt{1 - v^2/c^2} \sim \frac{\sqrt{s}}{m_n}$$

- **Electric field in cms frame of nucleus,**

$$\mathrm{E} = \frac{Ze}{R^2}\hat{\mathrm{r}}$$

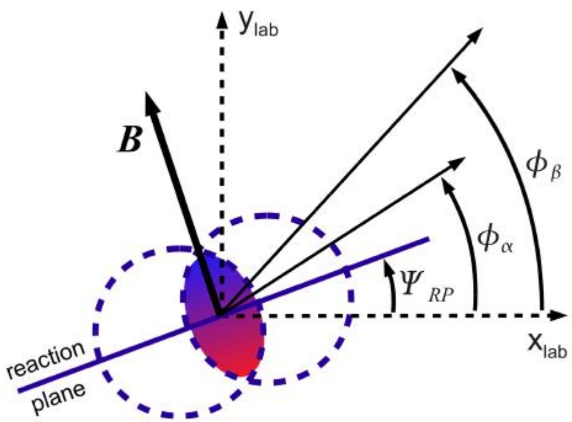
- **Boost to Lab frame (v_z= 0.99995 c for 200GeV),**

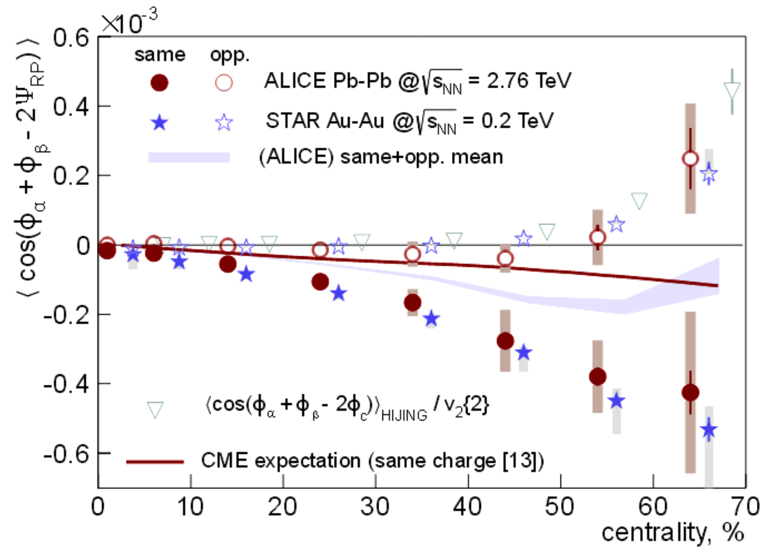
Scale of strong interaction

$$\mathrm{B} = -\gamma \mathbf{v}_z \times \mathrm{E} \rightarrow eB \rightarrow 2\gamma v_z \frac{Ze^2}{R^2} \sim \boxed{1.3 m_\pi^2} \sim 2.6 \times 10^{18}\ \mathrm{Gs}$$

Kharzeev, McLerran, Warringa (2008), Skokov (2009), Deng & Huang (2012), Bloczynski, Huang, Zhang, Liao (2012); many others ……

Charge Separation Effects in HIC





$$\langle\cos(\phi_\alpha + \phi_\beta - 2\Psi_{\mathrm{RP}})\rangle = \langle\cos\Delta\phi_\alpha \cos\Delta\phi_\beta\rangle - \underline{\langle\sin\Delta\phi_\alpha \sin\Delta\phi_\beta\rangle}$$

P-violation term

H.Z.Huang's talk
X.G. Huang's talk

STAR Collab., PRL 103, 251601 (2009); PRC 81, 054908(2010)
ALICE Collab., PRL 110, 012301 (2013).

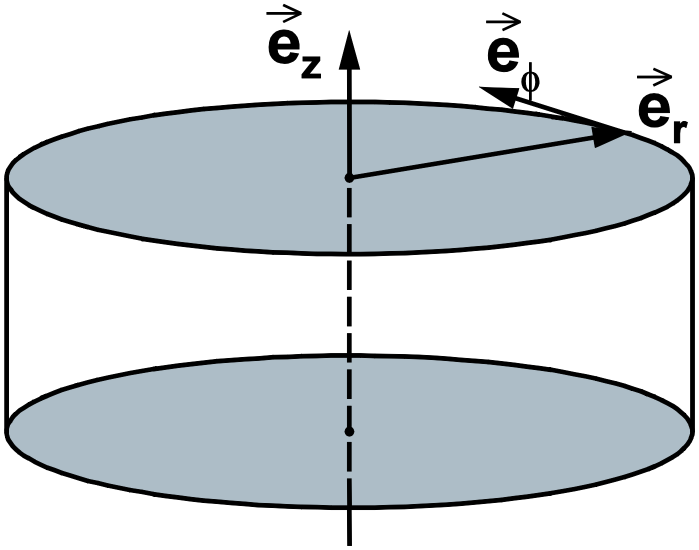
The interpretation of STAR and ALICE data is under debate. The mechanism behind the Charge Separation Effect is still inconclusive.

EM fields of a moving charge in vacuum

A positive charge moves in z direction

Lienard-Wiechert form

$$
\begin{aligned}
E_r &= \frac{Q}{4\pi} \cdot \frac{\gamma x_T}{[\gamma^2(z-vt)^2 + x_T^2]^{3/2}} \\
E_z &= \frac{Q}{4\pi} \cdot \frac{\gamma (z-vt)}{[\gamma^2(z-vt)^2 + x_T^2]^{3/2}} \\
B_\phi &= \frac{Q}{4\pi} \cdot \frac{v\gamma x_T}{[\gamma^2(z-vt)^2 + x_T^2]^{3/2}} = vE_r \\
E_\phi &= 0 \\
B_{r,z} &= 0
\end{aligned}
$$



As t→∞, E and B decay as $E_r \sim B_\phi \sim \frac{1}{t^3}$

EM fields in conducting medium

In conducting medium with conductivity σ,

E and B can have inducting contribution

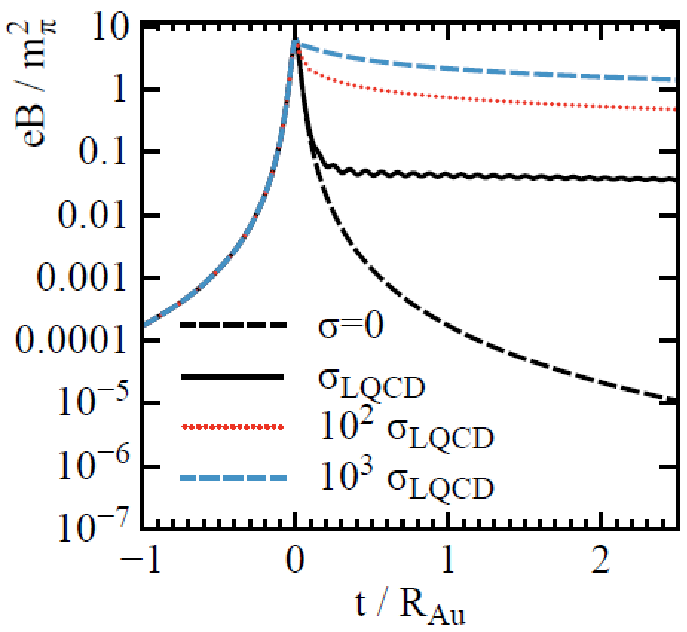
and make fields last longer

$$E_r \sim B_\phi \sim \frac{1}{t^2}$$

Also we have to include

the effect of chiral magnetic

conductivity σ_χ



Mclerran and Skokov, 2013; Tuchin, 2014

Maxwell equations in conducting medium

In conducting medium with σ and σ_χ

$$
\begin{aligned}
\nabla \cdot \mathbf{E} &= \frac{\rho_{\mathrm{ext}}}{\epsilon}, \\
\nabla \cdot \mathbf{B} &= 0, \\
\nabla \times \mathbf{E} &= -\partial_t \mathbf{B}, \\
\nabla \times \mathbf{B} &= \partial_t \mathbf{E} + \mathbf{J}_{\mathrm{ext}} + \sigma \mathbf{E} + \sigma_\chi \mathbf{B},
\end{aligned}
$$

External sources

Inducting current

Taking curl of last two Eqs

Source terms

$$
\begin{aligned}
\left(\nabla^2 - \partial_t^2 - \sigma \partial_t\right) \mathbf{B} + \sigma_\chi \nabla \times \mathbf{B} &= -\nabla \times \mathbf{J}_{\mathrm{ext}}, \\
\left(\nabla^2 - \partial_t^2 - \sigma \partial_t\right) \mathbf{E} + \sigma_\chi \nabla \times \mathbf{E} &= \frac{1}{\epsilon} \nabla \rho_{\mathrm{ext}} + \partial_t \mathbf{J}_{\mathrm{ext}}.
\end{aligned}
$$

Differential operators L

Formal solution in momentum space

Both Eqs have same structure, in momentum space

$$\begin{pmatrix} L & -i\sigma_\chi k_z & i\sigma_\chi k_y \\ i\sigma_\chi k_z & L & -i\sigma_\chi k_x \\ -i\sigma_\chi k_y & i\sigma_\chi k_x & L \end{pmatrix} \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} (\omega, \mathbf{k}) = \begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix} (\omega, \mathbf{k}),$$

$$L = \omega^2 + i\sigma\omega - k^2$$

The formal solution in momentum space

$$\mathbf{F}(\omega, \mathbf{k}) = \frac{1}{L^2 - \sigma_\chi^2 k^2} \left[L\mathbf{f}(\omega, \mathbf{k}) - i\sigma_\chi \mathbf{k} \times \mathbf{f}(\omega, \mathbf{k}) \right],$$

$$\mathbf{f}(\omega, \mathbf{k}) = \begin{cases} -i\mathbf{k} \times \mathbf{J}_{\text{ext}}(\omega, \mathbf{k}), & \text{for } \mathbf{B} \\ i\mathbf{k} \frac{\rho_{\text{ext}}(\omega, \mathbf{k})}{1 + i\sigma/\omega} - i\omega \mathbf{J}_{\text{ext}}(\omega, \mathbf{k}), & \text{for } \mathbf{E} \end{cases}$$

Formal solution in momentum space

Positive point charge moves in +z

$$\rho(t,\mathbf{x}) = Q\delta(x)\delta(y)\delta(z-vt),$$
$$\mathbf{J}(t,\mathbf{x}) = Qv\delta(x)\delta(y)\delta(z-vt)\mathbf{e}_z.$$

$$\rho(\omega,\mathbf{k}) = 2\pi Q\delta(\omega-k_z v),$$
$$\mathbf{J}(\omega,\mathbf{k}) = 2\pi Qv\delta(\omega-k_z v)\mathbf{e}_z.$$

momentum space

Magnetic fields in momentum space

additional constraint

$$\begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix}(\omega,\mathbf{k}) = -2\pi i Qv\frac{\delta(\omega-k_z v)}{L^2-\sigma_\chi^2 k^2}\begin{pmatrix} Lk_y - i\sigma_\chi k_x k_z \\ -Lk_x - i\sigma_\chi k_y k_z \\ i\sigma_\chi(k_x^2+k_y^2) \end{pmatrix}$$

$$\mathbf{B}(t,\mathbf{x}) = \int \frac{d\omega d^3\mathbf{k}}{(2\pi)^4} e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}}\mathbf{B}(\omega,\mathbf{k})$$

poles

(a) k_z integral to remove delta function; (b) ω integral on contour; (c) k_T integral

Formal solution in momentum space: poles

Four poles from

$$L^2(\omega, k_T) - \sigma_\chi^2 \frac{\omega^2}{v^2} - \sigma_\chi^2 k_T^2 = 0$$

Upper-half plane poles $\mathrm{Im}\omega_+ \quad > \quad 0$

$$\mathrm{Im}\omega_+ > 0 \quad \rightarrow \quad \int_{-\infty}^{\infty} d\omega e^{-i\omega(t-z/v)} \mathbf{B}(\omega, \mathbf{k}) \rightarrow \theta(z - vt)$$

Advanced contribution

Lower-half plane poles $\mathrm{Im}\omega_- \quad < \quad 0$

$$\mathrm{Im}\omega_- < 0 \quad \rightarrow \quad \int_{-\infty}^{\infty} d\omega e^{-i\omega(t-z/v)} \mathbf{B}(\omega, \mathbf{k}) \rightarrow \theta(vt - z)$$

Retarded contribution

Pole structure

Four poles

$$\omega_{s_1 s_2} \equiv \omega_{s_1} + s_2\sigma_\chi c_{s_1}^{(1)} + \sigma_\chi^2 c_{s_1}^{(2)}, \qquad (s_1, s_2 = \pm 1),$$

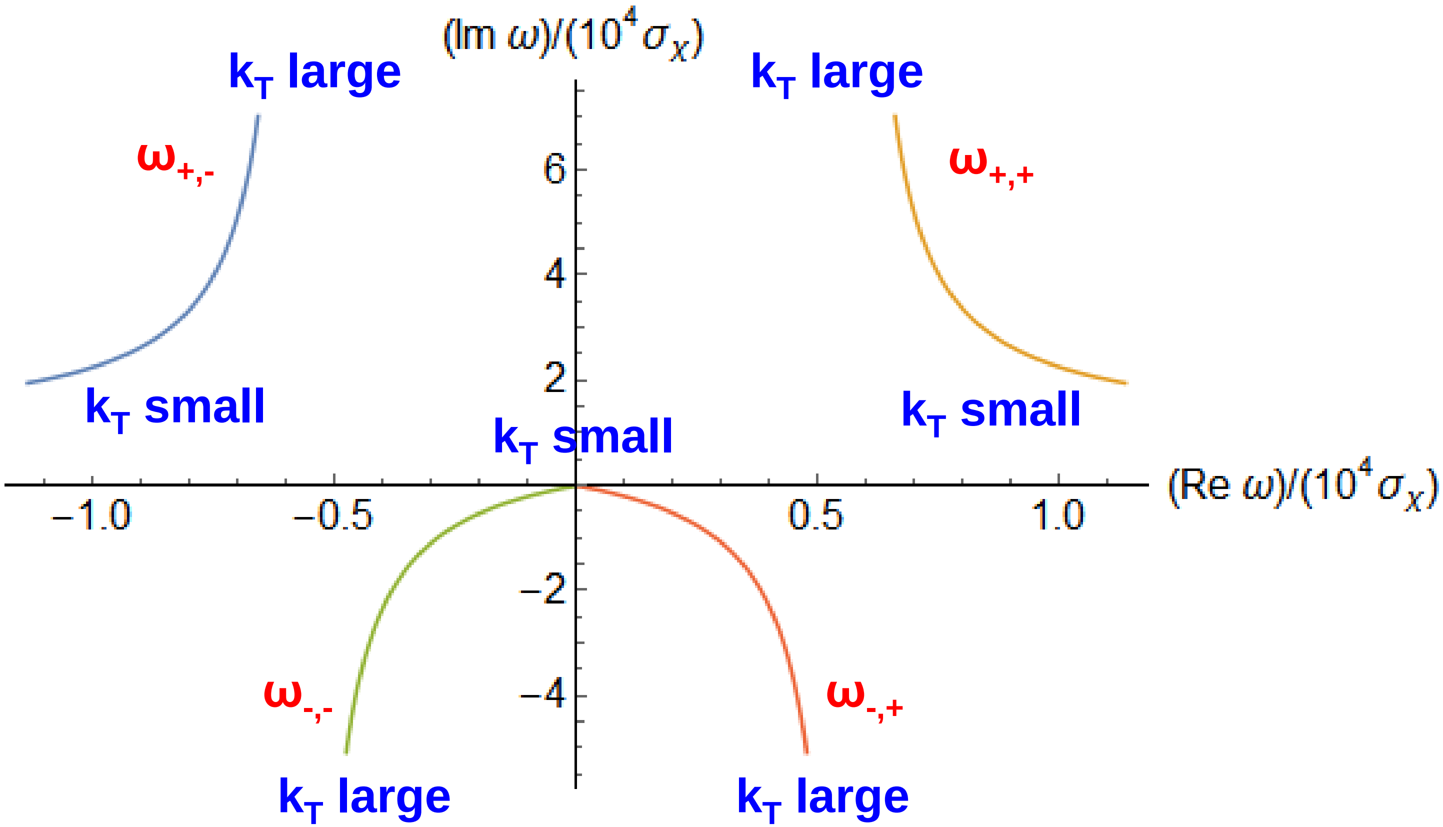
$$\omega_\pm \equiv iv\gamma\frac{1}{2}\left[v\gamma\sigma \pm \sqrt{(v\gamma\sigma)^2 + 4k_T^2}\right]$$

Two poles $\omega_{+,+}$, $\omega_{+,-}$ are in upper half-plane,

$\omega_{-,-}$ is in lower half-plane,

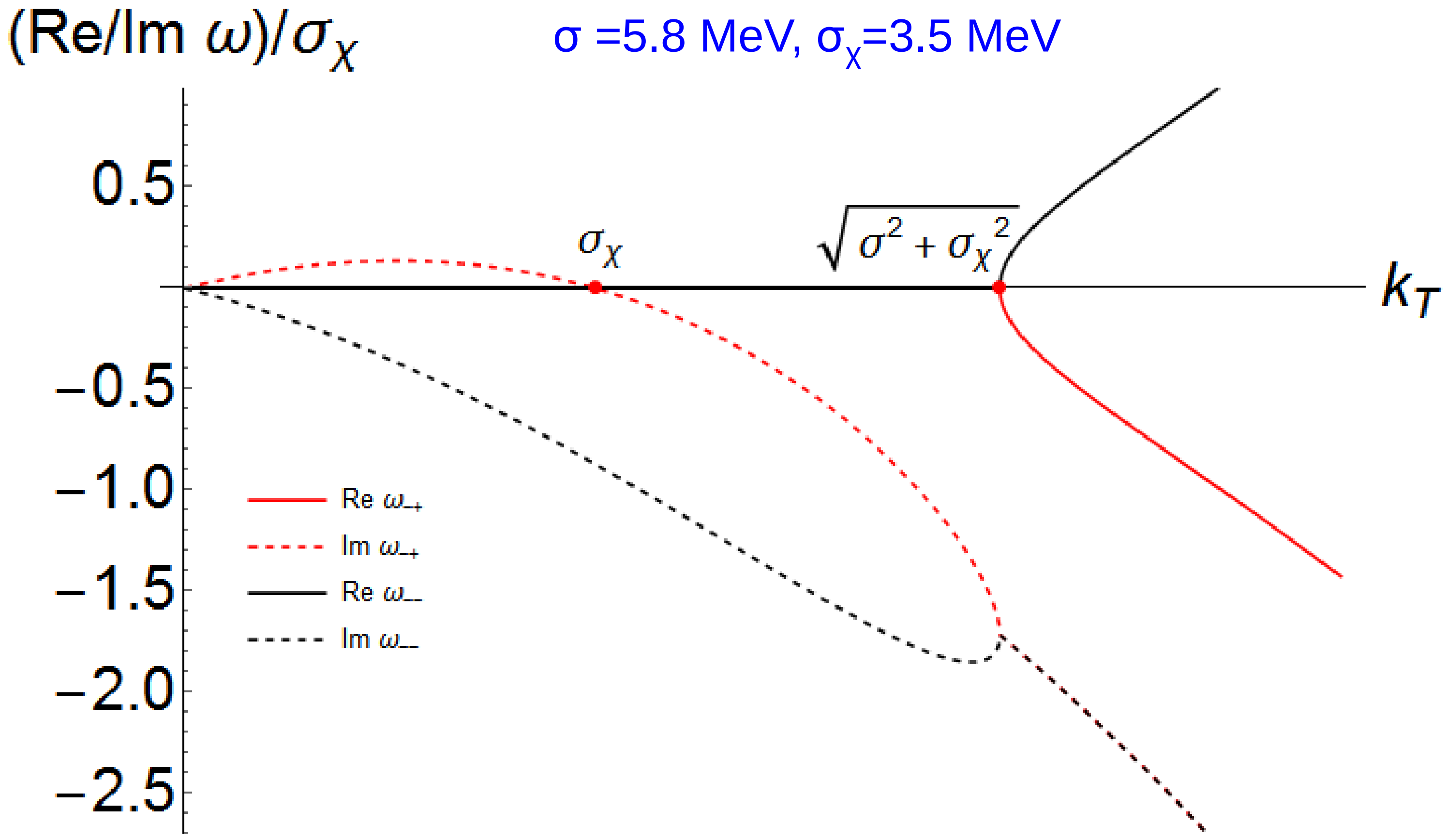
but $\omega_{-,+}$ is in upper ($k_T<\sigma_x$) or lower ($k_T>\sigma_x$) half-plane.

Pole straucture



Pole structure

σ =5.8 MeV, σ_χ=3.5 MeV



Magnetic field: algebraic form

Algebraic solutions for magnetic fields

$$B_\phi(t,\mathbf{x}) = \frac{Q}{4\pi}\cdot\frac{v\gamma x_T}{\Delta^{3/2}}\left(1+\frac{\sigma v\gamma}{2}\sqrt{\Delta}\right)e^{A}$$

Gursoy, Kharzeev, Rajagopal 1401.3805

$$B_r(t,\mathbf{x}) = -\sigma_\chi\frac{Q}{8\pi}\cdot\frac{v\gamma^2 x_T}{\Delta^{3/2}}\left[\gamma(vt-z)+A\sqrt{\Delta}\right]e^{A}$$

$$B_z(t,\mathbf{x}) = \sigma_\chi\frac{Q}{8\pi}\cdot\frac{v\gamma}{\Delta^{3/2}}\left[\gamma^2(vt-z)^2\left(1+\frac{\sigma v\gamma}{2}\sqrt{\Delta}\right)+\Delta\left(1-\frac{\sigma v\gamma}{2}\sqrt{\Delta}\right)\right]e^{A}$$

Our new result:
Li, Sheng, QW, 1602.02223

where we use

$$\Delta \equiv \gamma^2(z-vt)^2+x_T^2 = \mathbf{x}'\cdot\mathbf{x}'$$

$$A \equiv \frac{1}{2}(\sigma v\gamma)\left[\gamma(vt-z)-\sqrt{\Delta}\right]$$ <0

$$\mathbf{x}' = (x_T, z') = (x_T, \gamma(z-vt))$$

Electric field: algebraic form

Algebraic solutions for electric fields

$$E_\phi = \sigma_\chi \frac{Q}{8\pi} \frac{v^2\gamma^2 x_T}{\Delta^{3/2}} \left[\gamma(vt - z) + A\sqrt{\Delta}\right] e^A$$

$E_\phi = -vB_r$

γ>>1

$$E_r = \frac{Q}{4\pi}\left\{\frac{\gamma x_T}{\Delta^{3/2}}\left(1 + \frac{\sigma v \gamma}{2}\sqrt{\Delta}\right) - \frac{\sigma}{v x_T}\left[1 + \frac{\gamma(vt - z)}{\sqrt{\Delta}}\right]\right\} e^A$$

$$E_z = \frac{Q}{4\pi}\left\{-e^A \frac{1}{\Delta^{3/2}}\left[\gamma(vt - z) + A\sqrt{\Delta} + \frac{\sigma\gamma}{v}\Delta\right] + \frac{\sigma^2}{v^2} e^{-\sigma(t - z/v)}\Gamma(0, -A)\right\}$$

Our new result:
Li, Sheng, QW, 1602.02223

where we use

$$\Delta \equiv \gamma^2(z - vt)^2 + x_T^2 = \mathbf{x}' \cdot \mathbf{x}'$$

$$A \equiv \frac{1}{2}(\sigma v \gamma)\left[\gamma(vt - z) - \sqrt{\Delta}\right]$$ <0

Retarded and Advanced fields

The exponential factor exp(A) plays a major role. The advanced field is exponentially suppressed.

$$
\begin{aligned}
A_{\mathrm{Adv}} &= \frac{1}{2}(\sigma v\gamma)\left[\underbrace{\gamma(vt-z)}_{<0} - \sqrt{\gamma^2(z-vt)^2+x_T^2}\right] < 0 \\
A_{\mathrm{Ret}} &= \frac{1}{2}(\sigma v\gamma)\left[\underbrace{\gamma(vt-z)}_{>0} - \sqrt{\gamma^2(z-vt)^2+x_T^2}\right] < 0 \\
A_{\mathrm{Adv}} - A_{\mathrm{Ret}} &= -\sigma v\gamma^2|vt-z| \quad \Longrightarrow \quad \frac{\mathrm{Advanced}}{\mathrm{Retartded}} = \exp(-\sigma v\gamma^2|vt-z|)
\end{aligned}
$$

This indicates the time reversal (T) symmetry is broken: retarded solution dominates in conducting medium. This is dissipation effect from σ:

electric energy → heat

Causality and instability

Charged particle moves in z-direction, exp(A):

(1) v<1, A_{adv} is finite

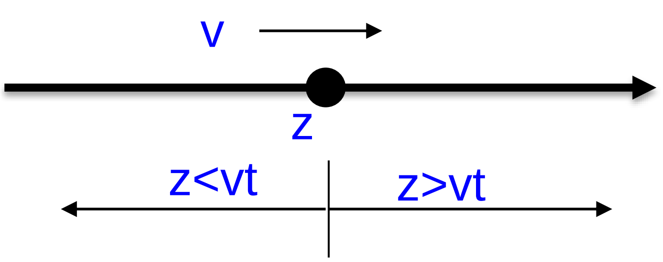
there is advanced part

(2) v=1 and γ=∞, we have

$A_{adv}=-\infty \rightarrow \exp(A_{adv})=0$

no advanced part!!

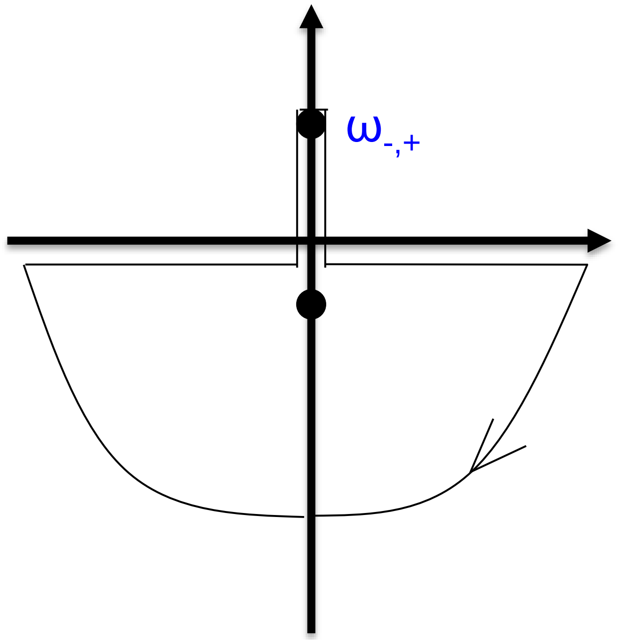
fields cannot move faster than particle



$$\begin{aligned} A_{\rm adv} &= \frac{1}{2}(\sigma v\gamma)[\gamma(vt-z) \\ &\quad -\sqrt{\gamma^2(z-vt)^2+x_T^2}] \\ &< 0 \end{aligned}$$

Causality and instability

- **Instability occur when $k_T<\sigma_x$, then $\omega_{-,+}$ is in upper half-plane, leading to instable mode.**
- **But if $R<1/\sigma_x$, the $1/k_T>R$ which is not realistic.**



For small σ_x and finite system size R, instable mode can be avoided

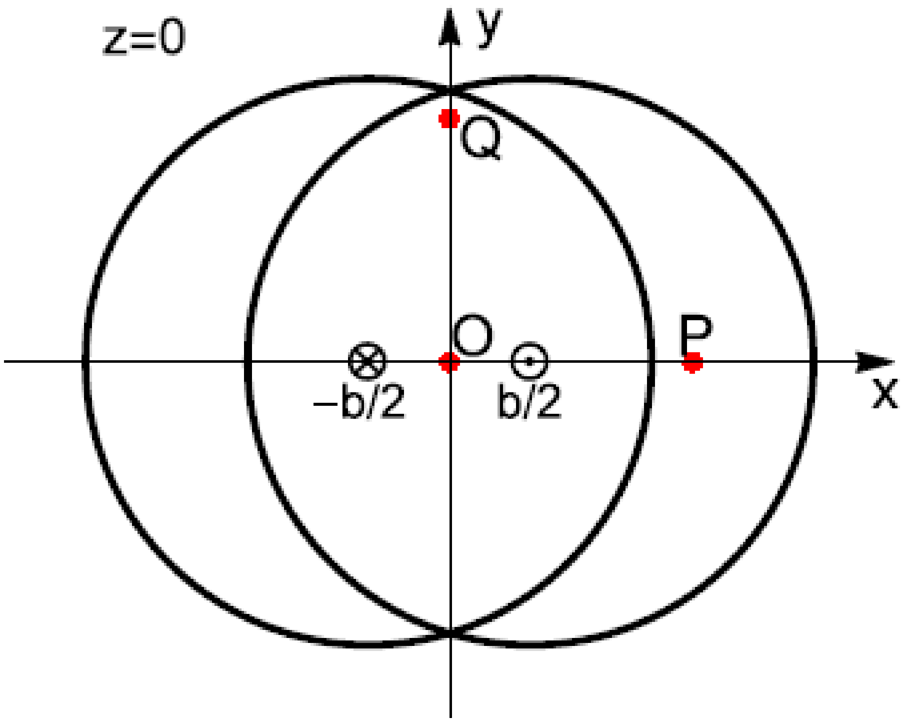
Magnetic field at relativistic limit

Relativistic limit v=1

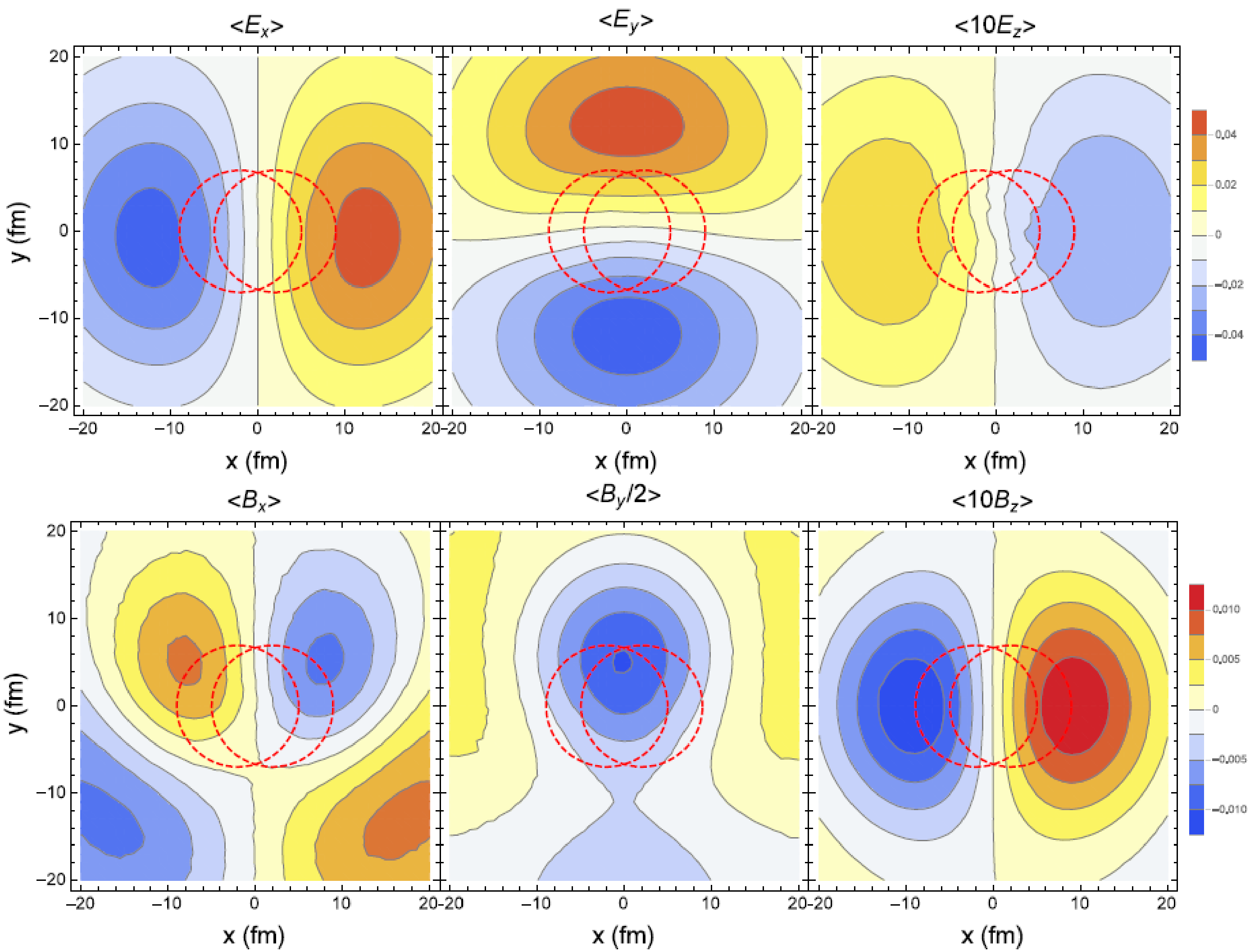
$$
\begin{aligned}
B_r(t,\mathbf{x}) &= \theta(t-z)Q\frac{x_T}{8\pi(t-z)^2}\exp\left[-\frac{\sigma x_T^2}{4(t-z)}\right] \\
&\times\left\{\sigma\sin\left[\frac{\sigma_\chi x_T^2}{4(t-z)}\right]-\sigma_\chi\cos\left[\frac{\sigma_\chi x_T^2}{4(t-z)}\right]\right\} \overset{\sigma_\chi=0}{\Longrightarrow} 0 \\
B_\phi(t,\mathbf{x}) &= \theta(t-z)Q\frac{x_T}{8\pi(t-z)^2}\exp\left[-\frac{\sigma x_T^2}{4(t-z)}\right] \\
&\times\left\{\sigma\cos\left[\frac{\sigma_\chi x_T^2}{4(t-z)}\right]+\sigma_\chi\sin\left[\frac{\sigma_\chi x_T^2}{4(t-z)}\right]\right\} \\
B_z(t,\mathbf{x}) &= \theta(t-z)Q\frac{1}{4\pi(t-z)}\exp\left[-\frac{\sigma x_T^2}{4(t-z)}\right] \\
&\times\left\{-\sigma\sin\left[\frac{\sigma_\chi x_T^2}{4(t-z)}\right]+\sigma_\chi\cos\left[\frac{\sigma_\chi x_T^2}{4(t-z)}\right]\right\} \overset{\sigma_\chi=0}{\Longrightarrow} 0
\end{aligned}
$$

Fields in nuclear collisions

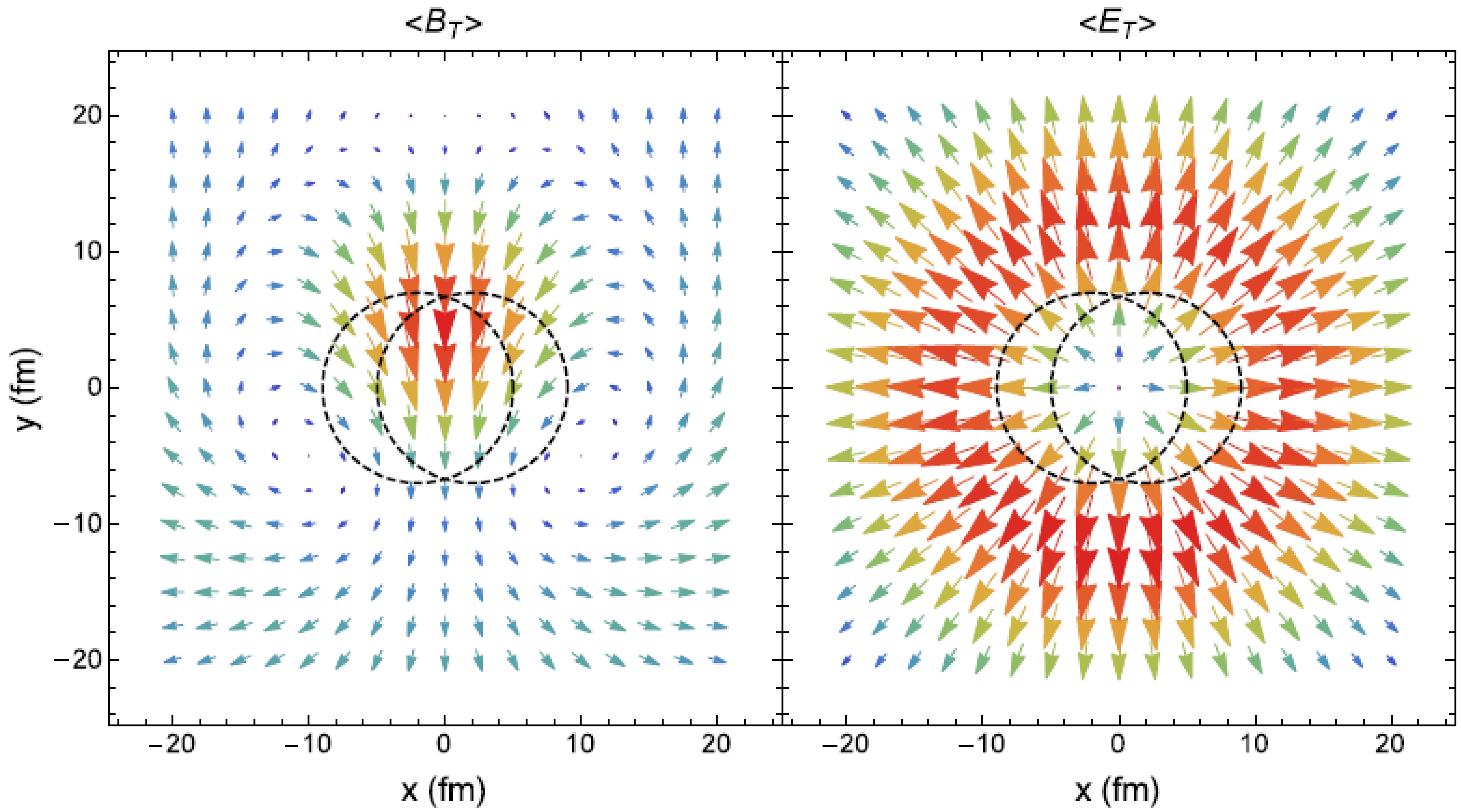
Collision geometry



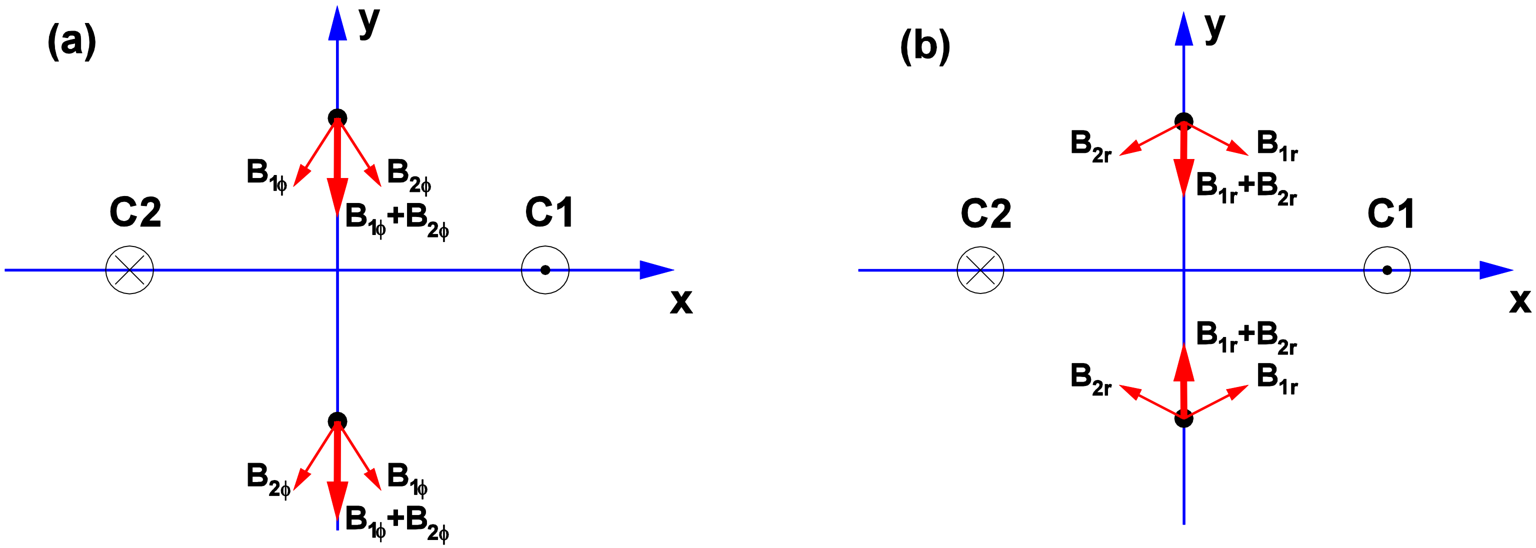
Fields in nuclear collisions (z=0, t=2 fm)



Fields in nuclear collisions (z=0, t=2 fm)



Why asymmetry in y for magnetic fields?



Azimuthal components
Same sign

Radial components
Different sign
▸ ▸ **effects of σ_X**

Quantum Kinetic Approach in Wigner function

- To describe dynamics of chiral fermions, we have to explicitly know their helicity (equivalently p), therefore we need to know information of (t,x,p), that's why we use kinetic approach
- Classical kinetic approach: f(t,x,p)
- Quantum kinetic approach: W(t,x,p)

4D Wigner Function

Gauge invariant Wigner operator/function

Vasak, Gylassy, Elze,
Annals Phys. 173 (1987) 462-492

$$W(x,p) = <:\hat{W}(x,p):>$$

$$\hat{W}_{\alpha\beta}(x,p) = \int \frac{d^4y}{(2\pi)^4} e^{-ip\cdot y} \bar{\psi}_\beta\left(x+\frac{1}{2}y\right) \mathcal{P}U\left(A, x+\frac{1}{2}y, x-\frac{1}{2}y\right) \psi_\alpha\left(x-\frac{1}{2}y\right)$$

Gauge link $\mathcal{P}U\left(A, x+\frac{1}{2}y, x-\frac{1}{2}y\right) \equiv \mathcal{P}\mathrm{Exp}\left(-iey^\mu \int_0^1 ds A_\mu\left(x-\frac{1}{2}y+sy\right)\right)$

Dirac equation in electromagnetic field

$$[i\gamma^\mu D_\mu(x) - m]\,\psi(x) = 0, \qquad \bar{\psi}(x)\left[i\gamma^\mu D_\mu^\dagger(x) + m\right] = 0$$

Quantum Kinetic Equation for Wigner function for massless fermion in homogeneous electromagnetic field

$$\gamma_\mu\left(p^\mu + \frac{1}{2}i\nabla^\mu\right) W(x,p) = 0$$

phase space derivative

$$\nabla^\mu \equiv \partial_x^\mu - QF^{\mu\nu}\partial_\nu^p$$

Wigner functions

- **For massless and collisionless fermions in constant EM field**

$$\gamma_\mu \left(p^\mu + \frac{1}{2} i \nabla^\mu \right) W(x,p) = 0.$$

- **Wigner function decomposition in 16 generators of Clifford algebra**

$$W = \frac{1}{4}\left[\mathscr{F} + i\gamma^5 \mathscr{P} + \gamma^\mu \mathscr{V}_\mu + \gamma^5\gamma^\mu \mathscr{A}_\mu + \frac{1}{2}\sigma^{\mu\nu}\mathscr{S}_{\mu\nu} \right]$$

scalar p-scalar vector axial-vector tensor

$$j^\mu = \int d^4p \mathscr{V}^\mu, \qquad j_5^\mu = \int d^4p \mathscr{A}^\mu, \qquad T^{\mu\nu} = \int d^4p p^\mu \mathscr{V}^\nu$$

Vasak, Gyulassy and Elze, Annals Phys. 173, 462 (1987);
Elze, Gyulassy and Vasak, Nucl. Phys. B 276, 706(1986).

Wigner functions

- **P. Zhuang and U. Heinz, Ann.Phys. 245, 311-338(1996)**

Relativistic Quantum Transport Theory for Electrodynamics*

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We investigate the relationship between the covariant and the three-dimensional (equal-time) formulations of quantum kinetic theory. We show that the three-dimensional approach can be obtained as the energy average of the covariant formulation. We illustrate this statement in scalar and spinor QED. We especially emphasize the importance of constraint equations in the three-dimensional formulation and explicitly derive via the energy averaging method a complete set of kinetic equations, which contains the BGR equations by Bialynick-Birula *et al.* as a subset.

Solution of Wigner function

- **The solutions to $(F_{\mu\nu})^1$ and $(\partial_x)^1$ encodes a lot of information !!**

$$\mathscr{J}_s^\rho(x,p) = \mathscr{V}^\rho + s\mathscr{A}^\rho$$

$$\mathscr{J}^\rho_{(0)s}(x,p) = p^\rho f_s \delta(p^2)$$

$$\mathscr{J}^\rho_{(1)s}(x,p) = -\frac{s}{2}\tilde{\Omega}^{\rho\beta}p_\beta \frac{df_s}{dp_0}\delta(p^2) - \frac{s}{p^2}Q\tilde{F}^{\rho\lambda}p_\lambda f_s \delta(p^2)$$

- **where**

Fermi-Dirac Distr.

$$f_s(x,p) = \frac{2}{(2\pi)^3}\left[\Theta(p_0) f_F(p_0 - \mu_s) + \Theta(-p_0) f_F(-p_0 + \mu_s)\right]$$

$$\tilde{F}^{\rho\lambda} = \frac{1}{2}\epsilon^{\rho\lambda\mu\nu}F_{\mu\nu}$$

$$\mu_s = \mu + s\mu_5$$

$$\tilde{\Omega}^{\xi\eta} = \frac{1}{2}\epsilon^{\xi\eta\nu\sigma}\Omega_{\nu\sigma} \qquad \Omega_{\nu\sigma} = \frac{1}{2}(\partial_\nu u_\sigma - \partial_\sigma u_\nu)$$

Decoding the solution:
(1) CME , CVE, CSE and Chiral Anomly

- **Vector current** $j^{\mu} = nu^{\mu} + \xi\omega^{\mu} + \xi_B B^{\mu}$

CME: $\xi_B = \frac{1}{2\pi^2}\mu_5$ **CVE:** $\xi = \frac{1}{\pi^2}\mu\mu_5$

J.H. Gao, Z.T. Liang, S. Pu, QW, X.N. Wang, PRL 109, 232301(2012)

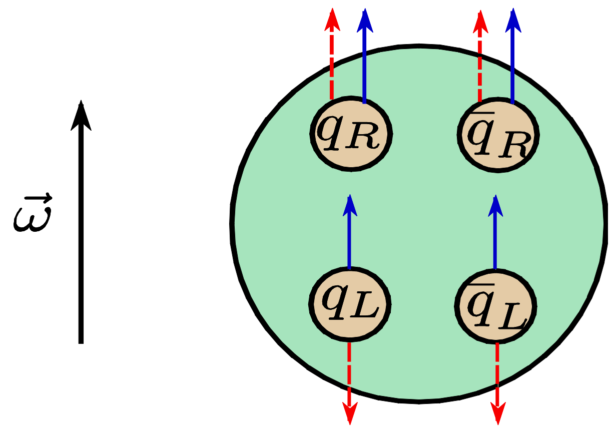
- **Axial current** $j_5^{\mu} = n_5 u^{\mu} + \xi_5\omega^{\mu} + \xi_{B5}B^{\mu}$

-Chiral separation effect:

$$\xi_{B5} = \frac{1}{2\pi^2}\mu,$$

-Local polarization effect

$$\xi_5 = \frac{1}{6}T^2 + \frac{1}{2\pi^2}\left(\mu^2 + \mu_5^2\right)$$



- **Conservation laws**

$$\partial_\mu T^{\mu\nu} = F^{\mu\nu} j_\mu \qquad \partial_\mu j^{\mu} = 0, \qquad \partial_\mu j_5^{\mu} = CE \cdot B.$$

Decoding the solution: (2) CKE from 4D to 3D and Berry Phase

- **Covariant Chiral Kinetic Equation in 4D (CCKE)**

$$\nabla_\mu \mathscr{J}_s^\mu = 0 \quad \Rightarrow \quad \delta\left(p^2\right)\left(\frac{dx^\mu}{d\tau}\partial_\mu^x f_s + \frac{dp^\mu}{d\tau}\partial_\mu^p f_s\right) = 0$$

$$\frac{dx^\mu}{d\tau} \equiv p^\mu + s\epsilon^{\mu\nu\alpha\beta} b_\nu F_{\alpha\beta}, \qquad \frac{dp^\mu}{d\tau} \equiv F^{\mu\nu} p_\nu - s\left(E \cdot B\right) b^\mu$$

Berry Curvature In 4D $\quad b^\mu \equiv -\dfrac{p^\mu}{p^2}$

- **Chiral Kinetic Equation in 3D**

$$\int dp_0 \nabla_\mu \mathscr{J}_s^\mu = 0 \quad \Rightarrow \quad \partial_t f_s + \frac{d\mathbf{x}}{dt} \cdot \nabla_{\mathbf{x}} f_s + \frac{d\mathbf{p}}{dt} \cdot \nabla_{\mathbf{p}} f_s = 0$$

$$\frac{d\mathbf{x}}{dt} \equiv \frac{\hat{\mathbf{p}} + s\left[(\hat{\mathbf{p}} \cdot \mathbf{\Omega})\mathbf{B} + \mathbf{E} \times \mathbf{\Omega}\right]}{1 + s\mathbf{\Omega} \cdot \mathbf{B}}, \qquad \frac{d\mathbf{p}}{dt} \equiv \frac{\mathbf{E} + \hat{\mathbf{p}} \times \mathbf{B} + s(\mathbf{E} \cdot \mathbf{B})\mathbf{\Omega}}{1 + s\mathbf{\Omega} \cdot \mathbf{B}}$$

J.W. Chen, S.Pu, QW, X.N. Wang, PRL 110 (2013) 262301
D.T. Son, N. Yamamoto, PRL 109 (2012) 181602
M.A. Stephanov,Y. Yin, PRL 109 (2012) 162001

Berry Curvature In 3D $\quad \mathbf{\Omega} = \dfrac{\hat{\mathbf{p}}}{2\mathbf{p}^2}$

Decoding the solution:
(3) Energy shift and magnetic Moment

- **Particle and energy density with** $\mu_s(x,\mathbf{p})$ $(\mu_s \equiv \mu + s\mu_5)$

$$n_s = \int d^3\mathbf{p} f_s + \int d^3\mathbf{p}\frac{s}{2E_p^2}(\mathbf{v}\cdot\mathbf{B})f_s - \int d^3\mathbf{p}\frac{s}{2E_p}(\mathbf{v}\cdot\mathbf{B})\frac{d}{dE_p}f_s$$
$$\approx \int \frac{d^3\mathbf{p}}{(2\pi)^3}\sqrt{\gamma_s}\left[f_F\left(E'_p - \mu_s(x,\mathbf{p})\right) - f_F\left(E'_p + \mu_s(x,\mathbf{p})\right)\right],$$

$$\epsilon_s = \int d^3\mathbf{p} E_p f_s + \int d^3\mathbf{p}\frac{s}{2E_p}(\mathbf{v}\cdot\mathbf{B})f_s - \int d^3\mathbf{p}\frac{s}{2}(\mathbf{v}\cdot\mathbf{B})\frac{d}{dE_p}f_s$$
$$\approx \int \frac{d^3\mathbf{p}}{(2\pi)^3}\sqrt{\gamma_s}E'_p\left[f_F\left(E'_p - \mu_s(x,\mathbf{p})\right) + f_F\left(E'_p + \mu_s(x,\mathbf{p})\right)\right],$$

- **Phase-space measure:** $\sqrt{\gamma_s} \equiv (1 + s\Omega\cdot\mathrm{B})$
- **Berry curvature:** $\Omega = \frac{\hat{\mathrm{p}}}{2\mathrm{p}^2}$

J.H.Gao, QW, PLB749 (2015) 542.

Decoding the solution:
(3) Energy shift and magnetic moment

- **Effective Energy of Chiral Fermion:**

$$E'_p = |\mathbf{p}| + \Delta E_B$$

- **Energy Shift:**

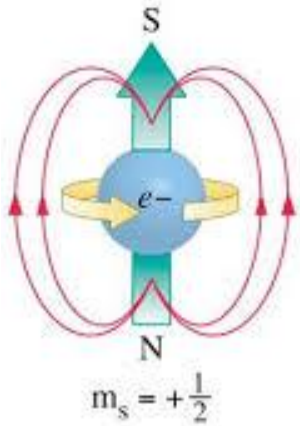
$$\Delta E_B \;=\; -\boldsymbol{\mu}_m \cdot \mathbf{B}$$

- **Magnetic moment of massless fermion:**

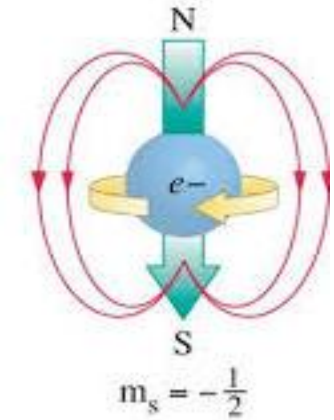
$$\boldsymbol{\mu}_m \;=\; \frac{Q}{|\mathbf{p}|}\mathbf{S}$$

$s = \pm 1$

- **Spin:** $\boldsymbol{S} \;=\; \frac{s}{2}\hat{\mathbf{p}}$



$$\boldsymbol{\mu}_m \;=\; \frac{Q}{\mathrm{m}}\mathbf{S}$$



Son, Yamamoto (2013); Manual, Torres-Rincon (2014); Satow, Yee (2014)

Is it the energy for quasi-particle?

- **Effective Energy of Chiral Fermion with magnetic energy shift:**

$$
\begin{aligned}
E'_p &= |\mathrm{p}| - \frac{sQ}{2|\mathrm{p}|^2}(\mathrm{p}\cdot\mathrm{B}) \\
\mathrm{v}'_p &= \nabla_{\mathrm{p}} E'_p = \hat{\mathrm{p}} + \frac{sQ}{2|\mathrm{p}|^2}(2\hat{\mathrm{p}}\hat{\mathrm{p}} - 1)\cdot\mathrm{B} \\
\mathrm{v}'_p \cdot \hat{\mathrm{p}} &= 1 + \frac{sQ}{2|\mathrm{p}|^2}(\hat{\mathrm{p}}\cdot\mathrm{B}) > 1 \qquad \text{for} \;\; sQ(\hat{\mathrm{p}}\cdot\mathrm{B}) > 0
\end{aligned}
$$

$sQ = \pm 1$

Superluminal

- **Can E'_p be regarded as quasi-particle dispersion relation? Is this a problem?**
- **In our formalism, we don't have such superluminal problem. It appears only when re-writing the number/energy density in a 'quasi-particle' way in weak field approximation**

Decoding the solution:
(4) Energy Shift from Spin-Vorticity Coupling

- **Particle and energy density with** $\mu_s(x,\mathbf{p})$

$$\begin{aligned} n_s &= \int d^3\mathbf{p} f_s - \int d^3\mathbf{p}\frac{s}{2}(\boldsymbol{\omega}\cdot\mathbf{v})\frac{d}{dE_p}f_s \\ &\approx \int \frac{d^3\mathbf{p}}{(2\pi)^3}\left[f_F\left(E'_p-\mu_s(x,\mathbf{p})\right)-f_F\left(E'_p+\mu_s(x,\mathbf{p})\right)\right], \end{aligned}$$

$$\begin{aligned} \epsilon_s &= \int d^3\mathbf{p} E_p f_s - \int d^3\mathbf{p}\frac{s}{2}(\mathbf{v}\cdot\boldsymbol{\omega})E_p\frac{d}{dE_p}f_s \\ &\approx \int \frac{d^3\mathbf{p}}{(2\pi)^3}E'_p\left[f_F\left(E'_p-\mu_s(x,\mathbf{p})\right)+f_F\left(E'_p+\mu_s(x,\mathbf{p})\right)\right], \end{aligned}$$

- **No phase-space measure, no Berry curvature**
- **Effective energy:**

$$\Delta E_\omega = -\boldsymbol{\omega}\cdot\mathbf{S}$$

$$E'_p = E_p + \Delta E_\omega$$

J.H.Gao, QW, PLB749 (2015) 542.

Another solution: Linear Response Theory for Wigner Functions

- **Expansion in $(F_{\mu\nu})^n$ [not in $(\partial_x)^n$!] , the first order equation:**

$$p^\mu \mathscr{J}^{(1)}_{s\mu} + \delta\Pi^\mu \mathscr{J}^{(0)}_{s\mu} = 0, \quad \partial^\mu_x \mathscr{J}^{(1)}_{s\mu} + \delta G^\mu \mathscr{J}^{(0)}_{s\mu} = 0,$$

$$\epsilon_{\mu\nu\rho\sigma}\left[\partial^\rho_x \mathscr{J}^{(1)\sigma}_s + \delta G^\rho \mathscr{J}^{(0)\sigma}_s\right] = -2s\left(p_\mu \mathscr{J}^{(1)}_{s\nu} - p_\nu \mathscr{J}^{(1)}_{s\mu}\right) - 2s\left[\delta\Pi_\mu \mathscr{J}^{(0)}_{s\nu} - \delta\Pi_\nu \mathscr{J}^{(0)}_{s\mu}\right]$$

- **Formal solution:** $\Delta \equiv \partial^p \cdot \partial_x$

$$\mathscr{J}^{(1)}_{s\mu} = -\frac{s}{2p\cdot\partial_x}\epsilon_{\mu\nu\rho\sigma}\partial^\nu_x\left[j_0\left(\frac{\Delta}{2}\right)F^{\rho\lambda}\partial^p_\lambda \mathscr{J}^{(0)\sigma}_s\right] + \frac{1}{p\cdot\partial_x}p_\mu j_0\left(\frac{\Delta}{2}\right)F^{\nu\lambda}\partial^p_\lambda \mathscr{J}^{(0)}_{s\nu}$$
$$-\frac{1}{2p\cdot\partial_x}\partial^\nu_x\left[j_1\left(\frac{\Delta}{2}\right)\left(F_{\mu\lambda}\partial^\lambda_p \mathscr{J}^{(0)}_{s\nu} - F_{\nu\lambda}\partial^\lambda_p \mathscr{J}^{(0)}_{s\mu}\right)\right]$$

- **Parity-odd part of the Wigner function in momentum space:**

$$\mathscr{J}^{(1)}_{s\mu}(k,p) = -i\frac{s}{2p\cdot k}\epsilon_{\mu\nu\rho\sigma}k^\nu p^\sigma A^\rho(k) j_0(\Delta)(k\cdot\partial_p)[f_s\delta(p^2)]$$

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Another solution: Chiral Magnetic Conductivity

- **Chiral Magnetic Conductivity:**

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$$\vec{j}(\omega,\mathbf{k}) = \int d^4p \left(\vec{\mathscr{J}}_{+}^{(1)} + \vec{\mathscr{J}}_{-}^{(1)} \right) = \sigma_\chi(\omega,\mathbf{k})\vec{B}$$

$$\sigma_\chi = \frac{\mathbf{k}^2-\omega^2}{16\pi^2|\mathbf{k}|^3}\int d|\mathbf{p}| f(|\mathbf{p}|) \sum_{t=\pm1} (2|\mathbf{p}|+t\omega) \ln\left[\frac{(\omega+i\epsilon+t|\mathbf{p}| - (\omega+|\mathbf{p}|)^2}{(\omega+i\epsilon+t|\mathbf{p}| - (\omega-|\mathbf{p}|)^2}\right]$$

- **HTL/HDL results from Wigner function:** $\omega, |\mathbf{k}| \ll |\mathbf{p}|$

$$\sigma_\chi(\omega,\mathbf{k}) = \sigma_\chi^{(0)} \left(1-\frac{\omega^2}{|\mathbf{k}|^2}\right)\left[1-\frac{\omega}{2|\mathbf{k}|}\ln\frac{\omega+|\mathbf{k}|}{\omega-|\mathbf{k}|}\right]$$

$$\sigma_\chi^{(0)} = \frac{1}{2\pi^2}\mu_5$$

M.Laine, JHEP 0510 (2005) 056;
D.Kharzeev, H.Warringa, PRD80 (2009) 034028;

Summary

- **We derived algebraic formula for electric and magnetic fields of a moving point charge in conducting medium with σ and σ_χ , which can be used in simulation.**
- **Due to σ_χ, in HIC, the symmetry of fields w.r.t. y-direction is broken.**
- **The time reversal symmetry is broken due to σ, advanced solution is highly suppressed: dissipation effect**
- **One-line solution to Wigner function encodes: (a) CME and CVE; (b) Covariant Chiral Kinetic Equation; (c) Berry phase and monopole in 4D; (d) Magnetic energy of chiral fermions; (e) Spin-vorticity coupling of chiral fermions**