

# Exploring the phase structure and dynamics of QCD

**Jan M. Pawłowski**

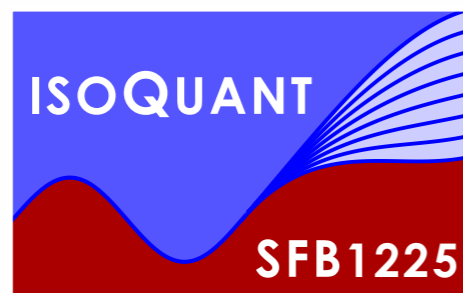
**Universität Heidelberg & ExtreMe Matter Institute**

**Wuhan, Juni 7<sup>th</sup> 2016**



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für Bildung  
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# Outline

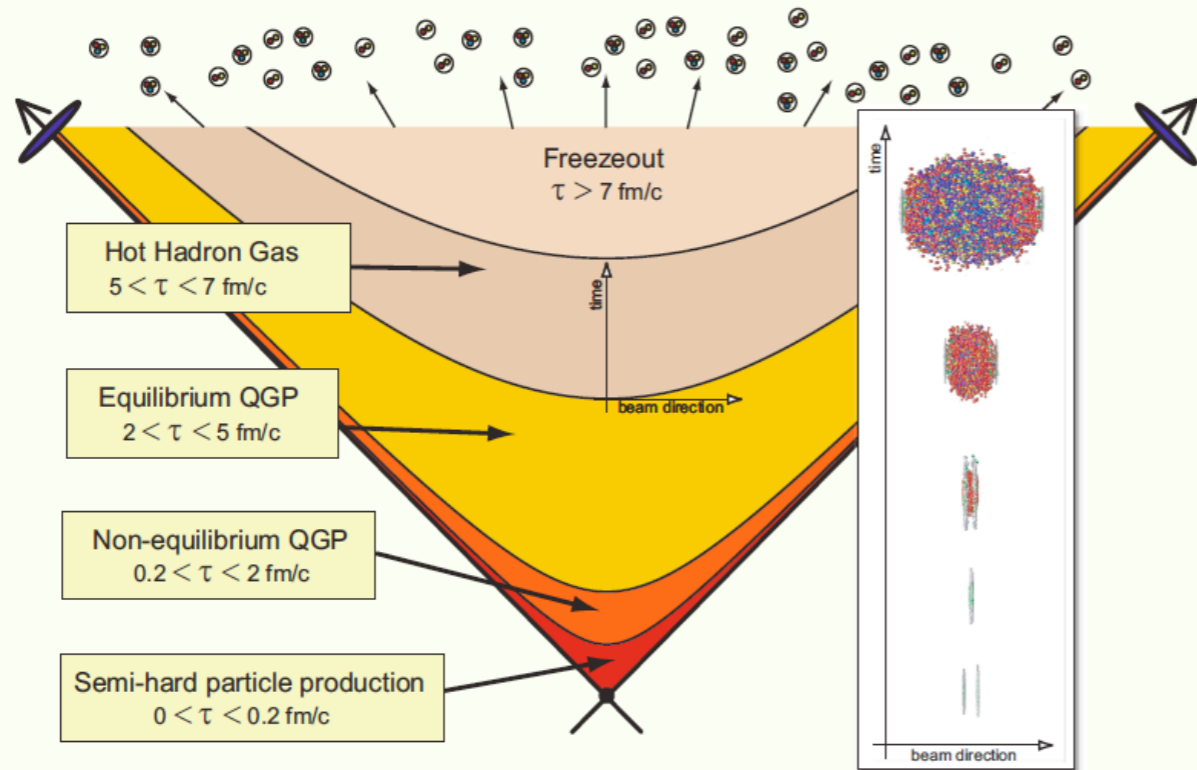
- **Introduction**
- **Confinement & transport**
- **Chiral symmetry breaking & the phase structure**
- **Summary & outlook**

# Outline

- **Introduction**
- **Confinement & transport**
- **Chiral symmetry breaking & the phase structure**
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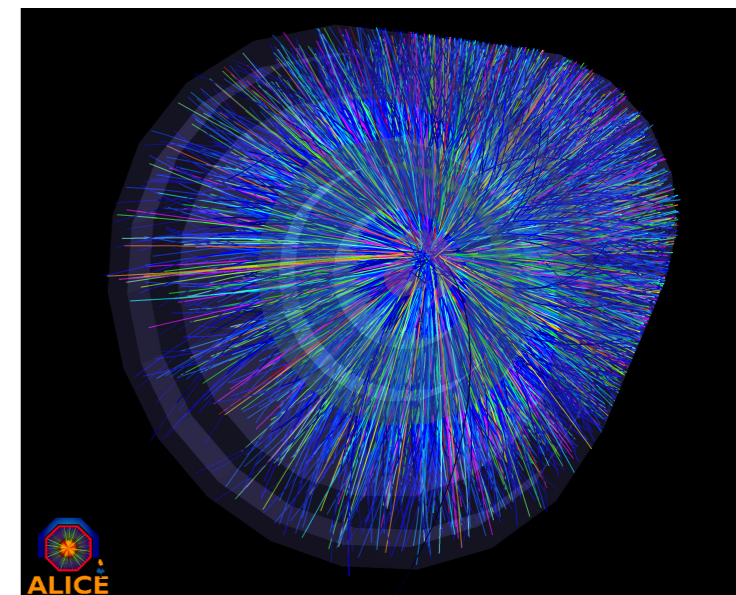
# Heavy ion collisions

Heavy-ion collision timescales and "epochs" @ RHIC

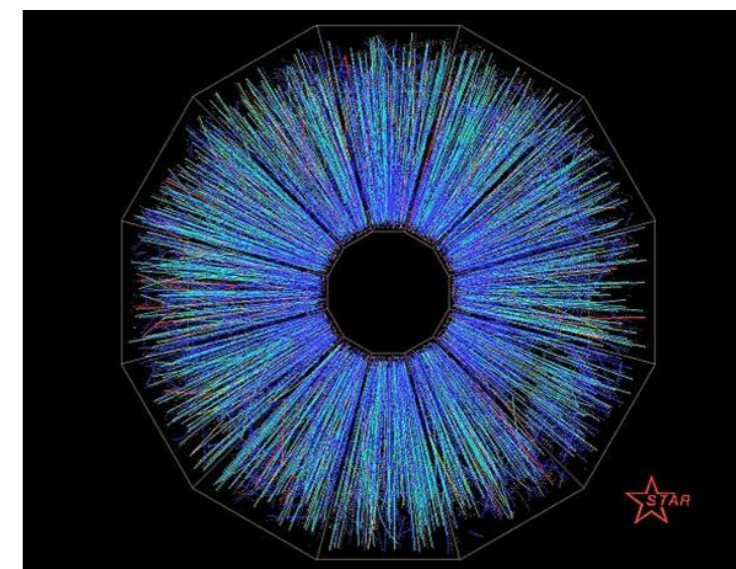


\*1 fm/c  $\simeq 3 \times 10^{-24}$  seconds

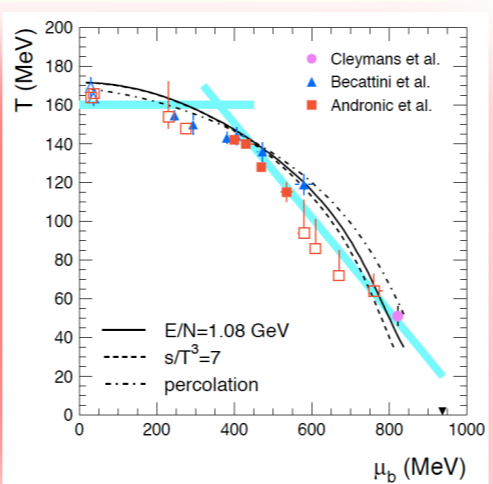
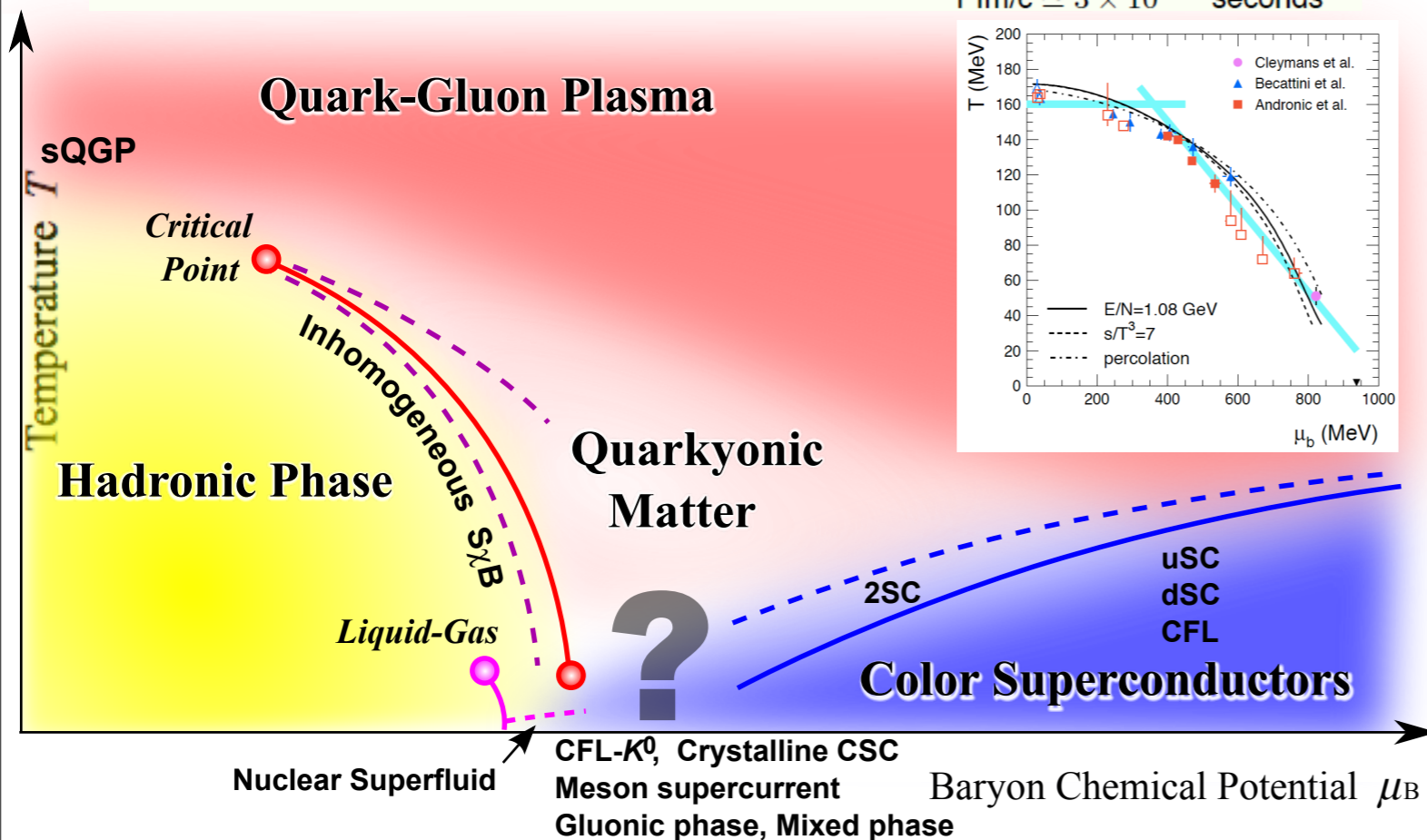
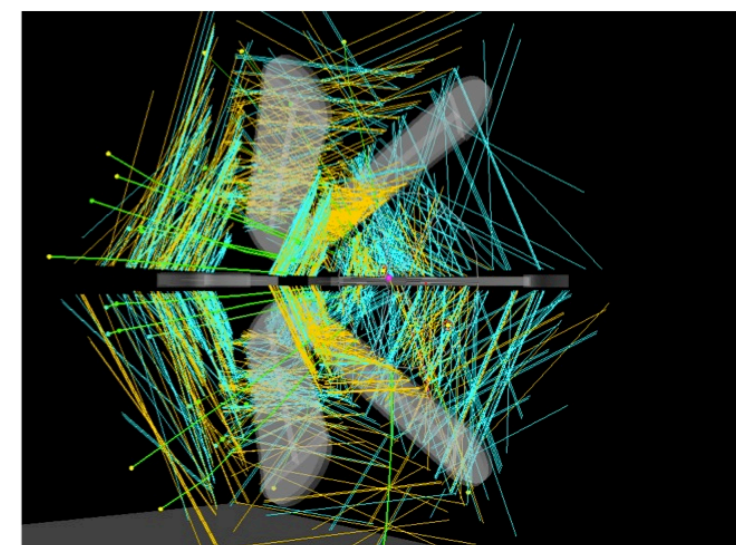
LHC



RHIC

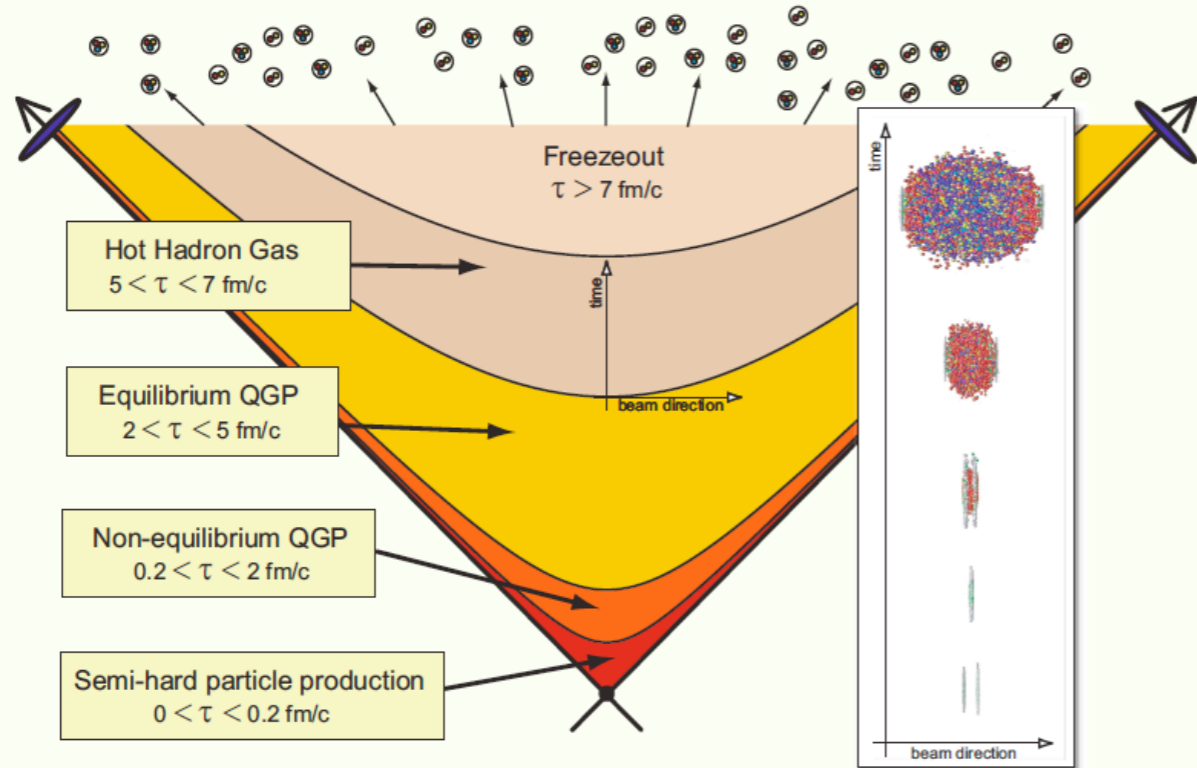


GSI



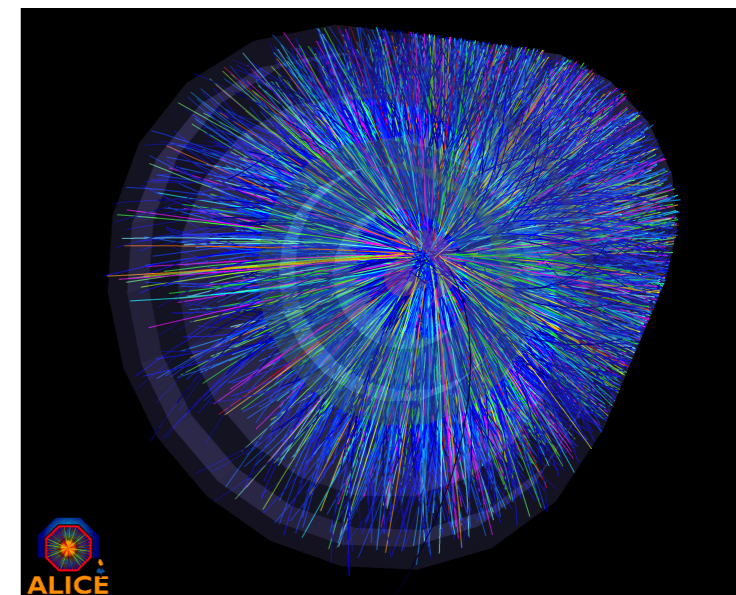
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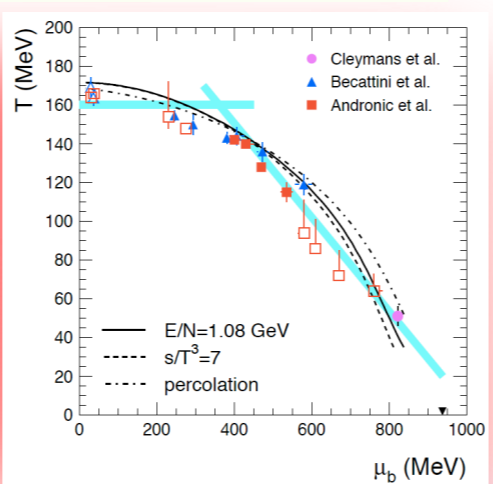
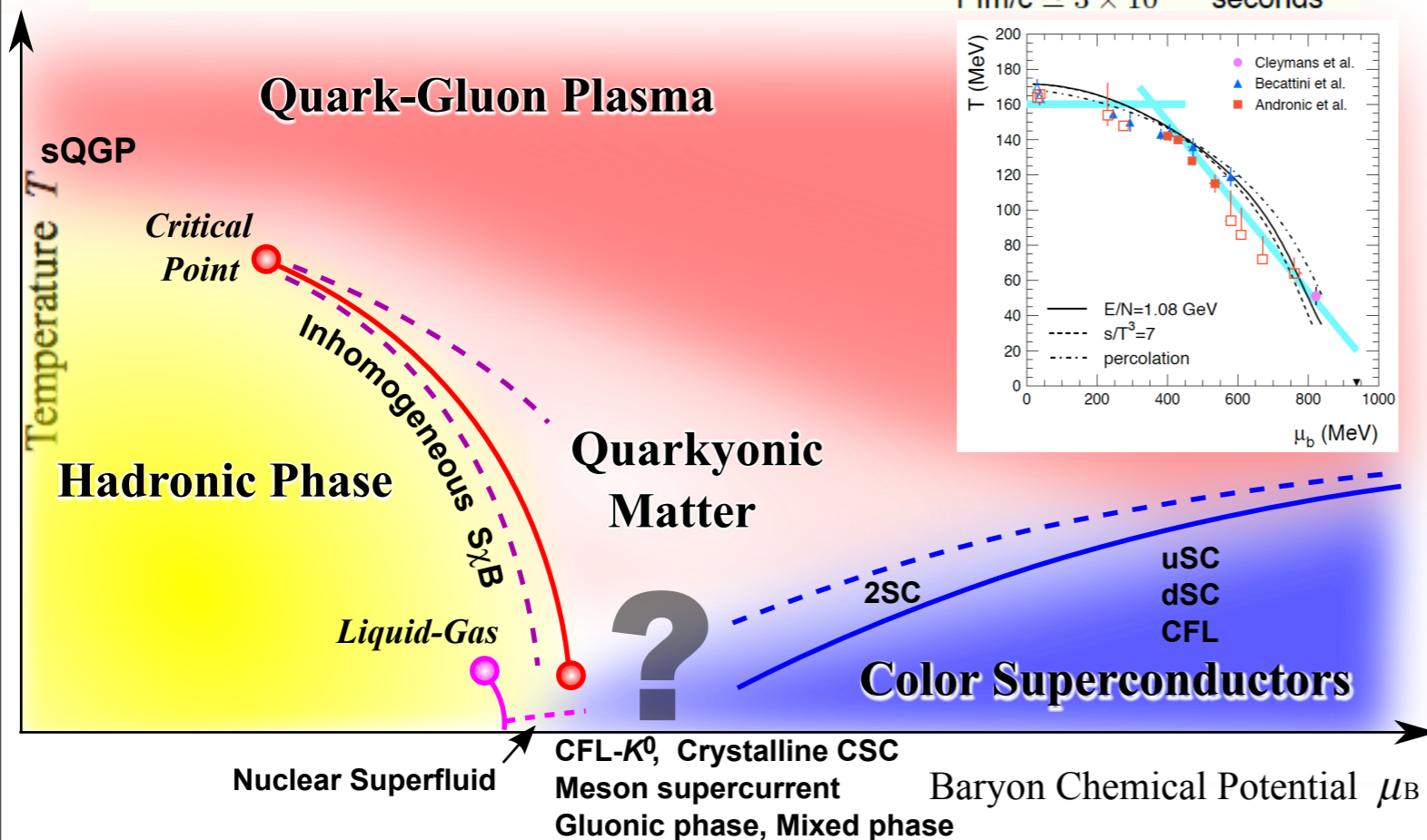
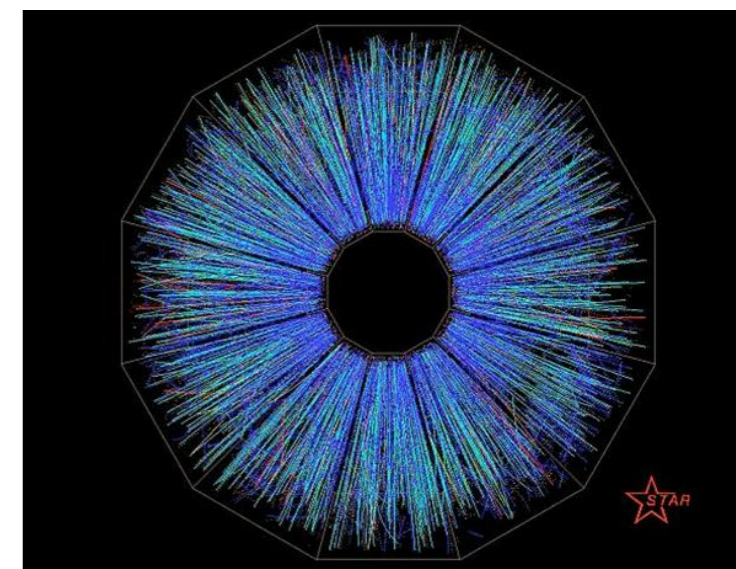


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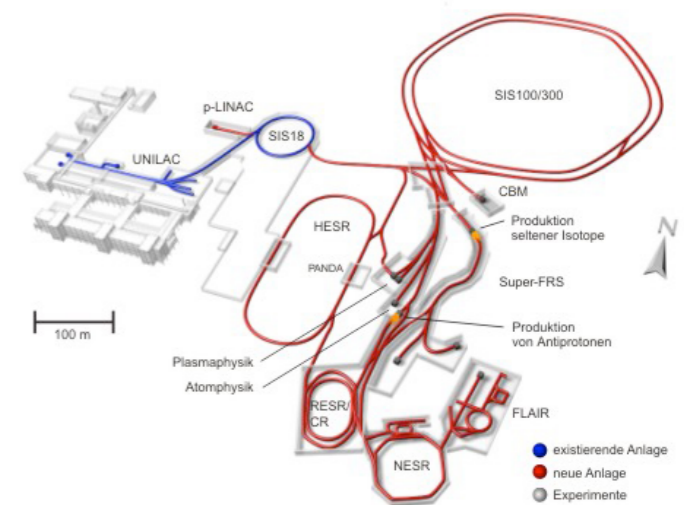
LHC



RHIC

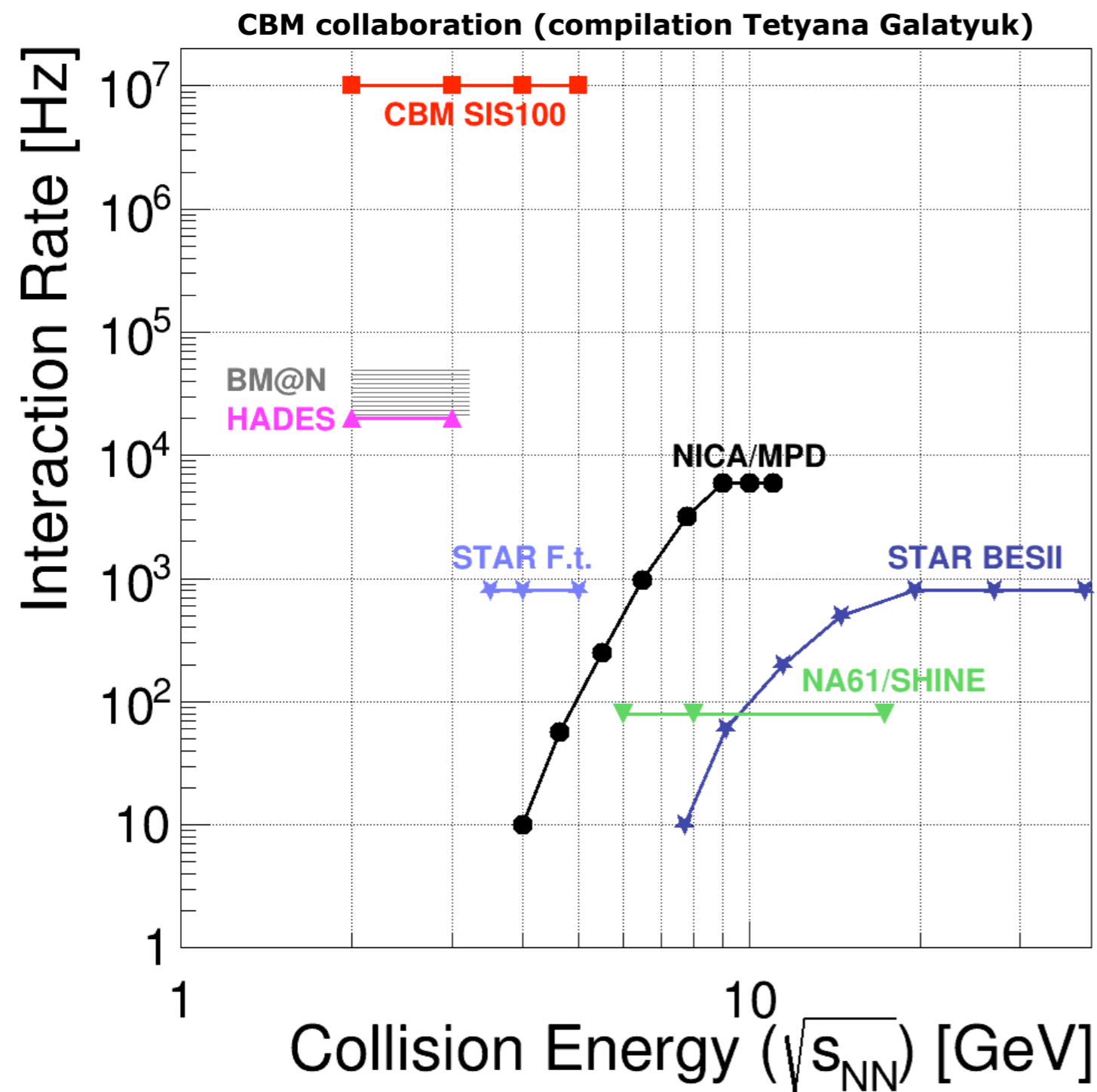


FAIR

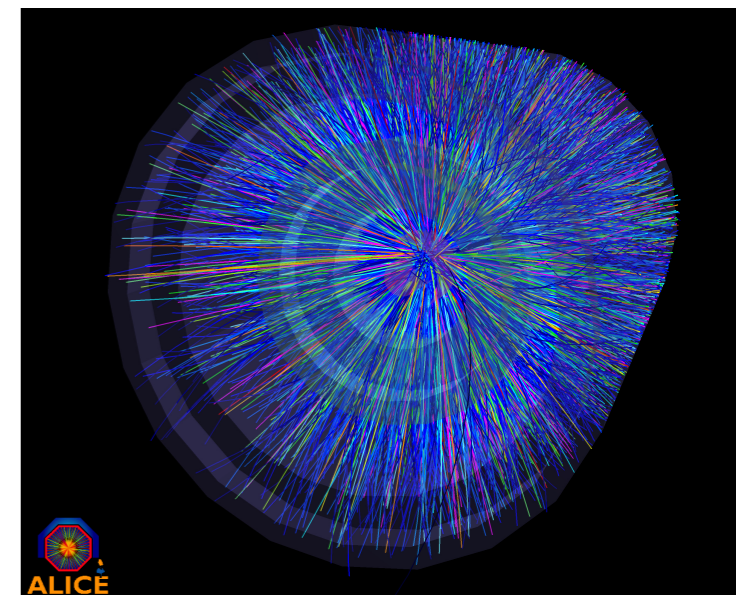


NICA

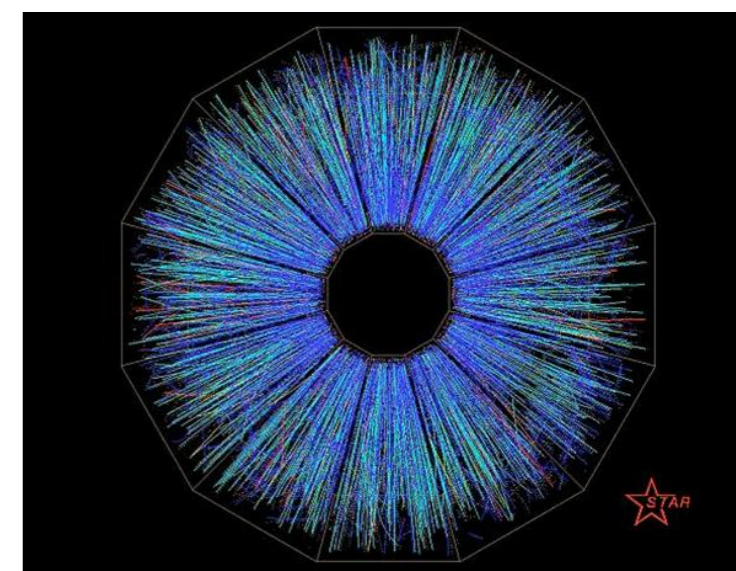
# Heavy ion collisions



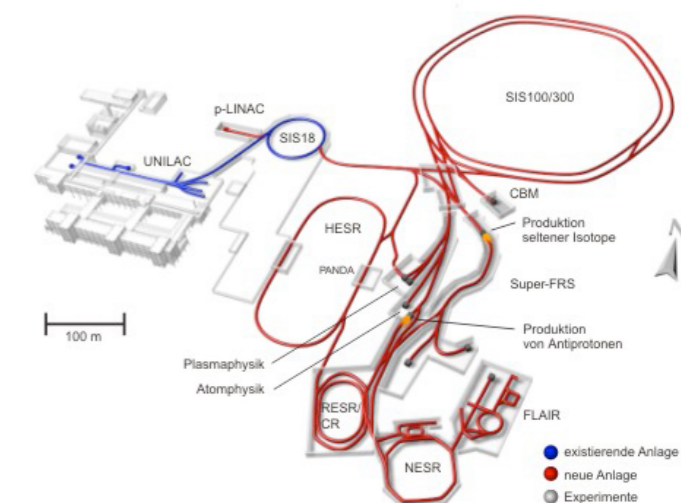
LHC



RHIC



FAIR



NICA

# Outline

- **Introduction**

- **Confinement & transport**

- **Chiral symmetry breaking & the phase structure**

- **Summary & outlook**

# Gluonic correlation functions

## Functional Renormalisation Group

$$\langle A A \rangle(p^2)$$

$$\partial_t \text{gluon}^{-1} = \text{gluon loop} - 2 \text{ghost loop} + \frac{1}{2} \text{ghost loop}$$



# Gluonic correlation functions

## Functional Renormalisation Group

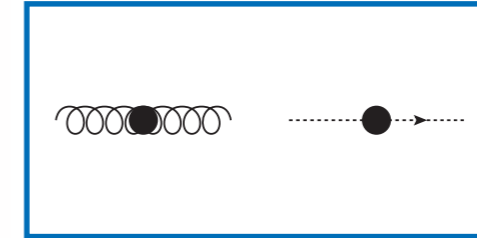
$$\partial_t \text{---}^{-1} = \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---}$$

$$\partial_t \text{---}^{-1} = \text{---} \text{---} \text{---} - 2 \text{---} \text{---} \text{---} + \frac{1}{2} \text{---} \text{---} \text{---}$$

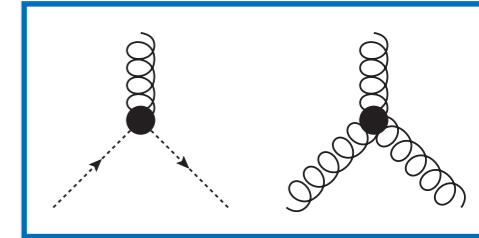
$$\partial_t \text{---} = - \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} + \text{perm.}$$

$$\partial_t \text{---} = - \text{---} \text{---} \text{---} + 2 \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} + \text{perm.}$$

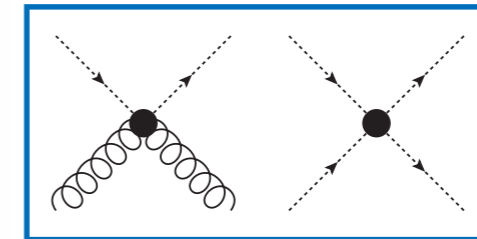
$$\partial_t \text{---} = - \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} + 2 \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} + \text{perm.}$$



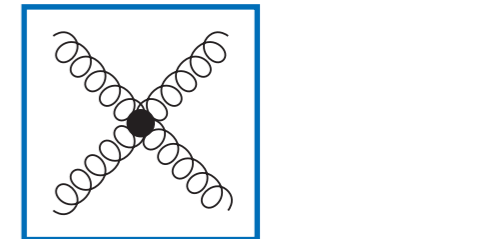
full. mom. dep.



full. mom. dep.  
classical tensor structures



mom. dep. needed by tadpoles  
full tensor basis

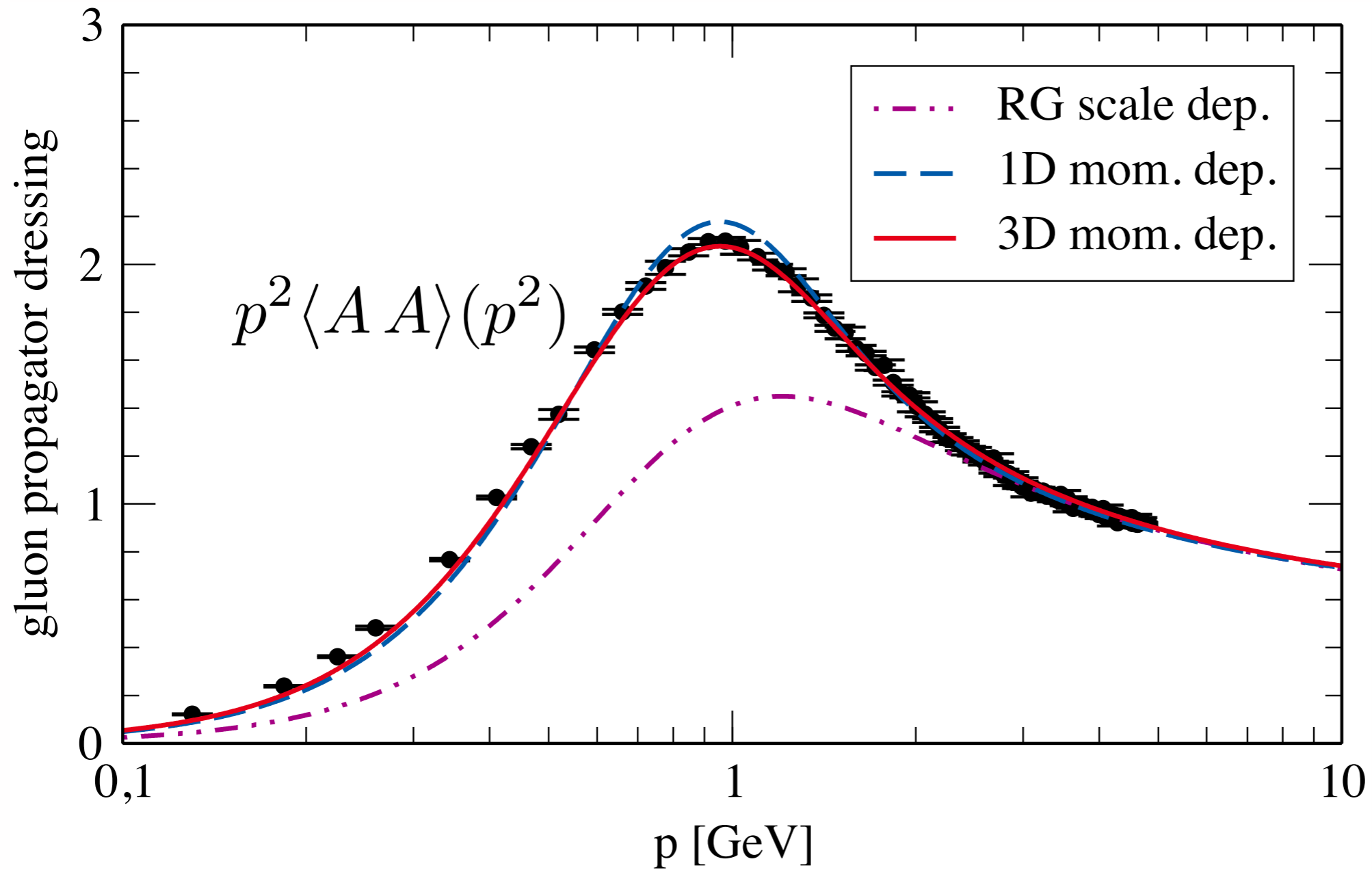


sym. point mom. dep. and  
mom. dep. needed by tadpole  
classical tensor structure

**Aiming at apparent convergence**

# Euclidean gluon propagator

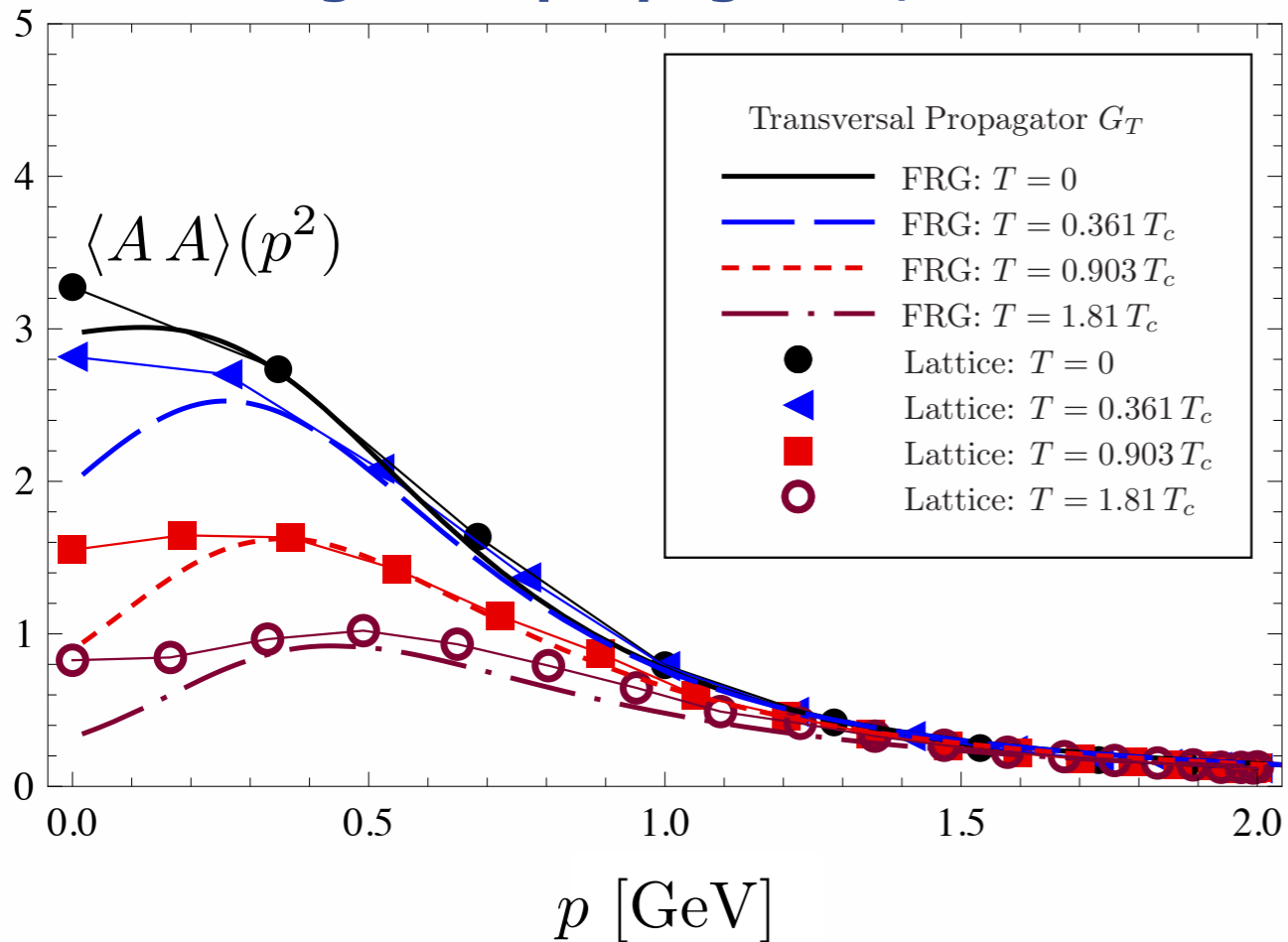
Functional Renormalisation Group



Aiming at apparent convergence

# Euclidean gluon propagator

## Yang-Mills propagators, finite T



Fister, JMP, arXiv:1112.5440

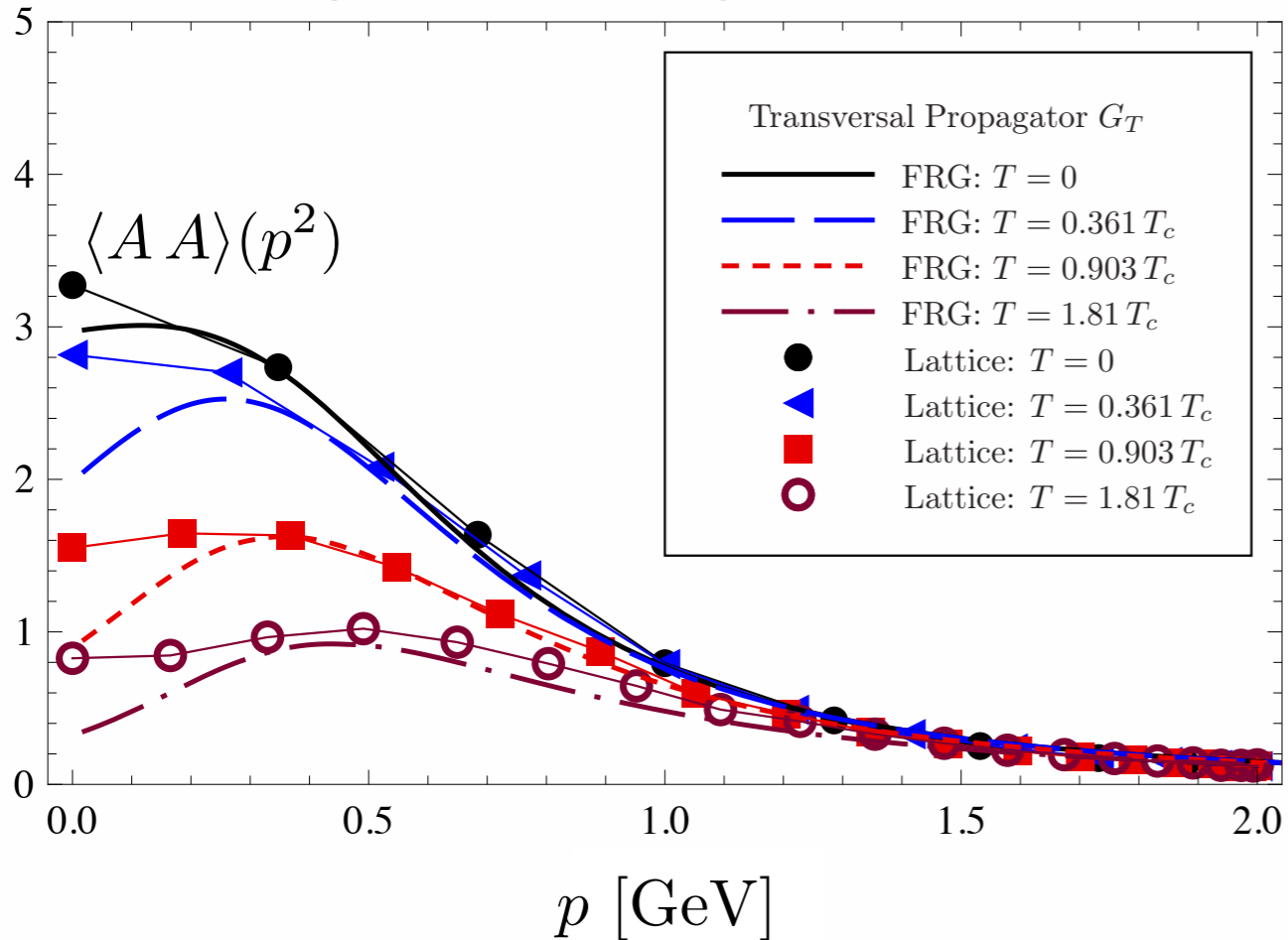
Lattice: Maas, JMP, Smekal, Spielmann, PRD 85 (2012) 034037

Approximations of infrared dynamics involved

up to date DSE: Cyrol, Huber, Smekal, EPJ C75 (2015) 102

# Euclidean gluon propagator

## Yang-Mills propagators, finite T

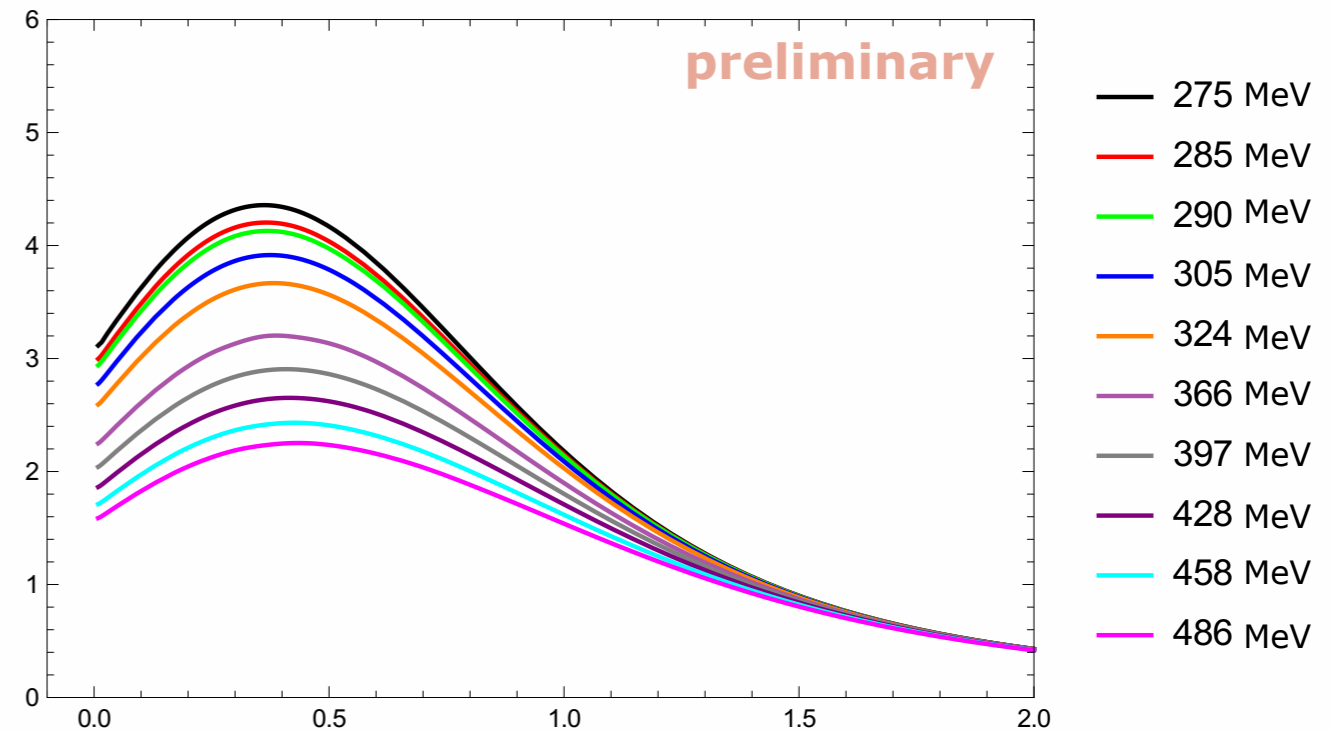
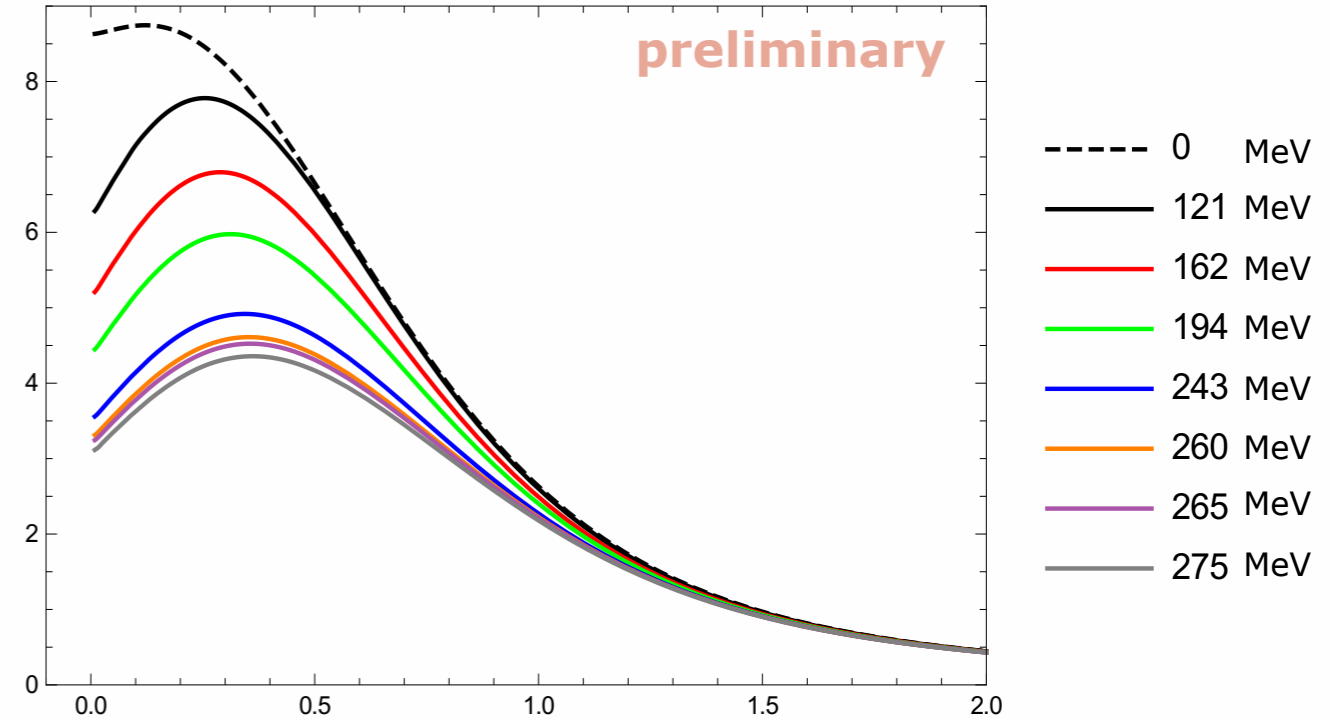


Fister, JMP, arXiv:1112.5440

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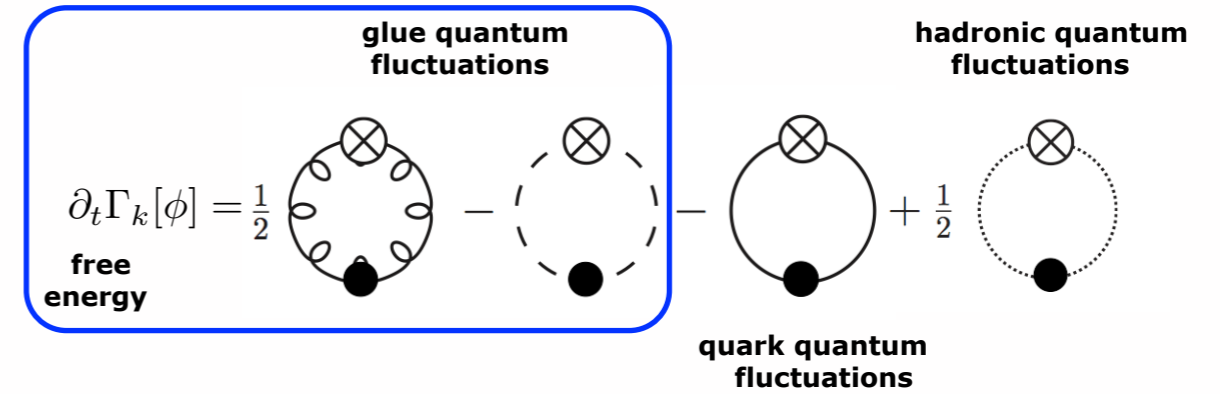
Cyrol, Mitter, JMP, Strodthoff, in preparation

# Confinement

FRG: Braun, Gies, JMP, PLB 684 (2010) 262

FRG, DSE, 2PI: Fister, JMP, PRD 88 (2013) 045010

$$L[A_0] = \frac{1}{N_c} \text{tr} \mathcal{P} e^{ig \int_0^\beta A_0(\mathbf{x})}$$

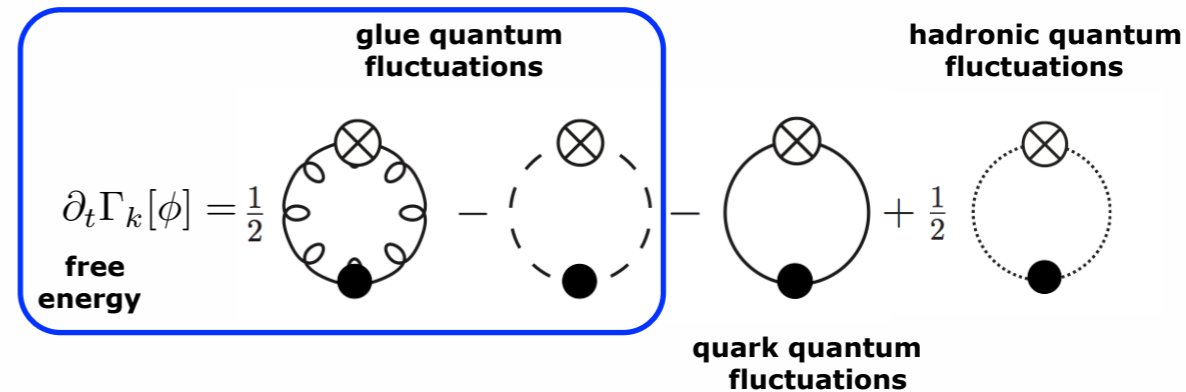


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$$L[A_0] = \frac{1}{N_c} \text{tr} \mathcal{P} e^{i g \int_0^\beta A_0(x)}$$

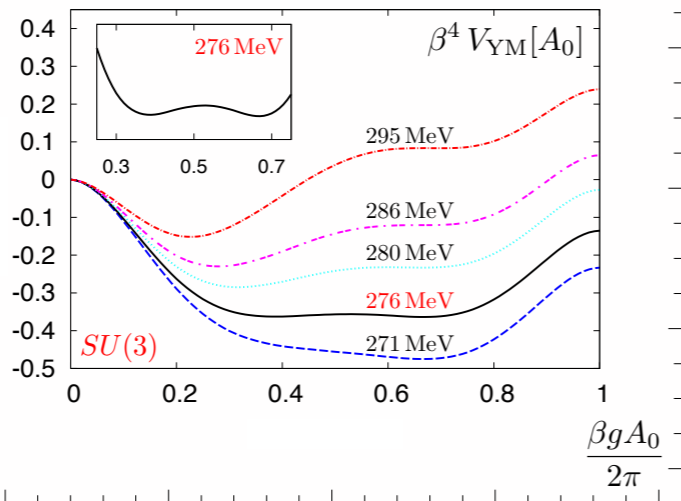


$$\mathcal{P} e^{i g \int_0^\beta A_0(x)} = e^{i\varphi}$$

Polyakov loop

$L(\langle \varphi \rangle)$

Polyakov loop Potential



0.90 0.95 1.00 1.05 1.10 1.15 1.20 1.25

$T/T_c$

$$T_c / \sqrt{\sigma} = 0.658 \pm 0.023$$

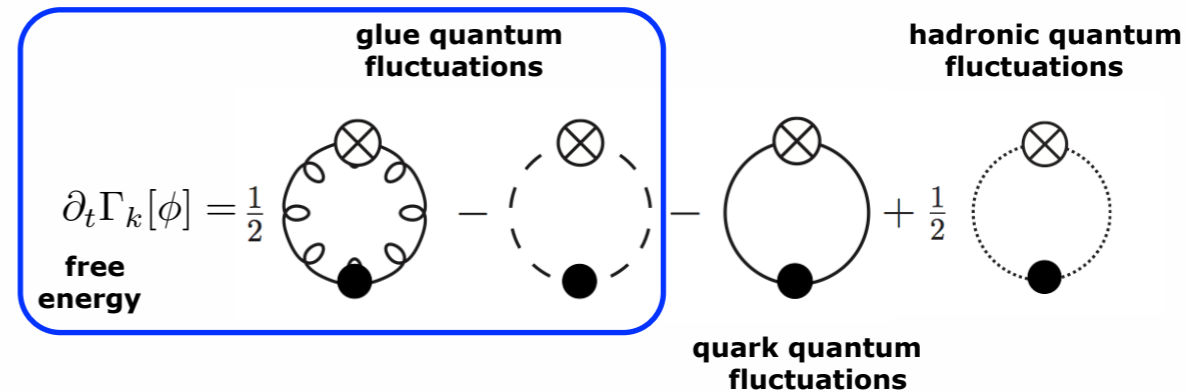
lattice :  $T_c / \sqrt{\sigma} = 0.646$

# Confinement

FRG: Braun, Gies, JMP, PLB 684 (2010) 262

FRG, DSE, 2PI: Fister, JMP, PRD 88 (2013) 045010

$$L[A_0] = \frac{1}{N_c} \text{tr} \mathcal{P} e^{i g \int_0^\beta A_0(x)}$$

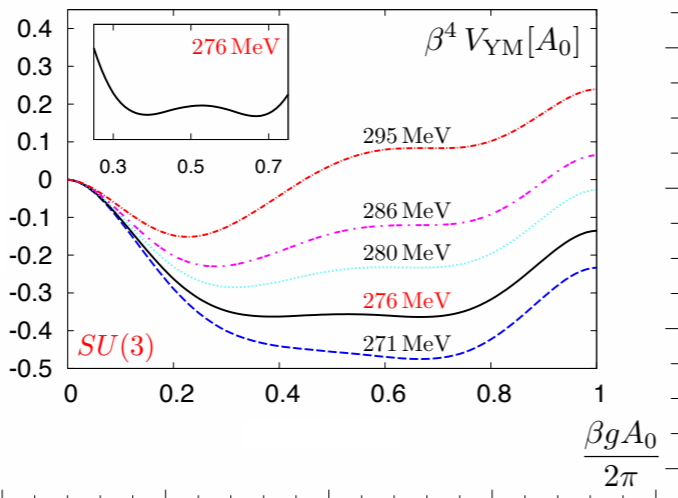


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Polyakov loop

$L(\langle \varphi \rangle)$

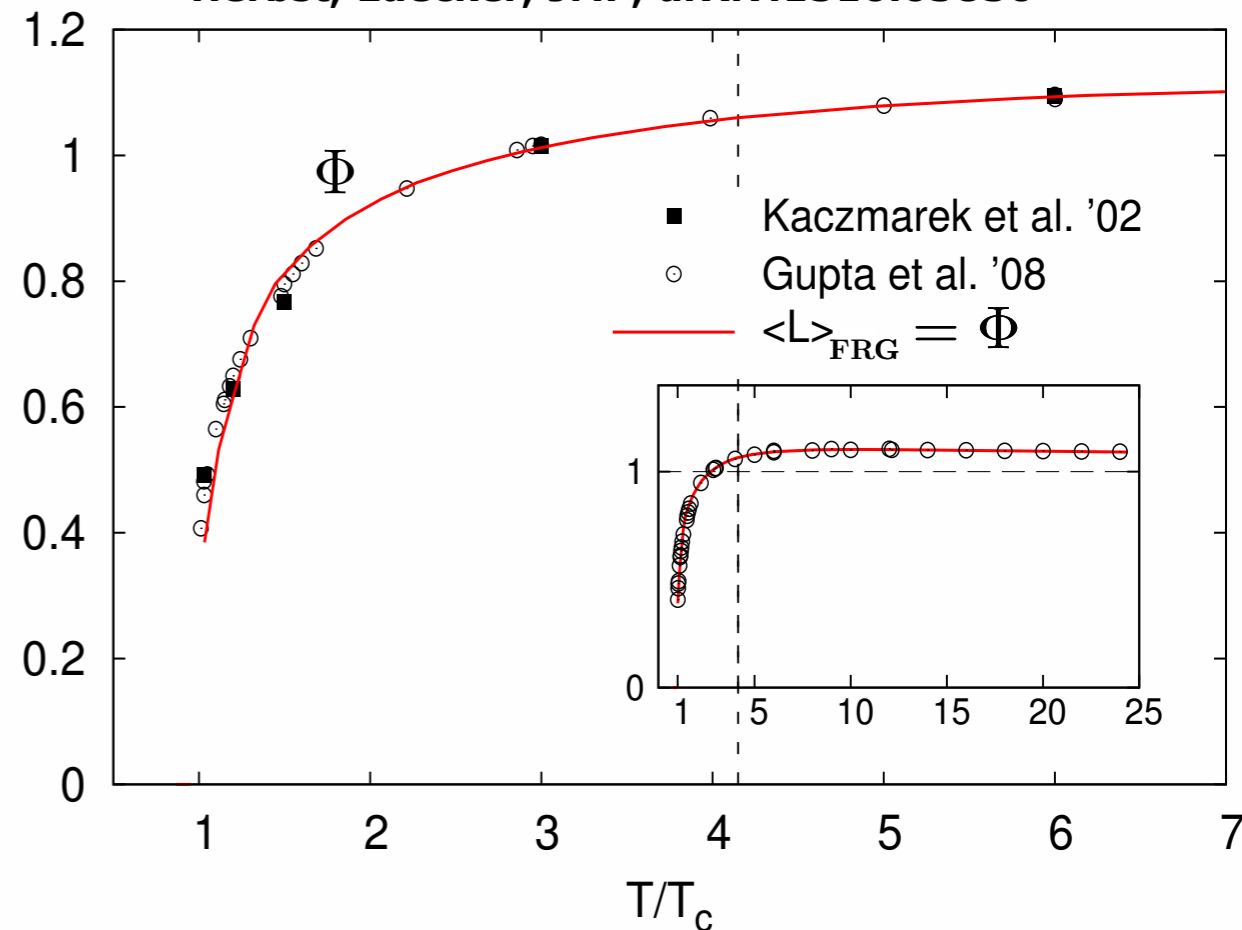
Polyakov loop Potential



0.90 0.95 1.00 1.05 1.10 1.15 1.20 1.25

$T/T_c$

Herbst, Luecker, JMP, arXiv:1510.03830



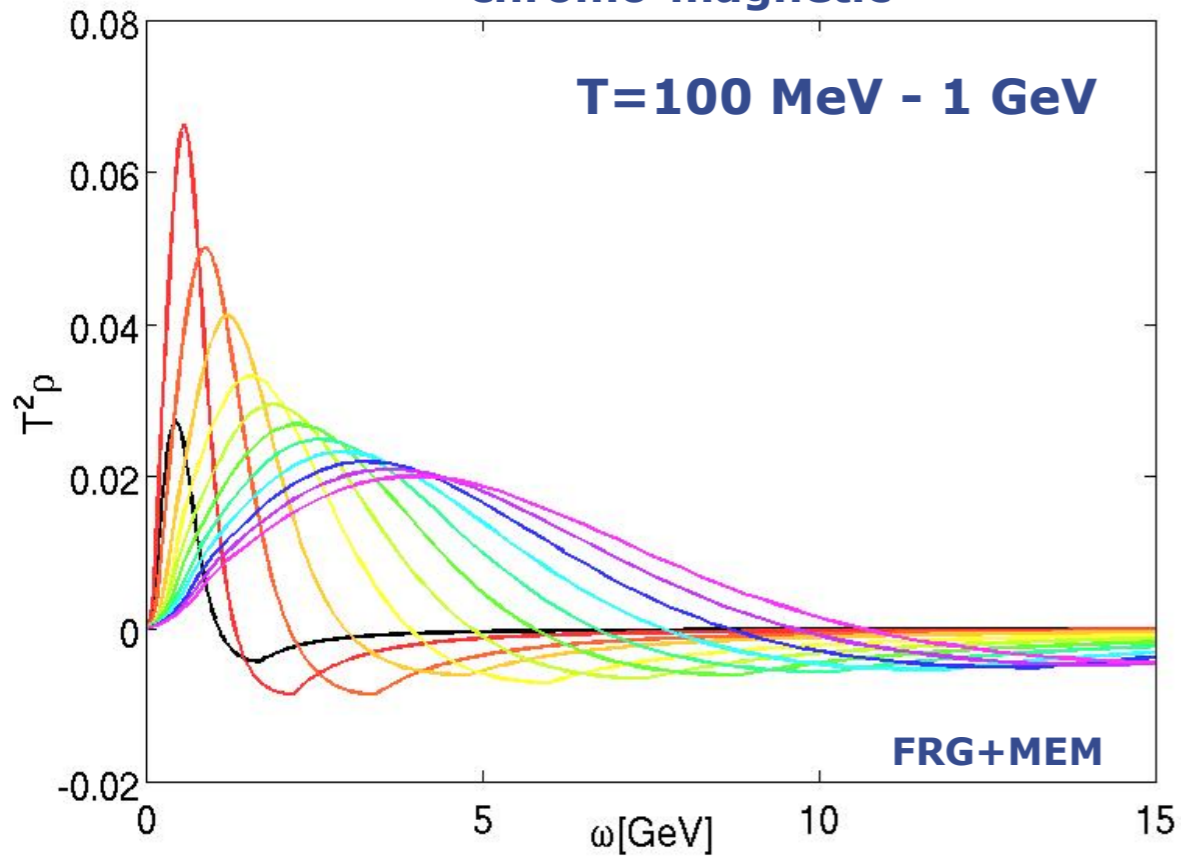
# Single particle spectral functions

$$\rho(p) = 2 \operatorname{Im} \langle A A \rangle_{\text{ret}}(p)$$



# Single particle spectral functions

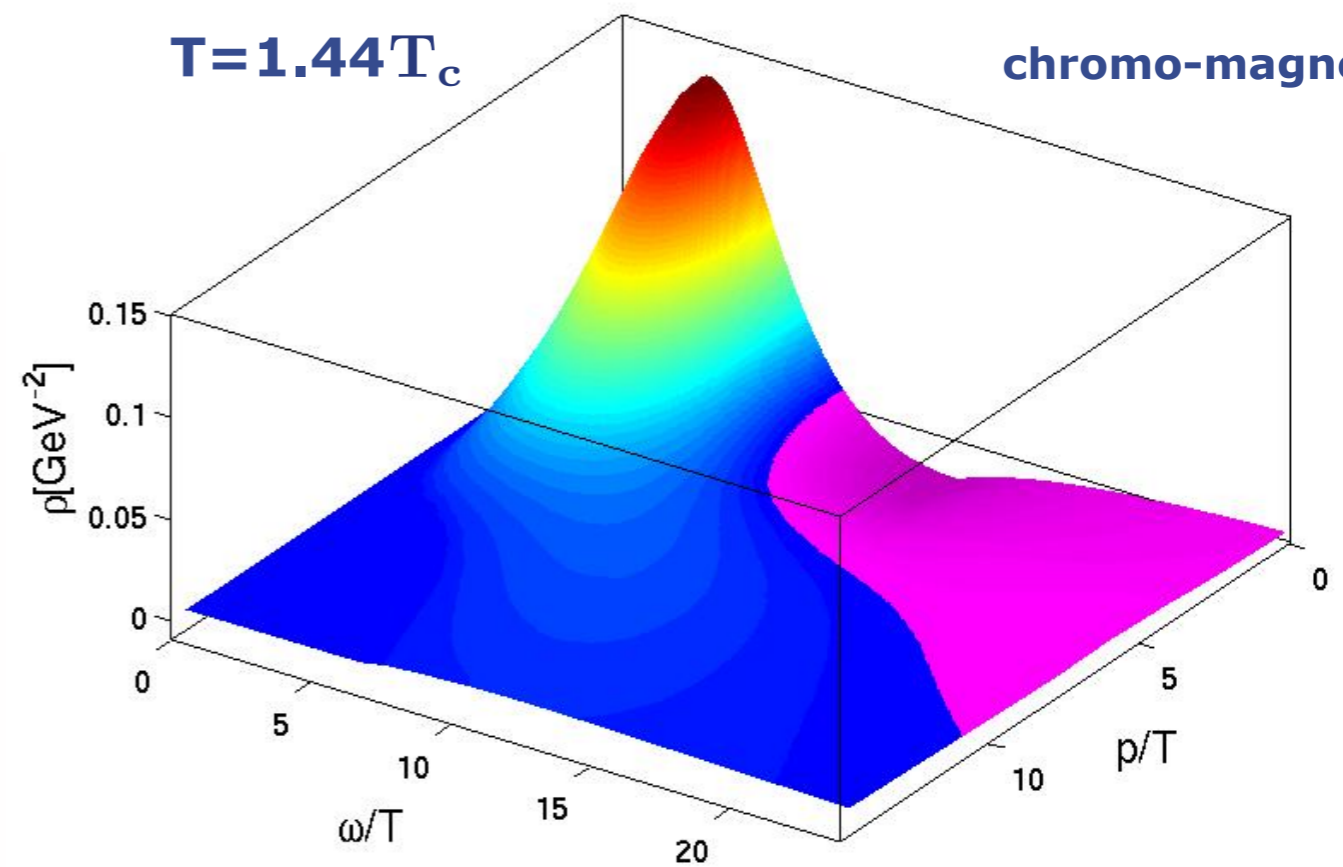
chromo-magnetic



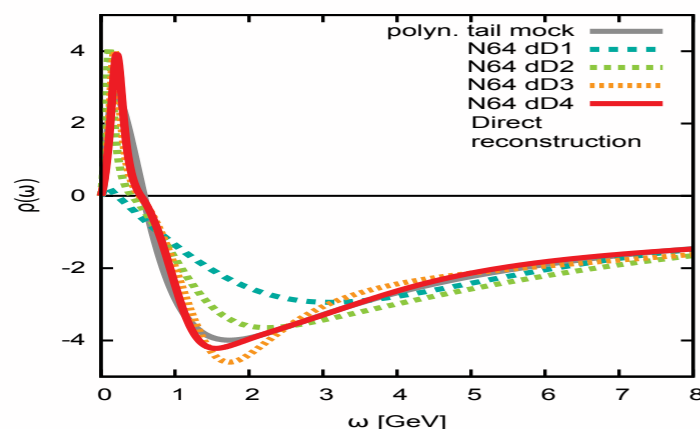
$$\rho(p) = 2 \text{Im} \langle A A \rangle_{\text{ret}}(p)$$

**T=1.44 T<sub>c</sub>**

chromo-magnetic



**Maximum Entropy Method**

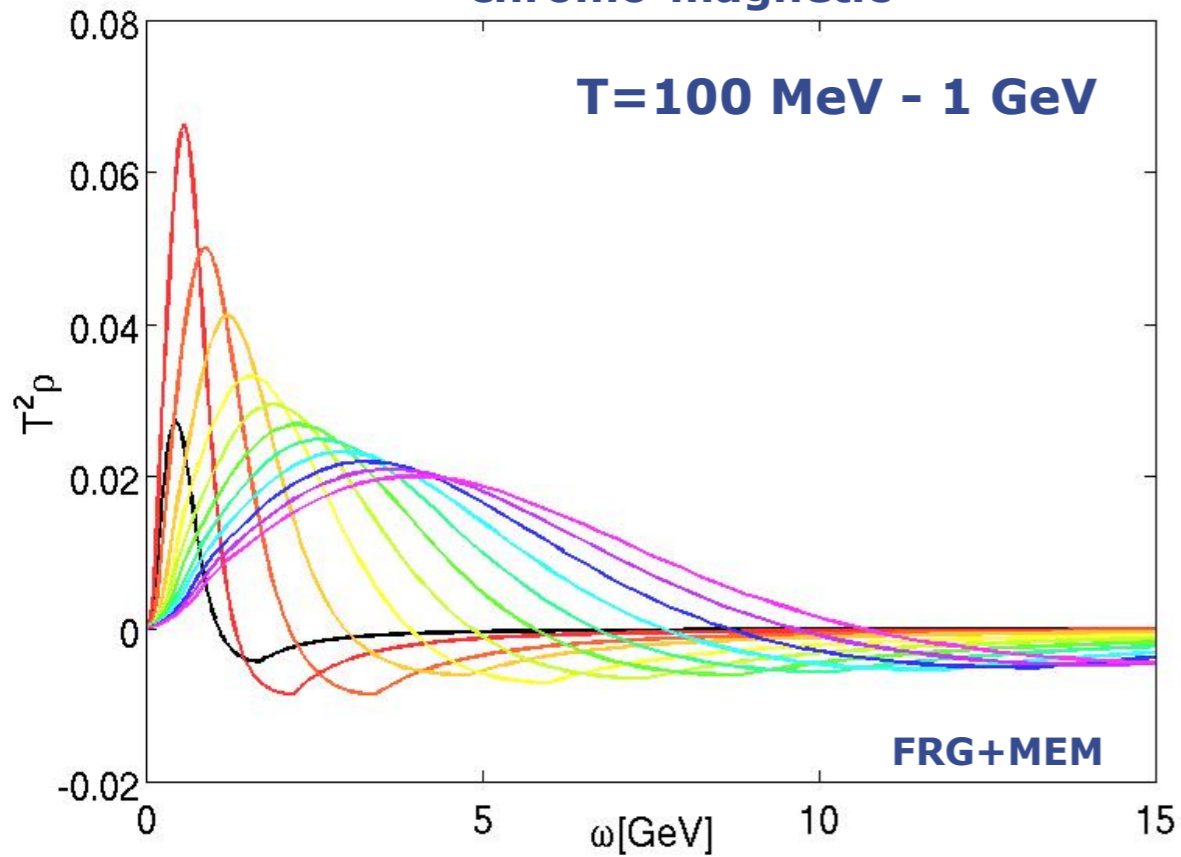


JMP, Rothkopf, work in progress

Haas, Fister, JMP, PRD 90 (2014) 9, 091501

# Single particle spectral functions

chromo-magnetic

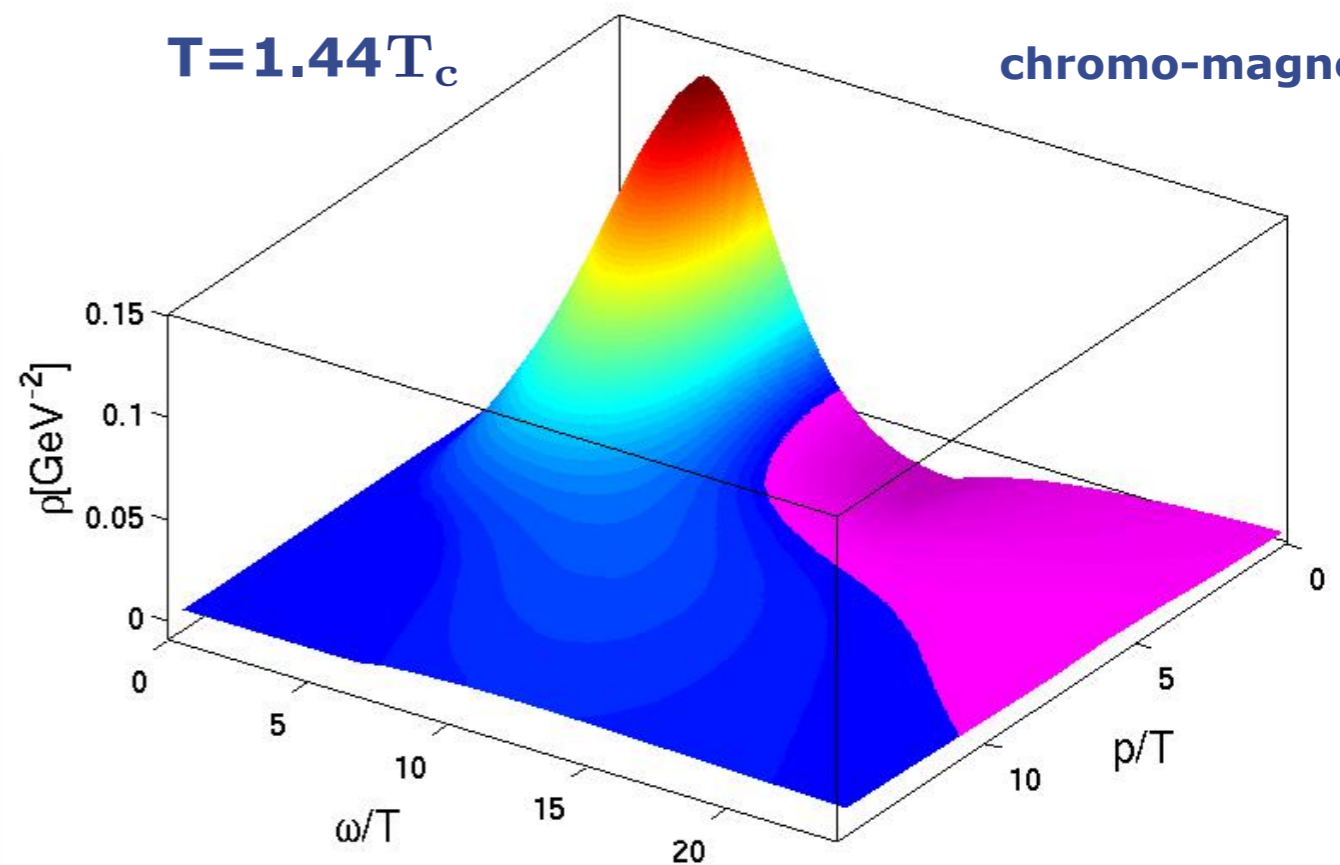


$$\rho(p) = 2 \text{Im} \langle A A \rangle_{\text{ret}}(p)$$

'Those are my methods (principles), and if you don't like them...well, I have others'  
 direct computation Groucho Marx

**T=1.44 T<sub>c</sub>**

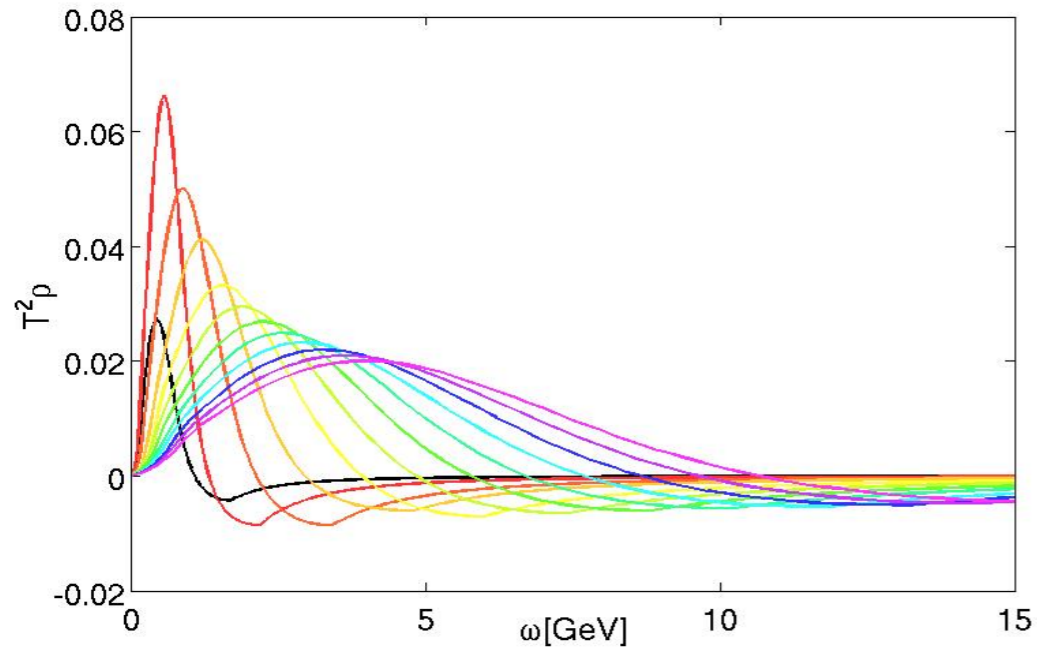
chromo-magnetic



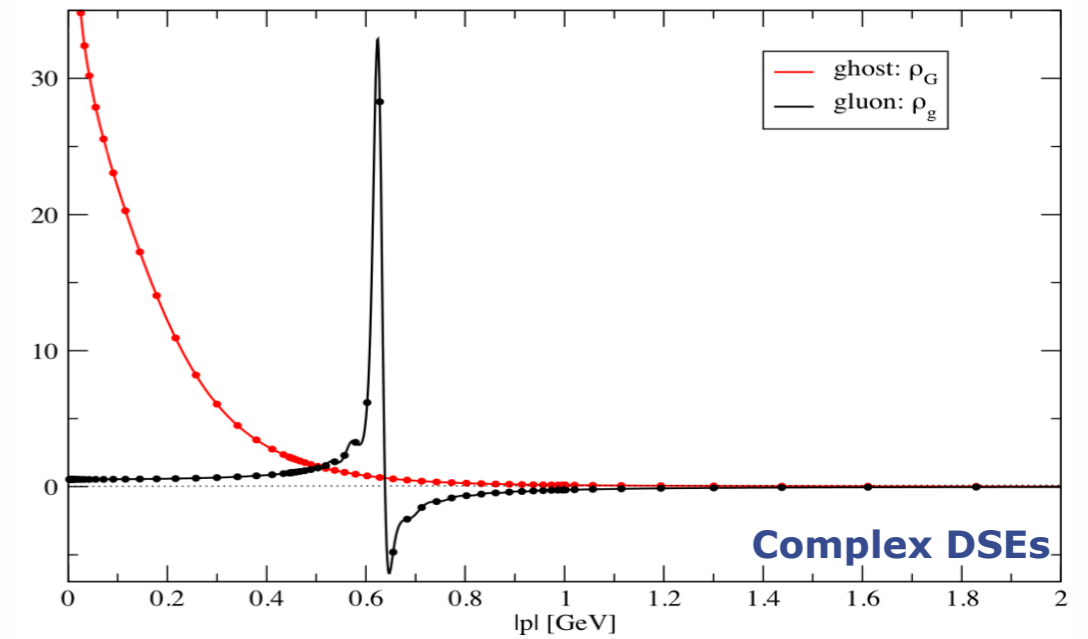
Haas, Fister, JMP, PRD 90 (2014) 9, 091501

# Transport

## gluon spectral functions

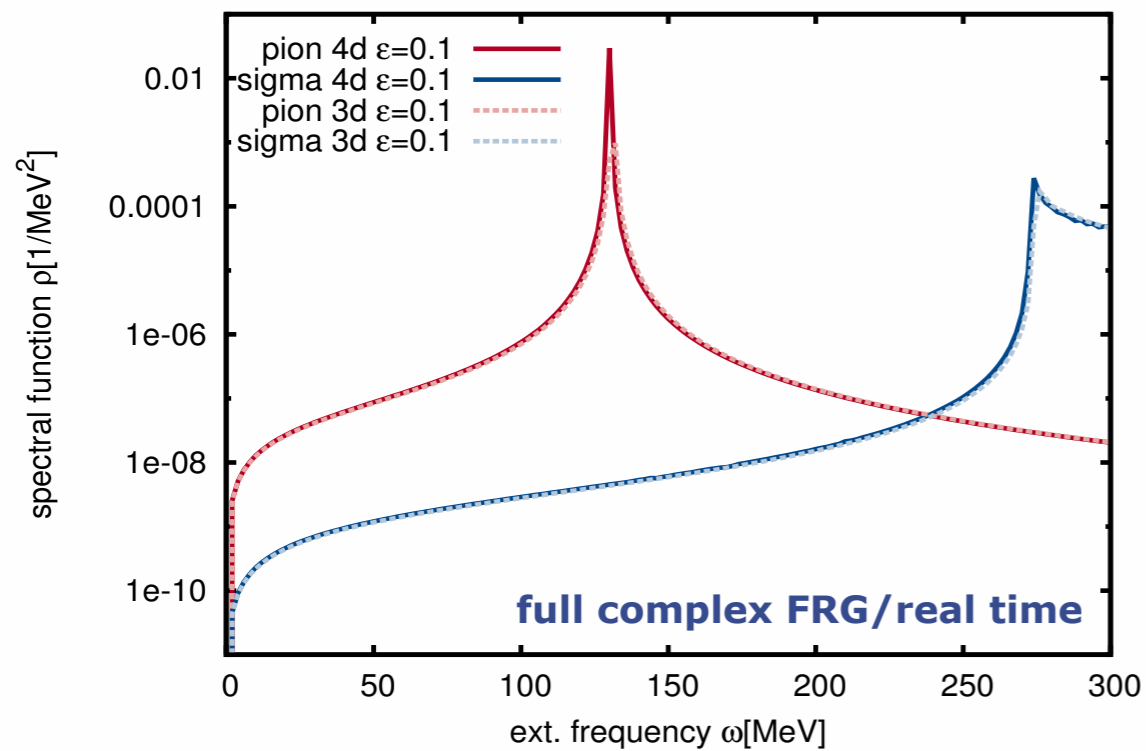


Haas, Fister, JMP, PRD 90 (2014) 9, 091501

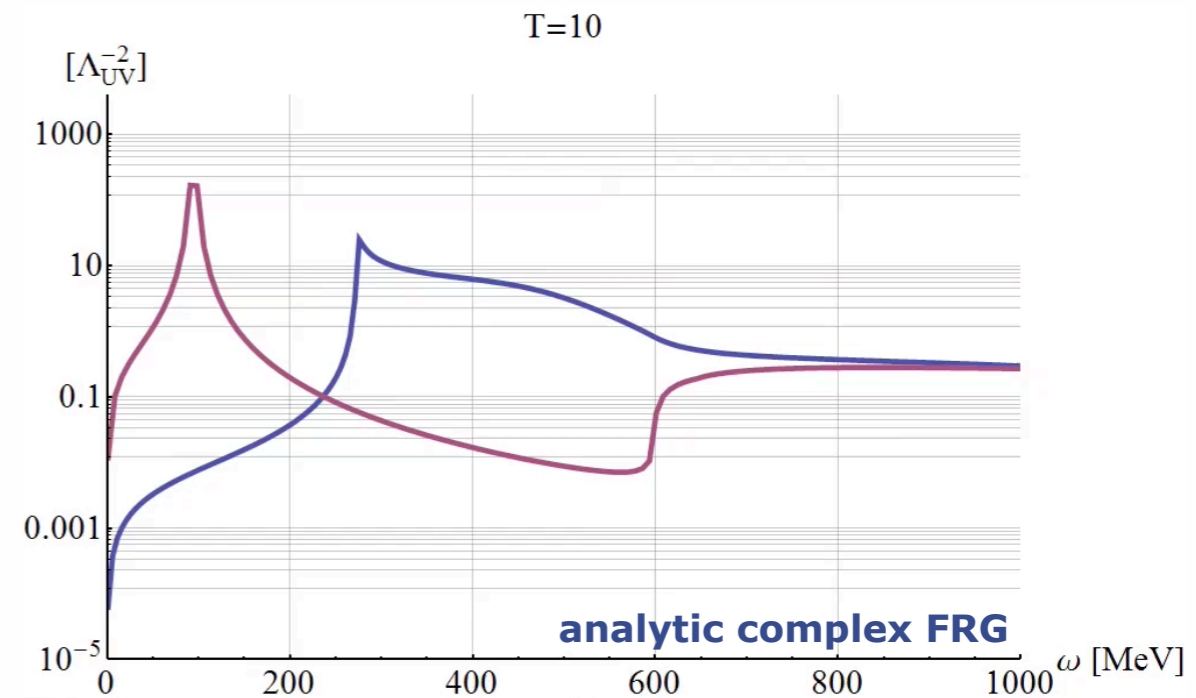


Strauss, Fischer, Kellermann, PRL 109 (2012) 252001

## pion and sigma spectral functions



JMP, Strodthoff, PRD 92 (2015) 094009



Tripolt, Strodthoff, von Smekal, Wamach, PRD 89 (2014) 034010  
Kamikado, Strodthoff, von Smekal, Wambach, EPJ C74 (2014) 2806

# Transport

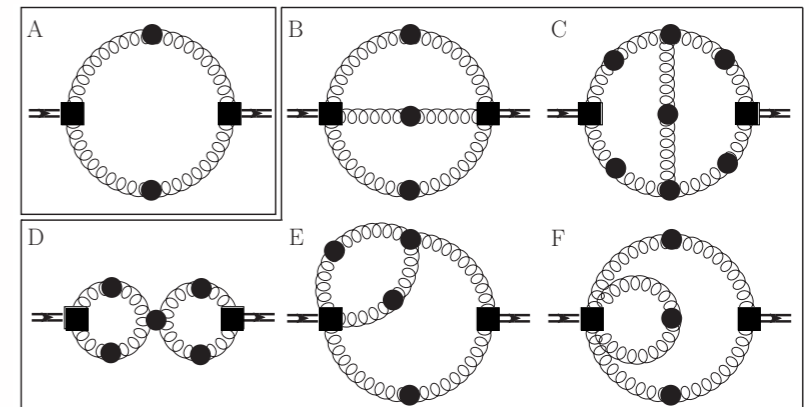
## transport coefficients

### Kubo relation

$$\eta = \frac{1}{20} \left. \frac{d}{d\omega} \right|_{\omega=0} \rho_{\pi\pi}(\omega, 0)$$

### '3-loop' exact functional relation for $\rho_{\pi\pi}$

#### 1 & 2-loop terms



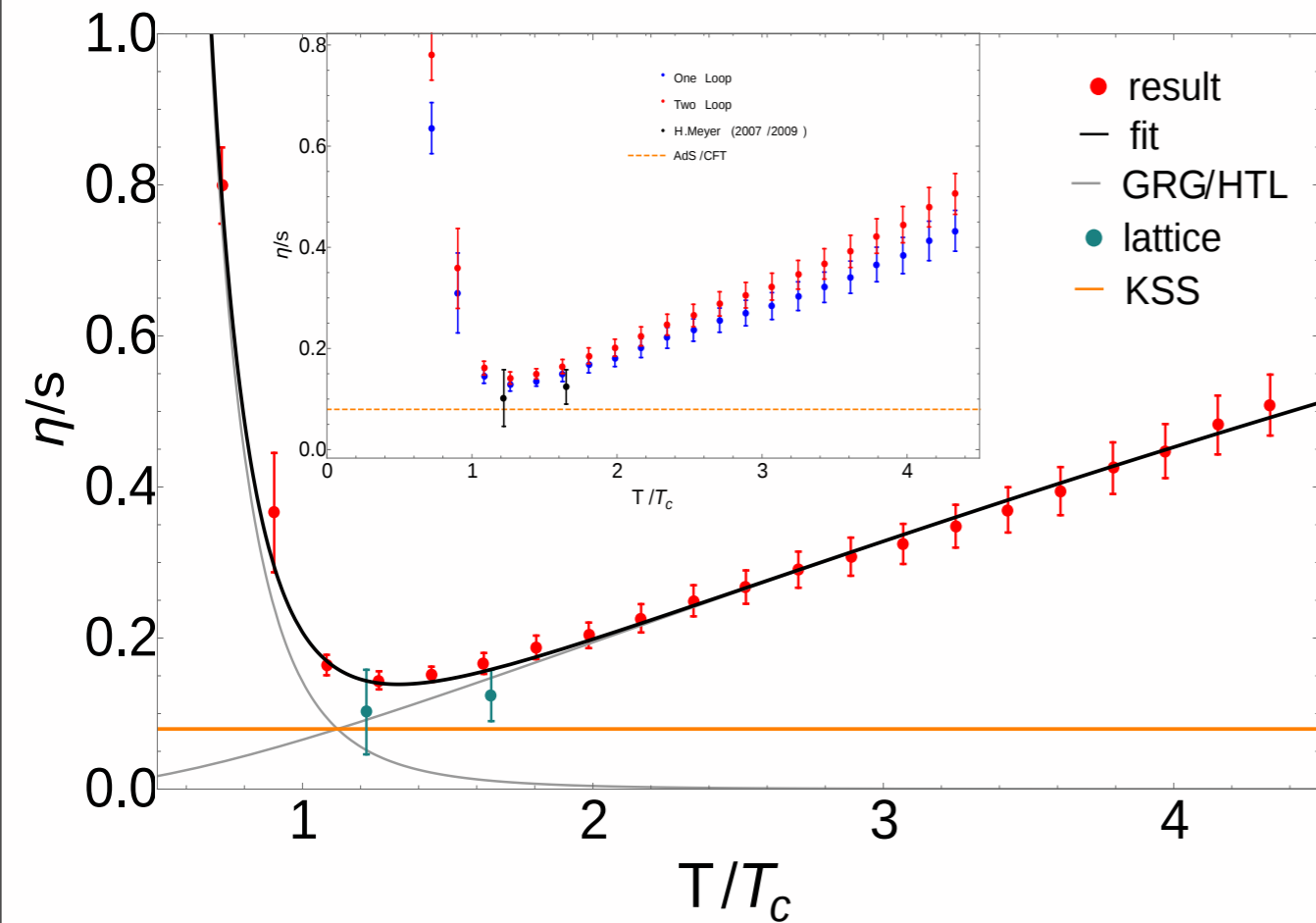
Haas, Fister, JMP, PRD 90 (2014) 9, 091501

Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 11, 112002

# Transport

## transport coefficients

### Yang-Mills viscosity over entropy ratio



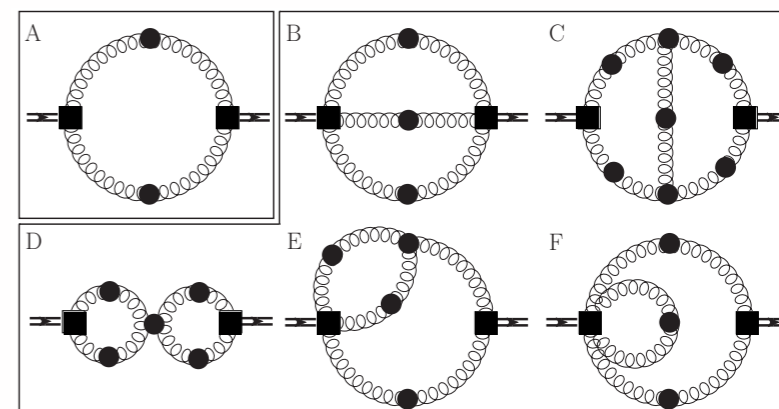
**Aiming at apparent convergence**

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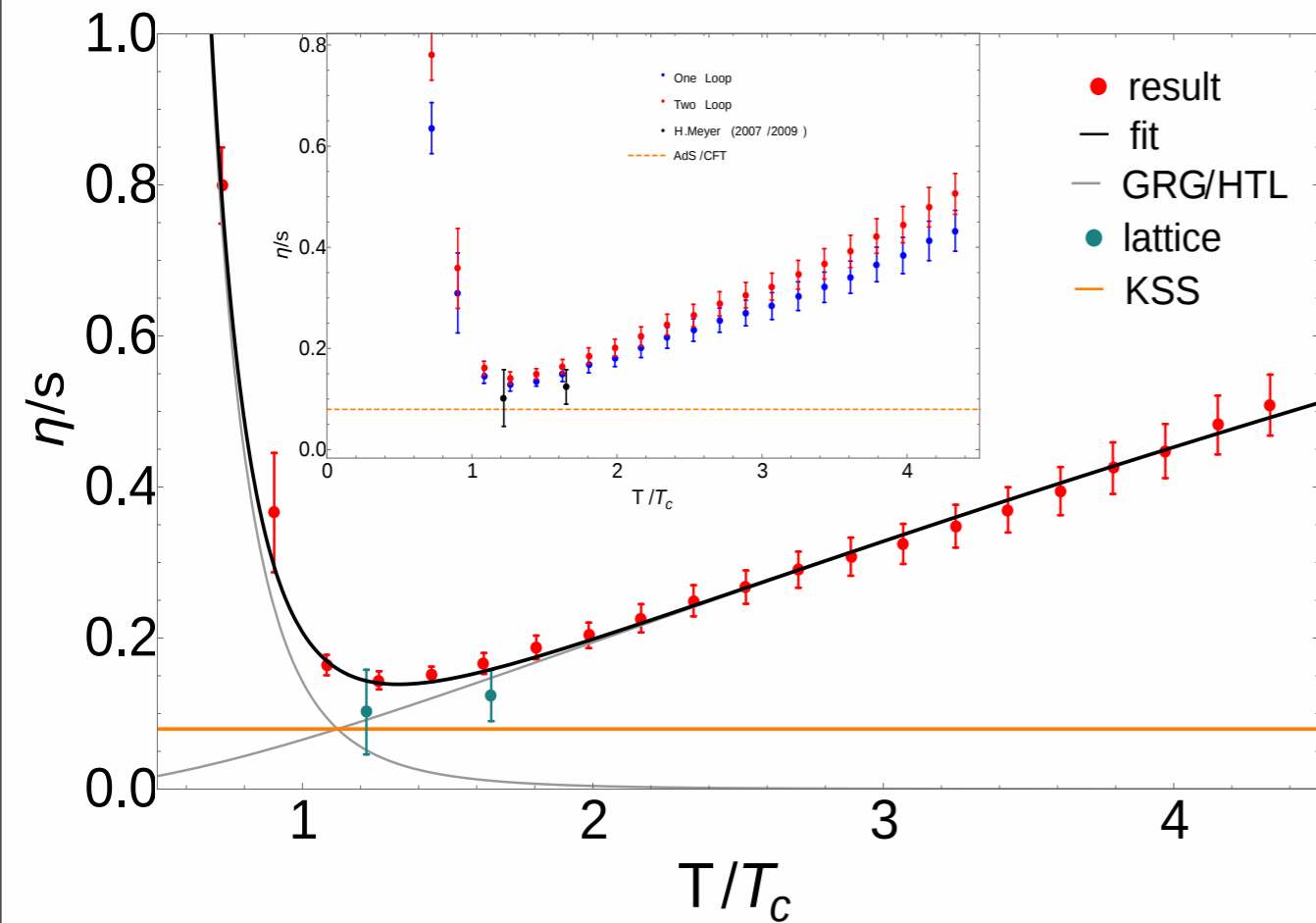
Haas, Fister, JMP, PRD 90 (2014) 9, 091501

Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 11, 112002

# Transport

## QCD - estimate for viscosity over entropy ratio

### viscosity over entropy ratio



$$\gamma_{\text{grg}} \approx 5$$

$$\gamma_{\text{qgp}} \approx 1.6$$

pure glue

$$a_{\text{qgp}} \approx 0.15$$

$$a_{\text{hrg}} \approx 0.14$$

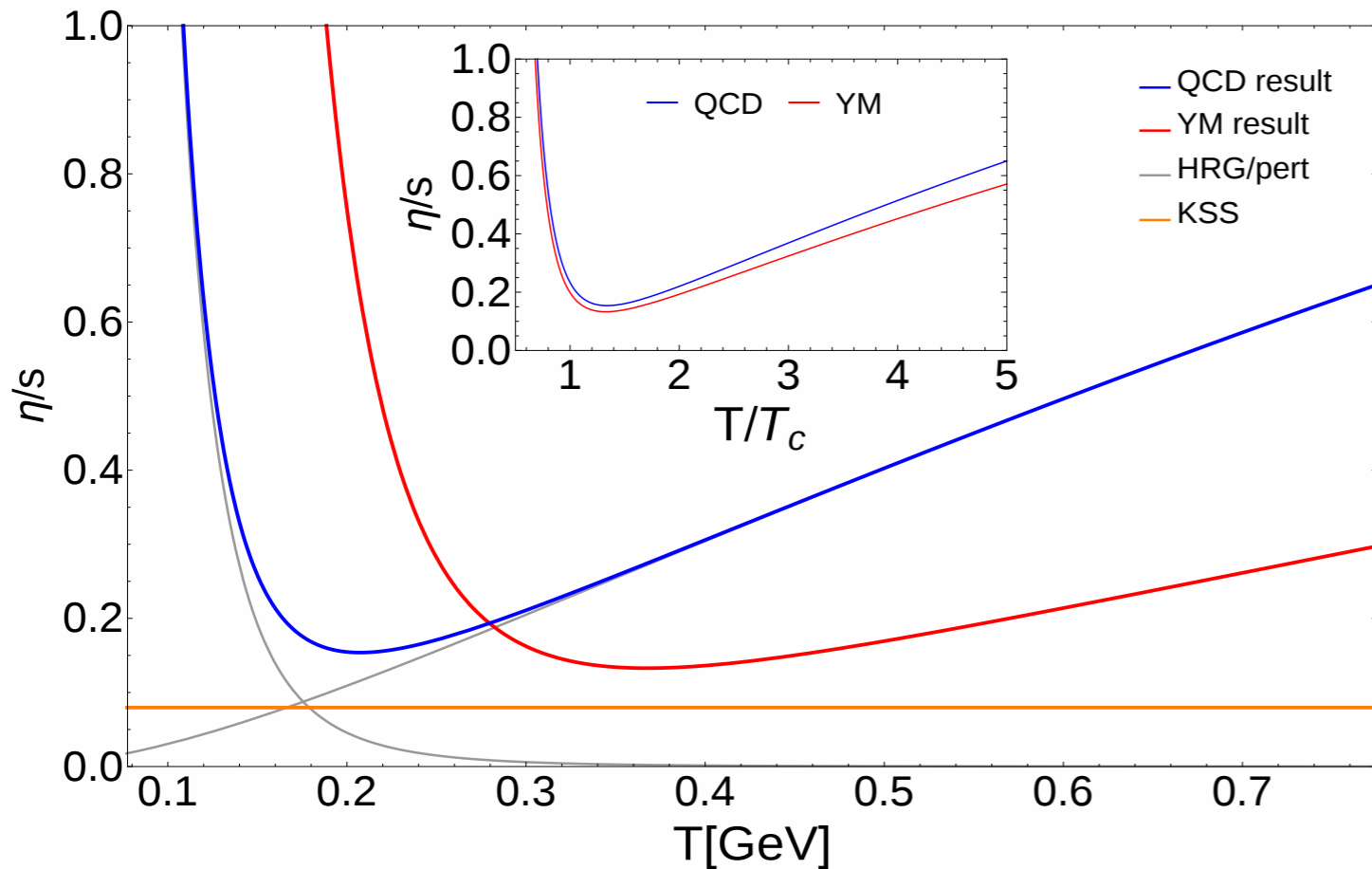
$$c \approx 0.66$$

$$\frac{\eta}{s}(T) = \frac{a_{\text{qgp}}}{\alpha_s^{\gamma_{\text{qgp}}}(cT/T_c)} + \frac{a_{\text{grg}}}{(T/T_c)^{\gamma_{\text{grg}}}}$$

# Transport

## QCD - estimate for viscosity over entropy ratio

### viscosity over entropy ratio



$$a_{\text{qgp}} \approx 0.2$$

$$a_{\text{hrg}} \approx 0.16$$

$$c \approx 0.79$$

**QCD**

$$\gamma_{\text{grg}} \approx 5$$

$$\gamma_{\text{qgp}} \approx 1.6$$

**pure glue**

$$a_{\text{qgp}} \approx 0.15$$

$$a_{\text{hrg}} \approx 0.14$$

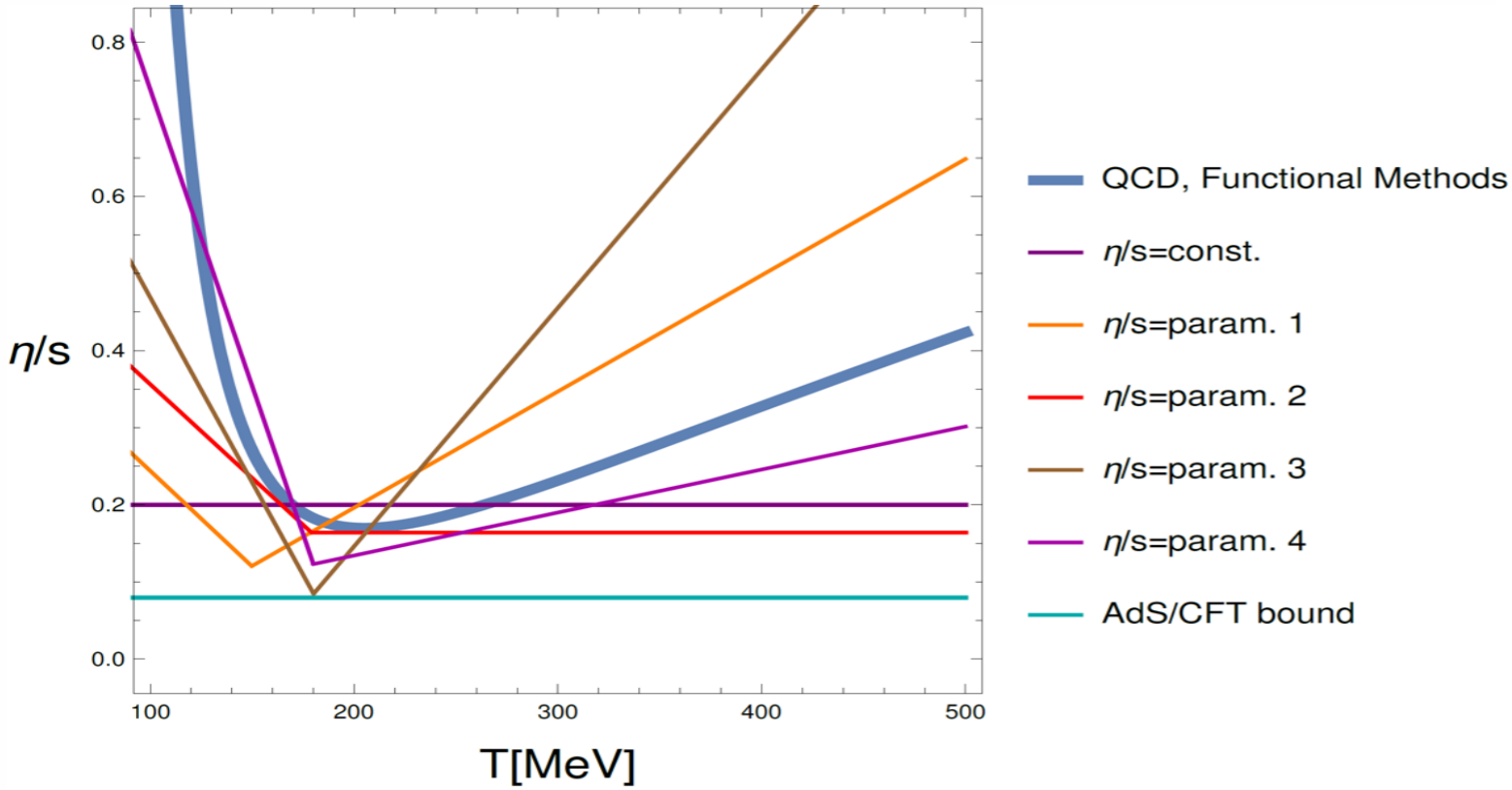
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# Transport

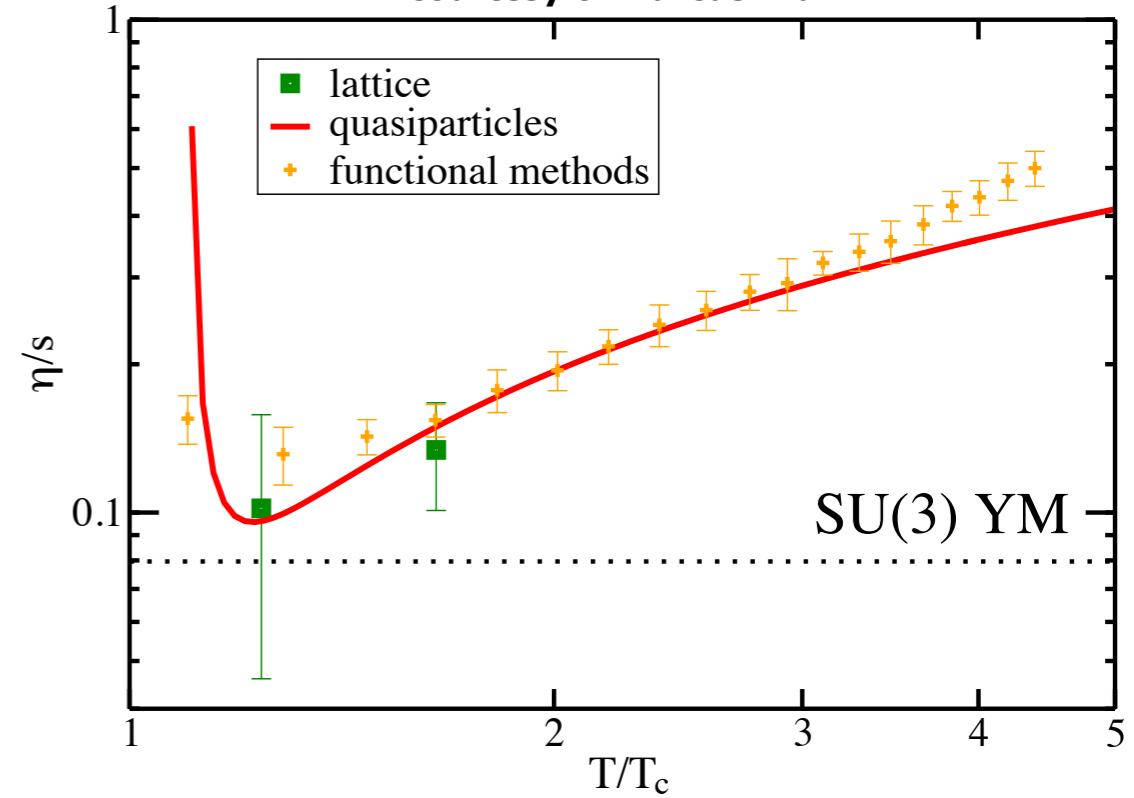
## QCD transport & transport models

courtesy of Nicolai Christiansen



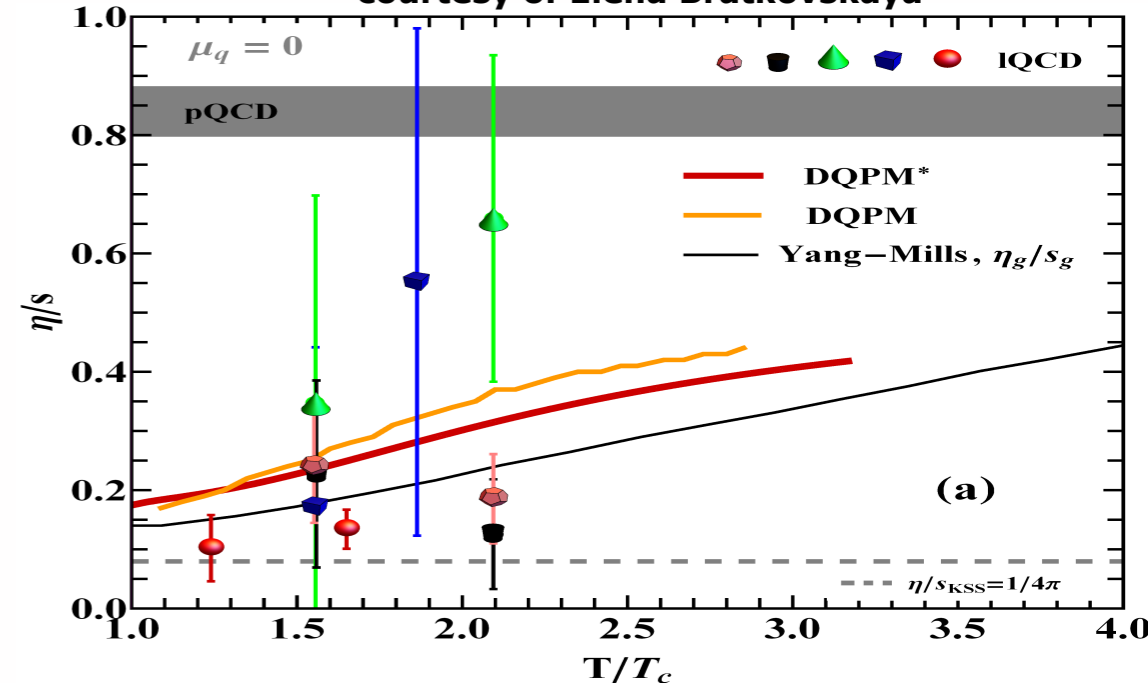
Niemi, Eskola, Paateleinen, PRC 93 (2016) 024907

courtesy of Marcus Bluhm



Bluhm, Kaempfer, Redlich, PRC 84 (2011) 025201

courtesy of Elena Bratkovskaya



Berrehrhah, Cassing, Bratkovskaya, Steinert, PRC 93 (2016) 044914

$$\frac{\eta}{s}(T) = \frac{a_{\text{qgp}}}{\alpha_s^{\gamma_{\text{qgp}}}(cT/T_c)} + \frac{a_{\text{grg}}}{(T/T_c)^{\gamma_{\text{grg}}}}$$

Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 11, 112002



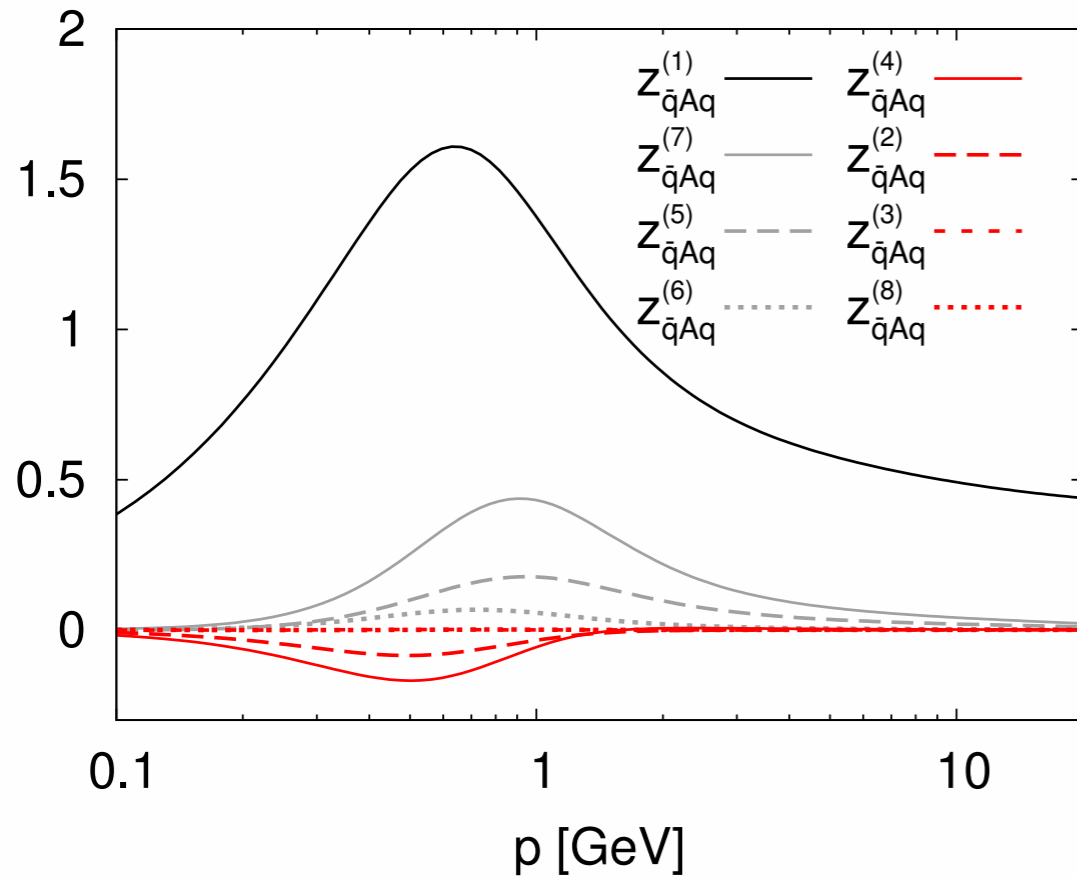
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- **Chiral symmetry breaking & the phase structure**
- **Summary & outlook**

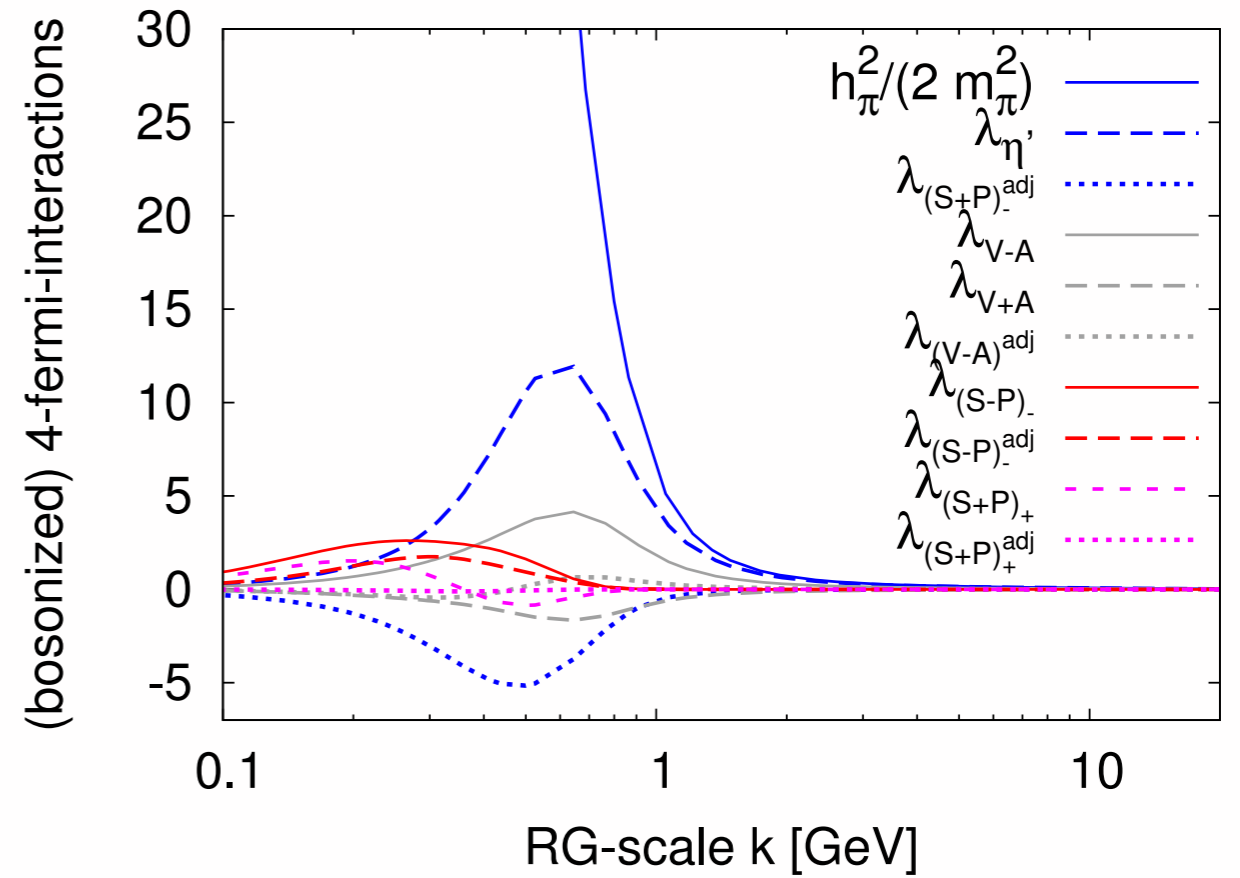
# Chiral symmetry breaking

## FRG-quenched QCD vs lattice-quenched QCD

quark-gluon vertex



four-fermi vertex



quark-gluon couplings

(bosonized) 4-fermi-interactions

see also

Williams, Eur.Phys.J. A51 (2015) 5, 57  
Williams, Fischer, Heupel, PRD 93 (2016) 034026

$N_f = 2$

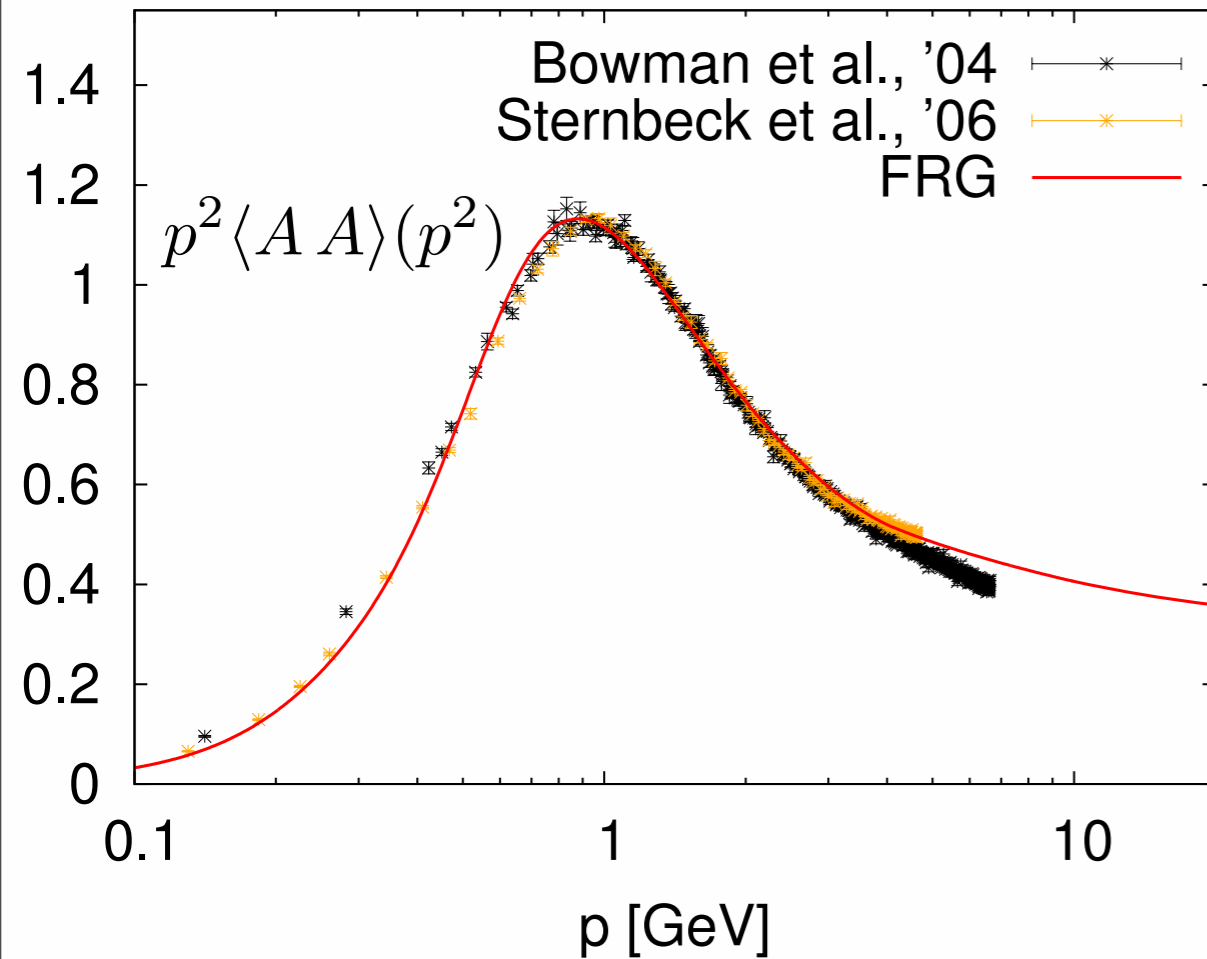
rapid convergence in covariant expansion scheme

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

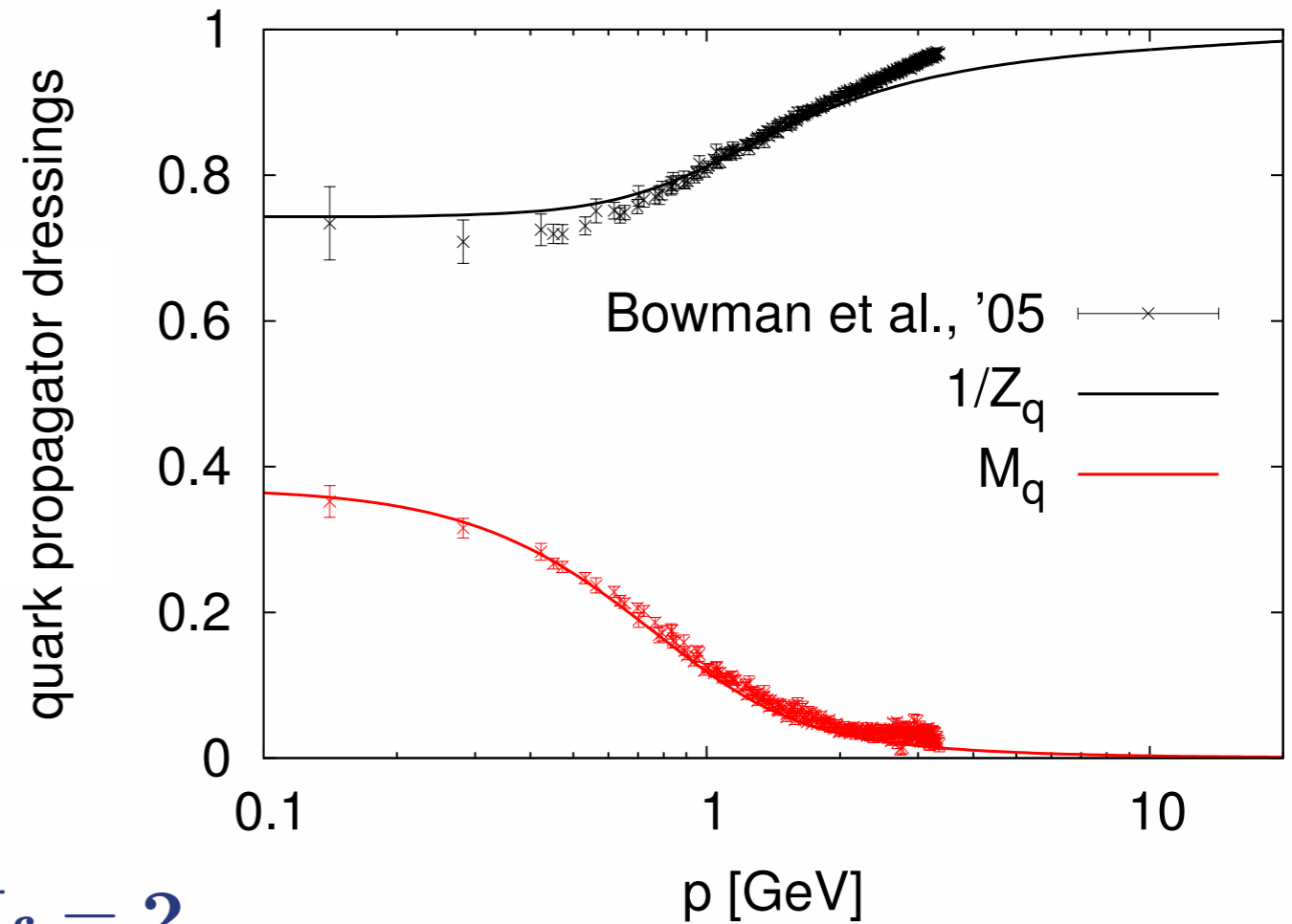
# Chiral symmetry breaking

## FRG-quenched QCD vs lattice-quenched QCD

quenched gluon dressing



quark propagator



$N_f = 2$

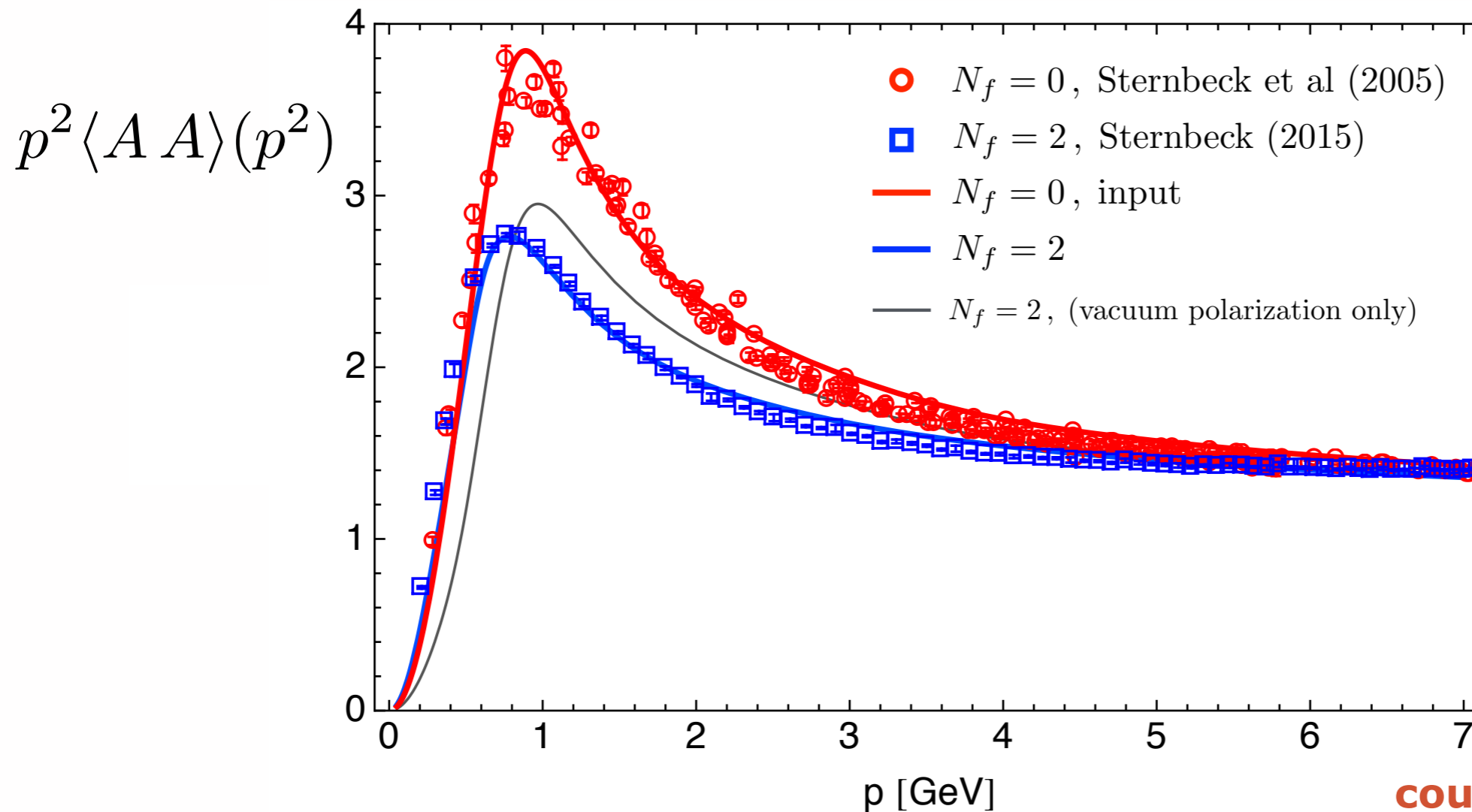
rapid convergence in covariant expansion scheme

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

# Chiral symmetry breaking

FRG-unquenched QCD vs lattice-unquenched QCD

unquenched gluon dressing



courtesy of F. Rennecke

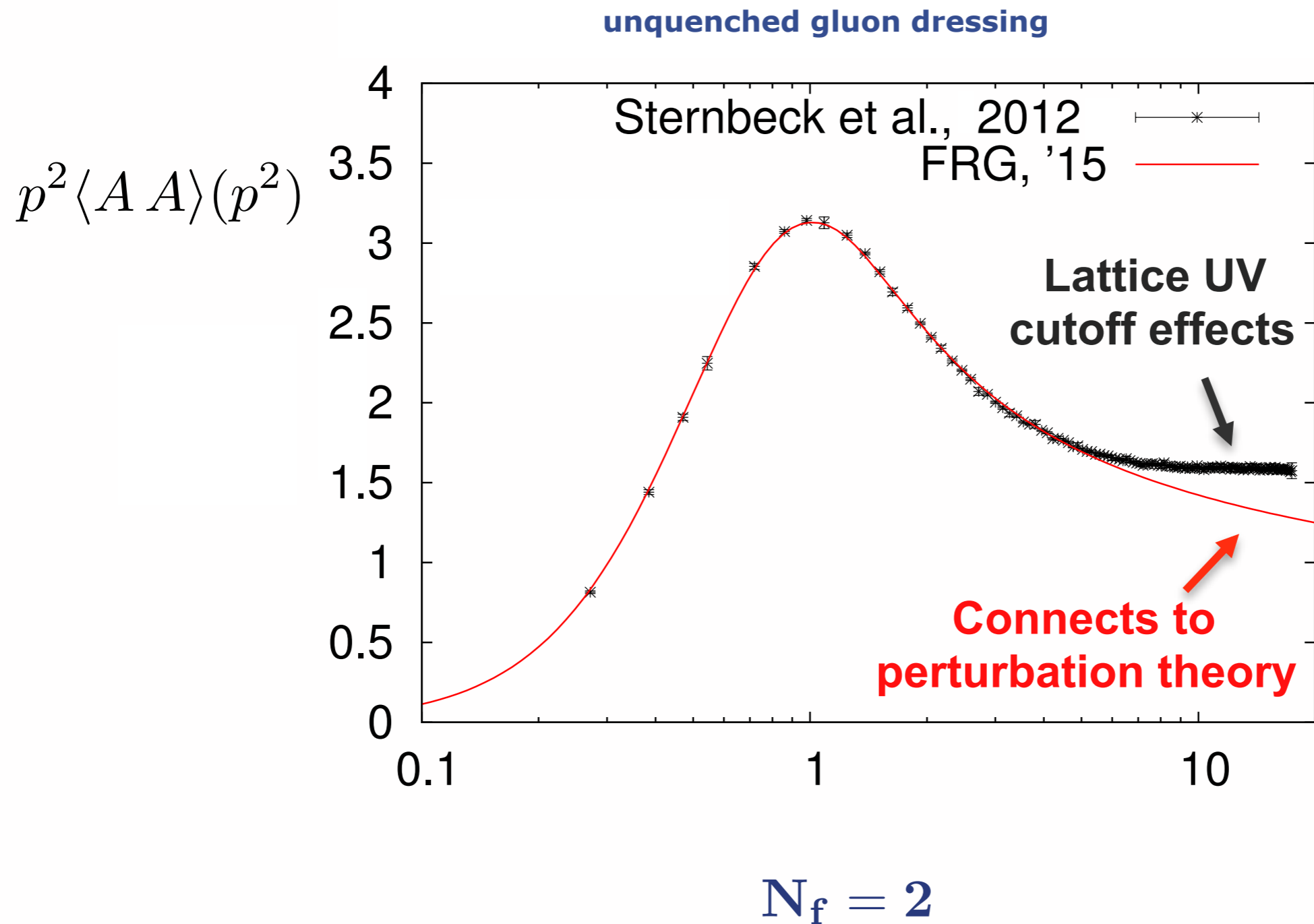
$N_f = 2$

Braun, Fister, Haas, JMP, Rennecke, arXiv:1412.1045

Rennecke, Phys.Rev. D92 (2015) 7, 076012

# Chiral symmetry breaking

## FRG-unquenched QCD vs lattice-unquenched QCD



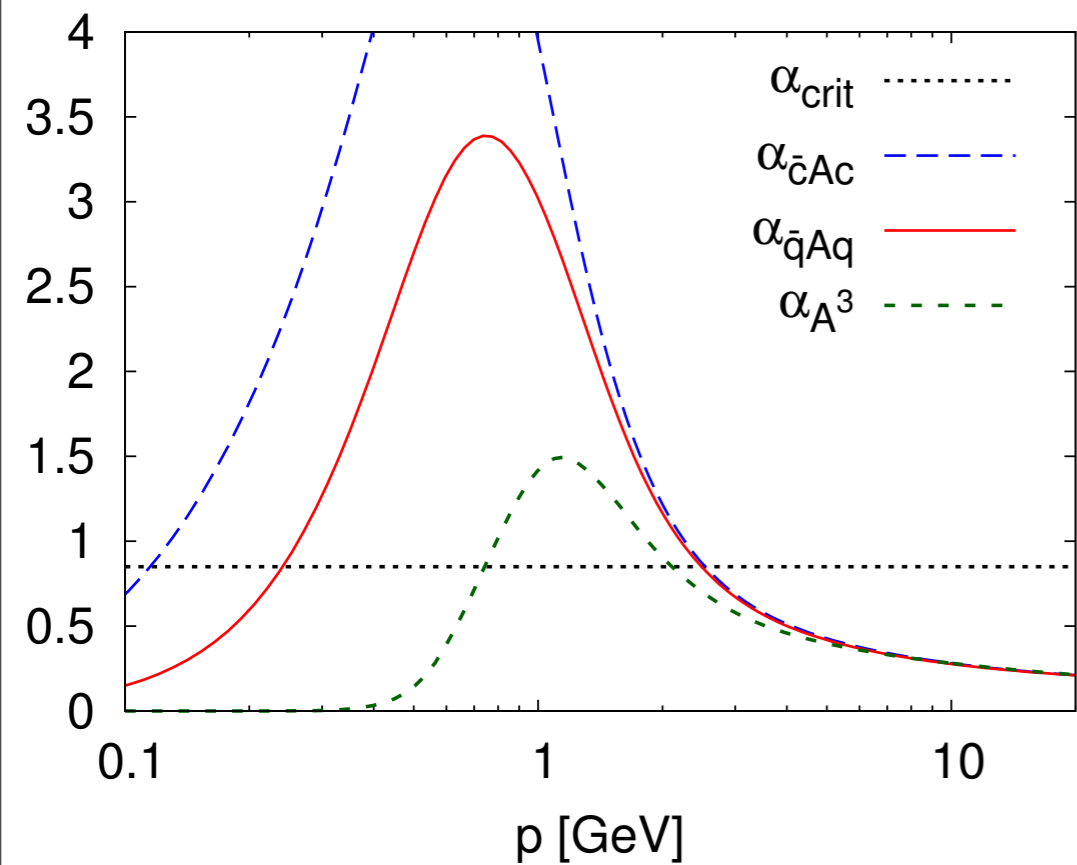
Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, in prep.

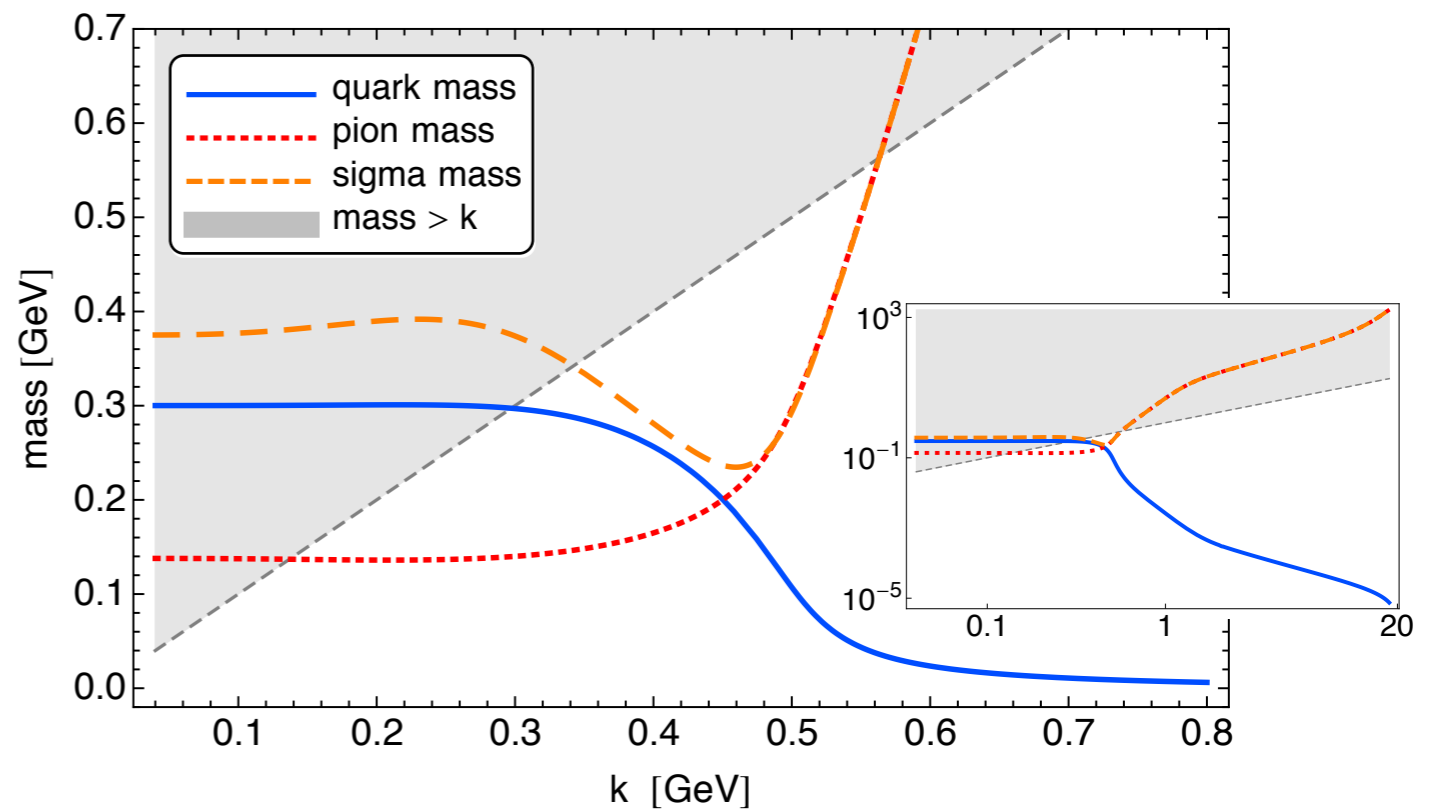
# QCD

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left( \text{loop with ghost} - \text{loop with quark} - \text{loop with gluon} + \frac{1}{2} \text{loop with pion} \right)$$

## Sequential decoupling of gluon, quark, sigma, pion fluctuations



Mitter, JMP, Strodthoff, PRD 91 (2015) 054035



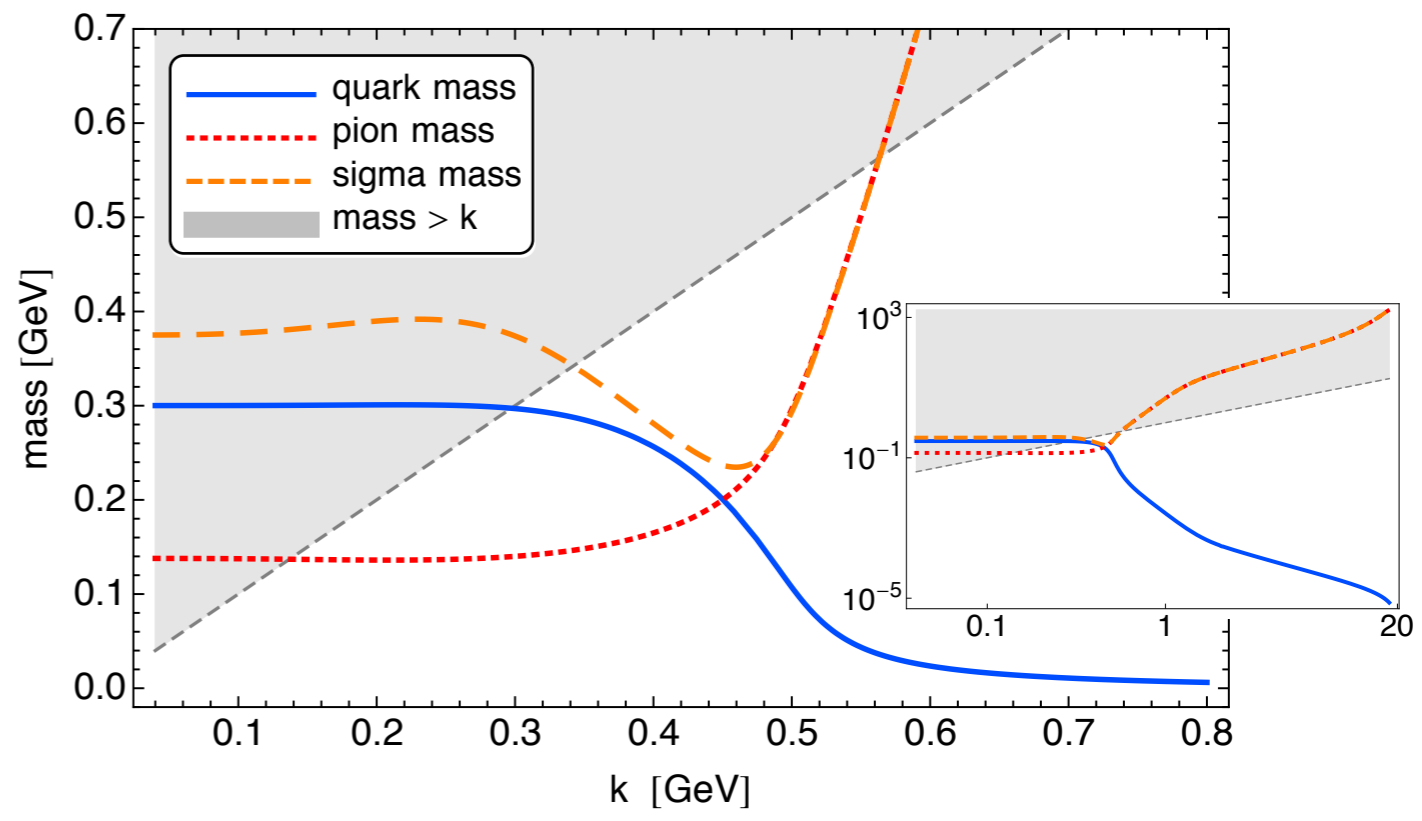
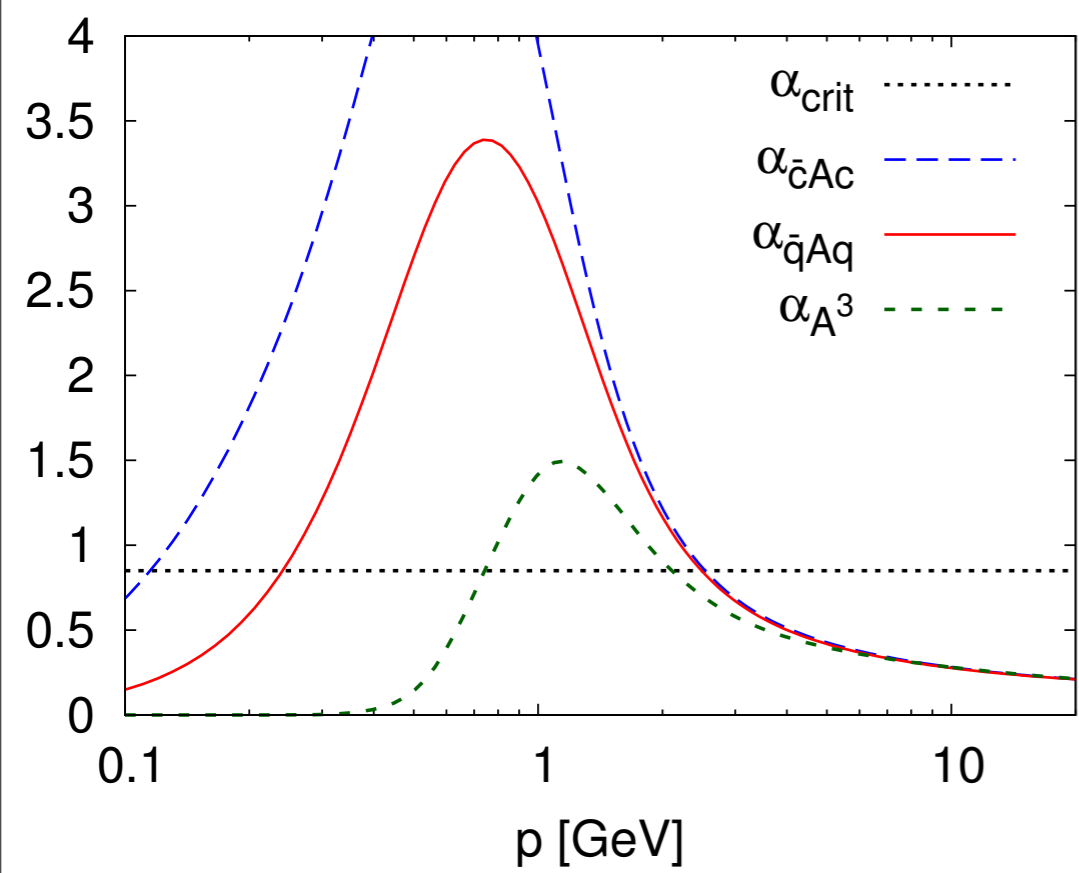
Braun, Fister, Haas, JMP, Rennecke, arXiv:1412.1045

Rennecke, Phys.Rev. D92 (2015) 7, 076012

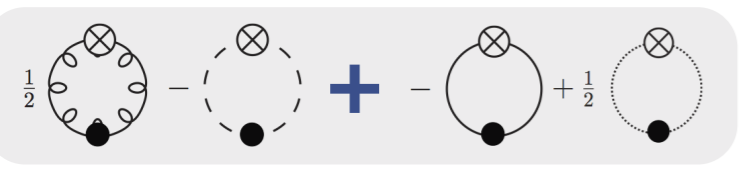
# QCD

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{[diagram 1]} - \text{[diagram 2]} - \text{[diagram 3]} + \frac{1}{2} \text{[diagram 4]}$$

## Sequential decoupling of gluon, quark, sigma, pion fluctuations



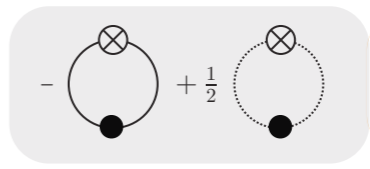
### PQM-model



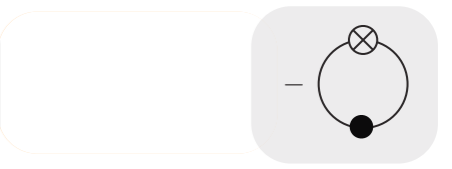
### PNJL-model



### QM-model



### NJL-model

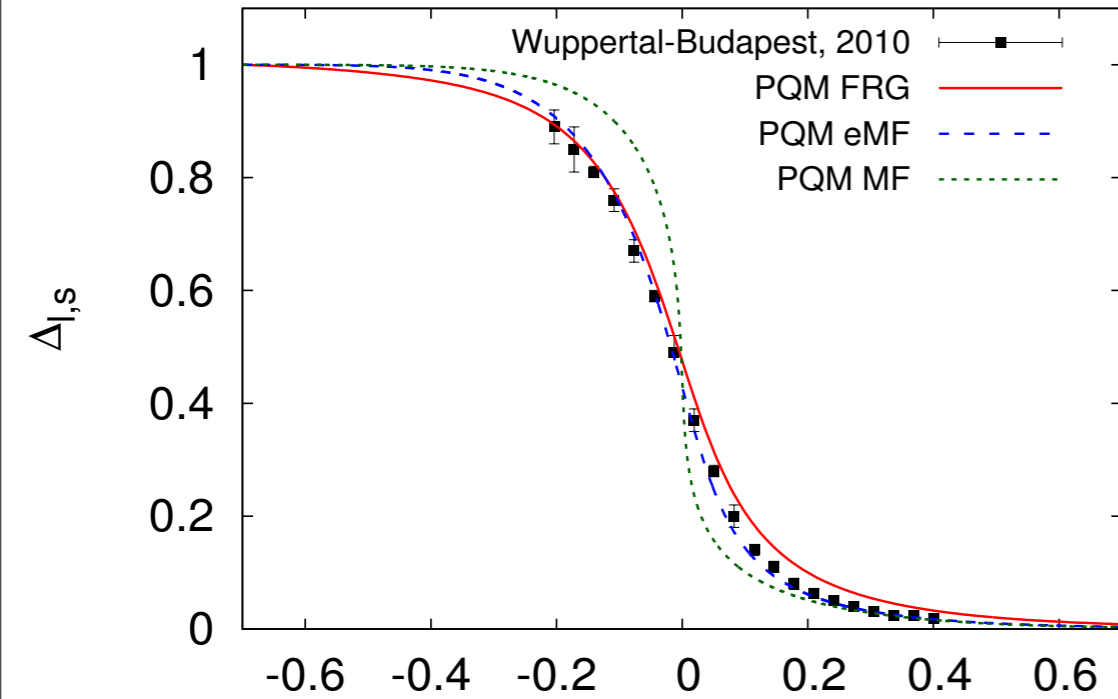


# Thermodynamics and condensates

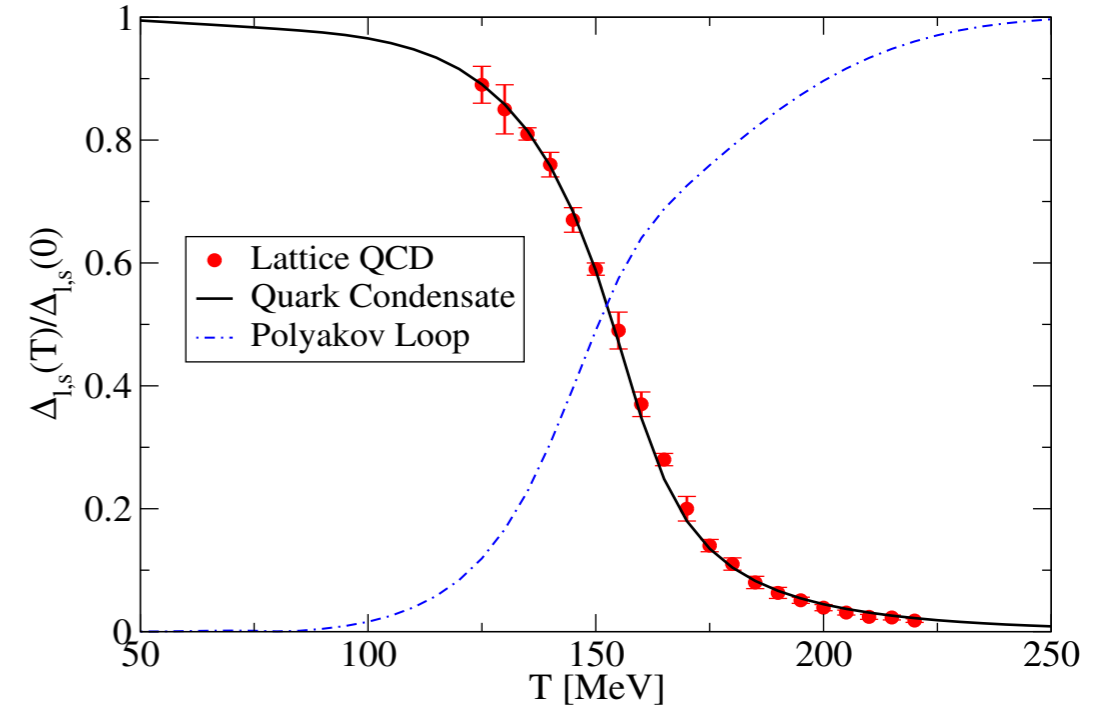
## 2+1 flavor QCD - enhanced PQM-model

## 2+1 flavor DSE

### reduced chiral condensate



Herbst, Mitter et al, PLB 731 (2014) 248-256



Fischer, Luecker, Welzbacher, PRD 90 (2014), 034022

Fischer, Fister, Luecker, JMP, PLB 732 (2014) 273-277

## Glue potential from QCD-computation with FRG

Braun, Haas, Marhauser, JMP, PRL 106 (2011) 022002

**Approximations of infrared dynamics involved**

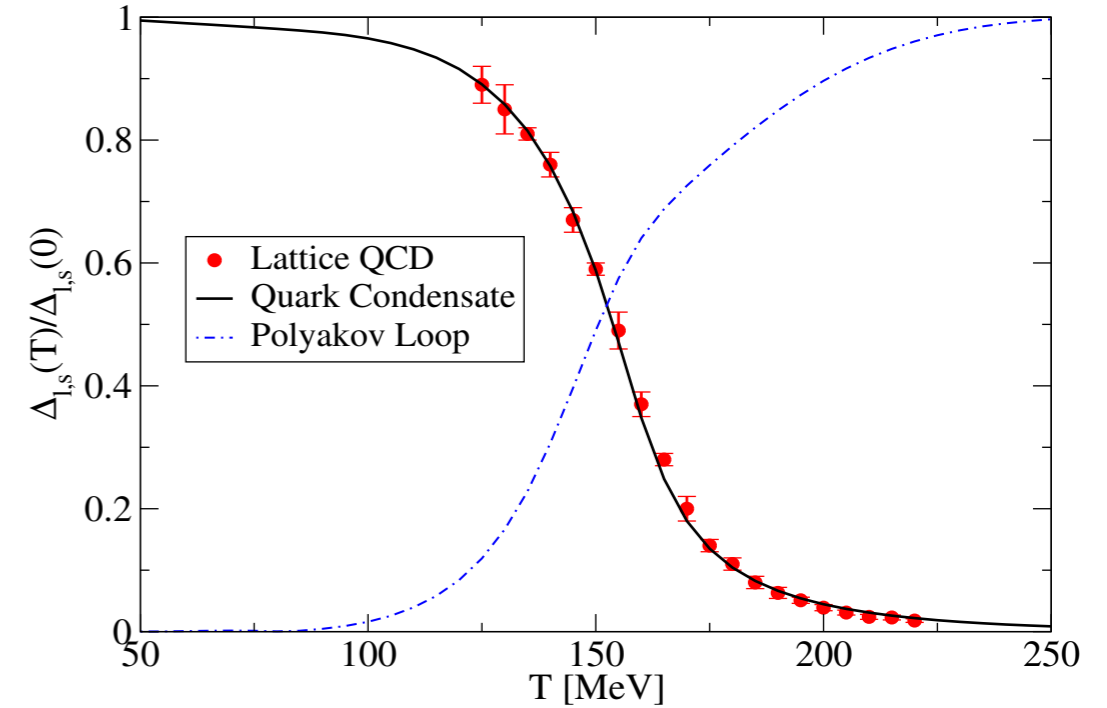
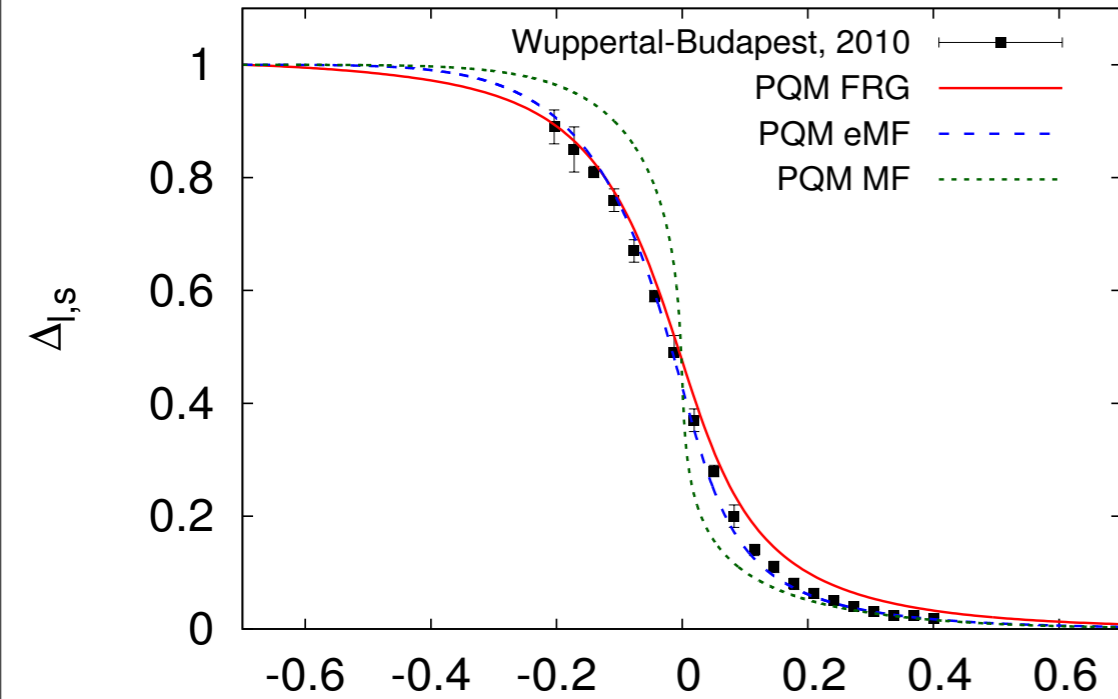


# Thermodynamics and condensates

2+1 flavor QCD - enhanced PQM-model

2+1 flavor DSE

reduced chiral condensate

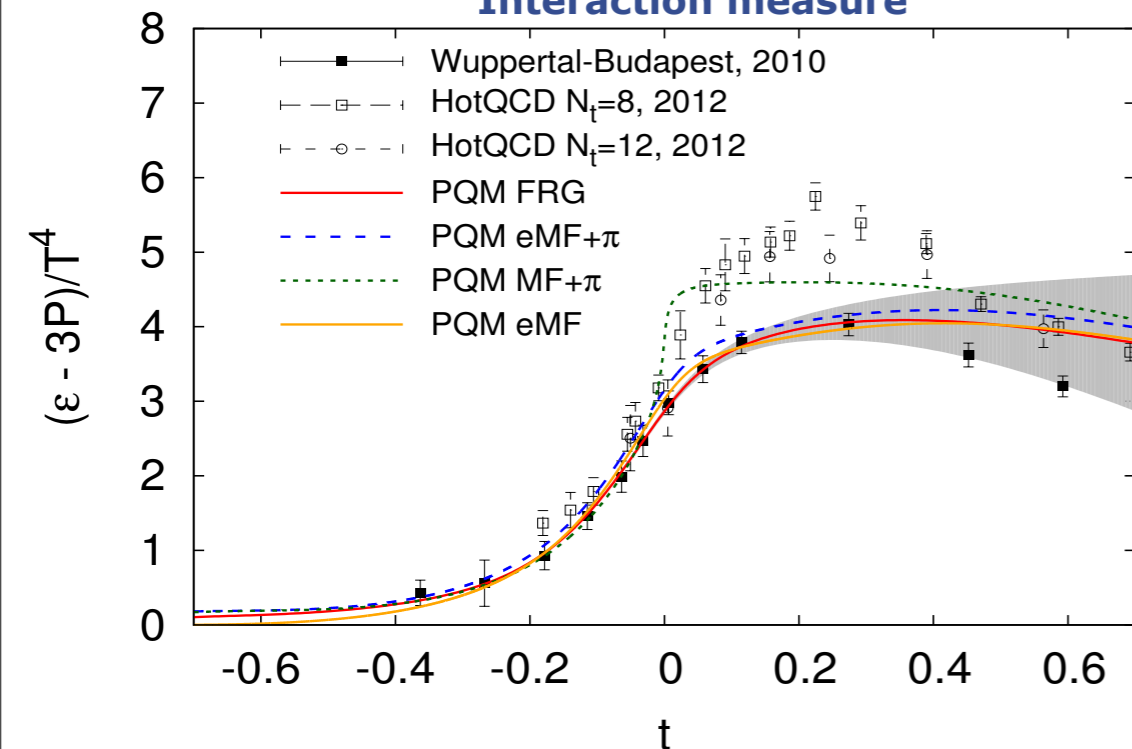


Herbst, Mitter et al, PLB 731 (2014) 248-256

Fischer, Luecker, Welzbacher, PRD 90 (2014), 034022

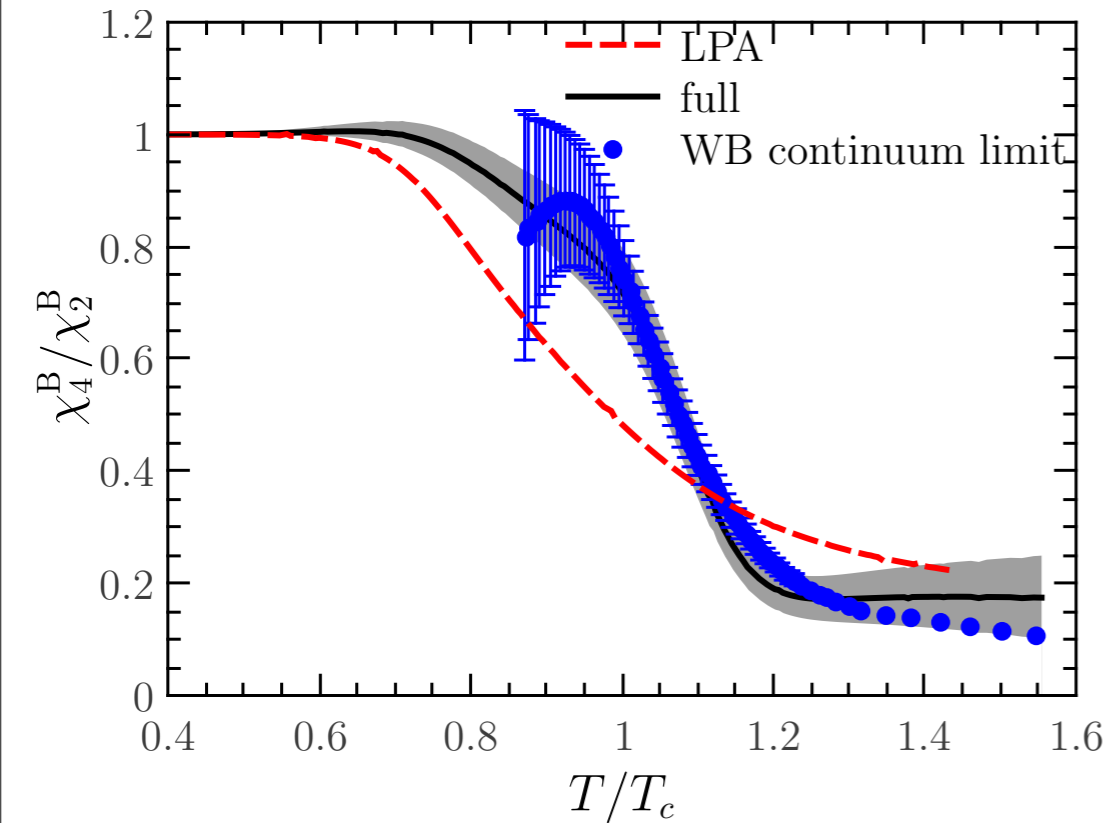
Fischer, Fister, Luecker, JMP, PLB 732 (2014) 273-277

Interaction measure



Shaded area:  
systematic error estimate  
due to low initial UV scale 1 GeV

# Fluctuations as a measure of confinement



Fu, JMP, PRD 92 (2015) 11, 116006

$$\chi_n^B = \frac{\partial^n}{\partial(\mu_B/T)^n} \frac{p}{T^4}$$

Skewness, Kurtosis

$$\sigma^2 = VT^3 \chi_2^B \quad S = \frac{\chi_3^B}{\chi_2^B \sigma} \quad \kappa = \frac{\chi_4^B}{\chi_2^B \sigma^2}$$

Karsch, Schaefer, Wagner, Wambach, PLB 698 (2011) 256

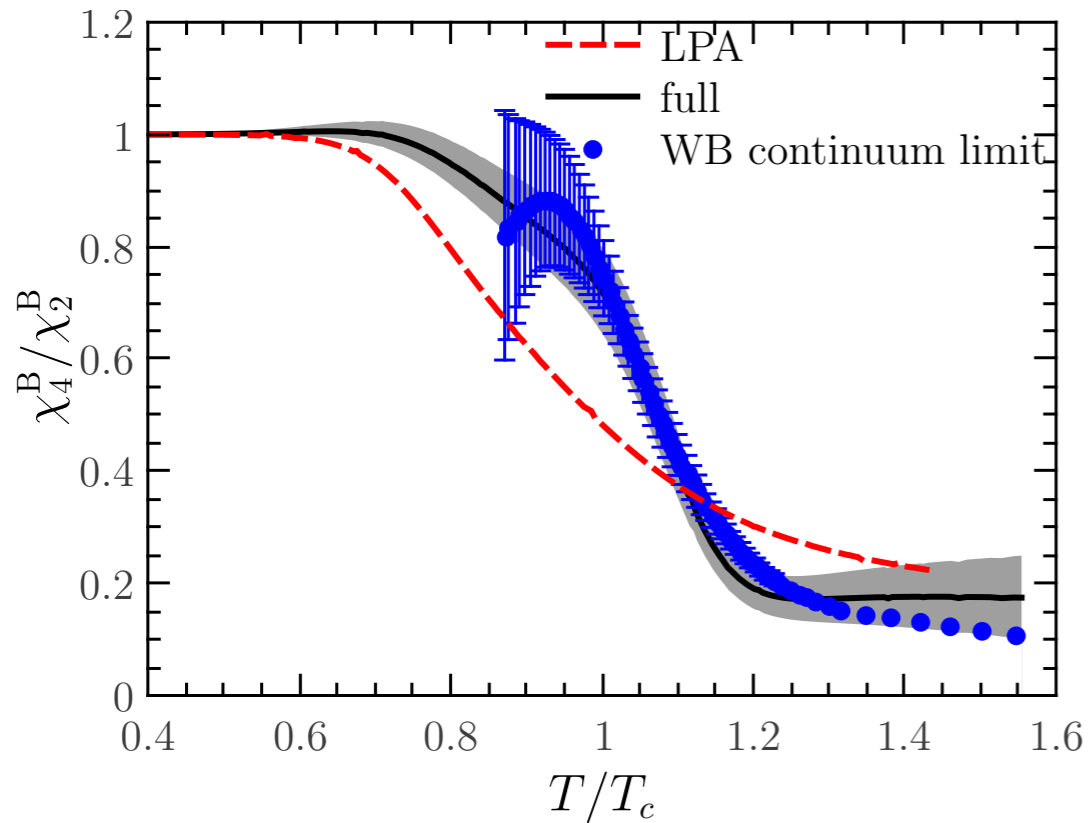
Friman, Karsch, Redlich, Skokov, EPJ C71 (2011) 1694

Schaefer, Wagner, PRD 85 (2012) 034027

Skokov, Friman, Redlich, PRC 88 (2013) 034911

Almasi, Friman, Redlich, arXiv:1601.0078

# Fluctuations as a measure of confinement

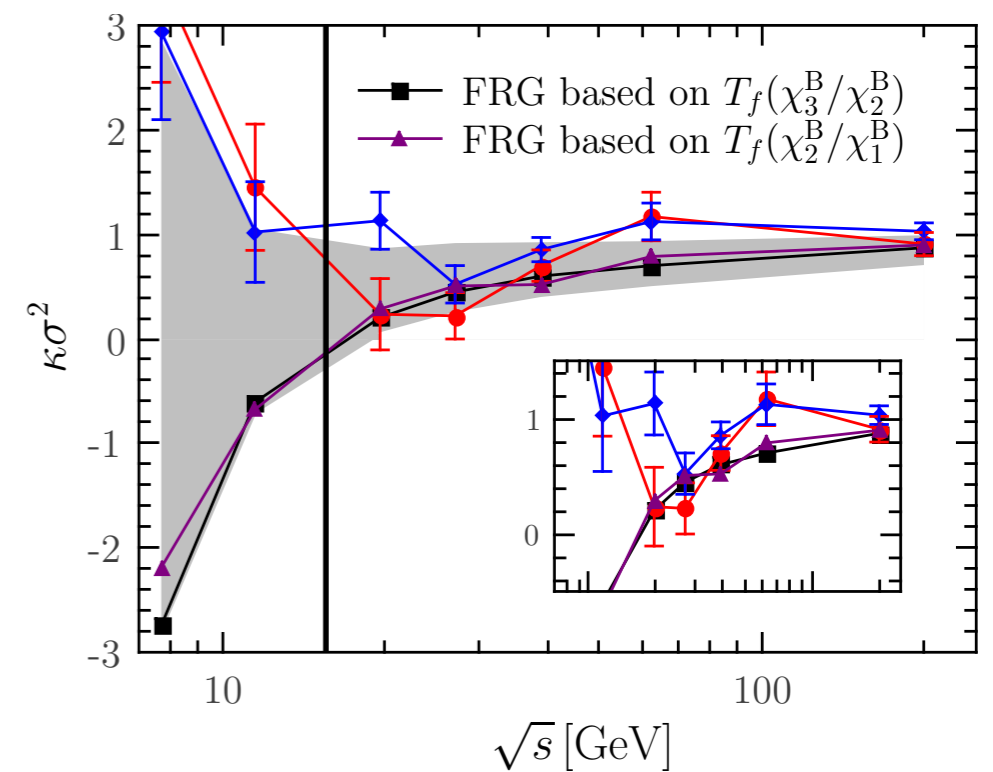
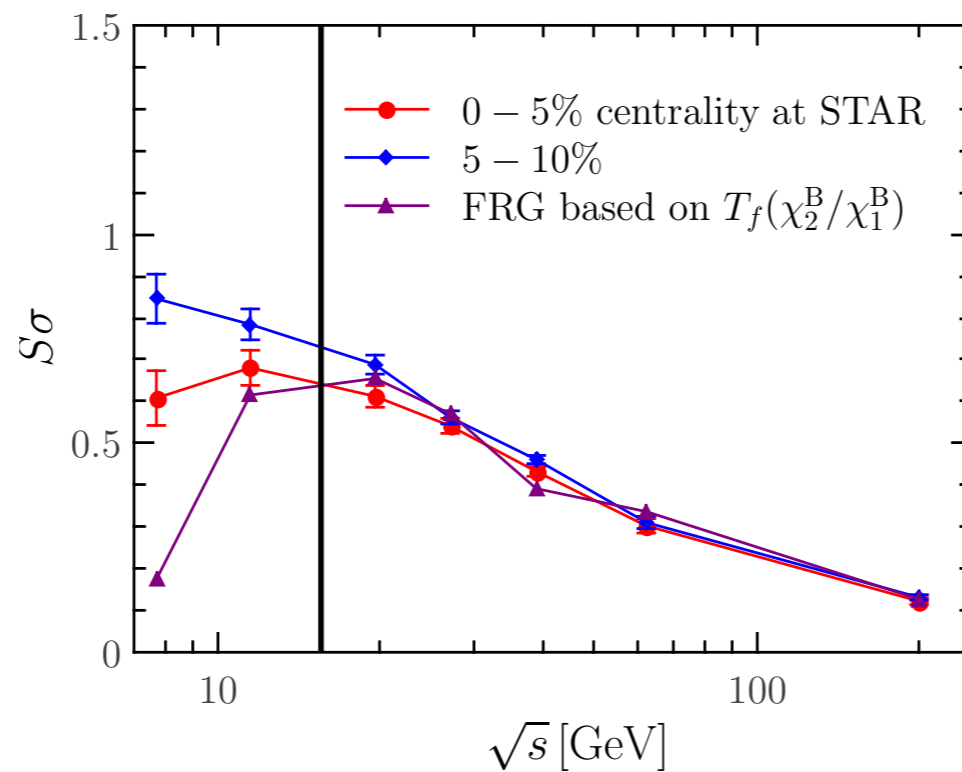


Fu, JMP, PRD 92 (2015) 11, 116006

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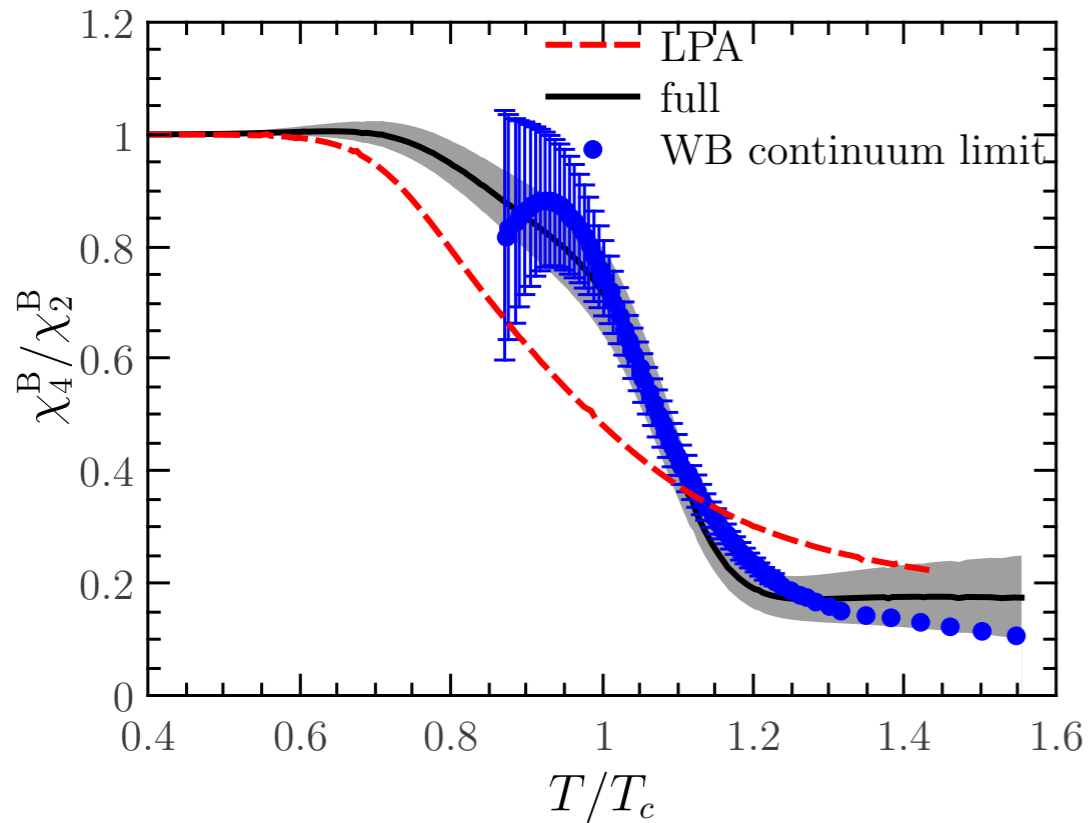
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Fu, JMP, arXiv:1512.08461

# Fluctuations as a measure of confinement



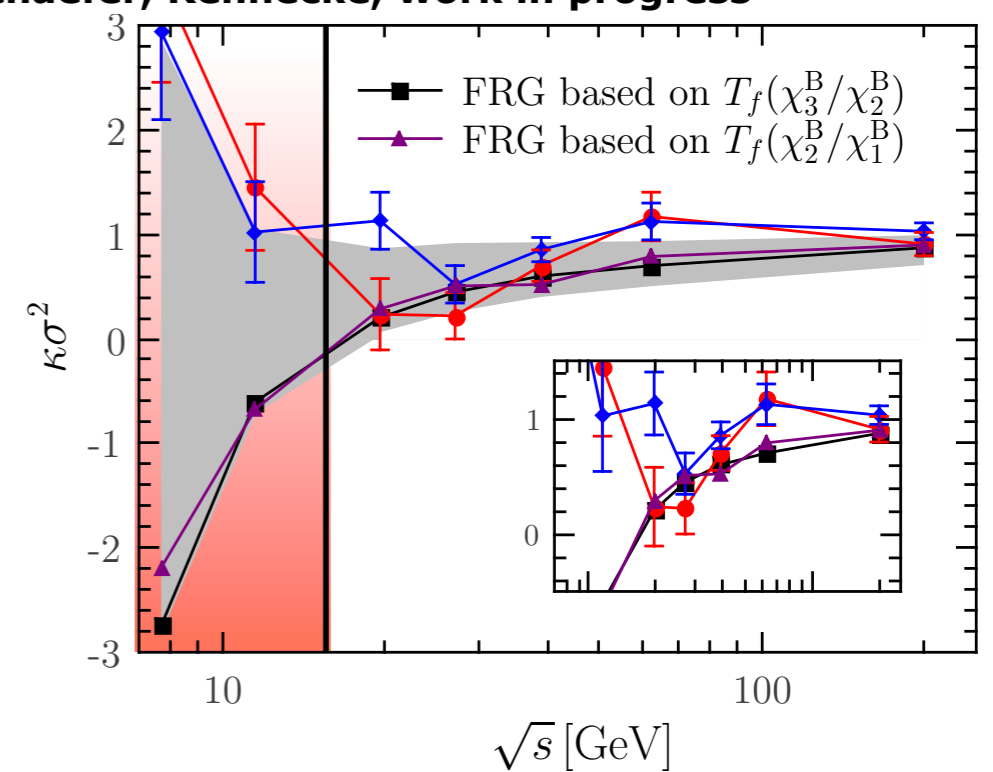
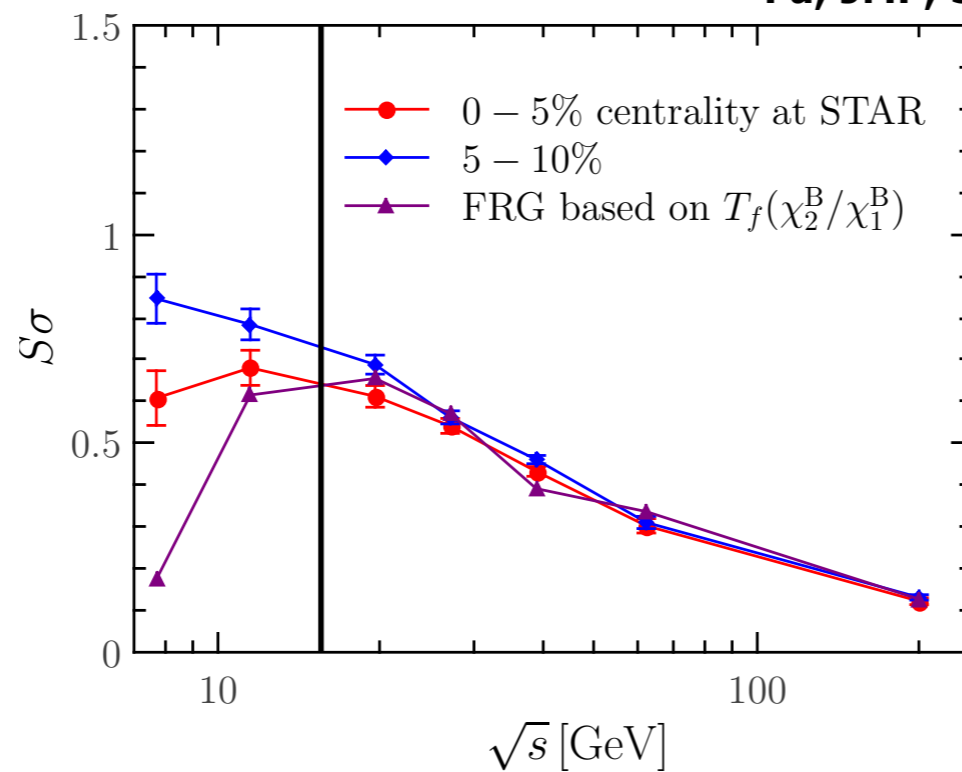
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Fu, JMP, PRD 92 (2015) 11, 116006

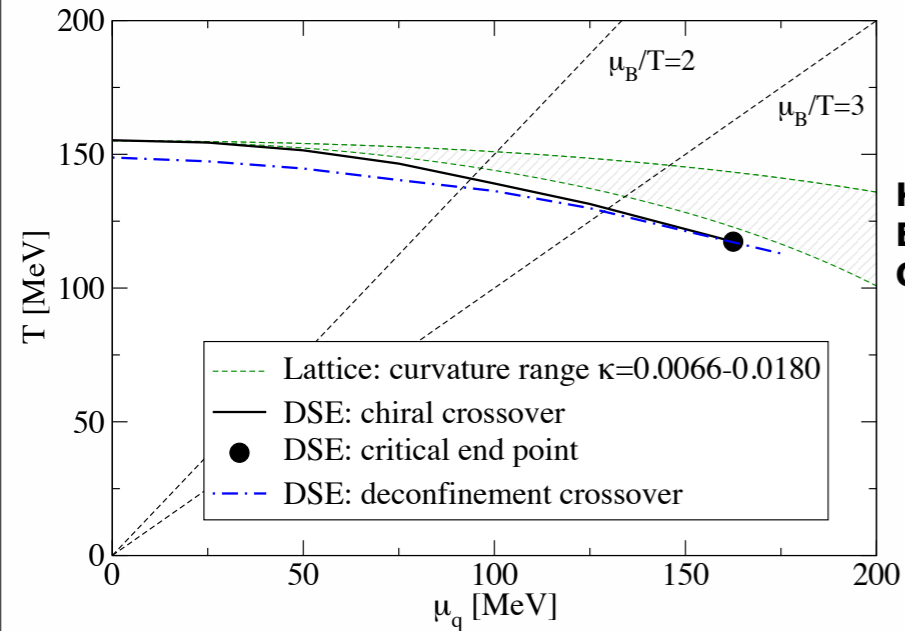
Fu, JMP, Schaefer, Rennecke, work in progress



Fu, JMP, arXiv:1512.08461

# Phase structure at finite density

Phase diagram of 2+1 flavor QCD



**Kaczmarek et al. '11**  
**Endrodi, Fodor, Katz, Szabo '11**  
**Cea, Cosmai, Papa '14**

Fischer, Fister, Luecker, JMP, PLB732 (2014) 248

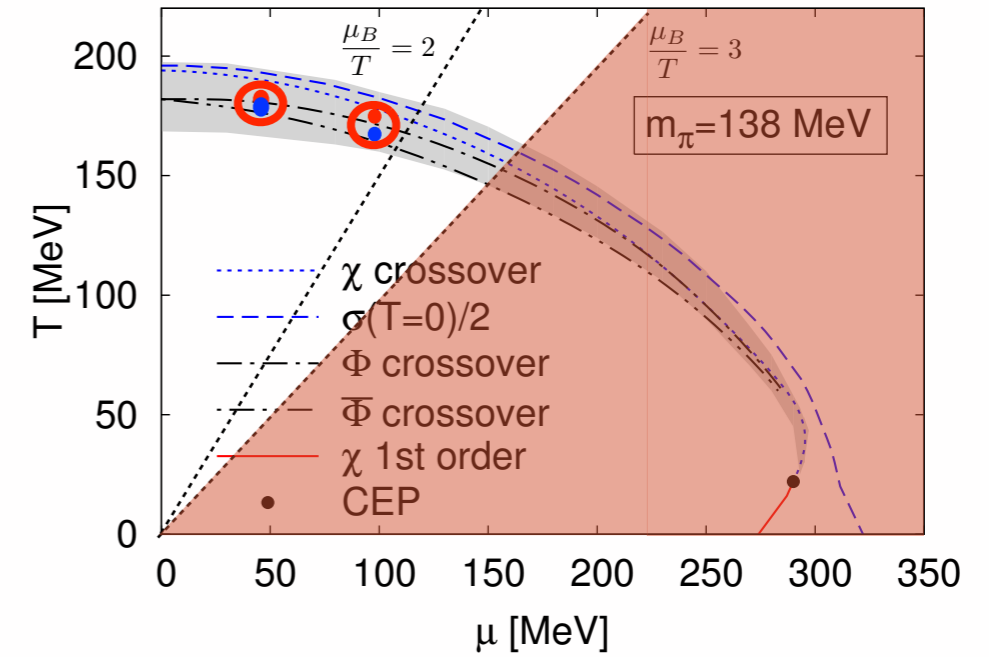
Fischer, Luecker, Welzbacher, PRD 90 (2014) 034022

Eichmann, Fischer, Welzbacher, PRD 93 (2014) 034013

## Chiral phase structure

Qin, Chang, Chen, Liu, Roberts, PRL 106 (2011) 172301

Phase diagram of QCD-enhanced 2-flavor PQM-model



Herbst, JMP, Schaefer, PLB 696 (2011) 58-67  
 PRD 88 (2013) 1, 014007

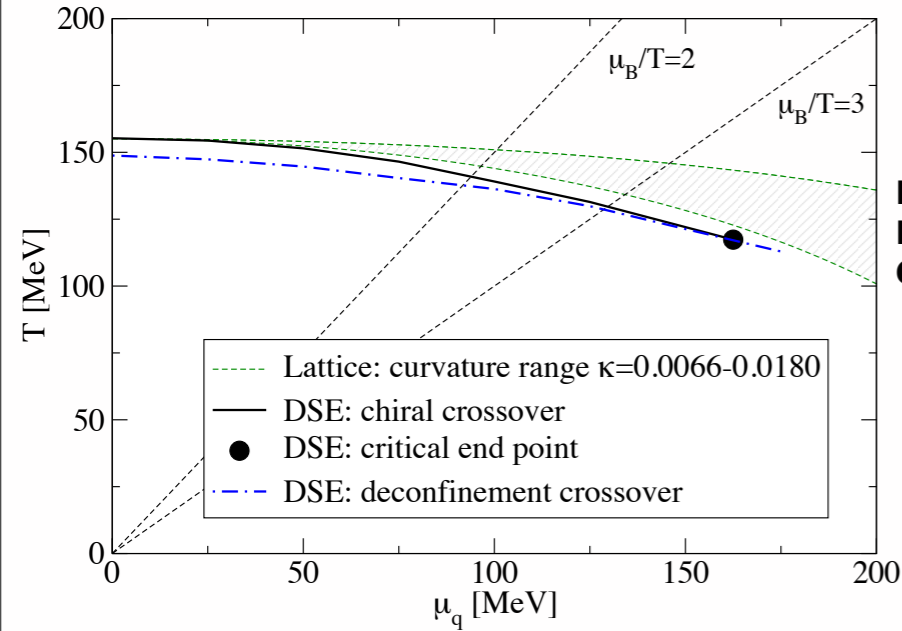


FRG QCD results at finite density

Haas, Braun, JMP '09, unpublished

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Phase diagram of 2+1 flavor QCD



Kaczmarek et al. '11  
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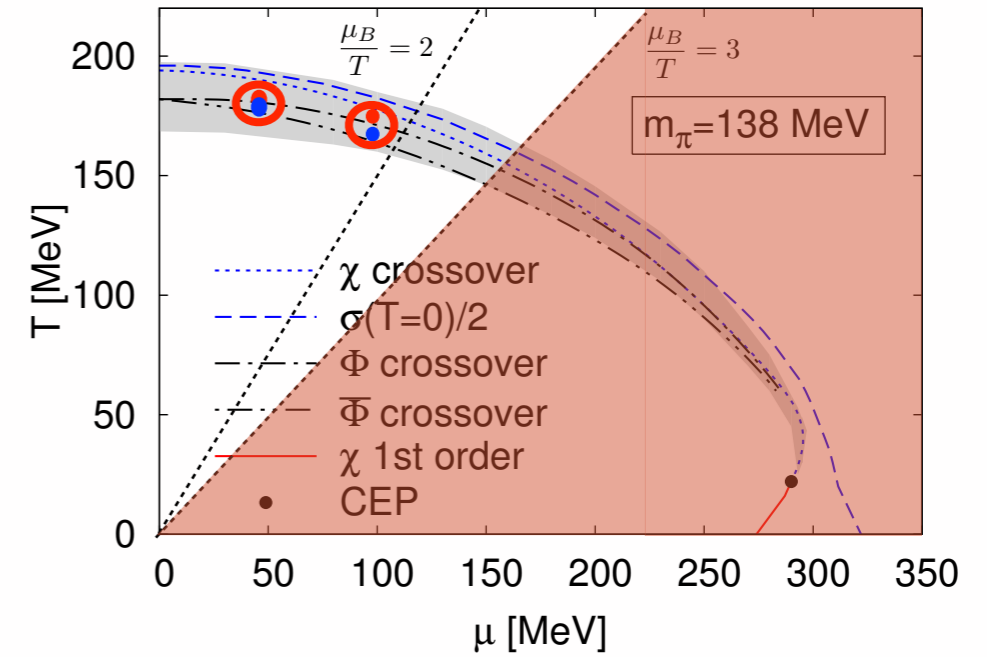
Fischer, Luecker, Welzbacher, PRD 90 (2014) 034022

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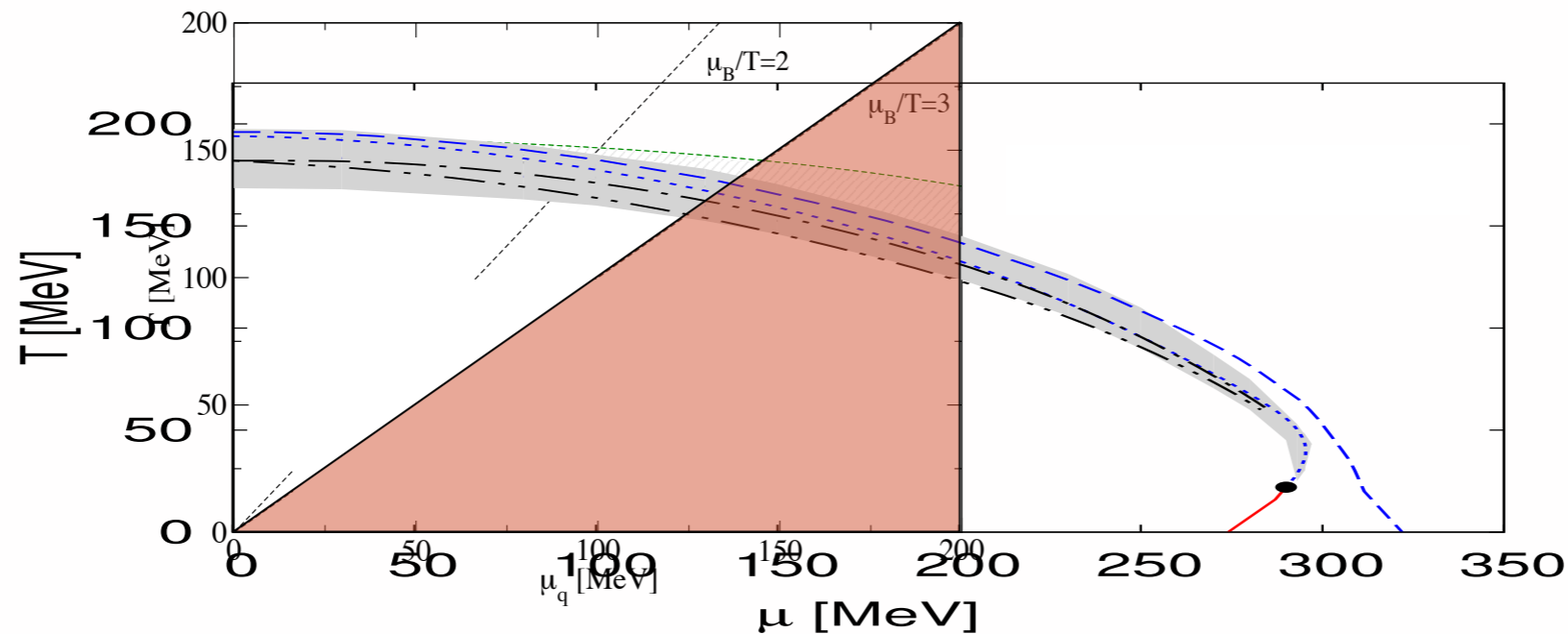
Herbst, JMP, Schaefer, PLB 696 (2011) 58-67  
 PRD 88 (2013) 1, 014007



FRG QCD results at finite density

Haas, Braun, JMP '09, unpublished

Comparison with 2 flavor vs 2+1 flavor scale matching of  $T_c$



# Summary & Outlook

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- **Chiral Symmetry Breaking and Confinement**
- **Phase Structure and Transport**

# Summary & Outlook

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- **Chiral Symmetry Breaking and Confinement**
- **Phase Structure and Transport**
- **Towards quantitative precision**
- **Baryons, high density regime & CEP, dynamics**
- **Hadronic properties**
  - **hadron spectrum & in medium modifications**
  - **low energy constants**