QCD Phase Structure III, CCNU, Wuhan

Anomalous transport effects in heavy-ion collisions

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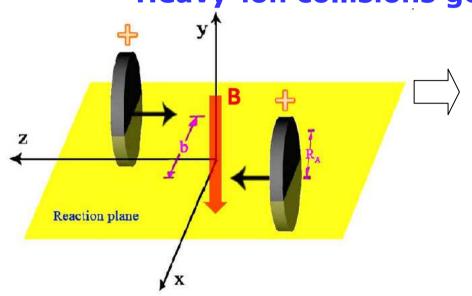
Outline

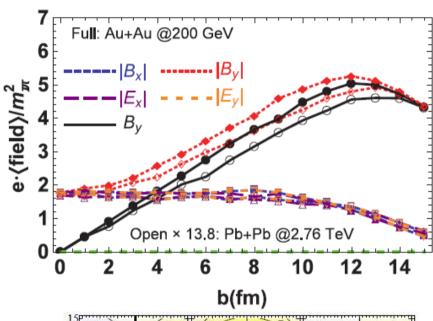
- ☐ Electromagnetic (EM) fields and vorticity in heavyion collisions
- EM-field induced anomalous transports (chiral magnetic effect, etc.)
- □ Vorticity induced anomalous transports (chiral vortical effect, etc.)
- summary

EM fields and vorticity

EM fields in heavy-ion collisions

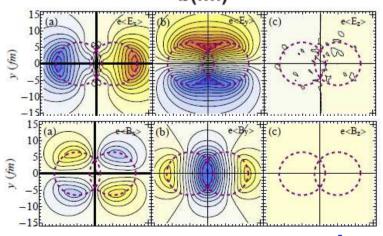
Heavy-ion collisions generate EM fields





- Very strong fields, eB~10^18-20 G
- Event-by-event fluct.
- Event averaged eB:

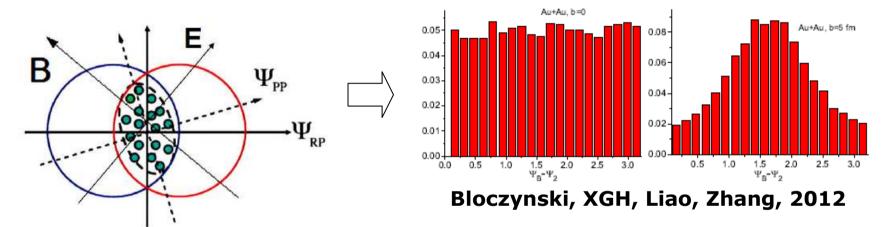
$$e\langle B_y \rangle \propto \frac{\sqrt{s}}{2m_N} \frac{Z}{A^{2/3}} \frac{b}{2R_A} m_{\pi}^2$$
, for $b < 2R_A$



EM fields in heavy-ion collisions

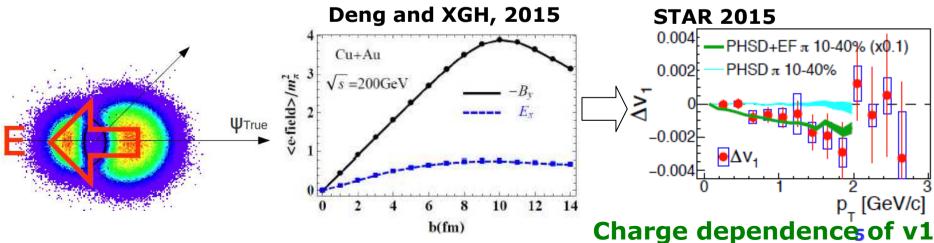
Heavy-ion collisions generate EM fields

Fluctuating azimuthal direction



Hirono and Hirano, 2012

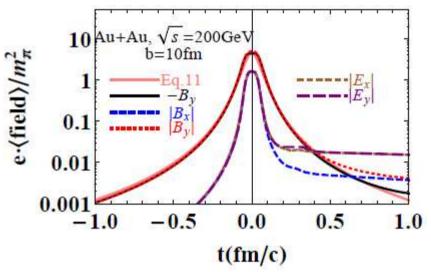
Strong in-plane E field in Cu + Au collisions



EM fields in heavy-ion collisions

Heavy-ion collisions generate EM fields

Time evolution of the B field (insulating medium)



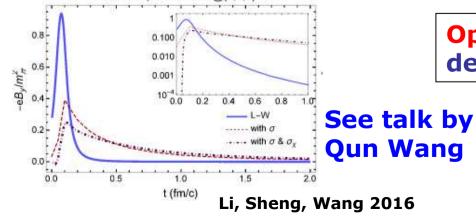
Well fitted by

$$\langle eB_y(t)\rangle \approx \frac{\langle eB_y(0)\rangle}{(1+t^2/t_B^2)^{3/2}}$$

Life time of B field

$$t_B \approx R_A/(\gamma v_z) \approx \frac{2m_N}{\sqrt{s}} R_A$$

But QGP_Ais ideally conducting: lattice of Ding et al $\sigma \approx (1/3)C_{\rm EM}T$

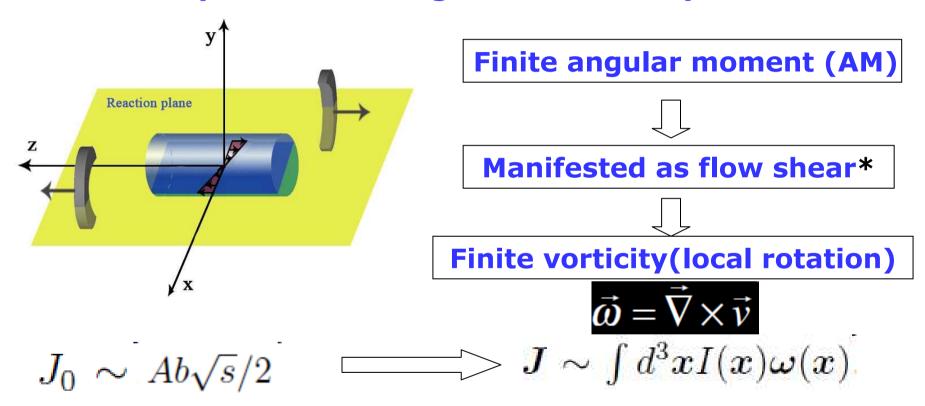


Open question: what is the time dependence of B field?

Pengfei's suggestion: high pt charmonium v2 sensitive to lifetime of B

Vorticity in heavy-ion collisions

Heavy-ion collisions generate vorticity



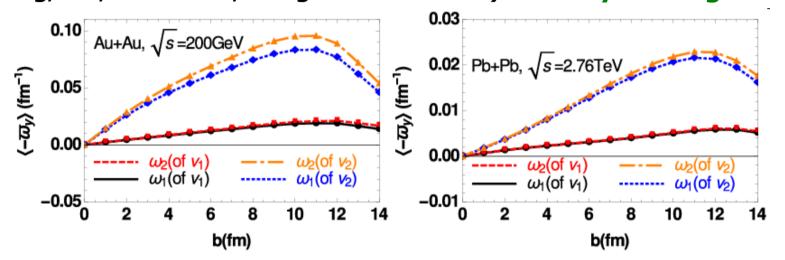
 $I(x) \sim [x^2 - (x \cdot \hat{\omega})^2] \varepsilon(x)$ is the moment of inertia density

J_0 is about 10^6 for RHIC Au+Au @ 200 GeV, system volume is ~ fm^3, very large AM density

^{*}For low energy collision, the system after collision may be globally rotating

Vorticity in heavy-ion collisions

Heavy-ion collisions generate vorticity
The detailed simulations of vorticity come out very recently
(Jiang, Lin, Liao 2016; Deng and XGH 2016) Talk by Jinfeng Liao

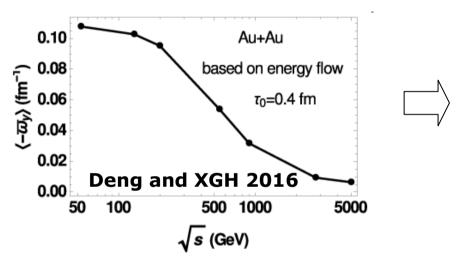


- •Vorticity of energy flow at RHIC at b=10 fm is 10^22 Hz. (Fastest man-made rotation via laser light ~ 10^7 Hz (Arita et al Nat.Comm. 2013))
- •RHIC: Take T~300 MeV, T*vorticity~10^4 MeV^2 comparable to magnetic field eB~10^4 MeV^2. But at LHC, T*vorticity is much smaller than eB.
- •At b<2R_A, increase with b; then drops. Anglular momentum of the overlapping region has a similar behavior.

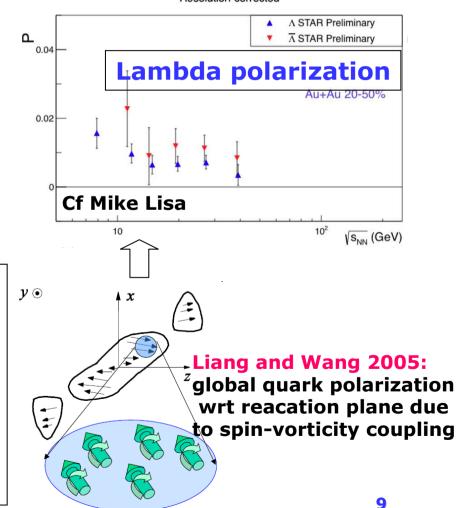
Vorticity in heavy-ion collisions

Heavy-ion collisions generate vorticity

Vorticity at mid-rapidity has very nontrivial energy dependence



- •Reason: higher energy: more AM carried by finite rapidity particles; mid-rapidity closer to Bjorken boost invariant; larger moment of inertia
- •Indicates stronger vortical effect at lower energy (BES is ideal for vortical effect)



EM-field induced anomalous transports

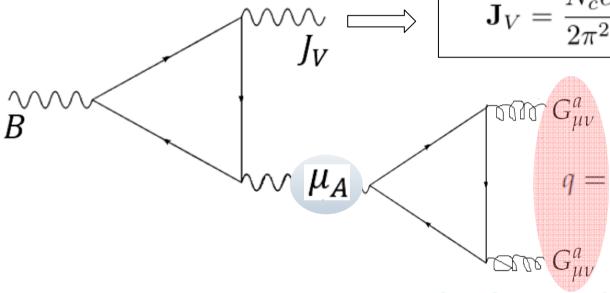
Chiral magnetic effect

B fields can monitor nontrivial topology of QCD

QED VVA triangle anomaly



$$\mathbf{J}_V = \frac{N_c e}{2\pi^2} \mu_A \mathbf{B}$$



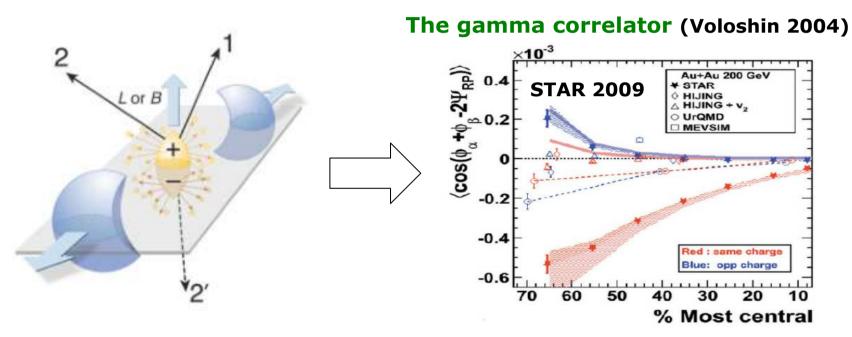
$$\frac{g^2}{32\pi^2} \int d^4x G^a_{\mu\nu} \tilde{G}^{\mu\nu}_a$$



- **QCD VVA triangle anomaly**
- **Parity-odd transport**
- Time-reversal even, no dissipation
- Fixed by anomaly coefficient, universal

Chiral magnetic effect

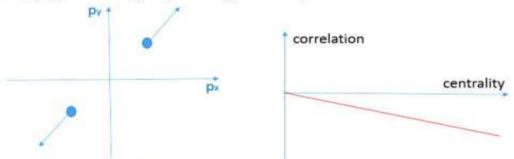
Phenomenology of CME in heavy-ion collisions: Event-by-event charge separation wrt. reaction plane



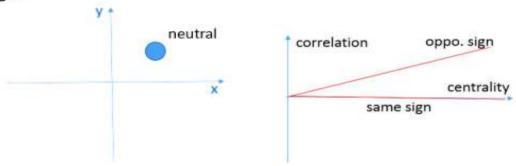
- Positive opposite-sign correlation, negative same-sign correlation
- Increase with centrality = B increases with centrality
- Nearly independent of collision energy = B * t_B ~ constant
- Delta gamma disspears for low energy = no chiral symmetry res.

Chiral magnetic effect

- Other sources of $\langle \cos(\phi_{\alpha} + \phi_{\beta}) \rangle$:
 - Transverse momentum conservation(Pratt etal 2011, Liao etal 2011): $\langle \cos(\phi_{\alpha} + \phi_{\beta}) \rangle \approx -v_2/N$, charge independent.



Local charge conservation(Pratt and Schlichting, 2011): $\langle \cos(\phi_{\alpha} + \phi_{\beta}) \rangle \propto v_2/N$ for opposite sign $\langle \cos(\phi_{\alpha} + \phi_{\beta}) \rangle \approx 0$ for same sign.

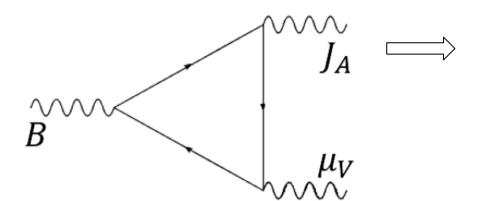


Dipole fluctuation(Teaney and Yan 2010), clustering correlation (Wang, 2010),...

Main challenge: how to subtract the background effects

Chiral magnetic wave

Same QED VVA triangle anomaly, but with V and A interchanged



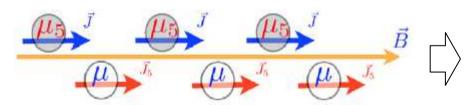
Chiral separation effect (CSE)

$$\mathbf{J}_A = \frac{N_c e}{2\pi^2} \mu_V \mathbf{B}$$

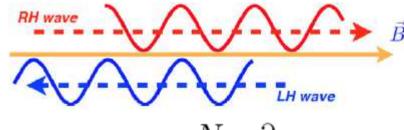
Son and Zhitnitsky 2004, Metlitski and Zhitnitsky 2004

CME + CSE give gapless wave modes: chiral magnetic wave (Kharzeev and Yee 2010)

$$v_R = \frac{N_c e}{4\pi^2} \frac{\partial \mu_R}{\partial n_R}$$



(Courtesy: Jinfeng Liao)

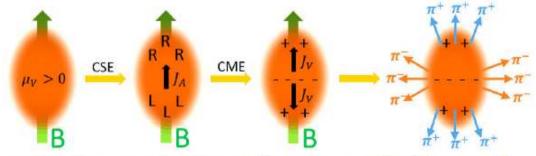


$$v_L = -\frac{N_c e}{4\pi^2} \frac{\partial \mu_L}{\partial n_L}$$

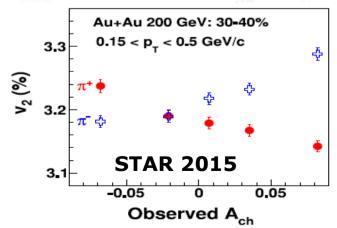
Chiral magnetic wave

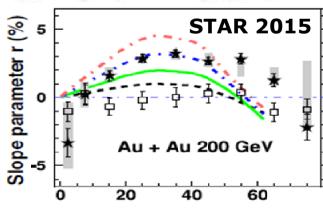
Phenomenology of CMW in heavy-ion collisions: Elliptic flow splitting of charged pions (Burnier, Kharzeev, Liao, Yee 2011)

Intuitive picture



► CMW $\Rightarrow v_2(\pi^-) \neq v_2(\pi^+)$: $v_2(\pi^-) - v_2(\pi^+) \approx rA_{\pm}$: linear approx. in net charge asymmetry $A_{\pm} = (N_+ - N_-)/(N_+ + N_-)$





Chiral magnetic wave

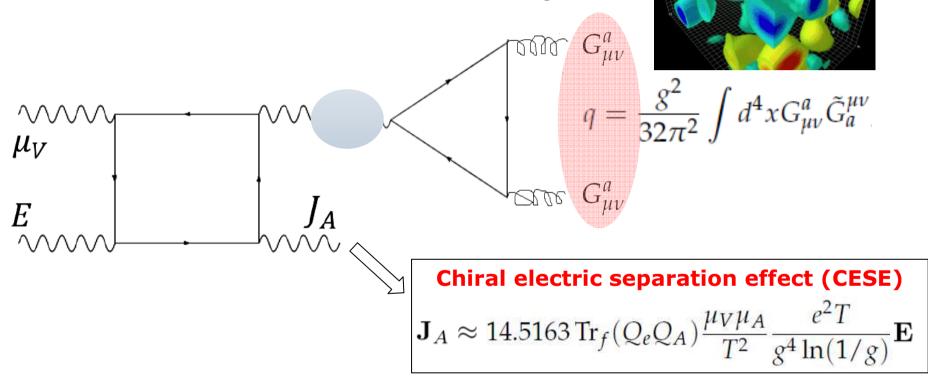
- Other sources of $v_2(\pi^{\pm})$ difference.
 - Quark transport and coalescence (Dunlop, Lisa, and Sorensen 2011; Campbell and Lisa 2013):

Neutron rich $\Rightarrow v_2(d) > v_2(u)$ (assume transported quarks v_2 is larger than produced quarks) $\Rightarrow v_2(\pi^- = d\bar{u}) > v_2(\pi^+ = \bar{d}u)$

- Hadronic mean-field potential (Xu,Chen, Ko, and Lin 2012):
 Works at low-energy collision. Neutron rich ⇒ repulsive (attractive) potential for
 π⁻(π⁺)⇒ π⁻(π⁺) is push-forward (pull-back) in in-plane ⇒ v₂(π⁻) ≥ v₂(π⁺)
 Electric field effect (Deng and XGH, 2012; Stephanov and Yee, 2013):
- Electric field effect (Deng and XGH, 2012; Stephanov and Yee, 2013): Electric field points outwards in y direction \Rightarrow positive (negative) charges move outwards(inwards) in y direction \Rightarrow charge quadrupole $\Rightarrow v_2(\pi^-) > v_2(\pi^+)$
- Local charge conservation and rapidity cut(Bzdak and Bozek 2013): $v_2(|\eta| \sim 1) < v_2(|\eta| \sim 0) \Rightarrow$ consider a positive-negative charge pair, if at $\eta = 1$ and the positive charge inside and negative charge outside the rapidity window then A>0 and the corresponding v_2 is smaller than that when the pair is inside the rapidity window $\Rightarrow v_2(\pi^+)_A < v_2(\pi^+)_{A=0} \Rightarrow$ at small A $v_2(\pi^+)_A = v_2(\pi^+)_{A=0} \#A$. Similarly, $v_2(\pi^-)_A = v_2(\pi^-)_{A=0} + \#A$
- Viscous hydrodynamics combined with finite isospin effectmay also explain the data (Hatta, Monnai, Xiao 2015)

... ...

Electric field induced anomalous transport



- P-odd, C-odd, T-odd transport (may be dissipative)
- Non-universal (receive perturbative correction)

- Collective modes by CESE, CME, and CSE.
- ▶ The complete electromagnetic response of a chiral matter:

$$j_V^{\mu} = \sigma E^{\mu} + \frac{e}{2\pi^2} \mu_A B^{\mu},$$

 $j_A^{\mu} = \sigma_5 E^{\mu} + \frac{e}{2\pi^2} \mu_V B^{\mu}.$

- Coupled evolution of vector and axial currents leads to several collective modes (XGH and Liao, PRL110(2013)232302):
 - If $\mathbf{B} = B\hat{z}$ and $\mathbf{E} = 0$: two Chiral magnetic waves along \mathbf{B}

$$\omega = \pm \sqrt{(v_{\chi}k_z)^2 - (e\sigma_0/2)^2} - i(e\sigma_0/2)$$

If $\mathbf{B} = 0$ and $\mathbf{E} = E\hat{z} + A$ -background: two Chiral electric waves

$$\omega = \pm \sqrt{(v_e k_z)^2 - (e\sigma_0/2)^2} - i(e\sigma_0/2)$$

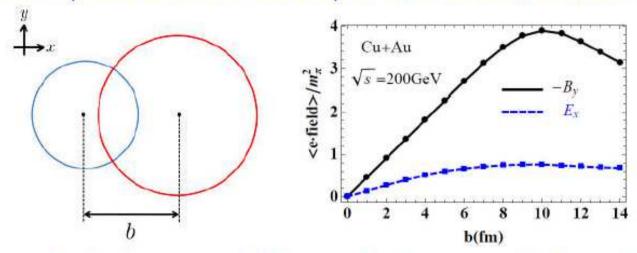
If $\mathbf{B} = 0$ and $\mathbf{E} = E\hat{z} + V$ -background: one Vector density wave and one Axial density wave along E-field

$$\omega_V = v_v k_z - ie\sigma_0,$$

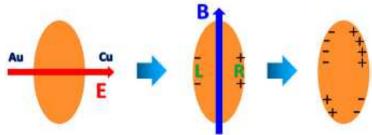
$$\omega_A = v_a k_z$$

▶ These collective excitations transport chirality and charge, and leads to novel charge azimuthal distribution ⇒

Possible implication: Recall that in-plane E-field in AuCu collisions.

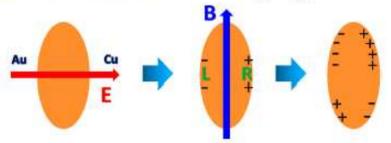


In-plane dipole due to usual Ohm conduction + out-of-plane dipole due to CME + quadrupole due to CESE and CME in Cu + Au collisions.

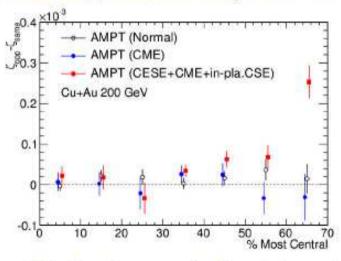


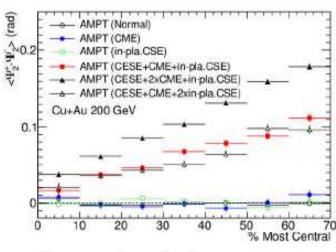
$$f_1(q,\phi) \propto 1 + 2v_1^0 \cos(\phi - \psi_1) + \frac{2qd_E \cos(\phi - \psi_E)}{2qd_B \cos(\phi - \psi_B)} + 2\chi qd_B \cos(\phi - \psi_B) + 2v_2^0 \cos[2(\phi - \psi_2)] + 2\chi qh_B \cos[2(\phi - \psi_c)] + \text{higher harmonics}$$

Signals for CESE in Cu + Au: $\zeta_{\alpha\beta} = \langle \cos[2(\phi_{\alpha} + \phi_{\beta} - 2\psi_{\rm RP})] \rangle$ and Ψ_2^q (the event-plane for hadrons of charge q).



 $\Delta \zeta = \zeta_{opp} - \zeta_{same}$ and $\Delta \Psi = \langle |\Psi_2^+ - \Psi_2^-| \rangle$ sensitive to CESE, survive final interaction(Ma and XGH, PRC 91(2015)054901)





Possible backgrounds for $\Delta \zeta = \zeta_{opp} - \zeta_{same}$: local charge conservation, chiral magnetic wave. Need more studies.

Vorticity induced anomalous transport

Chiral vortical effect

A modified "relativistic Larmor theorem"

$$e\mathbf{B} \sim 2\mu_V \boldsymbol{\omega}$$

$$\mathbf{J}_V = \frac{N_c e}{2\pi^2} \mu_A \mathbf{B}$$
 \longrightarrow $\mathbf{J}_V = \frac{N_c \mu_V \mu_A}{\pi^2} \boldsymbol{\omega}$

Chiral magnetic effect

Vector chiral vortical effect

This naïve mapping does not work for axial current. The calculation gives the vorticity induced axial current:

$$\mathbf{J}_A = N_c \left(\frac{T^2}{6} + \frac{\mu_V^2 + \mu_A^2}{2\pi^2} \right) \boldsymbol{\omega}$$

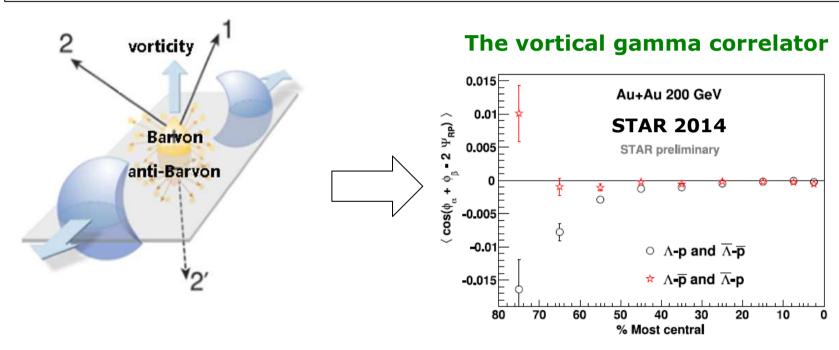
Axial chiral vortical effect

- Related to chiral anomaly and gravitational anomaly
- Universal (receive no perturbative correction*)

* T^2 term may have perturbative correction

Chiral vortical effect

Phenomenology of vector CVE in heavy-ion collisions: Event-by-event baryon separation wrt. reaction plane



- Positive opposite-sign correlation, negative same-sign correlation
- Increase with centrality = vorticity increases with centrality

Back ground effects: e.g., transverse momentum conservation, local baryon conser.

Chiral vortical wave

The vortical analogue of chiral magnetic wave

$$\vec{J}_A = \left(\frac{T^2}{6} + \frac{\mu^2 + \mu_5^2}{2\pi^2}\right) \vec{\omega}, \qquad \vec{J}_V = \frac{\mu\mu_5}{\pi^2} \vec{\omega}$$

$$V \qquad J_V \qquad V \qquad J_V \qquad V \qquad V \qquad \Delta$$

$$J_A \qquad A \qquad J_A \qquad A \qquad J_A \qquad \Delta$$
Courtesy: Yin Jiang

• A new collective mode. To reveal its dispersion we use continuity eq.

$$\partial_t n_{L,R} + \nabla \cdot \vec{J}_{L,R} = 0$$

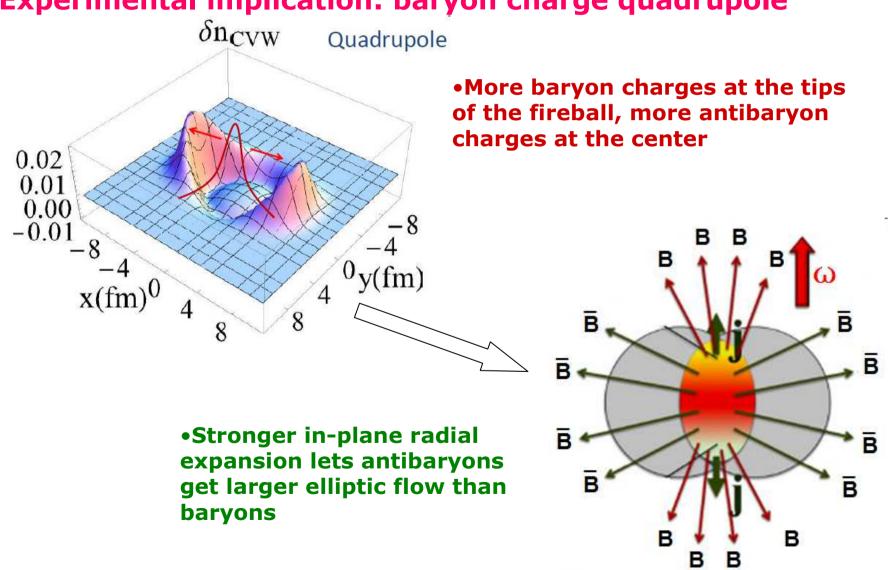
•Substitute CVE currents. Obtain Burgers wave equation which is linearized to normal wave equation

$$\partial_t n_{L,R} = \pm \frac{\omega \alpha^2}{\pi^2} \partial_{\chi} (n_{L,R}^2) \Longrightarrow \pm \frac{2\omega \alpha^2}{\pi^2} n_0 \partial_{\chi} (n_{L,R})$$

$$\alpha = \frac{\partial \mu}{\partial n} \sim \text{inverse baryon susceptiblity}$$
CVW velocity

Chiral vortical wave

Experimental implication: baryon charge quadrupole



Chiral vortical wave

Lambda-anti-Lambda v_2 splitting

Chemical potential shift of quark of flavor f (leading order in q):

$$\delta\mu_f \propto 2q_{\Omega}^f \cos(2\phi_s)$$

$$q_{\Omega}^f = [\int dx dy (\delta n_f) \cos(2\phi_s)]/[\int dx dy (\delta n_f)]$$
 ~ quadrupole moment

•Lambda (carries baryon charge but no electric charge: responses

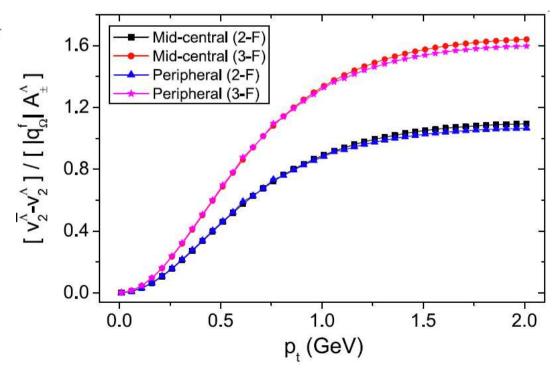
to CVW but not CMW)

$$\delta\mu_{\Lambda} \propto 2(q_{\Omega}^{u} + q_{\Omega}^{d} + q_{\Omega}^{s})\cos(2\phi_{s})$$

$$\Delta v_{2} = v_{2}^{\bar{\Lambda}} - v_{2}^{\Lambda} \propto |q_{\Omega}^{f}| A_{\pm}^{\Lambda}$$

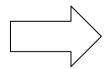
$$A_{\pm}^{\Lambda} = (N^{\Lambda} - N^{\bar{\Lambda}})/(N^{\Lambda} + N^{\bar{\Lambda}})$$

$$0.0 - 1$$



Summary

	E	В	ω
J_V	σ Ohm's law	$\frac{N_C e}{2\pi^2} \mu_A$ Chiral magnetic effect	$N_C \over \pi^2 \mu_V \mu_A$ Vector chiral vortical effect
J_A	$\propto \frac{\mu_V \mu_A}{T^2} \sigma$ Chiral electric separation effect	$\frac{N_Ce}{2\pi^2}\mu_V$ Chiral separation effect	$N_{C} \left(\frac{T^{2}}{6} + \frac{\mu_{V}^{2} + \mu_{A}^{2}}{2\pi^{2}} \right)$ Axial chiral vortical effect



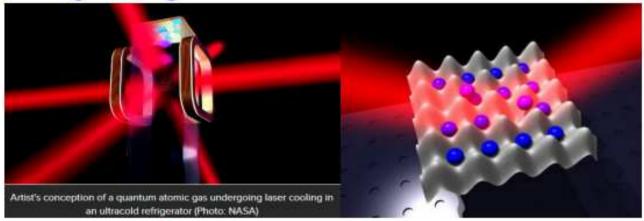
Collective waves: chiral magnetic wave, chiral electric waves, chiral vortical wave, vector and axial density wave,

Thank you!

Anomalous transport in cold atomic gases

Motivation

- ▶ The CME/CSE/CVE etc are masked by various backgrounds in HICs, it is hard to pin down and to explore their properties in HICs.
- Question: Is there any system that exhibits anomalous transport in a controllable way?
- Answer: Yes! One example is the Dirac or Weyl semimetal (Li, et al, 1412.6543 and many other recent experimental progresses).
- Here we propose another possibility: The cold atomic gases.
- Atomic gases experiments. 10^5-10^6 atoms put in magnetic trap or optical trap, and cooled down to nano Kelvin by using laser cooling or evaporating cooling



A lot of exciting low-temperature phenomena have been observed: superfluidity, Bose-Einstein condensation, BCS-BEC crossover, novel superfluid, polaron gases, ferromagnetisim,.....

Spin-orbit coupled atomic gases

In 2011, a new type of cold Bose gases generated in which the spin is coupled to the orbital motion of the atoms (Spielman et al 2011).

The single-particle Hamiltonian (Rashba-Dresselhaus SOC):

$$\mathcal{H} = \frac{\mathbf{p}^2}{2m} - \lambda \sigma_x p_y$$

- In 2012, same type spin-orbit coupling (SOC) for Fermi gases produced in MIT (Zwierlein group 2012) and in Shanxi(Zhang group 2012).
- Other types of SOC also possible, e.g., the Weyl SOC: (Spielman et al 2012)

$$\mathcal{H} = \frac{\mathbf{p}^2}{2m} - \lambda \sigma \cdot \mathbf{p}$$

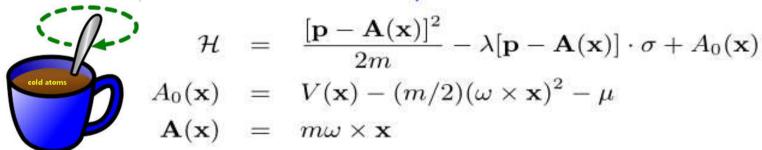
Now we show: there are CME and CSE in Weyl spin-orbit coupled Fermi gases.

Semiclassical equations of motion

• Consider the Weyl SOC, $\lambda \sigma \cdot \mathbf{p}$, in single atom Hamiltonian

$$\mathcal{H} = \frac{\mathbf{p}^2}{2m} - \lambda \mathbf{p} \cdot \sigma$$

- Along p, the spin has two projection which defines two helicities (we will call them chiralities as well), right-hand (project along p) and left-hand (project along −p).
- Consider atoms in a harmonic trap and let them rotate.



Integrate out the spin degree of freedom and at O(ħ) level: the semiclassical EOM(Niu 1998-)

$$\sqrt{G_c}\dot{\mathbf{x}} = \nabla_{\mathbf{k}}\varepsilon_c + c\hbar\mathbf{E} \times \Omega + c\hbar(\Omega \cdot \nabla_{\mathbf{k}}\varepsilon_c)\mathbf{B},
\sqrt{G_c}\dot{\mathbf{k}} = \mathbf{E} + \nabla_{\mathbf{k}}\varepsilon_c \times \mathbf{B} + c\hbar(\mathbf{E} \cdot \mathbf{B})\Omega$$

where $\mathbf{k} = \mathbf{p} - \mathbf{A}$ is the kinetic momentum, $\sqrt{G_c} = 1 + c\hbar \mathbf{B} \cdot \Omega$, $\mathbf{E} = -\nabla V(\mathbf{x})$ — effective E-field, $\mathbf{B} = 2m\omega$ —effective B-field, Ω —Berry curvature. $c = \pm$ for right- or left-hand.

Chiral anomaly

The kinetic equation reads (Son and Yamamoto 2012, Stephanov and Yin 2012, Gao, Wang, Pu, Chen, Wang 2012)

$$\partial_t f_c + \dot{\mathbf{x}} \cdot \nabla_{\mathbf{x}} f_c + \dot{\mathbf{k}} \cdot \nabla_{\mathbf{k}} f_c = I[f_c]$$

Direct calculation gives the U(1) chiral anomaly in current of chirality c:

$$\partial_t n_c + \nabla_{\mathbf{x}} \cdot \mathbf{j}_c = c(\mathbf{E} \cdot \mathbf{B}) \int \frac{d^3 \mathbf{k}}{(2\pi)^3} f_c \nabla_{\mathbf{k}} \cdot \Omega = c f_c(\mathbf{k}_0) \frac{W}{4\pi^2} \mathbf{E} \cdot \mathbf{B}$$

where W is the winding number of the Berry curvature.

Write down the current j_c explicitly:

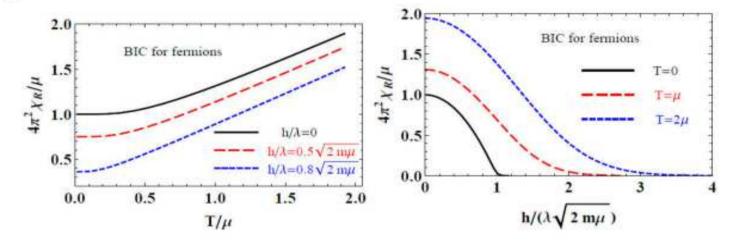
$$\mathbf{j}_{c} = \int \frac{d^{3}\mathbf{k}}{(2\pi)^{3}} f_{c} \nabla_{\mathbf{k}} \varepsilon_{c} + c\mathbf{E} \times \int \frac{d^{3}\mathbf{k}}{(2\pi)^{3}} \Omega f_{c}$$
$$+ c\mathbf{B} \int \frac{d^{3}\mathbf{k}}{(2\pi)^{3}} (\Omega \cdot \nabla_{\mathbf{k}} \varepsilon_{c}) f_{c}.$$

▶ The third term is B-induced currents:

$$\mathbf{j}_c^{\mathbf{B}-\mathrm{ind}} = \chi_c \mathbf{B}, \ \chi_c = c \int \frac{d^3 \mathbf{k}}{(2\pi)^3} (\Omega \cdot \nabla_{\mathbf{k}} \varepsilon_c) f_c$$

Chiral magnetic/separation effects

The B-induced conductivity χ_c for Fermi gas (XGH, Sci.Rep. 6, 20601 (2016))



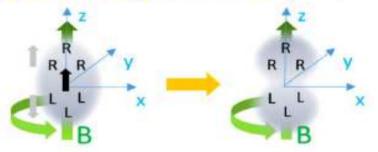
If there is parity-odd domains in the Fermi gases \Rightarrow $\mu_R = \mu + \mu_A, \mu_L = \mu - \mu_A \Rightarrow$

$$\mathbf{j}_{V}^{\mathbf{B}-\mathrm{ind}} \equiv \mathbf{j}_{R}^{\mathbf{B}-\mathrm{ind}} + \mathbf{j}_{L}^{\mathbf{B}-\mathrm{ind}} = \frac{\mu_{A}}{2\pi^{2}}\mathbf{B},$$
 $\mathbf{j}_{A}^{\mathbf{B}-\mathrm{ind}} \equiv \mathbf{j}_{R}^{\mathbf{B}-\mathrm{ind}} - \mathbf{j}_{L}^{\mathbf{B}-\mathrm{ind}} = \frac{\mu}{2\pi^{2}}\mathbf{B}$

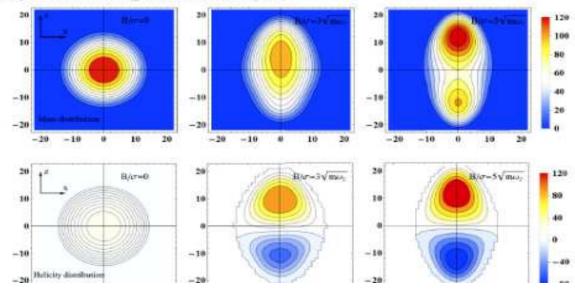
- These are exactly the chiral magnetic/speration effects!
- Question: how can produce parity-odd domins in Fermi gases?

Chiral dipole and mass quadrupole

Very like what happen in QGP, the CMW exists in SOC atomic gases, which transport chirality and mass (XGH, Sci.Rep. 6, 20601 (2016))

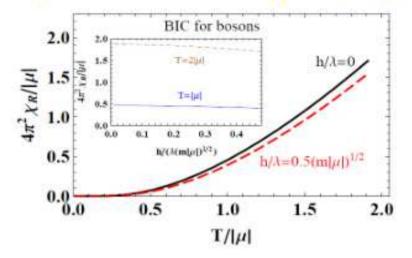


Unlike in QGP, the presence of trap will finally stop these transport currents and system reaches a equilibrium configuration where appear a mass quadrupole and chiral dipole. The mass quadrupole may be tested by light absorption images technique.

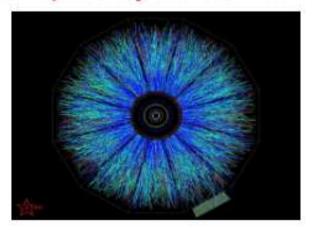


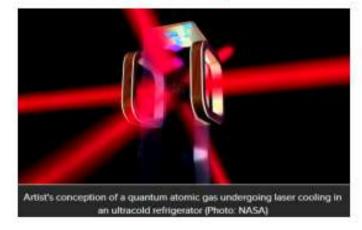
Link the hottest to the coldest

▶ The similar thing happens also in Bose gases, e.g., the BIC



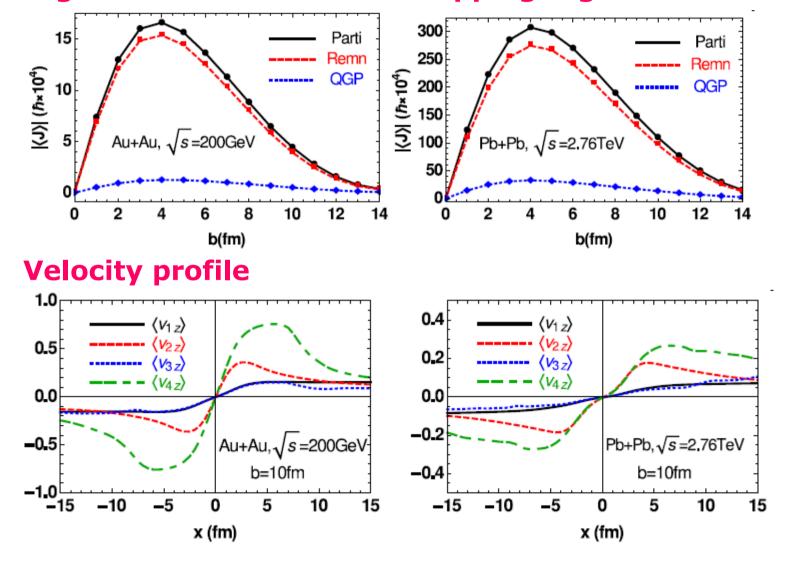
▶ The CME/CSE initiated in the study of the hottest matter, the QGP, can possibly be realized in the coldest matter, the cold atoms.





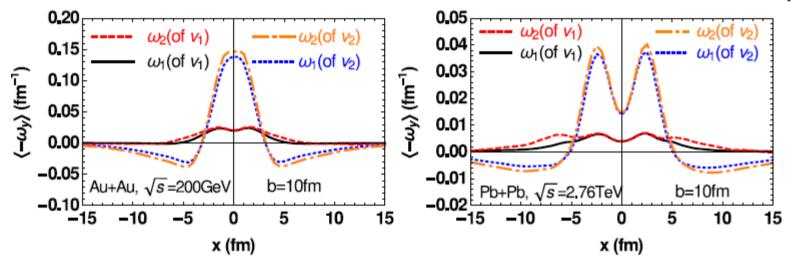
Backup

Angular momentum in overlapping region



Backup

Spacial distribution



Rapidity dependence

