Gubser flow and hydrodynamical fluctuations

Li Yan (严 力)

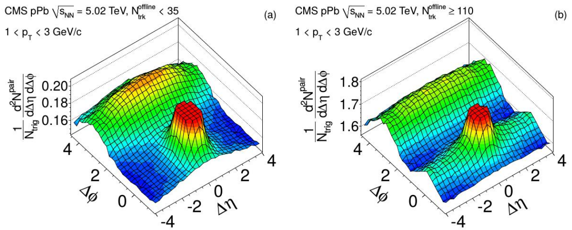
CNRS, Institut de Physique Théorique, Saclay

QCD Phase structure III, CCNU, June, 2016

with Hanna Grönqvist, JHEP03(2016)121

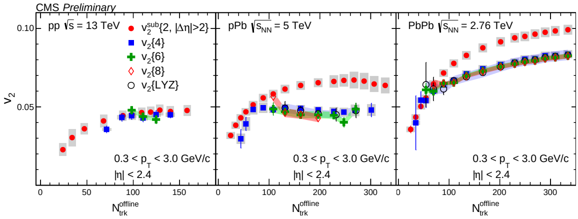
Medium collectivity in small colliding system: p+A, p+p, etc.

- Long-range multi-particle correlation ⇔ medium collective expansion (QGP).



(**CMS collaboration**)

- Long-range multi-particle correlation ⇔ flow v_n, and flow cumulants $v_n\{m\}$.



(**CMS collaboration**)

- Modeling medium exp. in small colliding systems – **hydro.?** (good in AA)

initial state + hydro. EoM & EoS + freeze-out

- Modeling medium exp. in small colliding systems – **hydro.?** (good in AA)

initial state + hydro. EoM & EoS + freeze-out

- But in small systems, applicability of hydro. is challenged,
    * convergence of gradient expansion – λ_{mfp}/L – Kn (**Niemi and Denicol**)
    * effects of thermal fluctuations are expected stronger.

- Modeling medium exp. in small colliding systems – **hydro.?** (good in AA)

initial state + hydro. EoM & EoS + freeze-out

- But in small systems, applicability of hydro. is challenged,
  - convergence of gradient expansion – λ_{mfp}/L – Kn (**Niemi and Denicol**)
  - effects of thermal fluctuations are expected stronger.

- Goal of this work : A preliminary analysis of the effect of thermal noise
  - How significant is the effect of thermal fluctuations in heavy-ion collisions?
  - In particular, effect of thermal fluctuations in small colliding systems?

Hydro. and thermal fluctuations in a fluid system

- Hydrodynamics – conservation of energy-momentum, etc.

$$T^{\mu\nu} = T^{\mu\nu}_{\rm ideal} + \Pi^{\mu\nu} \qquad \leftrightarrow \qquad d_\mu T^{\mu\nu} = 0 \quad \text{(Euler and equ. of continuity)}$$

especially from hydro EoM, one recognizes that

$$d_\mu(su^\mu) = -\frac{1}{T}\nabla_{(\mu}u_{\nu)}\Pi^{\mu\nu} \quad \rightarrow \quad \frac{dS}{dt} = -\int d^3x \frac{1}{T}\nabla_{(\mu}u_{\nu)}\Pi^{\mu\nu}$$

Navier-Stokes hydro. (1st order):

$$\Pi^{\mu\nu} = -2\eta\left[\frac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta}(d_\alpha u_\beta + d_\beta u_\alpha) - \frac{1}{3}\Delta^{\mu\nu}\Delta^{\alpha\beta}d_\alpha u_\beta\right] = -\eta\sigma^{\mu\nu}$$

where

$$\Delta^{\mu\nu} = u^\mu u^\nu + g^{\mu\nu}$$

* We ignore bulk viscosity ζ in our work.

Hydro. and thermal fluctuations in a fluid system

- EoM of a set of physical quantities $\{x_a\}$ in a thermal system

(**Landau and Lifshitz. J.Kapusta, B. Muller and M. Stephanov**)

$$\dot{x}_a = \underbrace{-\sum_b \gamma_{ab} X_b}_{\text{drag}} + \underbrace{y_a}_{\text{fluc.}} \quad \Leftrightarrow \quad \dot{S} = -\sum_a \dot{x}_a X_a$$

$$\text{Maximization of S} \;\Rightarrow\; \langle y_a(t_1) y_b(t_2) \rangle = (\gamma_{ab} + \gamma_{ba}) \delta(t_1 - t_2)$$

- Auto-correlations of thermal noise : fluctuation-dissipation
- For hydro., γ_{ab} is determined then by identifying

$$\dot{x} \to \Pi^{\mu\nu} \text{ and } X \to \frac{\Delta V}{T} \nabla_{(\mu} u_{\nu)}, \qquad \left(\frac{dS}{dt} = -\int d^3x \frac{1}{T} \nabla_{(\mu} u_{\nu)} \Pi^{\mu\nu} \right)$$

- Remarks:
  1. White noise – delta function $\delta() \sim \frac{1}{\Delta V \Delta t}$.
  2. Form of γ_{ab} corresponds to the detailed form of $\Pi^{\mu\nu}$, $\gamma_{ab} \sim$ dissipations.

- Hydrodynamics with thermal noise (Navier-Stokes hydro):

(**Landau and Lifshitz, J. Kapusta et. al.**)

$$T^{\mu\nu} = T^{\mu\nu}_{\rm ideal} + \Pi^{\mu\nu} + S^{\mu\nu}$$

where thermal fluctuation tensor $S^{\mu\nu}$ are introduced w.r.t. $\Pi^{\mu\nu}$

$$\langle S^{\mu\nu}(x)\rangle = 0$$

$$\text{Navier-Stokes: } \langle S^{\mu\nu}(x_1)S^{\alpha\beta}(x_2)\rangle = 4T\eta\Delta^{\mu\nu\alpha\beta}\delta^{(4)}(x_1 - x_2)$$

where

$$\Delta^{\mu\nu\alpha\beta} = \frac{1}{2}\left[\Delta^{\mu\alpha}\Delta^{\nu\beta} + \Delta^{\mu\beta}\Delta^{\nu\alpha}\right] - \frac{1}{3}\Delta^{\mu\nu}\Delta^{\alpha\beta}, \text{ and } \Delta^{\mu\nu\alpha\beta}d_\alpha u_\beta = \sigma^{\mu\nu}$$

* One-point functions of physical quantities are not affected.
* Thermal noise affects two-point correlations.

Solving hydro. with thermal noise

- Linearized hydro. EoM with thermal fluctuations around $d_\mu T_0^{\mu\nu} = 0$

$$T(x) = T_0(x) + \delta T(x)$$
$$\varepsilon(x) = \varepsilon_0(x) + \delta\varepsilon(x)$$
$$\mathcal{P}(x) = \mathcal{P}_0(x) + \delta\mathcal{P}(x)$$
$$u^\mu(x) = u_0^\mu(x) + \delta u^\mu(x)$$

therefore $d_\mu \delta T^{\mu\nu} = 0 \sim O(\delta)$,

$$\delta w D u_\alpha + w\delta u^\mu d_\mu u_\alpha + (Dw + w\partial\cdot u)\delta u_\alpha + \nabla_\alpha \delta\mathcal{P} + wD\delta u_\alpha + d_\mu(\delta\Pi_\alpha^\mu + S_\alpha^\mu) = 0$$
$$D\delta\varepsilon + \delta w\partial\cdot u + d_\mu(w\delta u^\mu) + w\delta u^\alpha D u_\alpha - u^\alpha d_\mu(\delta\Pi_\alpha^\mu + S_\alpha^\mu) = 0$$

Note that $\delta\Pi^{\mu\nu}$ is induced by δT, etc. (**C. Young, J. Kapusta et al.**)

- Beyond linear order, thermal noise becomes large, e.g., phase transition.
- Simplification can be made when hydro. is analytically solved:

Solving hydro. with thermal noise

- Linearized hydro. EoM with thermal fluctuations around $d_\mu T_0^{\mu\nu} = 0$

$$\begin{aligned} T(x) =& T_0(x) + \delta T(x) \\ \varepsilon(x) =& \varepsilon_0(x) + \delta\varepsilon(x) \\ \mathcal{P}(x) =& \mathcal{P}_0(x) + \delta\mathcal{P}(x) \\ u^\mu(x) =& u_0^\mu(x) + \delta u^\mu(x) \end{aligned}$$

therefore $d_\mu \delta T^{\mu\nu} = 0 \sim O(\delta)$,

$$\delta w D u_\alpha + w \delta u^\mu d_\mu u_\alpha + (Dw + w\partial \cdot u)\delta u_\alpha + \nabla_\alpha \delta\mathcal{P} + wD\delta u_\alpha + d_\mu(\delta\Pi_\alpha^\mu + S_\alpha^\mu) = 0$$
$$D\delta\varepsilon + \delta w \partial \cdot u + d_\mu(w\delta u^\mu) + w\delta u^\alpha D u_\alpha - u^\alpha d_\mu(\delta\Pi_\alpha^\mu + S_\alpha^\mu) = 0$$

Note that $\delta\Pi^{\mu\nu}$ is induced by δT, etc. (**C. Young, J. Kapusta et al.**)

- Beyond linear order, thermal noise becomes large, e.g., phase transition.
- Simplification can be made when hydro. is analytically solved:

Bjorken flow and Gubser's flow

Bjorken hydro. and thermal noise

- Boost invariance $\times$ translational invariance in the transverse plane
- Correlation of thermal noise in 1+1D Bjorken hydro,. (**J. Kapusta et. al., 2012**)

$$\langle S^{\mu\nu}(\tau_1,\xi_1)S^{\alpha\beta}(\tau_2,\xi_2)\rangle = \frac{8}{3\tau_1 A_\perp} T\eta\Delta^{\mu\nu}\Delta^{\alpha\beta}\delta(\tau_1-\tau_2)\delta(\xi_1-\xi_2)$$

1. $\delta(\vec{x}_{1\perp}-\vec{x}_{2\perp}) \to A_\perp$ characterizes transverse size of the system.
2. Tensor structure of $S^{\mu\nu}$ is factorized, due to the fact that $u^\mu=(1,0)$.

$$S^{\mu\nu} = w(\tau)f(\tau,\xi)\Delta^{\mu\nu},$$

such that the unknown scalar and dimensionless function $f(\tau,\xi)$

$$\langle f(\tau_1,\xi_1)f(\tau_2,\xi_2)\rangle = \frac{2\nu}{A_\perp w(\tau_1)\tau_1}\delta(\tau_1-\tau_2)\delta(\xi_1-\xi_2) \qquad \text{with } \nu = \frac{4}{3}\frac{\eta}{s}$$

3. Magnitude of thermal noise is constrained by (in addition to η/s):

$$(A_\perp w(\tau)\tau) \sim A_\perp \left(\frac{dE}{\tau d^2x_\perp d\xi}\right)\tau \sim \frac{dE_\perp}{dy} \sim \text{multiplicity}$$

Multiplicity more crucial than system size.

Hydro. with symmetry simplification – Gubser hydro.

- Gubser hydro., 2+1D (Gubser and Yarom, 2010)
  - Bjorken boost – indep. of spatial rapidity ξ
  - Rotational symmetry w.r.t. to beam axis – for p+A and ultra-central A+A
- Change coordinates $(\tau, r, \phi, \xi) \leftrightarrow (\rho, \theta, \phi, \xi)$ via the following mapping

$$\sinh\rho = -\frac{1 - q^2\tau^2 + q^2r^2}{2q\tau}\,, \qquad \tan\theta = \frac{2qr}{1 + q^2\tau^2 - q^2r^2}$$

ρ plays the role of 'time' in the new (with a 'hat') coordinate system.

$$\hat{u}^\mu = (1, 0, 0, 0)$$

- There are two parameters: $\hat{T}_0 \to$ multiplicity, $q \to$ transverse size.
- Non-trivial description of the radial expansion.

Gubser hydro. and thermal noise

- Correlation of thermal noise in 2+1D Gubser hydro., $(X \to (\rho, \theta, \phi, \xi))$

$$\langle \hat{S}^{\mu\nu}(\rho_1, \theta_1, \phi_1, \xi_1) \hat{S}^{\alpha\beta}(\rho_2, \theta_2, \phi_2, \xi_2) \rangle = \frac{2\nu \hat{T} \hat{s}}{\cosh^2 \rho_1 \sin \theta_1} \hat{\mathcal{P}}^{\mu\nu} \hat{\mathcal{P}}^{\alpha\beta} \delta(X_1 - X_2)$$

1. Tensor structure of $\hat{S}^{\mu\nu}$ is factorized, due to $\hat{u}^\mu = (1, 0, 0, 0)$.

$$\hat{S}^{\mu\nu}(\rho, \theta, \phi, \xi) = \hat{w}(\rho) \hat{f}(\rho, \theta, \phi, \xi) \hat{\mathcal{P}}^{\mu\nu} \,,$$

   and again we have the correlation of scalar function

$$\langle \hat{f}(\rho_1, \theta_1, \phi_1, \xi_1) \hat{f}(\rho_2, \theta_2, \phi_2, \xi_2) \rangle = \frac{2\nu}{\hat{w} \cosh^2 \rho_1 \sin \theta_1} \delta(X_1 - X_2)$$

2. For scalar function $\hat{f}(X)$, mode decomposition w.r.t. SO(3) symmetry leads to

$$\text{scalar modes: } \hat{f}(\rho, \theta, \phi, \xi) = \sum h(\rho) Y_{lm}(\theta, \phi) e^{i k_\xi \xi}$$

   and

$$\langle h(\rho_1) h(\rho_2) \rangle = \frac{2\nu}{\hat{w} \cosh^2 \rho_1} \delta(\rho_1 - \rho_2)$$

3. Magnitude of thermal noise is constrained by

$$\hat{w} \sim \hat{T}_0 \sim \text{multiplicity}$$

Multiplicity more crucial than system size.

Solve Gubser hydro. with thermal noise

- Decompose thermal fluctuations into modes – scalar and vector modes:

$$\delta\hat{T} = \hat{T}\sum \delta_l(\rho) Y_{lm}(\theta,\phi) e^{ik_\xi \xi}$$
$$\delta\hat{u}_i = \sum \left[v_{ls}(\rho)\partial_i Y_{lm}(\theta,\phi) e^{ik_\xi \xi} + v_{lv}(\rho)\Phi_{i(lm)}(\theta,\phi) e^{ik_\xi \xi} \right]$$
$$\delta\hat{u}_\xi = \sum v_{l\xi}(\rho) Y_{lm}(\theta,\phi) e^{ik_\xi \xi}$$

- EoM of each mode,

$$\tilde{\mathcal{V}}_l'(\rho) = -\tilde{\Gamma}(\rho, l, k_\xi)\tilde{\mathcal{V}}_l(\rho) + \tilde{\mathcal{K}}(\rho, k_\xi),$$

where prime denotes derivative w.r.t. ρ

$$\tilde{\mathcal{V}}_l(\rho) = \begin{pmatrix} \delta_l(\rho) \\ v_{ls}(\rho) \\ v_{l\xi}(\rho) \\ v_{lv}(\rho) \end{pmatrix}, \qquad \tilde{\Gamma} \text{ is a } 4\times 4 \text{ matrix}, \qquad \tilde{\mathcal{K}} = \begin{pmatrix} -\frac{2}{3}\tanh\rho h(\rho) \\ \frac{2\hat{T}}{3\hat{T}'}\tanh\rho h(\rho) \\ -\frac{ik_\xi \hat{T}}{\hat{T}+H_0\tanh\rho} h(\rho) \\ 0 \end{pmatrix}$$

1. Coupled EoMs in 3+1D.
2. Vector modes are decoupled, and NOT affected by thermal noise.
3. Note that thermal fluctuations are indep. of m.

Apply noisy Gubser hydro. to ultra-central Pb-Pb, p-Pb and p-p

- $\hat{T}_0$ and q determine the system.

	PbPb	pPb	pp
$\hat{T}_0$	7.3	3.1	1.7
$q^{-1}(fm)^{-1}$	4.3	1.1	1.1

Note that the strength of thermal fluctuations is fixed once $\hat{T}_0$ is given.

- Approximates system evolution of first several fm's
- $k_\xi = 0$ mode
  - Long rapidity range correlations, affected also by initial fluctuations.
  - Further simplification with v_ξ modes decoupled → 2 coupled equations.

$$(\delta\hat{T}, \delta\hat{u}_i) \longleftrightarrow (\delta T, \delta u_\tau, \delta u_r, \delta u_\phi)$$

- We will NOT discuss hadronization and freeze-out.

Apply noisy Gubser hydro. to ultra-central Pb-Pb, p-Pb and p-p

- $\hat{T}_0$ and q determine the system.

	PbPb	pPb	pp
$\hat{T}_0$	7.3	3.1	1.7
$q^{-1}(fm)^{-1}$	4.3	1.1	1.1

Note that the strength of thermal fluctuations is fixed once $\hat{T}_0$ is given.

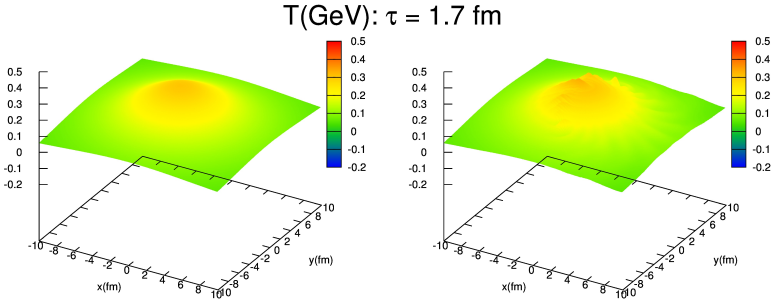
- Approximates system evolution of first several fm's
- $k_\xi = 0$ mode
  - Long rapidity range correlations, affected also by initial fluctuations.
  - Further simplification with v_ξ modes decoupled → 2 coupled equations.

$$(\delta\hat{T}, \delta\hat{u}_i) \longleftrightarrow (\delta T, \delta u_\tau, \delta u_r, \delta u_\phi)$$

- We will NOT discuss hadronization and freeze-out.
- Initial condition ?

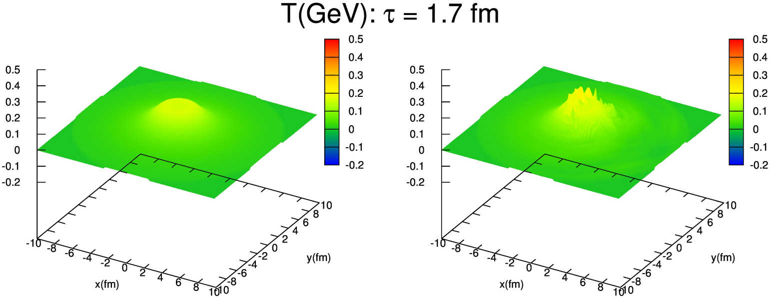
- $T(\tau, \vec{x}_\perp)$ with thermal noise, one random event

Pb-Pb



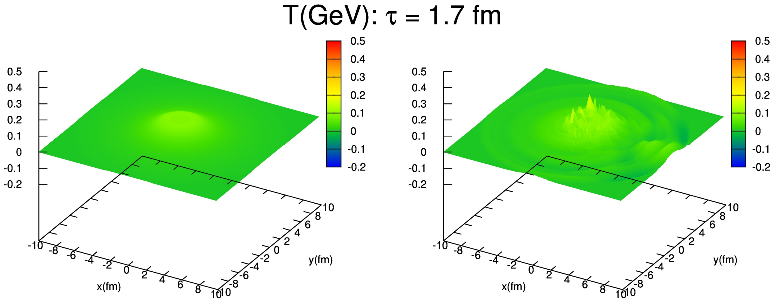
- $T(\tau, \vec{x}_\perp)$ with thermal noise, one random event

p-Pb



- $T(\tau, \vec{x}_\perp)$ with thermal noise, one random event

p-p



Two-point correlations – quantitative effects of thermal fluc.

- Two-point correlation of radial flow velocity

$$C_{u_r u_r}(\tau, \Delta\phi, r, \phi) = \langle u_r(\tau, r, \phi) u_r(\tau, r, \phi + \Delta\phi)\rangle - \langle u_{rb}(\tau, r)\rangle^2$$
$$= \underbrace{C^I_{u_r u_r}}_{\text{initial state fluc.}} + \underbrace{C^T_{u_r u_r}}_{\text{thermal fluc.}}$$

Can also be defined w.r.t. fluctuation modes δT.

- Initial condition : specify $\delta\hat{T}(\rho_0)$
  - Long-range correlation: initial state eccentricity of order m ε_m (single mode)

$$\frac{\delta\hat{T}(\theta, \phi, \rho_0, \xi)}{\hat{T}(\rho_0)} = -\sqrt{\Lambda_{ini}} \left[(-1)^m \frac{1}{\sqrt{2}} Y_{m,m}(\theta, \phi) + \frac{1}{\sqrt{2}} Y_{m,-m}(\theta, \phi) \right] ,$$

  - Short-range correlation: $\delta\hat{T}(\rho_0)$ as a Dirac delta function (all modes)

$$\frac{\delta\hat{T}(\rho_0, \theta, \phi, \xi)}{\hat{T}(\rho_0)} = \sqrt{\Lambda_{ini}} \times \frac{1}{\cosh\rho_0 \sin\theta} \delta(\theta - \theta_0)\delta(\phi - \phi_0) .$$

Two-point correlations – quantitative effects of thermal fluc.

- Two-point correlation of radial flow velocity

$$C_{u_r u_r}(\tau, \Delta\phi, r, \phi) = \langle u_r(\tau, r, \phi) u_r(\tau, r, \phi + \Delta\phi)\rangle - \langle u_{rb}(\tau, r)\rangle^2$$
$$= \underbrace{C^I_{u_r u_r}}_{\text{initial state fluc.}} + \underbrace{C^T_{u_r u_r}}_{\text{thermal fluc.}}$$

Can also be defined w.r.t. fluctuation modes δT.

- Initial condition : specify $\delta\hat{T}(\rho_0)$
  - Long-range correlation: initial state eccentricity of order m ε_m (single mode)

$$\frac{\delta\hat{T}(\theta, \phi, \rho_0, \xi)}{\hat{T}(\rho_0)} = -\sqrt{\Lambda_{ini}}\left[(-1)^m \frac{1}{\sqrt{2}} Y_{m,m}(\theta, \phi) + \frac{1}{\sqrt{2}} Y_{m,-m}(\theta, \phi)\right],$$

  - Short-range correlation: $\delta\hat{T}(\rho_0)$ as a Dirac delta function (all modes)

$$\frac{\delta\hat{T}(\rho_0, \theta, \phi, \xi)}{\hat{T}(\rho_0)} = \sqrt{\Lambda_{ini}} \times \frac{1}{\cosh\rho_0 \sin\theta}\delta(\theta - \theta_0)\delta(\phi - \phi_0).$$

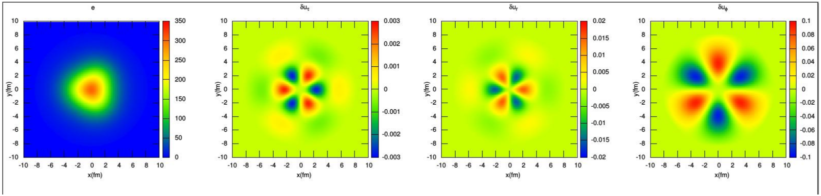
Λ_{ini} unknown, to be fixed by phenomenology.

- Initial eccentricity is responsible for harmonic flow v_n

$$\varepsilon_m = -\frac{\int d^2\vec{x}_\perp r^m e^{im\phi} e(x,y)}{\int d^2\vec{x}_\perp r^m e(x,y)}$$

- E.g., background with initial ε_3 : $Y_{3,3}$ and $Y_{3,-3}$

$e(x,y)$ δu_τ δu_r δu_ϕ

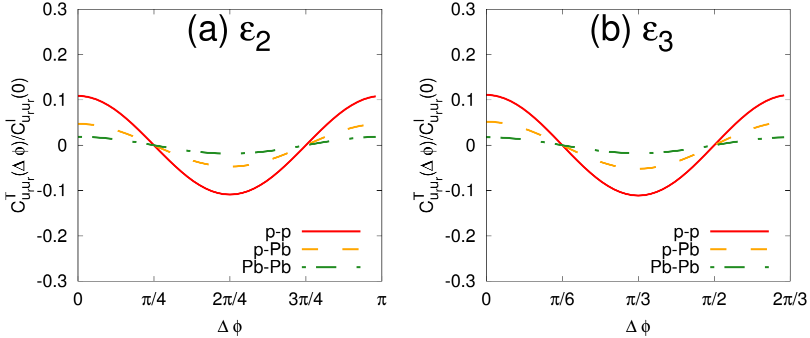


- Fix Λ_{ini} so that at $\tau = 0.6$ fm: $(m = 2, 3, 4, 5)$

$$\varepsilon_m(\text{Pb-Pb}) \approx 0.05\,, \quad \varepsilon_m(\text{p-Pb}) \approx 0.15\,, \quad \varepsilon_m(\text{p-p}) \approx 0.2\,.$$

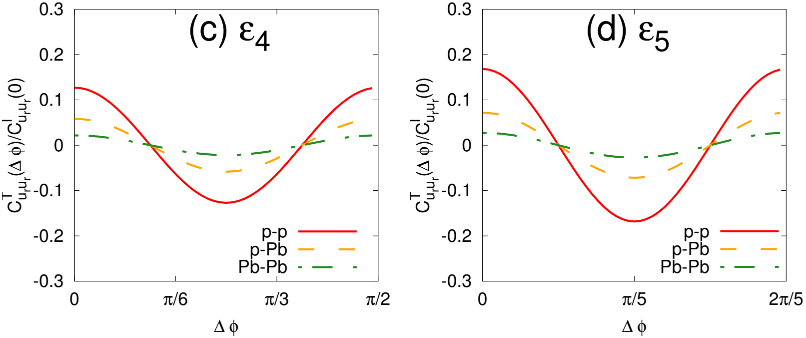
For long-range correlation

- $C^T_{u_r u_r}/C^I_{u_r u_r}$ with an initial ε_m vs. $\Delta\phi$ at $\tau = 2.5$ fm



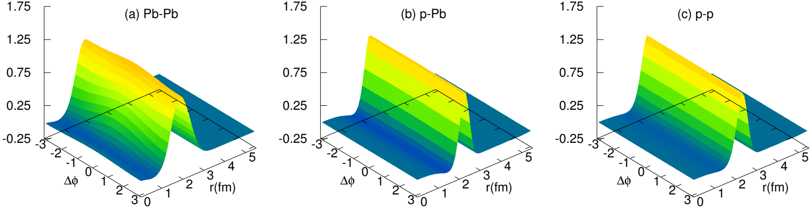
For long-range correlation

- $C^T_{u_r u_r}/C^I_{u_r u_r}$ with an initial ε_m vs. $\Delta\phi$ at $\tau = 2.5$ fm



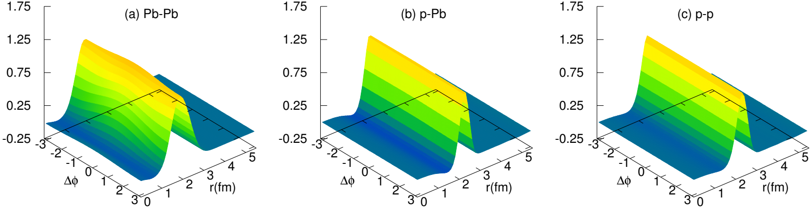
For short-range correlation

- $C^I_{u_r u_r}/C^I_{u_r u_r}(\Delta\phi = \pi)$ with initial Dirac delta vs. $(r, \Delta\phi)$ at $\tau = 2.5$ fm

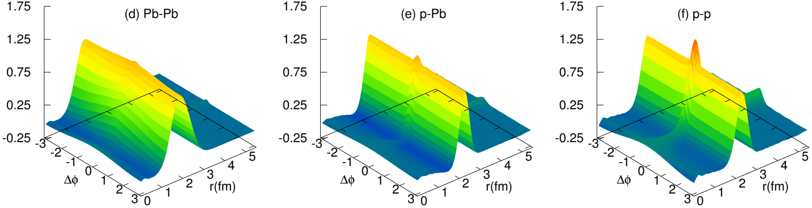


For short-range correlation

- $C^I_{u_r u_r}/C^I_{u_r u_r}(\Delta\phi = \pi)$ with initial Dirac delta vs. $(r, \Delta\phi)$ at $\tau = 2.5$ fm

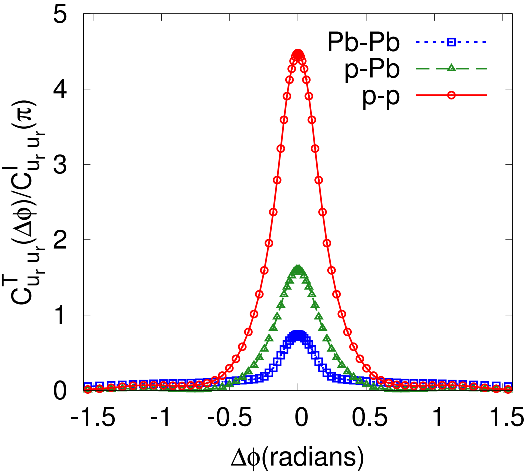


- $(C^I_{u_r u_r} + C^T_{u_r u_r})/C^I_{u_r u_r}(\Delta\phi = \pi)$ vs. $(r, \Delta\phi)$



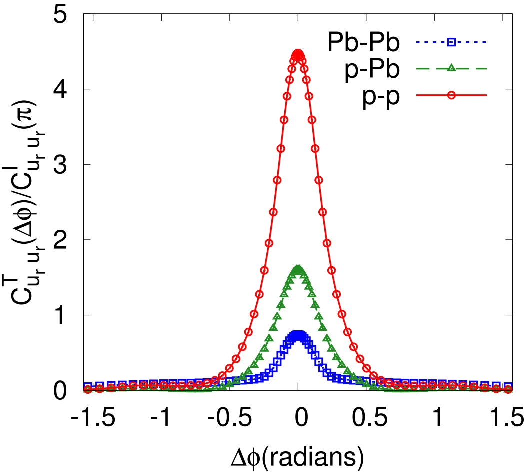
For short-range correlation

- $C^T_{u_r u_r}/C^I_{u_r u_r}(\Delta\phi = \pi)$ with initial Dirac delta vs. $\Delta\phi$ at $\tau = 2.5$ fm



For short-range correlation

- $C^T_{u_r u_r}/C^I_{u_r u_r}(\Delta\phi = \pi)$ with initial Dirac delta vs. $\Delta\phi$ at $\tau = 2.5$ fm



Related to the near-side peak structure in two-particle correlations.

Summary

Formulate and solve 2+1D Gubser hydro. with thermal fluctuations for HIC,

- Strength of hydrodynamical fluctuations is mostly controlled by multiplicity.
- Long-range correlations (evolution of anisotropy in medium):
  - No significant contributions from thermal fluc.
  - Clear trend of increasing effect of hydro. fluc. from Pb-Pb to p-p.
  - Clear trend of increasing effect of hydro. fluc. w.r.t. harmonic order.

$$\begin{cases} C^I(\Delta\phi, m) \quad \leftrightarrow \quad \text{suppressed stronger by } \eta/s \text{ for larger harmonic order} \\ \qquad\qquad\qquad\qquad\qquad (\textbf{cf. Gubser and Yarom, Shuryak and Staig}) \\ C^T(\Delta\phi, m) \quad \leftrightarrow \quad \text{irrespective of harmonic order} \end{cases}$$

- Short-range correlations (near-side): height and width incearse from Pb-Pb to p-p